Exogenous market changes analysis using artificial options volatility

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Research Article

Keywords: implied volatility, exogenous effects, artificial options, constant maturity, panel data, changepoint detection

Posted Date: June 20th, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3067943/v1

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Additional Declarations: No competing interests reported.
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Received: date / Accepted: date

Abstract This paper focuses on market changes due to exogenous effects. The standard implied volatility is shown to be insufficient for a proper detection and analysis of this type of risk. This is mainly because such changes are usually dominated by endogenous effects coming from a specific trading mechanism or a natural market dynamics. A unique methodological approach based on artificial options that always have a constant (over time) maturity is proposed and explicitly defined. The key principle is to use interpolated volatilities that can effectively eliminate instabilities due to the natural market dynamics while the changes caused by the exogenous causes stay preserved. Formal statistical tests for distinguishing significant effects are proposed under different theoretical and practical scenarios. Statistical theory, computational and algorithmic details, and comprehensive empirical comparisons together with a real data illustration are all presented in the paper.

Keywords implied volatility · exogenous effects · artificial options · constant maturity · panel data · changepoint detection

1 Introduction

It is a well-known fact, that the problem of analyzing financial markets relies on the ability to detect all kinds of sudden changes—changepoints—which randomly and repeatedly occur in the stock market over time. Some
changes are caused by the market itself, its natural dynamics, or various trading mechanisms. For instance, considering some specific financial contract, certain changes may occur due to a specific payoff structure of the contract either because of a fixed maturity or because the payoff structure is highly discontinuous, or both. This is also the case when studying the price of a fixed maturity bond that always converges to a nominal value when approaching the maturity. On the other hand, more important changes for practitioners and financial agents are those that are caused by different impulses due to human interactions (such as the recent COVID-19 outbreak, the President Biden canceling the permit for the Keystone XL pipeline, or the Russian attack in Ukraine). Unfortunately, using common market data and standard theoretical/methodological principles for a statistical analysis, these two types of changes can not be distinguished easily. Additional steps are needed in order to separate specific risk—the natural dynamics of the market—and systemic risk (mainly the changes caused by various external causes). Moreover, among these types of risk, we distinguish between those intrinsically connected to the structure of the financial instrument—so called endogenous effects—and those determined by some external cause—referred to as exogenous effects.

Focusing on the option markets and bearing in mind the exogenous effects that are of the main interest, we introduce a unique market analysis approach based on artificial options with a constant maturity over time. In general, the price of an option contract follows a specific dynamic when approaching the exercise date and this dynamic is assumed to be well described by the Black and Scholes model proposed in Black and Scholes (1973). However, the model postulates normally distributed returns and it also restricts the volatility to be constant with respect to strike values. Some modifications to account for non-normally distributed returns are proposed, for instance, in Corrado and Su (1997). In practice, however, the market agents adjust the former assumption changing the latter and an increasing volatility is quoted for strikes far from the current value of the underlying asset. This quoted volatility is known as the implied volatility (IV) and its characteristic convex shape is called the volatility smile. The scientific literature related to this topic is quite relevant from both the statistical and the financial viewpoint. Some characteristic mean-reverting behaviour is investigated in Ielpo and Guillaume (2010) and various smoothing approaches targeting optimal trading strategies are proposed and discussed in Appel (2003), Chong and Ng (2008), or Chio (2022). Advanced panel data approaches are discussed in Maciak (2019) while the most recent ideas utilize neural network models and machine learning techniques as, for instance, in Jang and Lee (2019). In all these works, as in many others, some pre-analysis is performed on raw observations in order to (a) smooth the discrete data or (b) remove the endogenous effects while studying the exogenous ones. As far as (a) is concerned, several methods have been proposed to generate a continuous IV curve with respected to the quoted strikes (see, for instance, Kahalé (2004), Fengler (2005), Benko et al. (2007), or a complete review in Homescu (2011)). Some recent applications of these smoothing techniques can be also found in Fengler (2012), Glaser and Heider...
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(2012), Fengler and Hin (2015), Ludwig (2015), and Kopa et al. (2017) but all of them assume some underlying smoothness of the unknown IV surface.

On the other hand, there are not so many studies regarding (b). Usually, the objective of the research is to capture and explain the endogenous dynamics of the IV smiles while removing the exogenous effects. However, it is still relevant in many practical applications to do the opposite: to remove the endogenous features that naturally affect the IV evolution and are strictly related to the financial structure of the option itself and to rather capture and model the exogenous effects pertained in the data as unnatural jumps or sudden breaks—so called changepoints. The main feature of the IV evolution over time is its increasing convexity when approaching the maturity date. When the option contract is close to its maturity, this adjustment becomes more relevant because even small fluctuations of the underlying value can cause that the option moves suddenly from an in-the-money (positive payoff) position to an out-of-the-money (zero payoff) position creating huge effects on the price of the option. Therefore, the market agents further adjust the model quoting a more convex IV smile. Alternatively, one could use a jump-diffusion process proposed by Jiang and Tian (2005) and so-called model-free implied volatilities suggested in Britten-Jones and Neuberger (2000) but the corresponding theoretical framework is rather more restrictive and empirical calculations tend to be more extensive and less intuitive. Nevertheless, in order to study the option price dynamics one needs to study the IV dynamics and to infer on the external causes affecting the IV dynamics it is firstly needed to (somehow) remove the natural dynamics of the market since it is evident that the convexity of the IV increases when the time progresses towards the maturity. One could argue that to remove the maturity effect it would be enough to consider options with a longer maturity. Unfortunately, this is difficult for several reasons: (1) For most of the companies (excluding American blue chips) long maturity options are not quoted; (2) Even if they are quoted the amount of different strikes is very limited and most strikes start to be available only at times close to maturity; (3) In any case, the liquidity of the options is not enough for the options far from maturity.

