Iterative Learning Control for Fractional order Nonlinear System with Initial Shift

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Abstract In this study, a closed-loop D^{α}-type iterative learning control(ILC) with a proportional D-type iterative learning updating law for the initial shift is applied to nonlinear conformable fractional system. First, the system with the initial shift is introduced. Then, fractional-order ILC (FOILC) frameworks that experience the initial shift problem for the path-tracking of nonlinear conformable fractional order systems are addressed. Moreover, the sufficient condition for the convergence of tracking errors is obtained in the time domain by introducing λ-norm and H$\ddot{o}$lder's inequality. Lastly, numerical examples are provided to illustrate the effectiveness of the proposed methods.

Keywords Nonlinear conformable fractional differential equations · Iterative learning control · Initial shift · H$\ddot{o}$lder's inequality

1 Introduction

Fractional differential calculus was originally developed in the 17th century, however, it has only been applied to the control field for a few decades [1,2]. Many systems in practical fields can be accurately described with the use of fractional derivatives and integrals [3]. Besides, numerous fractional order controllers have been applied in engineering areas [4–6]. Iterative learning control (ILC), which is used in control systems that repeat the same task via an unknown model during a finite duration, was first introduced by Uchiyama in Japanese [7]. Arimoto et al [8] further developed this method in English. Thereafter, ILC has achieved considerable development [9]

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and has been found to be a good alternative in practical application[10]. The combination of fractional differential calculus and ILC was first reported by Chen [11] who proposed a D^{α}-type control law analyzed in the frequency domain. FOILC has currently become a new topic and has received increasing attention [12-14] because it can enhance tracking performance in control systems.

To the best of our knowledge, Li et al [15] described a fractional-order linear system in state space form, and the convergent conditions for a D^{α}-type law were provided. In [16], the asymptotic stability of PD^{α}-type ILC for a fractional-order linear time invariant (LTI) system was studied. The convergence condition for open-loop P-type ILC for fractional-order nonlinear system was investigated in [17]. High-order fractional-order PID-type ILC strategies for a class of Caputo-type fractional-order LTI system was discussed in [18]. FOILC for both linear and nonlinear fractional-order multi-agent systems was applied to solve the consensus tracking problem in [19];For nonlinear and linear conformable fractional differential equations, Wang et al [20,21] provided standard analysis technique for standard open-loop P-type, D^{α}-type, and conformable $PI^{\alpha}D^{\alpha}$-type ILC; Unfortunately, it is supposed that the initial state is coincident with the desired initial state.

However, the aforementioned existing literature, FOILC laws generally assume that the initial state of a system must be strictly in accordance with the desired states. Therefore, the initial shift problem should be involved to extend the application of FOILC [22 - 24]. For example, Zhao [25] introduced an initial state learning scheme coupled with D^{α}-type ILC updating law to eliminate initial shift states in fractional order linear invariant systems. The possibility of application to the motion control of robot manipulators was discussed under specific conditions. Moreover, the D^{α}-type FOILC scheme was applied to a the fractional order linear system with different historical functions under the Riemann-Liouville definition [26], and the relationship between memory and convergent performance was highlighted. Lan [27] presented a P-type ILC scheme with initial state learning for a single-inputCsingle-output fractional order nonlinear system, and the asymptotic stability of PD^{α}-type ILC was studied by Li et al [28]. Li [29]discussed fractional order nonlinear systems with delay by P-type ILC scheme, initial state delay was considered. In addition, an adaptive generalized FOILC, which expands the practical applications of FOILC, was illustrated in fractional-order nonlinear systems [30].

In accordance with related work, a comfortable closed-loop D^{α}-type ILC with an initial state learning law was proposed in the current study to eliminate the influence of the initial shift in nonlinear comfortable fractional systems. Notably, sufficient conditions in the time domain were derived by introducing λ-norm and using H$\ddot{o}$lder's inequality. And the availability of this contribution is examined using two numerical examples.

The rest of the paper is organized in the following manner. Preliminary knowledge is briefly reviewed in Section 2. The nonlinear conformable fractional differential equations and the design of learning control are presented in detail in Section 3. Learning convergence is analyzed in Section 4. A number of simulations demonstrate the effectiveness of the theory in Section 5. Conclusions are drawn and suggestions for future work are provided in Section 6.

