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Dynamic modeling of thin-walled CFRP surface milling and the effect of chatter on surface quality

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Abstract

Milling a thin-walled curved structure is prone to chattering, and the quality of the machined surface cannot be guaranteed. For carbon fiber reinforced plastic (CFRP) parts with a strong anisotropic characteristic, the tool will contact different fiber directions, resulting in a large change in dynamic cutting force and more complex chatter. Thus, this study introduced the anisotropic and laminated characteristics of CFRP into the machining dynamic model and stability domain solution prediction in milling CFRP. Meanwhile, a dynamic cutting force model under the regenerating mechanism was first proposed by considering the instantaneous fiber cutting angle and the cutting edge length of the instantaneous contact micro-element with the same fiber layer. Moreover, the dynamic model for milling this CFRP workpiece was established based on the dynamic cutting force model and dynamic characteristics of the workpiece. The stability lobe diagram was drawn, and the stable cutting regions for milling CFRP were obtained. Also, experimental verifications were conducted, and the chattering and stable machining conditions were well predicted. Compared with the stable region obtained without considering the anisotropic characteristic of CFRP, the predicted maximum peak value of the stable region increased by 49.7%, proving the accuracy of the proposed method.

Keywords: CFRP; thin-walled curved structures; surface milling; fiber direction; chatter;
1. Introduction

Carbon fiber reinforced plastic (CFRP) has become the preferred material for high-end equipment manufacturing because of its superior characteristics such as lightweight, high strength, and good fatigue resistance [1-3]. The continuous improvement of the performance of aviation, aerospace, and other high-end equipment, such as rocket tanks and aircraft cylinder sections, requires the structural parts of CFRP equipment to develop towards large size, thin walls, and complex curved surfaces [4,5]. However, most complex curved CFRP components still need assembly and connection after forming. To meet the accuracy of the connection, it is necessary to perform surface milling on the connection and mating parts of the CFRP curved workpiece, and the surface quality requirements are very strict to ensure the service performance of high-end equipment [6-8]. Unfortunately, due to the complex structure and weak rigidity of the workpiece, the chatter often occurs under the continuous action of the cutting edge [9]. In addition, due to the anisotropic and laminated characteristics of CFRP [10,11], the chattering behavior is complex, and the vibration is difficult to control. Chatter could lead to irregular material removal, seriously affect the quality of the machined surface, and even directly cause a scrap of the workpiece. Therefore, it is urgent to suppress the chatter phenomenon during the surface milling of thin-walled CFRP and improve the surface machining quality.

Many studies have been conducted in the past few decades to reveal the chatter mechanism and find solutions to suppress the machining chatter. Currently, the self-excited vibration between the tool and the workpiece, i.e., regenerative chatter, is the most important chatter mode in thin-walled milling [12-14]. During milling, the vibration between the tool and the workpiece will lead to a change of cutting thickness and leave ripples on the machined surface, which in turn will affect the subsequent cutting process [15]. The dynamic cutting process can be mathematically expressed as a time-periodic
differential equation with a single delay [16]. Based on this, by predicting the dynamic milling force during the machining process, the dynamic equation of the milling system can be established, and the flap diagram of machining stability can be solved. In this way, the relationship between the critical axial cutting depth of chatter and the spindle speed can be established, and the chatter can be suppressed from the aspect of process parameters [17,18]. Initially, scholars established a single-degree-of-freedom dynamic model of thin-wall milling process dynamics by solving the dynamic milling force in the direction of machining vibration. The stability of the model was analyzed by the discretization method, and the machining stability region was obtained [19]. Y Ding et al. [20] also predicted the dynamic milling force in two directions during the machining process. They proposed a milling stability prediction method based on an integral equation and a numerical integral formula, and the validity of the method was verified through experiments. The above research starts from predicting the machining dynamic milling force, establishing a dynamic model, and predicting the machining stability. To improve the final prediction accuracy, improving the prediction accuracy of dynamic milling force is an important approach. Scholars analyzed the dynamic milling model of the thin-walled workpiece from two vertical directions of the machine-tool system and the workpiece system. Then, a compensation method was proposed based on inertia effect correction, and the stability domain of a thin-walled workpiece in high-speed milling was obtained [21]. Insperger et al. [22] analyzed the influence of tool runout on milling chatter frequency following the periodic system theory and verified the accuracy of theoretical analysis through a series of cutting experiments. Eksioglu C et al. [23] considered the influences of tool morphology, tool runout, and process damping in the unit force model to establish a distributed force model along with the axial depth of the cut. By applying the distributed force model to the tool and the thin-wall structure, the thin-wall vibration can be predicted accurately.
Based on improving dynamic milling force, considering multi-directional dynamic characteristics of the machining process is another important approach for improving the accuracy of stability region prediction. Zhang XW et al. [24] established a three-dimensional dynamic model of thin-walled parts in the milling process by considering the work-tool axis vibration and both the tool subsystem and workpiece subsystem, and the model achieved a good prediction effect. The above research on the thin-wall CFRP surface component provides an important basis for suppressing the milling chatter phenomenon. Most of the research focuses on the metal material, different material properties of CFRP, and homogeneous material, but the material properties of CFRP are different from those of homogeneous materials. The dynamic milling force and chatter characteristics of CFRP are greatly affected by fiber direction and lamination. As a result, the metal cutting theory cannot be simply applied to CFRP to suppress the chatter phenomenon.

