Tooth Profile Construction and Experimental Verification of Non-circular Gear Based on Double Arc Active Design

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Article

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Posted Date: June 17th, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3059041/v1

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Additional Declarations: No competing interests reported.
Tooth Profile Construction and Experimental Verification of Non-circular Gear Based on Double Arc Active Design

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Abstract: The design of a double arc pitch curve is a new non-circular gear tooth construction method. Compared with the higher-order elliptic method, although the double arc method has the advantage of low manufacturing difficulty, the problem, in the process of designing the double arc pitch curve for solving the number of teeth is non-integer teeth trade-off problem and nonlinear model solving the solution is local convergence, has not been solved. Based on the meshing characteristics of the gear in the limit position of the double arc method and the theory of the surface's relative curvature consistency, the pitch curve's characteristic parameters are selected actively. To avoid the design difficulties $y_1$ and $y_2$ of the numerical solution instability, the most optimal parameter solutions are obtained by evaluating the distribution characteristics of the keys of the nonlinear system of equations. They use orthogonal tests to determine the ideal tooth combinations; tooth profiles of non-circular gears are built based on the constructed tooth rounding handling rules. Trial manufacturing and hydraulic loading tests reveal that the product’s torque at the rated speed is around 2% higher than that of the foreign prototype.

Keywords: Non-circular gears; Non-linear equations; Data processing; Orthogonal experiments; Modelling and simulation analysis

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0 Introduction

The non-circular gear mechanism is a hydraulic power equipment of compact structure and high transmission torque. It uses nonflammable emulsion as the working media. [1-3] It is widely used in various machines providing power, such as non-circular gear power drilling tools, non-circular gear stamping mechanisms, and non-circular gear five-bar finger mechanisms. As shown in Fig. 1, the core structure of the non-circular planetary gear mechanism with variable center distance mainly relies on several planetary wheels, a sun wheel, and an inner gear ring.

For the design of such hydraulic motor non-circular gears, scholars at home and abroad have conducted much fruitful research, mainly including the higher-order elliptic and double-arc methods for designing the pitch curve.

Li et al. [4-6] proposed a variable center distance class of non-circular gears different from the fixed center distance of planetary and solar wheels, which laid a solid foundation for the study of the equation of the pitch curve of non-circular gears and revealed the relationship among the structure of the motor, displacement and the type of non-circular gear mechanism. Zhang et al. [7] studied a new design method based on a conjugate algorithm to establish the optimization algorithm of tooth profile, to quickly generate the tooth profile of a non-circular gear. Tang [8] studied the parametric design of a non-circular gear hydraulic pump, verified the 3-4 hydraulic motor, and proposed the design method of the 4-6 hydraulic motor for the first time. Liao [9] established the pitch curve design model to derive the tooth shape of the non-circular gear, which was machined by
the CNC cutting method. Liu [10] studied the eccentricity as the control pitch curve parameter, designed different pitch curves, and analyzed the influence law of abnormality on the operation of non-circular gears. For the first time in China, the Double arc pitch curve non-circular gear design method was proposed by Xu [11], which determined the geometric parameters of the non-circular gear teeth and compared them with foreign prototypes for performance verification. Although many researchers have studied non-circular gears, the design of a non-circular gear tooth profile is still complicated because the pitch curve of non-circular gear has continuously varying curvature, which brings great inconvenience to the promotion and use of such noncircular gear hydraulic motors.

In contrast, the double arc pitch curve design is superior to the higher order ellipse because the curvature of the pitch curve of non-circular gear changes only at the point where the double arcs meet, which indicates that the curvature of the double arc pitch curve changes only once in the cycle [12-13].