2 Preliminaries

In this section, some related mathematical definitions and properties are introduced, which will be applied in the following sections.

2.1 The λ-norm

Suppose $C(J,\mathbb{R}^n)$ is the space of vector-value continuous functions from $J \to \mathbb{R}^n$. Consider $C(J,\mathbb{R}^n)$ endued with λ-norm, and

$$\|x\|_\lambda = \sup_{t\in J}\left\{e^{-\lambda t}\|x\|\right\}, \;\; \lambda > 0 \tag{1}$$

2.2 Hölder's inequality [31]

Suppose that Ω is a measured space, $p,q \in [1,\infty)$ and satisfy $\frac{1}{p}+\frac{1}{q}=1$. If $f \in L^p(\Omega)$, and $g^p(\Omega)$, then

$$\int_\Omega |f(x)g(x)|dx \le \left(\int_\Omega |f(x)|^p dx\right)^{\frac{1}{p}} \left(\int_\Omega |g(x)|^q dx\right)^{\frac{1}{q}}$$

2.3 Conformable fractional order calculus

The definition of Fractional calculus contains Grünwald-Letnikov, Riemann-Liouville, Caputo's and other fractional derivatives [32,33]. In this paper, conformable fractional order calculus is defined and considered, the derivative of α-order function $f(t):[0,\infty) \to \mathbb{R}^n$ is defined as follows [34].

$$\begin{aligned} D^\alpha f(t) &= \lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon} \\ D^\alpha f(t_0) &= \lim_{t\to t_0^+} D^\alpha_{t_0} f(t) \end{aligned} \tag{2}$$

where$D^\alpha f(t)$ exists and is finite, $0<\alpha\le 1$, $t_0 \ge 0$, D^α presents fractional order derivative operator in $[0,t]$.

Besides, the conformable fractional integral is defined as

$$I^\alpha f(t) = \int_0^t f(\tau)\tau^{\alpha-1}d\tau \tag{3}$$

where I^α presents fractional order integral operator in $[0,t]$.

2.4 Solution of conformable fractional order function

According to the lemma 4 in [35], if the conformable fractional order function $D^{\alpha}x(t) = f(t)$ is continuous and given, the solution $x(t)$ of $f(t)$ with the initial value $x(0) = x_0$ satisfies

$$x(t) = x_0 + \int_0^t (f(\tau))\tau^{\alpha-1} d\tau \tag{4}$$

3 Problem Formulation and ILC Design

In order to solve the above problem, a class of fractional-order nonlinear system is described. The repetitive nonlinear conformable fractional differential system is considered as

$$\begin{cases} D_t^{\alpha} x_k(t) = F(t, x_k(t)) + Q(t)u_k(t) \\ y_k(t) = P(t)x_k(t) \end{cases} \tag{5}$$
$$t \in J := [0, T]$$

where $D_t^{\alpha}(0 < \alpha < 1)$ denotes conformable fractional derivative, $x_i(t) \in \mathbb{R}^n$, $y_i(t) \in \mathbb{R}^n$, $P(t) \in \mathbb{R}^{m\times n}$, $u_i(t) \in \mathbb{R}^r$, $Q(t) \in \mathbb{R}^{n\times r}$; $F : J \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous differentiable in t. $x_k(0)$ is an initial condition of state variable $x_k(t)$ for the kth iteration.

There exists a $L_F > 0$ such that satisfies

$$\| F(t, x_1(t)) - F(t, x_2(t)) \| \le L_F \| x_1(t) - x_2(t) \| \tag{6}$$

where $\forall t \in J, \forall x_1, x_2 \in \mathbb{R}^n$.