There are some rather exploratory approaches discussed in Guhathakurta, Bhattacharya, and Chowdhury (2010) or Marcaccioli, Bouchaud, and Benzaquen (2022) to distinguish between exogenous effects and endogenous effects as they are generally considered to be of different nature, however, unlike the aforementioned papers, we propose a formal statistical/inferential tool based on removing the endogenous effects caused by the market itself while allowing to focus directly on the exogenous effects—which has not been, to the best of our knowledge, done so far. There is a very popular volatility index VIX (see Knepper (2022) for further details) that estimates the implied volatility of options with an average expiration of 30 days but it aggregates multiple put/call options (over both, the maturities and the strikes) and it mainly serves as an exploratory tool to assess the overall market sentiment. Our primary focus, imbedded within an option specific interpolation instead, is to provide market agents with a valid stochastic tool to be able to correctly infer on a signifi-
cance of given option market changes due to some specific (well recognized) external stimulus. The corresponding market reaction can be either uncertain and it can be explained just by some random fluctuation while some other market changes are more essential and typically statistically significant and the methodology proposed in this paper can consistently distinguish between these two.

The detection of the *exogenous effects* within the IV smile dynamic is very important to understand the sentiment of the market agents since IV captures the expectations of the market about the evolution of the underlying in the next future. As the *changepoints* caused by the *exogenous effects* become more and more frequent in the last couple of years, a proper statistical analysis is needed here. The proposed idea moves the market analysis from the hypothesis that the implied volatility of the options of a given asset is able to capture the feeling and the view of financial agents about the future changes of the asset itself. The focus on the volatility of the *artificial options* also reduces the inter-day bias, which is further diminished using a carefully calibrated interpolation between the implied volatilities of options having consecutive maturities.

The rest of the paper is organized as follows: The principal idea of the paper—the *interpolated volatility* of the *artificial options*—is introduced in Section 2. A detailed description of the corresponding interpolation algorithm can be found there as well. Some formal mathematical and statistical theory regarding the changepoint detection procedure is provided in Section 3. Some sensitivity analysis, practical illustrations on real data, and empirical comparisons based on simulations are discussed in Section 4. Final remarks and conclusions are summarized in Section 5.

## 2 Interpolated volatility of artificial options

Let \( \{z_{itm}; i = 1, \ldots, N; t = 1, \ldots, T; m \in 1, \ldots, M\} \) represent implied volatility values for some underlying asset such that \( i \) stands for the option’s strike label, \( t \) is the observing day from the follow-up period, and \( m \) is the maturity dataset index. Such implied volatilities are well known for being non-stationary over time mainly due to the specific payoff structure of the market. Therefore, when focusing on *exogenous effects* in particular, one needs to firstly deal with this non-stationarity induced by the trading mechanism. For this purpose, we construct a new artificial dataset \( \{Y_{it}; i = 1, \ldots, N; t = 1, \ldots, T\} \) such that new data values will report, for each strike \( i \) and each observing day \( t \), an (artificial) implied volatility value \( Y_{it} \) of an *artificial option* having always a constant (over time) maturity of \( K \) days. The artificial option construction is based on a simple (weighted) linear interpolation across different IV datasets \( m \in \{1, \ldots, M\} \). For each day \( t \in \{1, \ldots, T\} \), the observed implied volatilities of the two options having the same strike and the corresponding maturities immediately before and immediately after the given day \( t \in \{1, \ldots, T\} \) are
interpolated together. The analytic formula can be expressed as

\[
Y_{it} = \frac{1}{(t+K)-t_b} z_{itm_b} + \frac{1}{(t-a)-(t+K)} z_{itm_a} \frac{1}{(t+K)-t_a} + \frac{1}{(t-a)-(t+K)}.
\]

(1)

where \(m_b\) is the maturity dataset index of the first option expiring before the time \(t + K\) (at the day \(t_b\)) and \(m_a\) is the maturity dataset index of the first option expiring after the time \(t + K\) (at the day \(t_a\)). Computational details are described in the Implied Volatility interpolation/Artificial Options—IVintAO Algorithm below. Recall, that unlike the popular VIX index (Kuepper 2022) the interpolation in (1) is so-called strike specific, computationally much simpler, and, moreover, it still provides the typical IV smile for every trading day \(t \in \{1, \ldots, T\}\).

For a brief illustration of the interpolation principle given by (1), using \(K = 35\) (i.e., artificial options with a constant 35 days maturity), let us use the call options data of Erste Group (see Section 4 for more details). The first day of the observation period \((t = 1)\) is July 16th, 2018. Thus, the artificial option will expire in \(t + 35\), i.e., August 20th, 2018, and the two real options that must be considered for the interpolation in (1) are the options with the maturity August 17th, 2018 (denoted by \(m_b\)) and the maturity September 21st, 2018 (denoted by \(m_a\)). The distance (in days) between the artificial maturity (August 20th, 2018) and the maturity of the first real option is \((t + 35) - t_b = 3\) days and the distance between the artificial maturity and the maturity of the second real option is \(t_a - (t + 35) = 32\) days. Therefore, the equation in (1) becomes

\[
Y_{it} = \frac{1}{3} z_{itm_b} + \frac{1}{32} z_{itm_a}.
\]