Given the characteristics of the nonlinear solution of the optimal local solution, the problem of computational accuracy, the actual design of the double-arc intersection point away from each other, and the double arc pitch curve non-integer tooth problem, it further increases the design cost. It prolongs the design cycle because it has to conduct many trial and error methods to make the two arcs of different radius intersections point at a smooth transition. A more complex nonlinear problem is the transmission relationship between the inner ring, planetary wheel, and sun wheel to meet certain constraints in the design process of non-circular gear [14-17]. Therefore, the pitch circle model, that is model for normal meshing gear pitch curves tangent, of non-circular gear based on the nonlinear programming model can be proposed, which requires satisfying the geometric and transmission relationships of non-circular gears.

The purpose of this paper is to actively design a double arc pitch curve non-circular gear hydraulic motor to make the double arc pitch curve intersection point position smooth transition and the number of teeth rounded on the section curve, which is based on the meshing properties of the double arc pitch curve method limit positions gear and
nonlinear solving distribution characteristics. The validity of the noncircular planetary
gear mechanism design was determined through simulation analysis and experimental
verification, which provides a new technical idea for the realization of the double arc non-
circular gear design through the non-circular gear hydraulic motor pitch circle model,
which makes the non-circular gear design cleverly avoid the design difficulties that are
the non-linear solution of $y_1$ and $y_2$ numerical instability and solve the problem of double
arc pitch curve non-integer number of teeth, and to ultimately take into consideration the
arbitrary location on the sun wheel, and inner ring pitch curve tangent during the pitch
curve design process, which helps to redefine the pitch curve of non-circular gear.

1 Design of double arc pitch curve

1.1 The model for nonlinear solution

The kind of non-circular gear construction and the selection of the double arc pitch curve
characteristics, which determine the type of hydraulic motor and its performance, are the
core parts of a hydraulic motor. Considering the most broadly used 4-6 noncircular
planetary gear mechanism as an example, the double arc pitch curve of the center wheel
and inner gear ring can be obtained by combining and arraying two arcs of different radii,
as shown in Fig. 2. Because many academics have discussed the nonlinear solution model
in detail [20], the geometric formulation of the non-circular gear pitch curve is easily
obtained in this study.

(a) Inner gear ring
Fig. 2. The non-circular gear pitch curve

\[ L_{DF} + L_{FE} = K\pi m/2 \]  
(2)

\[ x_1 + r_1 = x_2 + r_2 + 2r_3 \]  
(3)

\[ y_1 - r_1' = y_2 - r_2' + 2r_3 \]  
(4)

\[ L_{AC} = L_{DF} + L_{FP} \]  
(5)

\[ \angle FCO_3 = \angle O_2 FO \]  
(6)

\[ x_2 + r_2 < y_1 - r_1' \]  
(7)

\[ r_1, r_1', r_2, r_2' > 5m \]  
(8)

The shape of the pitch curve can be uniquely determined by the derivation of the above equation.

1.2 Local optimal solution processing for nonlinear solution models

The iterative technique is often used method for solving nonlinear equations. The iterative process, along with the simple iterative approach, the Newton iterative method, and the string-cutting method, is one of the fundamental computing methods [19]. Based on the above deducing process of the pitch curve for solving nonlinear equations, the nonlinear model can only find the optimal local solution because the development of computer technology limits the nonlinear resolution. Thus it is still possible to find the exact answer. In this paper, we explore a data analysis strategy to achieve this aim: minimizing singular values in the pitch curve design process, since depending on the standard nonlinear solution approach makes creating a globally optimal solution challenging. By using varied beginning values, different local optimum solutions are found. The distribution properties of the keys of the nonlinear equations are investigated by solving the nonlinear
equations through a large number of constant repetitions to locate the substantially global optimum solutions by covering the local optimal solutions.

(1) Take a noncircular planetary gear mechanism with a modulus of $m=0.75$ as an example and solve for optimized values of the design variables.

(2) Assuming that the result of the calculation is $X = [x_1 \ y_1 \ r_1 \ x_2 \ y_2 \ r_2]$, the lower bound of the solution domain is $[0 \ 0 \ 0 \ 0 \ 0 \ 0]$, the upper bound of the solution domain is $[10^3 \ 10^3 \ 10^3 \ 10^3 \ 10^3 \ 10^3]$, and the initial value of solution domain is any value.