According to section 2.4, the solution of $x_k(t)$ in equation (5) can be derived as follows

$$x_k(t) = x_{k0} + \int_0^t (F(\tau, x_k(\tau)) + Q(\tau)u_k(\tau))\tau^{\alpha-1} d\tau \tag{7}$$

Let $y_d(t)$ be a continuous differentiable desired function on J and $u_d(t)$ be a expectation control variable. If the desired initial value is $x_d(0) = x_{d0}$, and $y_d(t) = P(t)x_d(t)$, where

$$x_d(t) = x_{d0} + \int_0^t (F(\tau, x_d(\tau)) + Q(\tau)u_d(\tau))\tau^{\alpha-1} d\tau \tag{8}$$

For simplification, it is defined

$$\begin{aligned} \delta u_k(t) &= u_d(t) - u_k(t) \\ \delta x_k(t) &= x_d(t) - x_k(t) \\ e_k(0) &= y_d(0) - y_k(0) \end{aligned} \tag{9}$$

The ILC updating law with initial state learning is defined as follows,

$$x_{k+1}(0) = x_k(0) + Le_k(0) \tag{10}$$
$$u_{k+1}(t) = u_k(t) + \Lambda D_t^{\alpha} e_{k+1}(t) \tag{11}$$

where the subscript k is iterative index, $L \in R^{n\times r}$ and $\Lambda \in R^{r\times r}$ are the learning gains to be designed based on prior-knowledge about the system under investigation.

4 Convergence Analysis

Lemma 1 Consider the system (5) with a bounded reference $y_d(t)$ and arbitrary continuous initial state $x_k(0)$. Assuming that $\left\|[(I+P(t)L)]^{-1}\right\| < 1$ holds for all $t \in [0,\mathrm{T}]$, then the initial learning law guarantees that $\lim\limits_{k\to\infty} \|e_k(0)\|_\lambda = 0$

Proof

It follows equations (9) and (10), it obtains

$$x_{k+1}(0) - x_{d+1}(0) = \delta x_k(0) - Le_{k+1}(0) \tag{12}$$

Multiplying both sides of the above equation (12) by $P(t)$(defined in equation (5)),

$$P(t)\delta x_{k+1}(0) = P(t)\delta x_k(0) - P(t)Le_{k+1}(0) \tag{13}$$

It gets

$$e_{k+1}(0) = [(I+P(t)L)]^{-1} e_k(0) \tag{14}$$

where I denotes identity matrix. Taking the norm $\|\cdot\|$ of equation (14), it yields

$$\|e_{k+1}(0)\| \leq \left\|[(I+P(t)L)]^{-1}\right\| \|e_k(0)\| \tag{15}$$

Multiply $e^{-\lambda t}$ both sides of the above equation (15), it derives

$$e^{-\lambda t}\|e_{k+1}(0)\| \leq e^{-\lambda t}\left\|[(I+P(t)L)]^{-1}\right\| \|e_k(0)\| \tag{16}$$

Taking the λ-norm the above equation (16), it yields

$$\|e_{k+1}(0)\|_\lambda \leq \left\|[(I+P(t)L)]^{-1}\right\| \|e_k(0)\|_\lambda \tag{17}$$

Therefore, if $\| \ [(I+P(t)L)]^{-1} \| < 1$ is satisfied, $\lim\limits_{k\to\infty} \|e_{k+1}(0)\|_\lambda = 0$ can be concluded.

Lemma 2 For the system (5) with the D^α-type ILC schemes (11), suppose the assumption in **Lemma 1** is satisfied, and if

$$\rho_1 = (l + \|I + \Lambda P(t)Q(t)\|)^{-1} < 1$$

holds for all $t \in [0,T]$. Given an arbitrary initial input $u_k(0)$, $\lim\limits_{k\to\infty} \|e_k\|_\lambda = 0$ can be concluded.