(2)

where \(z_{itm_b}\) and \(z_{itm_a}\) are the corresponding raw implied volatility values obtained from the market (\(m_b\) is the index for the maturity dataset expiring in August 17th, 2018 and \(m_a\) is the index for the maturity dataset expiring in September 21st, 2018). The interpolation procedure is repeated for all available strikes \(i \in \{1, \ldots, N\}\) and all trading days \(t \in \{1, \ldots, T\}\) from the given observation period. The specific choice of \(K = 35\) takes into account the fact that a reasonable value of \(K\) should be in between 30 and 40 days. Any \(K < 30\) (and \(t\) being immediately after the expiration of the options of the given month) would make \(t + K\) smaller than the first consecutive maturity and, therefore, \(t_b\) would obviously not exist. On the other hand, any \(K > 40\) would generate a proxy being too far from the current day, not representing the sentiment of the market effectively while also considering \(t_a\) for which several strikes are not quoted yet. However, unlike the VIX index which—by default—produces options with the 30 day expiration, there is some flexibility in (2). Different choices of \(K\) are addressed in Section 4. For illustration, Figure 1 shows the Black-Scholes implied volatilities of Erste Group and the corresponding interpolated volatilities of the artificial options over the same follow-up around the days of a possible external stimulus—the tribunal trial
between Erste Group and Croatia. An inconsiderable role of the proposed interpolation is obvious: While the original IV structure (Fig.1(a)) is clearly non-stationary over time mainly due to the natural market dynamics driven by the trading mechanism and the option payoff structure (i.e., three spikes corresponding with the maturity dates—July 20th, August 17th, and September 21st) the IV structure of the artificial options in Fig.1(b) seems to be relatively stationary with only minor fluctuations. If some fluctuation is systematic at some specific time—which can be particularly observed between
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(a) Implied volatility of the Erste Group call options

(b) Interpolated volatility of the artificial options

Fig. 1 Strike specific panels of the implied volatility values of the Erste Group call options (top panel) and the corresponding interpolated volatility of the artificial options with the constant (over time) maturity of $K = 35$ days. Both panels are given over the same follow-up period of 50 trading days (from Monday, July 16th, 2018 till Friday, September 21st, 2018). Three maturities (July 20th, August 17th, and September 21st), implicitly present within the follow-up period, are represented by the red vertical (dashed) lines.

the trading days no.10 and no.18—it is believed to be, very likely, a result of an exogenous effect.

Analogous pattern can be also recognized in Fig.1(a) but the magnitude of the changes is rather negligible when being compared with the overall magnitudes of the three main spikes related to the expiration dates. Thus, it is clearly inevitable to somehow smooth the natural market dynamics firstly when a statistical analysis of the market changes due to exogenous effects is of some interest. The artificial options constructed in terms of the proposed interpolation in (1) filter the natural market dynamics and the related (rather non-essential) effects are effectively removed. The remaining volatility—that is supposed to reflects the exogenous effects—is, however, not effected and, moreover, the magnitudes of the changes become stronger when compared with the
overall variability of the artificial options volatility. Therefore, the statistical analysis performed in terms of a formal statistical test will have much more power to detect significant changes related to the external causes.

3 Changepoint tests for exogenous effects

Many different statistical approaches are proposed in literature to deal with a so-called changepoint problem when detecting significant changes in some underlying probabilistic model—see Csörgő and Horváth (1997) for an overview. The approach discussed in this paper is based on a nice property of the proposed Implied Volatility interpolation/Artificial Options—IVintAO algorithm which takes the original non-stationary (and typically skewed) IV values and interpolates them into a set of artificial options volatilities which already seem to form a stationary structure. This is crucial from the theoretical point of view when formulating the underlying stochastic model. On the other hand, considering the applicational viewpoint and bearing also in mind different arguments of different practitioners, it may be appropriate to distinguish three different (but practically very similar) scenarios when performing the formal statistical test. We briefly address all of them, however, specific details are only given for the last one which is, as we believe, the most appropriate one (while explicit arguments are also provided to justify such statement).

Assuming some form of stationarity, the artificial options volatility can be represented by a formal underlying probabilistic model

$$Y_{it} = \mu_i + \delta_i \mathbb{1}\{t > \tau\} + \varepsilon_{it},$$

for the set of strikes $i = 1, \ldots, N$ observed within the follow-up period $t = 1, \ldots, T$. Such model is also known as a panel data model. Parameters $\mu_i \in \mathbb{R}$ for $i = 1, \ldots, N$ in (3) are panel specific mean parameters (i.e., unknown true strike specific volatilities) which may change at some unknown time point $\tau \in \{1, \ldots, T\}$. The changepoint location is common for all panels (i.e., the same external cause affects the market) however, the panel specific change-point magnitudes $\delta_i \in \mathbb{R}$ may differ (i.e., the resulting effect of the change depends on the strike value) while also allowing for a situation where only some proportion of the panels is subjected to the change (meaning that $\delta_i = 0$ for some $i \in \{1, \ldots, N\}$). Each panel specific mean parameter (for some fixed $i \in \{1, \ldots, N\}$) equals to $\mu_i$ before the change and it becomes $\mu_i + \delta_i$ after the change. The errors $\varepsilon_i = [\varepsilon_{i1}, \ldots, \varepsilon_{iT}]^\top$ can be seen as panel-specific disturbances (explicit theoretical details are provided later).

Statistically speaking, the whole problem of detecting the changepoint within the given panels can be formulated in terms of a statistical test of the null hypothesis

$$H_0 : \tau = T,$$

against a general alternative

$$H_1 : \tau < T \quad \exists k \in \{1, \ldots, N\} \quad \text{such that } \delta_k \neq 0.$$
While the formulation of the null and alternative hypothesis is rather simple, particular details regarding the statistical test itself may differ a lot—depending mainly on the theoretical assumptions imposed on the number of panels $N \in \mathbb{N}$ and the length of the follow-up period $T \in \mathbb{N}$.