Fig. 3 Nonlinear solution of numerical distribution characteristics
The distribution of data points is shown, in Fig. 3, by the characteristic parameters in the process of repeated calculations several times. It can be found that four of the six values have a clear line of clustering of the solution, and it can be determined that the four values, except for the value points of \( y_1 \) and \( y_2 \) by excluding the singular values and the value points that do not match the constraints of the solution. The value of \( y_2 \) fluctuates too much, and the density of its attributes is a maximum between 450 and 900 in Fig. 3(d). Although the numerical distribution of \( y_1 \) is disorderly, the value’s range of variation is relatively narrow in Fig. 3(c).

2 Based on the active design of knotted circle model building

2.1 Selection of non-integer and integer teeth

The design of the pitch curve of the non-circular gear is the core of the non-circular equipment hydraulic motor. The non-circular gear hydraulic motor provides continuous drive through the geometric relationship between the planetary wheel, the internal gear ring, and the sun wheel to determine the meshing relationship between the gears and the hydraulic motor volume changes during the rotation of the non-circular gear. The final calculation result is \( X = [14.8790 \ 32.7749 \ 10.2336 \ 11.1665 \ 468.5719 \ 6.3664] \) by the conventional nonlinear solution used for non-circular gear in previous designs. The number of teeth of the double arc pitch curve \( r_1 \) is 27.2896, a non-integer number of teeth. Because non-integer teeth are non-standard, the tooth profile design calculates the tooth profile data points through MATLAB and then imports the data points into CAD to draw the non-standard tooth profile. The numerical simulation of the non-integer tooth shape is shown in Fig. 4.

![Fig. 4 Tooth shape of non-integer teeth](image-url)
As shown in Fig. 4, the alphabet a denotes its integer teeth, and the alphabet b means its fractional teeth. It was found that its non-integer teeth were more pointed at the point of intersection of the double arc with the top part of the integer teeth and that there was a larger arc of internal concavity on the lower side of its teeth, which seriously affected the regular meshing of the gears and caused the non-circular gears to jam during the meshing process.

Because non-circular gear designs exist for non-integer teeth, the precision of the tooth form is dependent on the computer's calculation accuracy via numerical simulation of the proposed tooth shape. Non-circular gears with non-integer teeth on the arc-shaped noncircular gear pitch curve complicate tooth design and noncircular gear manufacturing because non-integer teeth on the inner ring and sun wheel part can only be replaced by cylindrical gear approximation, resulting in significant tooth shape errors. The magnitude of the pitch curve entirely determines the number of teeth on the pitch curve. If the double arc pitch curve non-circular gear is assumed to be an integer gear pitch circle, the hydraulic motor's design difficulties and production cost considerably decrease. As a result, the integer tooth pitch curve design serves as the foundation for the double arc non-circular gear design.

2.2 Pitch circle model of non-circular gears

A simplified method of noncircular gear design is proposed in this paper, in which \( r'_1 \) and \( r'_2 \) replace the original \( y_1 \) and \( y_2 \) to form a new double arc pitch curve. Given the preceding, the pitch curve design is the critical issue of non-circular gear design, the key point of which is the active treatment at the intersection of the double arc, which ensures a smooth transition of the intersection position and the pitch radius of the double circle arc pitch curve for integer teeth. Because \( y_1 \) and \( y_2 \) are shown to be numerically unstable during non-linear solutions by non-linear solutions and data analysis, a non-circular gear pitch curve pitch circle model design is proposed in this article. The non-circular gear knotted circle model has the benefit of effectively avoiding the complex structure points \( y_1 \) and
\( y_2 \), requiring just \( X = [x_1 \ r'_1 \ r_1 \ x_2 \ r'_2 \ r_2] \), which considerably increases design efficiency and minimizes manufacturing difficulties to reduce industrial costs.