Proof

It follows from the aforementioned equations (5-10)

$$\begin{aligned}
e_{k+1}(t) &= P(t)(x_{d+1}(t) - x_{k+1}(t)) \qquad (18)\\
&= P(t)\left\{\delta x_{k+1}(0) + \int_0^t \left(F(\tau,x_d(\tau)) - F(\tau,x_{k+1}(\tau)) + Q(\tau)\delta u_{k+1}(\tau)\tau^{\alpha-1}\right)d\tau\right\}\\
&= e_{k+1}(0) + P(t)\left\{\int_0^t \left(F(\tau,x_d(\tau)) - F(\tau,x_{k+1}(\tau)) + Q(\tau)\delta u_{k+1}(\tau)\tau^{\alpha-1}\right)d\tau\right\}\\
&= e_{k+1}(0) + P(t)\left\{\int_0^t \left(F(\tau,x_d(\tau)) - F(\tau,x_{k+1}(\tau)) + Q(\tau)(\delta u_k(t) - \Lambda D_t^\alpha e_{k+1}(t))\tau^{\alpha-1}\right)d\tau\right\}
\end{aligned}$$

From the equation (11), the $u_{k+1}(t)$ can be yield

$$\begin{aligned} &\delta u_{k+1}(t) = \delta u_k(t) - \Lambda D_0^{\alpha} e_{k+1}(t) \\ &= \delta u_k(t) - \Lambda D_t^{\alpha}(y_d(t) - y_{k+1}(t)) \\ &= \delta u_k(t) - \Lambda D_t^{\alpha}(P(t)\delta x_{k+1}(t)) \\ &= \delta u_k(t) - \Lambda D_t^{\alpha} P(t)\delta x_{k+1}(t) - \Lambda P(t) D_{\alpha}^{0} \delta x_{k+1}(t) \\ &= \delta u_k(t) - \Lambda D_t^{\alpha} P(t)\delta x_{k+1}(t) - \Lambda P(t)((F(t,x_d(t)) - F(t,x_{k+1}(t))) + Q(t)\delta u_{k+1}(t)) \\ &= \delta u_k(t) - \Lambda \dot{P}(t) t^{\alpha-1} \delta x_{k+1}(t) - \Lambda P(t)((F(t,x_d(t)) - F(t,x_{k+1}(t))) + Q(t)\delta u_{k+1}(t)) \end{aligned} \tag{19}$$

Taking the norm $\|\cdot\|$ on both sides of the above equation (19), it obtains

$$\begin{aligned} &\|I + \Lambda P(t)Q(t)\| \, \| \, \delta u_{k+1}(t) \, \| \\ &\leq \|\delta u_k(t)\| + \left\|\Lambda \dot{P}(t) t^{\alpha-1} \delta x_{k+1}(t)\right\| + \|\Lambda P(t)\| \, \|(F(t,x_d(t)) - F(t,x_{k+1}(t)))\| \\ &\leq \|\delta u_k(t)\| + \left\|\Lambda \dot{P}(t) t^{\alpha-1} + \Lambda P(t) L_F\right\| \, \|\delta x_{k+1}(t)\| \\ &\leq \|\delta u_k(t)\| + h \, \|\delta x_{k+1}(t)\| \end{aligned} \tag{20}$$

where

$$h = \sup_{t\in[0,T]} \left\|\Lambda \dot{P}(t) t^{\alpha-1} + \Lambda P(t) L_F\right\|$$

and

$$\begin{aligned} &\delta x_{k+1}(t) \\ &= \delta x_{k+1}(0) + \int_0^t (F(\tau,x_d(\tau)) - F(\tau,x_{k+1}(\tau)) + Q(\tau)\delta u_{k+1}(\tau))\tau^{\alpha-1} d\tau \\ &\leq \delta x_{k+1}(0) + L_F \int_0^t \delta x_{k+1}(\tau)\tau^{\alpha-1} d\tau + \int_0^t Q(\tau)\delta u_{k+1}(\tau)\tau^{\alpha-1} d\tau \end{aligned} \tag{21}$$