Recently, some scholars have studied the cutting chatter of CFRP based on the metal chatter theory. For example, based on the traditional single-degree-of-freedom milling dynamics model, K Kecik et al. [25] considered the influence of fiber resistance on the cutting force and predicted the machining stability with numerical methods. By introducing vibration between the tool and the workpiece to suppress the chatter phenomenon, the quality of the processed surface was improved. Rafal Rusinek et al. [26] proposed a chatter prediction method based on the measurement of cutting force components, and they established the lobe diagram in the stability domain of tool speed and cutting depth in the plane milling process. Then, they decomposed the cutting force time series by the HHT method, proposed a new chatter index, and improved the detection accuracy of CFRP milling chatter. Hou Y [27] established the milling force model of CFRP one-way plate, described the change law of milling force in different fiber directions, analyzed the influence of fiber direction on the chatter stability of
milling UD-CFRP, and proposed a new method for predicting the surface quality of milling UD-CFRP. The above research conducted dynamic modeling for the CFRP milling system and predicted the stability region of CFRP milling, which provided an important reference for suppressing the chatter phenomenon in the milling of CFRP thin-walled surface workpieces.

The dynamics model of the milling system becomes complicated when a ball-end milling cutter is adopted to mill the surface of a CFRP thin-walled curved workpiece. However, little work has been devoted to solving the problem. Specifically, due to the anisotropy of CFRP, the cutting edge acts on different fiber directions at different times, and there is a large fluctuation within the layer. In this case, the cutting force feedbacks each other with cutting thickness, and the calculation accuracy of dynamic cutting force is low. Besides, CFRP has laminated characteristics, the ball-end milling cutter has a complex spiral cutting edge structure, the cutting edge at the same time is at different fiber direction layers, and the instantaneous cutting force on the same cutting edge is complex and discontinuous, making it more difficult to calculate the dynamic cutting force. During the surface milling of a CFRP thin-walled curved workpiece, the rigidity of the workpiece is weak, the dynamic characteristics of the workpiece change greatly, and the chattering phenomenon is more complex. Therefore, the accuracy of the model can be guaranteed by considering more degrees of freedom in more directions. Therefore, according to the structure and material properties of the CFRP thin-walled curved workpiece, analyzing the chatter mechanism, predicting the dynamic milling force and establishing an accurate dynamic model in the milling process is significant for guiding the selection of reasonable process parameters.

In this study, a complete dynamic modeling and stability prediction scheme is proposed for milling CFRP thin-walled curved workpieces with a ball-end milling cutter. The dynamic cutting force
of CFRP with a regeneration mechanism was predicted by considering the change in the instantaneous fiber cutting angle and the length of the cutting edge in instantaneous contact with the same fiber laminated micro-element. Then, a dynamic model considering the three-direction dynamic characteristics of a flexible CFRP workpiece was established to predict the machining stability region, and the stability lobe diagram was obtained. Finally, the accuracy of the machining stability region was verified by experiments. The study results provide a theoretical reference for selecting process parameters to suppress the chatter phenomenon in surface milling of CFRP thin-walled curved workpieces.

2. CFRP dynamic milling force prediction model

The cutting force of the CFRP component is affected by fiber direction and lamination, and the variation law of the dynamic cutting force is complicated. Therefore, this study established a static milling force prediction model by considering the instantaneous cutting angle and laminated characteristics of the CFRP fiber, and the milling force coefficient was solved. Since the milling force coefficient varies with the fiber cutting angle, the overall dynamic cutting force was obtained by considering the dynamic cutting thickness caused by the regeneration mechanism.