After extensive verification and analysis, it was discovered that the non-integer tooth rounding on the pitch curve and the smooth transition of the double arc intersection are at the core of solving the design of the double arc pitch curve hydraulic motor. The Model for normal meshing gear pitch curves tangent can meet the tangency of the double arc at the intersection point, resulting in a smooth transition of the knotted curve while solving the non-integer tooth rounding problem. The approach meets the operating requirement of keeping the pitch curve tangent during accurate non-circular gear operation, as shown in Fig. 5.

According to Fig. 5, the third edge position is directly related to \( y_2 \), and the processing of the work dramatically affects the final performance of the non-circular gear. Through these three edge positions, the design of the double arc curve can significantly simplify the design process.

![Diagram](image-url)
(a) the first engagement position
The first limit position satisfies the tangency of the pitch curve, and the second is at the point where the double arcs meet, which is not considered for the time being. Only the third limit position must be calculated, considering that the planetary wheel and the inner gear ring are tangential.

The value of $y_2$ is taken several times with different values, but the value of $y_1$ is fixed, considering that the value of $y_1$ is relatively stable compared to that of $y_2$. The study found that as the value of $y_2$ decreases, the amount of interference between the sun wheel and the planetary wheel gradually decreases. When other values are fixed, the value of $y_2$ decreases to a specific value to ensure that the third limit position of the non-circular gear pitch curve is tangent to each other, as shown in Figure 6.

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**Fig. 5** Three limit positions of non-circular gears

(b) the second engagement position

(c) the third engagement position
Studies have found that it changes with the $y_2$ value from the initial interference to the later tangent by continuously repeating the geometric mapping of the third limit positions using different $y_2$ values, as shown in Table 1. However, according to the graphical analysis of the data above, the discount of $y_2$ should be limited to 300 or more. It is a fact that is not allowed in the design of non-circular gears. Suppose the value of $y_2$ is less than 300 or greater than 1000. In that case, the remaining five variables will be out of the stable range values, causing various numerical defects in the solution process, thus making the values deviate from the reasonable range values. To avoid interference, the constraints of equation (9) should be established by the principle of relative curvature consistency of the surface, which satisfies the requirement of tangency of non-circular gear pitch curves [18].

$$\Delta k_{n1}\Delta k_{n2} - (\tau_1)^2 \geq 0 \quad (9)$$

$[\Delta k_{ni}]$ - The difference of the normal curvature of the curve along the two perpendicular directions ($i=1,2$).
\[ \tau_1 \] - The difference in the short-range deflection rate of the curve along the two perpendicular directions.

Regarding gear design and manufacturing, gears are designed with integer teeth instead of non-integer teeth to reduce costs. The number of teeth of the arcs of the inner ring \( r_i \) and \( r'_i \) is actively rounded based on the value of the nonlinear solution and the tangency constraint of the pitch curve. A part of the pitch circle of an integer gear is used as a double arc pitch curve, which requires rescaling based on known \( X \) values and satisfying the constraints of the geometry of non-circular equipment, as shown in Figure 7.

![Fig. 7 Schematic diagram of the solution of the inner gear ring of the knotted circle model](image)

Illustrated Fig. 7 shows three parameters of the inner ring \( x_1, y_1, \) and \( r_i \), redesigned by the pitch circle model, where \( r_{11}, r'_{11}, \) and \( x_{11} \) become the core data of the internal ring design.

\[
\Delta d = r_1 - r_{11} \tag{10}
\]
\[
x_{11} = x_1 + \Delta d \tag{11}
\]

\( \Delta d \) - The difference after taking an integer number of teeth for the double arc pitch curve.

\( x_{11} \) - The \( x_i \) of the pitch circle model

\( r_{11} \) - The \( r_i \) of the pitch circle model

\( r'_{11} \) - The \( r'_i \) of the pitch circle model

Calculating the pitch circle model parameters depends on the geometric and transmission relationships.