It yields

$$\begin{aligned} &\|\delta x_{k+1}(t)\| \\ &\leq \|\delta x_{k+1}(0)\| + L_F \int_0^t \|\delta x_{k+1}(\tau)\| \, \tau^{\alpha-1} d\tau + \int_0^t \|Q(\tau)\delta u_{k+1}(\tau))\| \, \tau^{\alpha-1} d\tau \\ &\leq \|\delta x_{k+1}(0)\| + \int_0^t \|Q(\tau)\delta u_{k+1}(\tau))\| \, \tau^{\alpha-1} d\tau \mathrm{e}^{\int_0^t L_F \tau^{\alpha-1} d\tau} \\ &\leq \|\delta x_{k+1}(0)\| + \int_0^t \|Q(\tau)\delta u_{k+1}(\tau))\| \, \tau^{\alpha-1} d\tau \mathrm{e}^{L_F \frac{t^{\alpha}}{\alpha}} \\ &\leq \|\delta x_{k+1}(0)\| + \int_0^t \mathrm{e}^{\lambda\tau} \tau^{\alpha-1} d\tau \, \|Q(t)\| \, \|\delta u_{k+1}\|_{\lambda} e^{L_F \frac{t^{\alpha}}{\alpha}} \end{aligned} \tag{22}$$

From the equation (22), for any given $\alpha > 0$, the existence of $p > 1$ makes $\alpha > \frac{1}{p}$. Then, it can be seen that $\exists q > 0$ makes $\frac{1}{q} + \frac{1}{p} = 1$, By using Hölder′s inequality[21]

$$\begin{aligned}
&\int_0^t e^{\lambda\tau}\tau^{\alpha-1}d\tau \qquad (23)\\
\leq &\left(\int_0^t e^{\lambda\tau p}d\tau\right)^{\frac{1}{p}}\left(\int_0^t \tau^{(\alpha-1)\times q}d\tau\right)^{\frac{1}{q}}\\
\leq &\frac{e^{\lambda t}}{\sqrt[p]{p}\sqrt[p]{\lambda}}\left(\frac{t^{q\alpha-q+1}}{q\alpha-q+1}\right)^{\frac{1}{q}}\\
\leq &\frac{1}{\lambda}e^{\lambda t}\left(\frac{T^{\alpha-\frac{1}{p}q}}{\sqrt[q]{q\alpha-q+1}}\right),(\lambda \geq 1)
\end{aligned}$$

Now, substituting (23) into (22), it gets

$$\|\delta x_{k+1}(t)\| \leq \|\delta x_{k+1}(0)\| + \frac{1}{\lambda}e^{\lambda t}\frac{T\alpha-\frac{1}{p}}{\sqrt[q]{q\alpha-q+1}}\|Q(t)\|\,\|\delta u_{k+1}\|_\lambda e^{L_F\frac{T^\alpha}{\alpha}} \qquad (24)$$

It can be obtained

$$\begin{aligned}
&\|I+\Lambda P(t)Q(t)\|\;\|\;\delta u_{k+1}(t)\;\| \qquad (25)\\
\leq &\|\delta u_k(t)\| + \left\|\Lambda\dot{P}(t)t^{\alpha-1}\delta x_{k+1}(t)\right\| + \|\Lambda P(t)\|\,\|(F(t,x_d(t))-F(t,x_{k+1}(t)))\|\\
\leq &\|\delta u_k(t)\| + \left\|\Lambda\dot{P}(t)t^{\alpha-1}+\Lambda P(t)L_F\right\|\,\|\delta x_{k+1}(t)\|\\
\leq &\|\delta u_k(t)\| + h\,\|\delta x_{k+1}(0)\| + h\frac{1}{\lambda}e^{\lambda t}\frac{T\alpha-\frac{1}{p}}{\sqrt[q]{q\alpha-q+1}}\|Q(t)\|\,\|\delta u_{k+1}\|_\lambda e^{L_F\frac{T^\alpha}{\alpha}}
\end{aligned}$$

Multiplying both sides of the above equation (25) by $e^{-\lambda t}$ and taking the λ-norm, it has

$$\begin{aligned}
&e^{-\lambda t}\|I+\Lambda P(t)Q(t)\|\;\|\;\delta u_{k+1}(t)\;\| \qquad (26)\\
\leq &e^{-\lambda t}\|\delta u_k(t)\| + e^{-\lambda t}h\,\|\delta x_{k+1}(0)\| + h\frac{1}{\lambda}\frac{T\alpha-\frac{1}{p}}{\sqrt[q]{q\alpha-q+1}}\|Q(t)\|\,\|\delta u_{k+1}\|_\lambda e^{L_F\frac{T^\alpha}{\alpha}}
\end{aligned}$$