2.1. Static milling force prediction considering CFRP material characteristics

There are many types of milling tools, and the ball milling cutter is developed based on the end milling cutter. Because of its large effective blade angle range and strong self-adaptability, the ball milling cutter is mainly used for surface processing to achieve high precision and efficient processing. The geometrical structure of the working part of the ball-end milling cutter is relatively complex. Two
coordinate systems are needed to define the geometric parameters of the cutter and express the position and state of the cutter in the milling process. Thus, the tool coordinate system XYZ and the workpiece coordinate system FCN are established, respectively. In the tool coordinate system, the geometric center of the ball head is the origin of the coordinate system O, the Z-axis is along the vector direction of the tool axis, the X axis and Y axis are in the transverse direction, and the tool radius changes along the tool axis. In the workpiece coordinate system, F represents the feed direction, N represents the normal direction of the workpiece surface, and C represents the transverse feed direction. The geometric parameters of the ball head part of the ball head milling cutter are shown in Fig. 1. For any point of cutting edge $j$ under the tool coordinate system, the parameter equation can be expressed as:

$$
\begin{align*}
X_{c,j} &= R \sin \kappa \cos \psi_j \\
Y_{c,j} &= R \sin \kappa \sin \psi_j \\
Z_{c,j} &= R(1 - \cos \kappa)
\end{align*}
$$

(1)

where $\kappa$ is the axial immersion angle; $\psi_j$ is the immersion angle, $\psi_j = \theta + (j-1)2\pi / N - \varphi(\kappa)$; $\theta$ is the tool rotation angle, $\theta = 2\pi \times n \times t / 60$; $\varphi$ is the helix lag angle, $\varphi = (R - R \cos \kappa) \tan \beta_0 / R$, and $\beta_0$ is the maximum helical angle of the cutter.

The fiber direction angle is denoted as $\lambda$. In the process of CFRP milling, the angle formed by fiber direction and cutting speed is defined as the fiber cutting angle $\beta$, as shown in Fig. 2. The cutting edge of the tool will experience different fiber cutting angles in each rotation cycle. The relationship between the fiber cutting angle and tool rotation angle is as follows:

$$\beta = \begin{cases} 
\lambda - \theta , & \theta \leq \lambda \\
180^\circ - (\theta - \lambda) , & \theta > \lambda
\end{cases}$$

(2)

The milling force model established by Altintas [28] is widely applied and studied. In this model,
the milling force is regarded as a function of the milling area and the contact length between the workpiece and the milling blade in the machining process. The milling blade is dispersed into many micro-elements, and the force analysis of each micro-element is conducted. Then, the whole force in the machining process is obtained through the efforts of the micro-cutting edge. When milling CFRP, the cutting force is considered to change with the fiber cutting angle. Therefore, the micro-element milling force at any point on the milling edge can be expressed as:

\[
\begin{align*}
    dF_t &= K_{tc}(\beta) \cdot h \cdot db + K_{te}(\beta) \cdot ds \\
    dF_r &= K_{rc}(\beta) \cdot h \cdot db + K_{re}(\beta) \cdot ds \\
    dF_a &= K_{ac}(\beta) \cdot h \cdot db + K_{ae}(\beta) \cdot ds
\end{align*}
\]  

(3)

where, \(dF_t\) is the tangential milling force coefficient; \(dF_r\) is the radial milling force coefficient; \(dF_a\) is the axial milling force coefficient; \(K_{tc}(\beta), K_{rc}(\beta),\) and \(K_{ac}(\beta)\) are tangential force coefficients, radial tangential force coefficients, and axial tangential force coefficients, respectively; \(K_{te}(\beta), K_{re}(\beta),\) and \(K_{ae}(\beta)\) are tangential edge force coefficient, radial edge force coefficient, and axial edge force coefficient, respectively; \(h\) is the instantaneous cutting thickness, \(db\) is the cutting width, and \(ds\) is the cutting micro length.