**Scenario 1: $T \to \infty$ and $N$ is fixed**

A rather simple and straightforward method can be applied when treating the whole problem within a multivariate time series context. The dimensionality is determined by the number of panels and the follow-up period is assumed to tend to infinity. An asymptotically consistent statistical test is proposed, for instance, in Horváth, Kokoszka, and Steinebach (1999). The test can properly take into account the spatial dependence between the panels and, also, the dependence structure over time. On the other hand, when analysing market changes due to specific exogenous effects, practitioners usually focus on short periods closely before and after the event that is assumed to trigger the change and, therefore, the assumption on $T \to \infty$ is slightly impractical. Nevertheless, the test can be still applied and some empirical comparisons are also provided in Section 4.

**Scenario 2: $T \to \infty$ and $N \to \infty$**

More complex methodological framework is elaborated, for instance, in Horváth and Hušková (2012) or Chan, Horváth, and Hušková (2013) where both—the number of panels $N \in \mathbb{N}$ as well as the observational period length $T \in \mathbb{N}$ are assumed to tend to infinity (such that $N/T^2 \to c \neq 0$). However, the results rely on the assumption that the individual panels are independent observations while the error terms within the panels form causal linear processes. Some limited dependent structure in terms of a common stochastic factor among the panels may be assumed but it is, in our opinion, not that much realistic for the market scenarios considered in practice. Some finite sample drawbacks of this approach can be also seen in empirical comparisons in Section 4. Nevertheless, the formal statistical test is very analogous to the third scenario below which effectively detects exogenous effects assuming the given artificial IV structure.

**Scenario 3: $T$ is fixed and $N \to \infty$**

Bearing in mind the true character of the financial markets, it is reasonable to assume that the error vectors $\varepsilon_i = [\varepsilon_{i1}, \ldots, \varepsilon_{iT}]^T$ are neither independent nor identically distributed. A strong mixing condition among the panels is postulated to reflect the fact that more distant strikes have less dependent implied volatility than the volatilities of two strikes close to each other (see Section 4.2 for some empirical justification). There is also no specific form of stationarity being assumed within the panels and some heteroscedasticity across the panels is also allowed. This accounts for the situations where the strikes close to
the at-the-money position are expected to have smaller volatility. Different
approaches under various theoretical assumptions can be used (see, for instance,
Andrews (1993), Csörgő and Horváth (1997), Horváth, Horváth and Hušková
(2008), Shao and Zhang (2010), or PeÅ¡ta and PeÅ¡tovÁ¡ (2018)) but we rely
on the approach presented in Maciak, PeÅ¡ta, and PeÅ¡tovÁ¡ (2020) where
two competitive self-normalized test statistics are defined as
\[
Q_N(T) = \max_{t=1, \ldots, T-1} \max_{s=1, \ldots, t} \frac{|L_N(t, T)|}{|L_N(s, t)| + \max_{s=t+1, \ldots, T-1} |R_N(s, t)|} \tag{6}
\]
and
\[
S_N(T) = \sum_{t=1}^{T-1} \frac{L^2_N(t, T)}{\sum_{s=1}^{t} L^2_N(s, t) + \sum_{s=t+1}^{T-1} R^2_N(s, t)} \tag{7}
\]
where \(L_N(s, t) := \sum_{i=1}^{N} \sum_{r=s}^{T} (Y_{ir} - \bar{Y}_{it})\) and \(R_N(s, t) := \sum_{i=1}^{N} \sum_{r=s+1}^{T} (Y_{ir} - \bar{Y}_{it})\). Moreover, \(\bar{Y}_{it}\) is the average of the first \(t\) observations in the panel \(i\) and \(\bar{Y}_{it}\) is the average of the last \(T - t\) observations in the panel \(i\), i.e., \(\bar{Y}_{it} = \frac{1}{t} \sum_{s=1}^{t} Y_{is}\) and \(\bar{Y}_{it} = \frac{1}{T-t} \sum_{s=t+1}^{T} Y_{is}\).

Under some standard regularity conditions and the assumptions listed be-
low, the statistical test based on the test statistics in (6) or (7) can be proved
to be consistent. The distribution of the test statistics under the null hypoth-
esis in (4) is given by the next theorem. For further theoretical and technical
details we refer to Maciak, PeÅ¡ta, and PeÅ¡tovÁ¡ (2020).

**Assumption A** The random error vectors \(\varepsilon_i = [\varepsilon_{i1}, \ldots, \varepsilon_{iT}]^\top\), for \(i = 1, \ldots, N\), from a zero mean \(\alpha\)-mixing sequence, such that the mixing coefficients \(\alpha(i)\) satisfy \(\sum_{i=1}^{\infty} (\alpha(i))^{1/(2+\gamma)} < \infty\), for some \(\gamma > 0\) and, also, \(\sum_{i \in N} E[|\varepsilon_{i,t}|^{2+\gamma}] < \infty\), for all \(t \in \{1, \ldots, T\}\).

**Assumption B** Let there exists a positive definite matrix \(A\), such that
\[
A = \lim_{N \to \infty} \frac{1}{N} \text{Var} \left\{ \sum_{i=1}^{N} \frac{1}{s} \sum_{s=1}^{T} \varepsilon_{is} \right\}. \tag{8}
\]