And

$$\begin{aligned}
&\left(h\frac{1}{\lambda}e^{L_F\frac{T^\alpha}{\alpha}}\frac{T\alpha-\frac{1}{p}}{\sqrt[q]{q\alpha-q+1}}\|Q(t)\| + \|I+\Lambda P(t)Q(t)\|\right)\|\;\delta u_{k+1}\;\|_\lambda \qquad (27)\\
&\leq \|\delta u_k\|_\lambda + h\|\delta x_{k+1}(0)\|_\lambda
\end{aligned}$$

where

$$l = h\frac{1}{\lambda}e^{L_F\frac{T^\alpha}{\alpha}}\frac{T\alpha-\frac{1}{p}}{\sqrt[q]{q\alpha-q+1}}\|Q(t)\|$$

and

$$\begin{aligned}&\| \delta u_{k+1}\|_\lambda \\ \le&[l+\|I+\Lambda P(t)Q(t)\|]^{-1}\|\delta u_k\|_\lambda + h[l+\|I+\Lambda P(t)Q(t)\|]^{-1}\|\delta x_{k+1}(0)\|_\lambda \\ \le&[l+\|I+\Lambda P(t)Q(t)\|]^{-1}\|\delta u_k\|_\lambda \\ &+h[l+\|I+\Lambda P(t)Q(t)\|]^{-1} \parallel [I+P(t)L]^{-1} \parallel \|\delta e_k(0)\|_\lambda\end{aligned} \tag{28}$$

where $\rho_1{=}[l+\|I+\Lambda P(t)Q(t)\|]^{-1}$, $\rho_2 = h[l+\|I+\Lambda P(t)Q(t)\|]^{-1} \parallel [I+P(t)L)]^{-1} \parallel$

Based on **Lemma 1**, the $\|\delta e_k(0)\|_\lambda$ is bounded. Thus, it possible to exist a constant ζ sufficiently small enough, it satisfies that $\|\delta e_k(0)\|_\lambda \le \zeta\|\delta u_k\|_\lambda$, and the equation can be written as

$$\| \delta u_{k+1}\|_\lambda \le (\rho_1+\varsigma)\|\delta u_k\|_\lambda \tag{29}$$

where $\varsigma = \rho_2\zeta$. It possible to exist λ sufficiently large that satisfies $\rho_2\zeta \to 0$; Hence it exists $\max \sup\limits_{0\le t,\tau\le \mathrm{T}} \rho_1 < 1$, together with **Lemma 1**, it satisfies $\| \delta u_{k+1}\|_\lambda \le \rho_1\|\delta u_k\|_\lambda$. Therefore, $\lim\limits_{k\to\infty}\|e_k\|_\lambda = 0$ can be concluded.

5 Simulation

In this section, numerical examples are presented to test the effectiveness of the designed methods. The following simulations are performed for the fractional-order nonlinear system.

Example 1 Consider the first fractional-order nonlinear system

$$\begin{aligned}&D_0^{0.5}x_k(t) = 0.6[x_k(t)]^2+0.5u_k(t)\\ &y_k(t) = x_k(t)\end{aligned} \tag{30}$$

The iterative learning control laws are chosen

$$x_{k+1}(0) = x_k(0)+0.5e_k(0) \tag{31}$$

And

$$u_{k+1}(t) = u_k(t)+D^{0.5}e_{k+1}(t) \tag{32}$$

where the system state is $x(t)$, and the desired trajectory is $y_d(t) = 12t^2(1-t)$, the initial control is $u_0(t) = 0$ and with initial condition $x_k(0) = 0.5$.