According to the coordinates of any point on the cutting edge curve, the microelement cutting edge length in Eq. (3) can be obtained:

\[
dS = \sqrt{\left(X_{c_j}^\prime \right)^2 + \left(Y_{c_j}^\prime \right)^2 + \left(Z_{c_j}^\prime \right)^2} d\kappa
\]

\[
= R \sqrt{1 + \sin^2 \kappa \tan^2 \beta_0} d\kappa
\]  

(4)

In Eq. (3), the cutting width of a micro-element can be expressed as:

\[
db = \frac{dz}{\sin \kappa} = \frac{dR(1 - \cos \kappa)}{\sin \kappa} = Rd\kappa
\]

(5)

Based on the micro-cutting thickness calculation model proposed by Lee and Altinas [28] to describe the horizontal feed of the ball-end milling cutter, the influence of tool attitude on the cutting
thickness is considered and shown in Fig. 3. The micro-cutting thickness is:

$$h = f_z \cos \alpha \sin \psi \sin \kappa$$  \hspace{1cm} (6)

where $\alpha$ is the tool rake angle.

According to the relationship between the workpiece feed direction and the rotation direction of the tool, the milling methods can be divided into forward milling and reverse milling. Under different milling methods, the expression forms of entry angle and exit angle are different, which are mainly related to the radial cutting depth $a_e$ and the milling radius $R$ of the cutter. For forward milling and reverse milling, the entry angle and exit angle can be expressed as:

Down milling:

$$\begin{cases}
\phi_{st} = \arccos(a_e / R - 1) \\
\phi_{ex} = \pi
\end{cases}$$ \hspace{1cm} (7)

Up milling:

$$\begin{cases}
\phi_{st} = 0 \\
\phi_{ex} = \arccos(1 - a_e / R)
\end{cases}$$ \hspace{1cm} (8)

When the tool axis is parallel to the surface normal, the maximum cutting position angle is:

$$\kappa_{\text{max}} = \arccos((R - a_p)/R)$$ \hspace{1cm} (9)

When the tool is tilted, both the rake angle $\alpha$ and roll angle $l$ are considered, and the maximum rake angle is:

$$\chi_{\text{max}} = \arccos(\cos(\alpha) \cos(l))$$ \hspace{1cm} (10)

Therefore, when both the rake angle and the roll angle exist, the maximum cutting position angle of the tool is:
\[ \kappa_{\text{max}} = \arccos\left(\frac{R - a_p}{R}\right) + |\chi_{\text{max}}| \]  

(11)

Due to the laminated property of CFRP, the same cutting edge will be used to cut different fiber layers at the same time, and the fiber cutting angle of different fiber direction layers at the same time also differs. To simplify the calculation, the materials of the same fiber direction layer are superimposed, and the cutting edge contact length is denoted as \( S_j \). The simplified model is presented in Fig. 4. In the ball-end milling process, there will be multiple milling micro-elements on the same milling edge at the same time. However, due to different positions of the milling micro-elements, the milling force of the milling edge \( j \) can be obtained by the integral method, and the global milling force in the X, Y, and Z directions can be obtained as follows:

\[
\begin{align*}
F_{xj} &= \int_{Z_{j1}}^{Z_{j2}} (dF_{x,0})dz + \int_{Z_{j1}}^{Z_{j2}} (dF_{x,45})dz + \int_{Z_{j1}}^{Z_{j2}} (dF_{x,90})dz + \int_{Z_{j1}}^{Z_{j2}} (dF_{x,135})dz \\
F_{yj} &= \int_{Z_{j1}}^{Z_{j2}} (dF_{y,0})dz + \int_{Z_{j1}}^{Z_{j2}} (dF_{y,45})dz + \int_{Z_{j1}}^{Z_{j2}} (dF_{y,90})dz + \int_{Z_{j1}}^{Z_{j2}} (dF_{y,135})dz \\
F_{zj} &= \int_{Z_{j1}}^{Z_{j2}} (dF_{z,0})dz + \int_{Z_{j1}}^{Z_{j2}} (dF_{z,45})dz + \int_{Z_{j1}}^{Z_{j2}} (dF_{z,90})dz + \int_{Z_{j1}}^{Z_{j2}} (dF_{z,135})dz 
\end{align*}
\] 

(12)

2.2. Dynamic milling force model allowing for regeneration mechanism

In the process of milling CFRP thin-walled curved surfaces, vibration is generated by the regenerative mechanism between the dynamic cutting thickness and the cutting force. The dynamic cutting force will cause a displacement to affect the cutting thickness in the milling process, and in turn, the cutting thickness will affect the dynamic cutting force, i.e., the whole change process is a closed-loop dynamic interaction process. The undeformed cutting thickness \( h \) on the cutting edge consists of static and dynamic parts: the static part is formed by the feed motion of the tool, which does not affect the dynamic analysis of the regeneration mechanism and is generally ignored in the stability
analysis; the dynamic part is caused by the vibration displacement of the tool and the workpiece under the action of cutting force. The dynamic displacement can be defined as the displacement difference between the current tool tooth and the last tool tooth in the previous period, as shown in Fig. 5. Then, the dynamic cutting thickness in three directions can be represented as:

\[
\Delta x = x(t) - x(t - T) \\
\Delta y = y(t) - y(t - T) \\
\Delta z = z(t) - z(t - T)
\]