The latter \( x_{11}, r_{11}, \) and \( r'_{11} \) are replaced by \( x_i, r_i, \) and \( r'_i \) to facilitate subsequent calculations and reading. First, the ray with an angle of 30° is made from the origin. Then
draw a circle with the connection point between $x_1$ and $r_1$ as the center, $O_1O'_1$ is the sum of the radii of the double arcs $r_1$ and $r'_1$, as the radius to make an intersection with the ray, whose distance from the intersection point to the origin is the length of $y_1$. The meeting and source are connected from Fig. 7(a) to Fig. 7(b). The three basic parameters $x_1$, $y_1$, and $r_1$ of the internal gear ring can be obtained through the pitch circle model, which can be used as the basis to achieve the design requirements of a smooth transition of the intersection point of the double arc pitch curve and the number of teeth to be rounded.

Similarly, the solar wheel can be treated in this way.

The final determination of $X = [x_1 \ r'_1 \ r_1 \ x_2 \ r'_2 \ r_2]$ parameters through the model for normal meshing gear pitch curves tangent, and using the geometric constraints can be found all the values of $X = [x_1 \ y_1 \ r_1 \ x_2 \ y_2 \ r_2]$.

In the following process of solving for the integer teeth based on the model for normal meshing gear pitch curves tangent, the problem of rounding up or down for non-integer teeth is verified using orthogonal experiments. The number of teeth for each section curve is determined based on the results of the orthogonal experiments, which are discussed in the next section.

After comparing with the original data, the maximum change in the $X$ value of the knotted circle model was 1.7%, and the minimum was 0.0332%. The model for normal meshing gear pitch curves tangent further reduces its design and manufacturing costs. It shortens the design cycle by rounding the number of teeth on the pitch curve and smoothly transitioning the double arc intersection point. It ensures the correctness of the nonlinear solution. The tangency of the non-circular gear pitch curve is verified by equation (9).

3 Modeling and simulation of non-circular gears

3.1 Determination of basic gear parameters

The double arc pitch curve is created, and the $X$ values are determined based on the model for normal meshing gear pitch curves tangent. The gear parameters of non-circular gears are shown in Table 2. After selecting the gear parameters, the corresponding pitch curves and non-circular gear tooth profiles were made through CAXA software. Then three limit
positions were verified to ensure that the non-circular gear pitch curves were tangent at any work during the design process and that the designed gears did not show tooth profile interference at each limit position.

The numerical geometry mapping of $X = [x_1 y_1 r_1 x_2 y_2 r_2]$ after the model for normal meshing gear pitch curves tangent shows no interference in the tooth profile's first and third limit position meshing positions, proving that the meshing gear requirements are met. That ensures the stability of the alternate meshing of the sun wheel, planetary wheel, and inner gear ring, laying the foundation for the later modeling and simulation, as shown in Figure 8.

Table 2. Noncircular gear parameters

<table>
<thead>
<tr>
<th>Gear parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>0.75</td>
</tr>
<tr>
<td>$Z_1$</td>
<td>44.00</td>
</tr>
<tr>
<td>$Z_2$</td>
<td>66.00</td>
</tr>
<tr>
<td>$Z_3$</td>
<td>10.00</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>20.00</td>
</tr>
<tr>
<td>$h_a^*$</td>
<td>0.70</td>
</tr>
<tr>
<td>$c^*$</td>
<td>0.25</td>
</tr>
</tbody>
</table>

(a) The first engagement positions
The third engagement positions

Fig. 8 Tooth profile limit position diagram

The drawing, modeling, and assembly of non-circular gears were implemented through CAXA and UG software, as shown in Figure 9.