**Theorem 1** Let Assumptions (A) and (B) hold. Then, under the null hypoth-
esis in (4), it holds that
\[
Q_N(T) \xrightarrow{D} \max_{t=1, \ldots, T-1} \frac{|X_t - \frac{1}{T} X_T|}{\max_{s=1, \ldots, t} |X_s - \frac{1}{T} X_T| + \max_{s=t+1, \ldots, T-1} |Z_s - \frac{T-s}{T-1} Z_t|}
\]
and
\[
S_N(T) \xrightarrow{D} \sum_{t=1}^{T-1} \frac{(X_t - \frac{1}{T} X_T)^2}{\sum_{s=1}^{t} (X_s - \frac{1}{T} X_T)^2 + \sum_{s=t+1}^{T-1} (Z_s - \frac{T-s}{T-1} Z_t)^2},
\]
for \(Z_t = X_T - X_t\), where \([X_1, \ldots, X_T]^\top\) is a multivariate normal random
vector with the zero mean vector and the covariance matrix \(A\).
Practically speaking, the test can be easily performed using the asymptotic distribution stated in the theorem above. Monte Carlo simulations or bootstrap approaches can be used as an alternative to mimic the distribution of interest and to obtain the corresponding quantiles. Once the null hypothesis in (4) is rejected practitioners and financial agents are usually interested in having a consistent estimate of the changepoint location $\tau \in \{1, \ldots, T-1\}$. A simple and straightforward estimator is proposed in PeÅ¡ta, PeÅ¡tovÁ¡, and Maciak (2019) while, unlike many other changepoint estimators suggested in the literature, it does not suffer from any common boundary issues when the true changepoint is located at the beginning or at the end of the follow-up period. The estimate for $\tau \in \{1, \ldots, T\}$ is defined as

$$\hat{\tau}_N = \arg \max_{t=1,\ldots,T} U_N(t),$$

(8)

where

$$U_N(t) = \begin{cases} \frac{1}{2(N-t)} \sum_{i=1}^{T} \sum_{u=t+1}^{T} (Y_{iu} - Y_{iv})^2 & t < T; \\ \frac{1}{2(T-1)} \sum_{i=1}^{T} \sum_{u=2}^{T} (Y_{iu} - Y_{iv})^2 & t = T. \end{cases}$$

Further theoretical details can be found in PeÅ¡ta, PeÅ¡tovÁ¡, and Maciak (2019).

One could argue that the underlying assumption of just one changepoint within the given follow-up period in the model in (3) is not realistic as market changes usually occur frequently and multiple “shocks” are typically observed within a consecutive series. However, the proposed statistical test in Scenario 3 does not require that $T \to \infty$ and, moreover, the follow-up period $T \in \mathbb{N}$ can be even arbitrarily short (as short as 2–3 days). Therefore, referring to a practical applicability of the proposed methodology, the market agents can just focus on any follow-up period around some specific external cause and if some significant change is detected by (6) or (7) the follow-up period is split into two parts—before and after the change and the whole mechanism is applied on both disjoint intervals again. On the other hand, if no changepoint is detected in some particular follow-up period, no further splitting is needed.

### 4 Empirical investigations

In this section we tackle down some finite sample particularities which may be considered important for practitioners and, also, we illustrate the proposed methodology using a real data example. The empirical performance of various changepoint scenarios is investigated via an extensive simulation study.

#### 4.1 Erste Group call options

The real data—the Black-Scholes model implied volatilities for the call options of Erste Group quoted in the EUREX Deutschland market—were downloaded
from Thomson Reuters Datastream. A follow-up period of 50 tradings days is considered\(^1\) starting on Monday, July 16th, 2018, ending on Friday, September 21st, 2018. The follow-up period is long enough to implicitly include some changes due to the natural market dynamics when approaching any of the three maturity dates (there are three consecutive maturities explicitly included in the follow-up period—July 20th, August 17th, and September 21st—to demonstrate the role of the proposed interpolation) and, also, possibly some changes caused by external causes because the company underwent a tribunal trial (Erste Group against Croatia (ARB 17/49), which is still a pending dispute) at that time. The first tribunal meeting took place in August 10th, 2018 while the first procedural order was issued in August 20th, 2018. Both events are typically considered to be important enough to have some serious impact on the market of the underlying asset and both dates are, therefore, intentionally included in the considered follow-up period. There are 11 strike specific panels equidistantly spanned between 30 Euros and 40 Euros (with a step of 1 Euro). The number of panels is limited in this example by the fact that the strikes must be quoted in all three maturity datasets but we rather consider this to be a technical issue that could be overcome in practice.

The raw IV values for the call options of Erste Group and the corresponding interpolated volatilities of the artificial options (for \( \hat{K} = 35 \)) are both visualized in Figure 1. Three maturity dates (July 20th, August 17th, and September 21st) implicitly present within the given observation period are visualized by the red vertical lines. The IV changes caused by the natural market dynamics are clearly visible in the top panel (Figure 1(a)) while the panel below (Figure 1(b)) provides more insight about the IV smile as it adapts to miscellaneous exogenous effects after the so called “getting-close-to-maturity” effect is removed by the proposed interpolation.

Applying the changepoint test on the raw IV values (from Figure 1(a)) both test statistics, (6) and (7), detect a significant changepoint (max-type test statistics \( Q_N(T) = 1.0897 \) with the corresponding critical value 0.8204 and the \( p \)-value 0.0059; sum-type test statistic \( S_N(T) = 3.0476 \), critical value 2.1747, \( p \)-value 0.0144). The changepoint estimated in terms of (3) yields \( \hat{\tau}_N = 4 \) which is July 20th, 2018. This perfectly corresponds with the first maturity date. Thus, the detected changepoint is clearly related with the natural stock market dynamics. Applying the same testing procedure on both sides from the estimated changepoint location, another significant change (on the right side of \( \hat{\tau}_N \)) is detected by both test statistics. The estimated changepoint location is August 17th, 2018 which again corresponds with the maturity date. No other changes are tested significant. Thus, both detected (significant) changes are clearly due to the natural market dynamics (see also Figure 3(a) for a visualization of the test statistics performance).

On the other hand, using the interpolated volatilities of the artificial options instead, the results become different and more informative. Again, both

\(^1\) Various option markets of different companies over a whole range of follow-up periods were considered with more-or-less the same results and analogous conclusions. The Erste Group case was chosen as one illustrative example.
test statistics detect a significant changepoint (max-type test statistic $Q_N(T) = 3.0171$ with the corresponding critical value 1.6541 and the $p$-value 0.0001; sum-type test statistic $S_N(T) = 7.9775$, critical value 7.4855, $p$-value 0.0230) but the location of the changepoint is different: $\hat{\tau}_N = 10$ (July 30th, 2018). This changepoint is mostly caused by some external event.