In this case, it can be calculated that

$$\left\|\frac{1}{(I+P(t)L)}\right\|{=}\frac{2}{3}\ ,\ \ \rho_1{=}\frac{1}{(l+\|I+\Lambda P(t)Q(t)\|)} < 0.5 < 1$$

The simulation results are demonstrated in Fig 1 - Fig 3

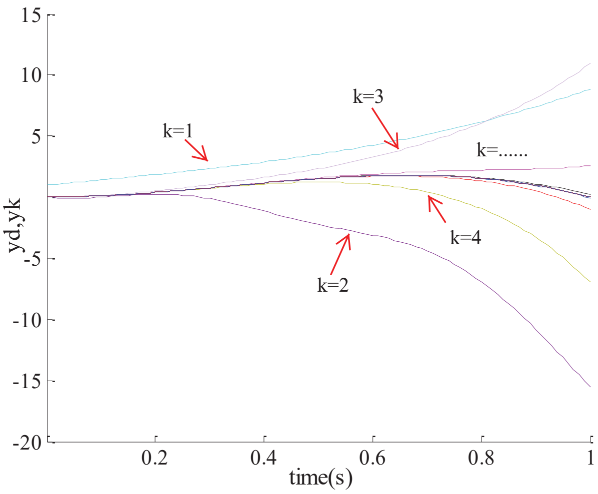

Fig. 1 The tracking process of Example 1 under ILC law

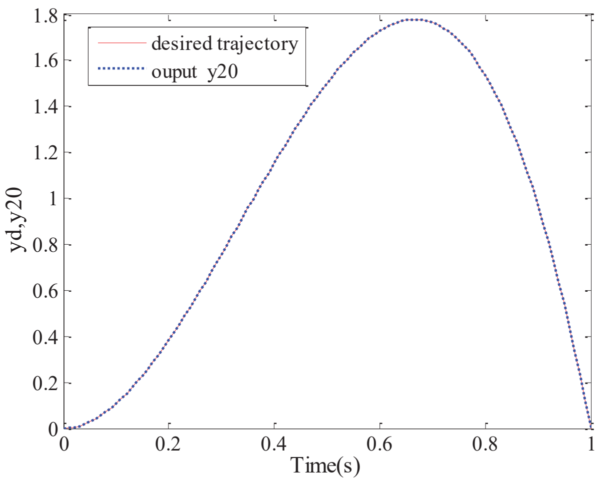

Fig. 2 The tracking result of Example 1 under ILC law

Example 2 Consider the second fractional-order nonlinear system

$$\begin{cases} D_0^{0.5}x_k(t) = 0.5[\sin x_k(t)]^2 + u_k(t) \\ y_k(t) = x_k(t) \end{cases} \tag{33}$$

The iterative learning control laws are chosen

$$x_{k+1}(0) = x_k(0) + 0.3e_k(0) \tag{34}$$

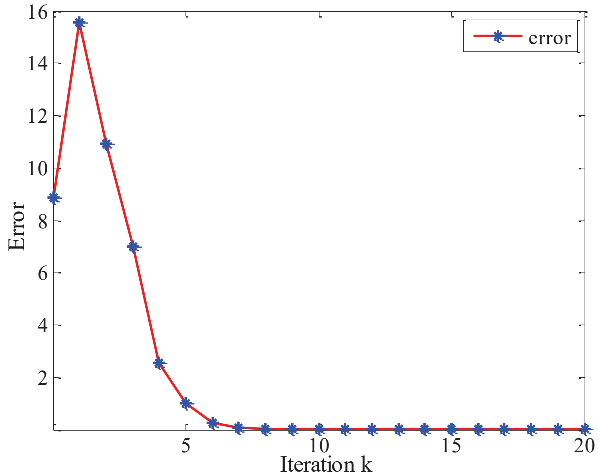

Fig. 3 Change of maximum absolute value of error

And

$$u_{k+1}(t) = u_k(t) + 2D^{0.6}e_{k+1}(t) \tag{35}$$

where the system state is $x(t)$, and the desired trajectory is $y_d(t) = 6sin(t)$, the initial control is $u_0(t) = 0$ and with initial condition $x_k(0) = 0.6$.

In this case, it can be calculated that

$$\left\| \frac{1}{(I+P(t)L)} \right\| = \frac{10}{13}\ ,\ \ \rho_1 = \frac{1}{(l+\|I+\Lambda P(t)Q(t)\|)} < \frac{5}{8} < 1$$

The simulation results are demonstrated in Fig.4 - Fig.6

Therefore, form the aforementioned simulations, it can be concluded that proposed laws with initial state laws perform well. Moreover, it can be seen that after iteration, they all arrive at the reference trajectory under the desired precision.