(a) the inner gear  (b) the center wheel

(c) the planetary wheel  (d) Noncircular Gear

Fig. 9 3D model of a non-circular gear

3.2 Orthogonal experiments of non-circular gears based on the pitch circle model
Discussing the problem of rounding non-integer teeth on different pitch curves based on the model for normal meshing gear pitch curves tangent under a nonlinear solution requires 24 experiments, significantly increasing design and manufacturing costs. The orthogonal experimental design is a method to study multiple factors and levels based on orthogonality to select some representative points from the comprehensive test, characterized as "uniformly dispersed and comparable."

In the process of trial rotation, the non-circular gear jamming phenomenon is expected in non-circular tooth assembly because the planetary wheel force suddenly becomes significant in operation to a particular position. Establishing a quantitative criterion to evaluate the peak forces on planetary wheels for non-circular gear teeth rounding is necessary. To select the optimal combination of teeth, the planetary rotation is allowed to run for one week, and the meshing force of the gears is measured at any moment. At the same time, the increase of the sum of two peak meshing forces of the planetary wheels in adjacent cycles is used as an evaluation indicator. Equation (12) is used as the evaluation index for the optimization results of this simulation.

Evaluation indicators for this orthogonal experiment.

\[
a = \frac{|s_i - s_{i-1}|}{s_i} \quad (i = 2 \ldots 6) \tag{12}
\]

\[
P = \sum_{i=2}^{i=6} a \tag{13}
\]

s_i - The sum of the two maximum forces in one cycle through the tooth at the point of intersection of the double arc

a - Increment of adjacent cycles.

P - Engagement force coefficient – the ratio of incremental engagement force of adjacent cycles.

Based on the nonlinear solution, the final result was \( X = [14.8790, 32.7749, 10.2336, 11.1665, 468.5791, 6.3664] \). The non-circular gear hydraulic motor has four arcs to draw the corresponding tooth shape, whose results on any angles are \( z_1 = 27.2896, z'_1 = 30.3008, z_2 = 16.98, \) and \( z'_2 = 1115.39 \) respectively. Because the number of teeth of \( z_2 \) is 16.98,
incredibly close to 17, the teeth of \( z_2 \) are directly rounded. In the subsequent design process, there are three arcs on which the number of teeth needs to be rounded, with the number of teeth on the pitch curve of each hook rounded up or down. The factors and levels of the non-circular gear orthogonal experiments are shown in Table 3.

<table>
<thead>
<tr>
<th>Test levels</th>
<th>Test factors</th>
<th>( z )</th>
<th>( z' )</th>
<th>( z'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27</td>
<td>30</td>
<td>1115</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>28</td>
<td>31</td>
<td>1116</td>
<td></td>
</tr>
</tbody>
</table>

The experiment is a 3-factor 2-level, choosing the orthogonal experiment table as \( L4(2^3) \), requiring four experiments to do simulation experiments for four different combinations of tooth numbers on the pitch curve. The coefficient \( P \) of the engagement force for each group of measurement schemes can be obtained, as shown in Table 4.

<table>
<thead>
<tr>
<th>Number of tests</th>
<th>Test factors</th>
<th>( z )</th>
<th>( z' )</th>
<th>( z'' )</th>
<th>( P ) Engagement coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27</td>
<td>30</td>
<td>1115</td>
<td>1.294</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>27</td>
<td>31</td>
<td>1116</td>
<td>6.486</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>30</td>
<td>1116</td>
<td>1.394</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>28</td>
<td>31</td>
<td>1115</td>
<td>0.965</td>
<td></td>
</tr>
</tbody>
</table>

Table 4 shows that the engagement force coefficient of experiment 4 is the smallest, which indicates that the stability of the engagement force of the planetary wheel of experiment 4 is optimal. The range value \( R \) reflects the effect of the tooth number factor on the non-circular gear meshing force. The larger the range \( R \)-value, indicating that this factor has a more significant influence on the engagement force, is the primary factor. The second factor is the smaller the range \( R \)-value, meaning that this factor has less impact on the engagement force. Table 5 shows that the change in the number of \( z' \) teeth has the most significant
effect on the stability of the gear meshing power. The change in the number of \( z_1 \) teeth has a slightly smaller impact on the strength of the equipment meshing energy compared to \( z'_2 \), and the difference in the number of \( z'_1 \) teeth has a minor effect on the stability of the gear meshing force. The optimal combination of better levels corresponding to the number of teeth is shown in Table 5.