This becomes even more evident when repeating the whole testing procedure again considering either the right side from the previously detected changepoint or the left side respectively. Another significant changepoint is detected by both test statistics (max-type test statistics $Q_N(T) = 0.5820$, critical value 0.4932, $p$-value 0.0016; sum-type test statistics $S_N(T) = 4.9291$, critical value 4.8702, $p$-value 0.0494) while the estimated changepoint location is $\hat{\tau}_{N:2} = 16$ (August 8th, 2018). Finally, one more significant changepoint can be detected on August 23rd, 2018 ($\hat{\tau}_{N:3} = 29$) and no other significant changepoints are detected in addition (see Figure 2).

Looking back at important dates of the ongoing dispute between Erste Group and Croatia (ARB17/49), two out of three detected changepoints are immediately linked with the specific tribunal trial events. While the first changepoint ($\hat{\tau}_N = 10$, July 30th, 2018) is most likely also somehow related to the dispute, there seem to be no doubts regarding the other two changepoints. One ($\hat{\tau}_{N:2} = 17$, August 8th, 2018) is reflecting a sudden increase of uncertainty just before the first tribunal meeting and the other ($\hat{\tau}_{N:3} = 29$, August 20th, 2013) occurs right after the first procedural order when the situation at the market stabilized due to positive outcome of the meeting.

Thus, unlike the raw IV values where the only significant changepoints are related to the natural market dynamics—i.e., some volatility spikes when approaching the maturity dates—the interpolated volatility allows to detect significant changes which are clearly related to some more interesting exoge-
Fig. 3 Daily quantitative contributions to the test statistics defined in (6) and (7). The overall test statistic $Q_N(T)$ is defined as the maximum attained by the blue curve while $S_N(T)$ is given as the overall sum of all daily contributions. Black dotted lines correspond with the estimated changepoint locations defined in terms of the estimator in (3).

nous effects} instead. The interpolated volatility of the artificial options with the constant (over time) maturity is indeed a thoughtful tool for the analysis of the exogenous effects due to external causes which are otherwise practically undetectable as they are hidden by more accentuated changes created by the natural market dynamics.

4.2 Sensitivity analysis & residual inspection

Firstly, we briefly address some sensitivity issues related to different choices of the artificial maturity, $K \in \mathbb{N}$. As already mentioned, the specific choice of $K = 35$ takes into account the fact that any reasonable value should be in between of 30 and 40 days. Moreover, the choice of $K = 35$ reduces situations where no interpolation can be performed. For example, considering August 10th, 2018 and $K = 35$, the artificial option will expire on September 14th, 2018. Thus, the option expiring right before this artificial maturity is the one expiring on August 16th, 2018 while the option expiring right after
expires on September 20th, 2018. Both dates are sufficiently far from the artificial maturity which allows very effective interpolation. If, instead, \( K = 39 \), the artificial option would expire on September 19th, 2018 and the option expiring right after has the maturity September 20th, 2018 which is too close to the artificial maturity and some “getting-close-to-maturity” effect could introduce a bias in the interpolated volatilities.

Three specific choices for \( K \in \{30, 35, 40\} \) are particularly considered for an illustration. However, in general, small values of \( K \) seem to undersmooth the natural market dynamics (see Figure 4) but the most important structural breaks are still nicely preserved (significant changepoints are detected for the trading days no.10, 16, and 23). Larger values of \( K \) seem to oversmooth the natural market dynamics (see Figure 5) with only two significant changepoints being detected (trading days no.10 and 19). However, the overall conclusions made with respect to the exogenous effects are very similar for all three choices of \( K \in \{30, 35, 40\} \). The first changepoint is always detected at the same location—no matter what is the underlying maturity of the artificial options. The other changepoints depend on the amount of smoothness introduced by the interpolation algorithm. Thus, there is a typical statistical trade-off when determining the value of \( K \) for constructing the artificial options and the value of \( K = 35 \) seems to provide the best empirical results.

In the second part we provide some empirical investigation of the model based residuals for the Erste Group call options—used for the practical illustration in the previous section—in order to justify the theoretical assumptions postulated for Scenario 3 in Section 3. Crucial assumptions involve non-independent and non-identically distributed error terms which form a specific string mixing sequence—which can be also concluded from the estimated correlations in Table 1 and the partial auto-correlation plot in Fig.6(c). On the other hand, some heteroscedasticity among the strike specific panels is obvious again from Table 1 or, alternatively, from Fig.6(a)). The postulated assumption in Scenario 3 indeed seem to reflect the underlying stochastic nature of the market (interpolated) volatilities.

4.3 Simulation study

Finally, we empirically compare different testing approaches explicitly mentioned in Section 3. Three specific scenarios are considered: a) The follow-up period \( T \in \mathbb{N} \) tends to infinity but the number of panels \( N \in \mathbb{N} \) is kept fixed and the statistical test proposed in Horváth, Kokoszka, and Steinebach (1999) is applied; b) Both, the follow-up period as well as the number of panels tend do infinity and the test proposed in Horváth and Hušková (2012) is used; c) Eventually, the number of panels is assumed to tend to infinity while the follow-up period is fixed. The statistical test based either on the test statistic in (6) or the test statistic in (7) is performed and all the results are compared.

In order to closely mimic the real data example discussed above, relatively small values are considered for the follow-up period length and the number
Table 1 Residual based empirical estimates for the strike specific standard errors (over time) and the corresponding volatility correlations between two given strikes. The volatility of two distant strikes is clearly less dependent than the volatility of two strikes close to each other.