6 Conclusions

This study presents D^{α}-type ILC with D-type initial learning strategy for a class of nonlinear conformable fractional-order systems with the initial shift. Its major feature is that disturbance in the initial state at each iteration is eliminated by introducing an initial state learning scheme. Furthermore, the robust convergent analysis of tracking errors with respect to initial errors is derived by introducing H$\ddot{o}$lder's inequality. Lastly, numerical simulations are provided to validate the obtained theoretical results.

In the future, $PI^{\lambda}D^{\alpha}$-type ILC for general nonlinear fractional-order systems with repetitive properties will be researched. Moreover, when $PI^{\lambda}D^{\alpha}$-type ILC is applied to track the nonlinear fractional-order system, nonrepetitive uncertainties (such as

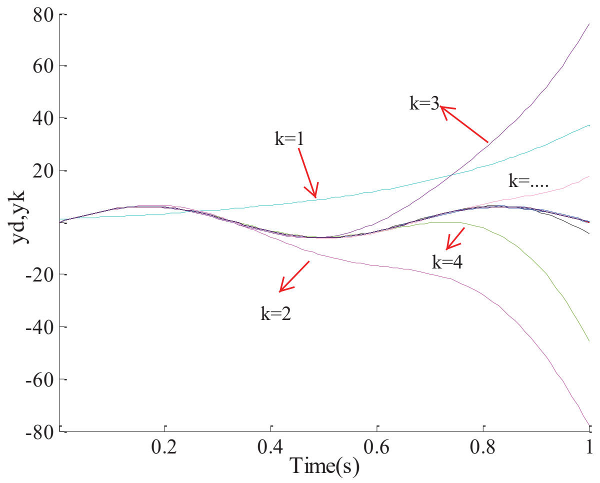

Fig. 4 The tracking process of Example 2 under ILC law

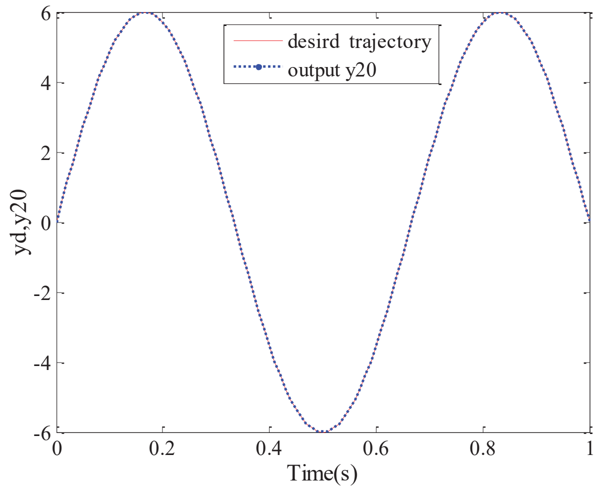

Fig. 5 The tracking result of Example 2 under ILC law

time delay, input saturation, and nonrepetitive desired trajectory) should be considered.

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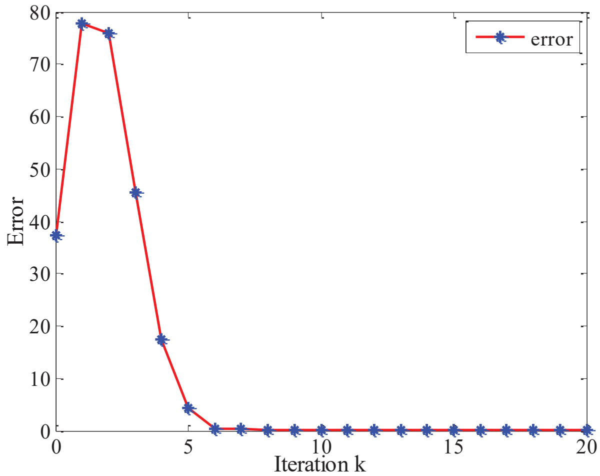

Fig. 6 Change of maximum absolute value of error

Conflict of interest

The authors declare that they have no conflict of interest.

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