<table>
<thead>
<tr>
<th>Test factors</th>
<th>range</th>
<th>Better level</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_1 )</td>
<td>2.216</td>
<td>28</td>
</tr>
<tr>
<td>( z'_1 )</td>
<td>2.292</td>
<td>30</td>
</tr>
<tr>
<td>( z'_2 )</td>
<td>2.659</td>
<td>1115</td>
</tr>
</tbody>
</table>

The most optimal combination of tooth numbers was obtained with \( z_1 = 28 \), \( z'_1 = 30 \), \( z_2 = 17 \), and \( z'_2 = 1115 \), as shown in the better levels in Table 5. By analyzing the range values, the degree of influence on the engagement force is from the largest to the smallest: \( z'_2, z'_1, z_1 \).

### 4 Conclusions

The non-circular gear design model drawing is machined by slow-walking EDM machine, and the finished product is shown in Figure 10. Figures 10, 1, 2, and 3 indicate the inner gear ring, planetary wheel, and sun wheel, respectively.

**Fig. 10** Self-made gear mechanism
After machining the non-circular gears, there is no jamming in rotation after assembly, and the Gear Backlash at any place tends to be the same; then, the performance is tested on the built-up test bench.

The test stand is shown in Figure 11. In the test bench, the emulsion pump is used as the power source, and the emulsion is used as the working medium to make the hydraulic motor rotate under a specific torque; the magnetic powder brake applies a sure torque at the output of the hydraulic engine according to the command sent by the speed collector; the torque speed sensor is installed on the magnetic powder brake to measure the torque and speed data in real-time, which is transmitted to the torque-speed power collector and displayed in real-time through the form of data. The data collected at the experimental scene were analyzed using the least squares method, as shown in Figure 12, and red lines indicate the fitted curves.
In Fig. 12, the small square indicates that the torque of the foreign prototype is 95N·m at 380 r/min, and the torque of the homemade gear mechanism data is 97N·m at the same speed, slightly higher than that of the foreign prototype. The actual curve variation trend of the torque-speed graph is reasonable, indicating that the research method proposed in this paper meets the basic design requirements.

The main conclusions of this paper are as follows:

1) In solving the nonlinear system of equations, the curvature consistency constraint is established to effectively determine the final relative optimal solution by finding the distribution law of the optimal local solution. This provides a new idea for the nonlinear solution of the noncircular gear model.

2) Based on the nonlinear solution of the non-circular gear double arc, the design difficulty of y1 and y2 is cleverly avoided through the design of the model for normal meshing gear pitch curves tangent, and orthogonal experiments are used to determine how the number of teeth on the double arc is chosen. The relative curvature of the surface and the geometric mapping method to verify the selection of the X value and the number of teeth to be rounded are reasonable, which further reduces the design and manufacturing
costs, ensures that the pitch curves are tangential and avoids interference during the gear meshing process.

3) Field processing experiments have demonstrated the design method of this thesis. The hydraulic performance experiments show that the self-made product can meet the requirements of its performance in use because there is no jamming in the gear meshing process, and the Gear Backlash at any position tends to be the same.

Author Contributions

All the authors contributed significantly to this work.

Funding

This work was supported by the National Natural Science Foundation of China (Grant nos. 51975185 and 51505129) and University’s Scientific Research Project (Grant no. ZQK202002).

Conflicts of Interest The authors have no conflicts of interest to declare that are relevant to the content of this article.

Availability of Data and Materials The data used to support the findings of this study are available from the corresponding author upon request..
References


Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- Table1.docx
- Table3.docx
- Table2.docx
- Table4.docx