<table>
<thead>
<tr>
<th>Strike(s) (in EUR)</th>
<th>Errors 31</th>
<th>Errors 32</th>
<th>Errors 33</th>
<th>Errors 34</th>
<th>Errors 35</th>
<th>Errors 36</th>
<th>Errors 37</th>
<th>Errors 38</th>
<th>Errors 39</th>
<th>Errors 40</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.0136</td>
<td>0.90</td>
<td>0.76</td>
<td>0.71</td>
<td>0.58</td>
<td>0.51</td>
<td>0.41</td>
<td>0.30</td>
<td>0.25</td>
<td>0.22</td>
</tr>
<tr>
<td>31</td>
<td>0.0112</td>
<td>0.93</td>
<td>0.90</td>
<td>0.80</td>
<td>0.71</td>
<td>0.59</td>
<td>0.45</td>
<td>0.39</td>
<td>0.33</td>
<td>0.28</td>
</tr>
<tr>
<td>32</td>
<td>0.0089</td>
<td>0.95</td>
<td>0.86</td>
<td>0.79</td>
<td>0.67</td>
<td>0.53</td>
<td>0.47</td>
<td>0.41</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>33</td>
<td>0.0072</td>
<td>0.95</td>
<td>0.89</td>
<td>0.78</td>
<td>0.64</td>
<td>0.56</td>
<td>0.46</td>
<td>0.39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>0.0047</td>
<td>0.97</td>
<td>0.91</td>
<td>0.81</td>
<td>0.73</td>
<td>0.61</td>
<td>0.52</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>35</td>
<td>0.0045</td>
<td>0.97</td>
<td>0.89</td>
<td>0.81</td>
<td>0.68</td>
<td>0.58</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>0.0052</td>
<td>0.97</td>
<td>0.91</td>
<td>0.78</td>
<td>0.67</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>0.0058</td>
<td>0.98</td>
<td>0.88</td>
<td>0.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>0.0071</td>
<td></td>
<td></td>
<td></td>
<td>0.94</td>
<td>0.86</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>0.0083</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.94</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>0.0109</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

The empirical levels of the statistical tests performing under the null hypothesis of no changepoint in the model (3) are summarized over 1000 Monte Carlo runs in Table 2 (considering the theoretical level $\alpha = 0.05$). Regarding the empirical power of the tests the following alternative hypothesis is considered: a common changepoint location is placed after the first third of the follow-up period and the corresponding panel-specific changepoint magnitudes are generated randomly from the uniform distribution over the interval $(0, \theta)$, for $\theta > 0$, such that the signal-to-noise ratio equals one. The empirical performance of the tests under the alternative hypothesis (again over 1000 Monte Carlo runs) is summarized in Table 3.

Recall, that all statistical tests used in this section (and also mentioned in Section 3) are asymptotic tests. The theoretical critical level $\alpha = 0.05$ should be achieved asymptotically (either for $T \to \infty$, or $N \to \infty$, or both). Considering rather small values for $T \in \mathbb{N}$ and $N \in \mathbb{N}$ the first type error probability is usually larger than expected but it seems to properly converge to the nominal level (at different rates in different scenarios). On the other hand, the empirical powers of the tests seem to be the best for the test statistics defined in (6) and (7) especially when considering rather short follow-up periods, small number of panels, and some dependence within the panels.
Table 2  Empirical levels of four statistical tests (for $\alpha = 0.05$) described in Section 3 applied for three different scenarios with respect to the length of the follow-up period $T \in \mathbb{N}$ and the number of panels $N \in \mathbb{N}$. The results are summarized over 1000 Monte Carlo repetitions.

<table>
<thead>
<tr>
<th>Simulation Setup</th>
<th>$T \to \infty$</th>
<th>$T \to \infty$</th>
<th>$T$ fixed</th>
<th>$N \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$ 10/20/50</td>
<td>0.24 0.65 1.00</td>
<td>0.94 0.90 0.81</td>
<td>0.97 0.99 0.99</td>
<td>0.96 0.98 0.98</td>
</tr>
<tr>
<td>20</td>
<td>0.95 0.99 1.00</td>
<td>0.99 0.97 0.58</td>
<td>0.99 1.00 1.00</td>
<td>0.99 1.00 1.00</td>
</tr>
<tr>
<td>50</td>
<td>0.99 1.00 1.00</td>
<td>1.00 1.00 0.97</td>
<td>1.00 1.00 1.00</td>
<td>1.00 1.00 1.00</td>
</tr>
<tr>
<td>$D_2$ 10/20/50</td>
<td>0.40 0.60 1.00</td>
<td>0.99 0.99 0.99</td>
<td>0.48 0.58 0.61</td>
<td>0.47 0.56 0.56</td>
</tr>
<tr>
<td>20</td>
<td>0.73 0.73 1.00</td>
<td>0.99 1.00 1.00</td>
<td>0.70 0.75 0.79</td>
<td>0.71 0.72 0.72</td>
</tr>
<tr>
<td>50</td>
<td>0.75 0.86 1.00</td>
<td>1.00 1.00 1.00</td>
<td>0.91 0.98 0.99</td>
<td>0.89 0.94 0.97</td>
</tr>
<tr>
<td>$D_3$ 10/20/50</td>
<td>0.70 0.88 1.00</td>
<td>0.60 0.52 0.36</td>
<td>0.77 0.86 0.88</td>
<td>0.78 0.83 0.83</td>
</tr>
<tr>
<td>20</td>
<td>0.72 0.82 1.00</td>
<td>0.76 0.54 0.29</td>
<td>0.91 0.97 0.99</td>
<td>0.90 0.94 0.97</td>
</tr>
<tr>
<td>50</td>
<td>0.87 1.00 1.00</td>
<td>0.96 0.89 0.29</td>
<td>0.99 1.00 1.00</td>
<td>0.99 1.00 1.00</td>
</tr>
</tbody>
</table>

Table 3  Empirical powers of four statistical tests described in Section 3 applied for three different scenarios with respect to the length of the follow-up period $T \in \mathbb{N}$ and the number of panels $N \in \mathbb{N}$. The results are summarized over 1000 Monte Carlo repetitions.

<table>
<thead>
<tr>
<th>Simulation Setup</th>
<th>$T \to \infty$</th>
<th>$T \to \infty$</th>
<th>$T$ fixed</th>
<th>$N \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_1$ 10/20/50</td>
<td>0.20 0.12 0.10</td>
<td>0.06 0.06 0.01</td>
<td>0.10 0.10 0.08</td>
<td>0.10 0.09 0.07</td>
</tr>
<tr>
<td>20</td>
<td>0.09 0.08 0.05</td>
<td>0.06 0.02 0.00</td>
<td>0.08 0.07 0.07</td>
<td>0.07 0.07 0.06</td>
</tr>
<tr>
<td>50</td>
<td>0.10 0.09 0.05</td>
<td>0.05 0.03 0.01</td>
<td>0.07 0.06 0.05</td>
<td>0.07 0.06 0.05</td>
</tr>
<tr>
<td>$D_2$ 10/20/50</td>
<td>0.03 0.00 0.01</td>
<td>0.07 0.07 0.03</td>
<td>0.21 0.19 0.19</td>
<td>0.20 0.19 0.17</td>
</tr>
<tr>
<td>20</td>
<td>0.08 0.01 0.01</td>
<td>0.08 0.07 0.06</td>
<td>0.13 0.13 0.12</td>
<td>0.14 0.13 0.11</td>
</tr>
<tr>
<td>50</td>
<td>0.10 0.07 0.03</td>
<td>0.08 0.06 0.09</td>
<td>0.10 0.09 0.07</td>
<td>0.09 0.08 0.06</td>
</tr>
<tr>
<td>$D_3$ 10/20/50</td>
<td>0.06 0.04 0.04</td>
<td>0.07 0.02 0.01</td>
<td>0.18 0.15 0.15</td>
<td>0.17 0.15 0.15</td>
</tr>
<tr>
<td>20</td>
<td>0.08 0.06 0.07</td>
<td>0.06 0.02 0.01</td>
<td>0.12 0.10 0.10</td>
<td>0.10 0.10 0.09</td>
</tr>
<tr>
<td>50</td>
<td>0.09 0.09 0.07</td>
<td>0.04 0.03 0.01</td>
<td>0.09 0.07 0.06</td>
<td>0.07 0.06 0.05</td>
</tr>
</tbody>
</table>

5 Conclusion

The implied volatility serves as a very common and popular tool for analyzing the options markets but there are also some obvious limitations. For instance, the natural behaviour of the IV smiles usually generates changes of considerably higher magnitudes than those being observed when the market adapts to some sudden external impulses. Therefore, when analyzing the IV smiles automatically, most of the detected changes are very likely to be only related to the underlying market dynamics and any detection of exogenous effects is almost impossible. However, practitioners and financial agents are typically interested in all kinds of external events that may or may not affect the riskiness (or the price) of the underlying asset. Therefore, we proposed a whole methodological approach aiming explicitly and exclusively at the analysis of
the market reactions caused by the external, *exogenous effects*. The key contribution of our paper is four-fold: First, the *artificial options* with a *constant maturity* over time are introduced as the key tool for the *exogenous effects* analysis; Second, the standard implied volatility is shown to be insufficient for a proper detection of the exogenous effects and the implied volatility of the artificial options is empirically proved to be an undepreciated surrogate being capable to eliminate the natural market dynamics while conveniently preserving all exogenous effects; Third, the implied volatility of the artificial options is constructed in a simple and straightforward way using a (weighted) linear interpolation of the raw implied volatilities while introducing only very mild aggregation of the existing information (almost no information loss); Finally, as a direct consequence, the changes due to the *exogenous causes* are emphasized and a formal consistent statistical test is proposed as a valid inferential tool to detect statistically significant market changes which can be usually directly linked by market experts to some specific external (usually man-made and well-recognized) events. Thus, the proposed methodology allows for an effective and efficient analysis of the market behaviour when focusing on the changes caused by the external causes rather than the natural market dynamics itself. The presented approach is simple and the analysis can be performed within a fully automatic (data-driven) procedure. The artificial options are easily constructed using the standard IV values available on various financial databases and the *IVintAO* algorithm explicitly described in this paper.

**Acknowledgement**

The work of MM and SV was partially supported by the Czech Science Foundation, GAČR No. 21-10768S. SV also acknowledges a partial support by the Italian Ministry of Education, University, and Research, MIUR-ex60% 2022 sci.rep. Sebastiano Vitali.

**Declarations**

The authors have no conflicts of interest to declare that are relevant to the content of this article.

**References**


Fig. 4 Artificial call options volatilities of Erste Group for a constant (over time) maturity of \( K = 30 \) days (top panel) and strike specific differences when compared with the artificial call options with the reference maturity of \( K = 35 \) days (bottom panel). The maturities are given by red dashed lines and three detected changepoints in blue.

Fig. 5 Artificial call options volatilities of Erste Group for a constant (over time) maturity of \( K = 40 \) days (top panel) and strike specific differences when compared with the artificial call options with the reference maturity of \( K = 35 \) days (bottom panel). The maturities are given by red dashed lines, two detected changepoints in blue.
Fig. 6 Strike specific standard error estimates with the minimum attained around the at-the-money position (left panel) and the corresponding auto-correlation and partial auto-correlation functions for the times points $t \in \{1, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$ (middle and right panel).