(13)

The instantaneous radial total cutting thickness of the cutter teeth can be expressed as:

\[
h_d = \left[ (\Delta x \sin \kappa + \Delta y \sin \kappa) \sin \psi - \Delta z \cos \psi \right]
\]

(14)

By introducing the dynamic cutting thickness of Eq. (14) into the cutting force model, the dynamic milling force is obtained as follows:

\[
\begin{bmatrix} F_{xj} \\ F_{yj} \\ F_{zj} \end{bmatrix} = G \cdot a_p \cdot \sum_{i=1}^{n} K_{nc}(\beta) \cdot \begin{bmatrix} K_{nc}(\beta) \\ 1 \\ K_{ac}(\beta) \end{bmatrix} \cdot \left[ (\Delta x \sin \psi + \Delta y \sin \psi) \sin \kappa - \Delta z \cos \kappa \right]
\]

(15)

where \( G \) is the micro-element milling force conversion matrix:

\[
G = \begin{bmatrix} -\sin \kappa \sin \psi & -\cos \psi & \cos \kappa \sin \psi \\ -\sin \kappa \cos \psi & \sin \psi & \cos \kappa \cos \psi \\ \cos \kappa & 0 & \sin \kappa \end{bmatrix}
\]

(16)

The establishment of dynamic milling force lays a foundation for stability analysis of thin-wall CFRP milling systems.

3. Modeling and stability analysis of CFRP dynamic milling system

CFRP thin-walled surface components have anisotropic characteristics. The vibration transmission differs in different directions, and the vibration law is more complicated than that of
homogeneous materials. Based on the dynamic milling force obtained in the previous section, which considers the variation of fiber cutting angle and the characteristics of stacking, and by considering the dynamic characteristics of CFRP thin-walled in three directions, an analytical model is established. This model reflects the cutting dynamic characteristics of CFRP, and it can be easily used in simulation analysis to predict the machining stability region.

When milling thin-walled CFRP, the following assumptions can be made:

1. The tool is a rigid body, and the workpiece is a flexible body;
2. The static elastic deformation of CFRP thin-walled curved parts is ignored;
3. When the workpiece is clamped, one side of the workpiece is clamped, and the other side is free to rotate;
4. The coupling effect in XY, YZ, and XZ directions is ignored.

As shown in Fig. 6, the milling dynamics model of thin-wall CFRP can be considered as a spring damping system with three degrees of freedom to approximate the actual machining situation. Different from metal materials, because of the influence of fiber direction, CFRP cutting edge cutting to different fiber directions will lead to different dynamic cutting forces and dynamic cutting thickness changes, chatter behavior is more complex. The 3D dynamic model can be simplified as:

\[
\begin{bmatrix}
M_{xx} & \frac{C_{xx}}{2} & \frac{K_{xx}}{2}
\end{bmatrix}
\begin{bmatrix}
x(t)
\end{bmatrix}
+ \begin{bmatrix}
C_{yy} & \frac{K_{yy}}{2}
\end{bmatrix}
\begin{bmatrix}
y(t)
\end{bmatrix}
+ \begin{bmatrix}
K_{zz}
\end{bmatrix}
\begin{bmatrix}
z(t)
\end{bmatrix}
= \begin{bmatrix}
F_x(t)
\end{bmatrix}
\]

where, \(M_{xx}, M_{yy},\) and \(M_{zz}\) are the modal mass of the CFRP workpiece; \(C_{xx}, C_{yy},\) and \(C_{zz}\) are the damping coefficients; \(K_{xx}, K_{yy},\) and \(K_{zz}\) are the stiffness of the CFRP workpiece, ignoring the structural modal coupling effect; \(F_x(t), F_y(t),\) and \(F_z(t)\) are components of the three-direction dynamic milling force in
X, Y, Z directions.

In Eq. (17), the three-dimensional dynamic milling force can be expressed as:

\[
\begin{aligned}
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} &= \sum_{j=0}^{N-1} \begin{bmatrix} F_{xj}(\psi) \\ F_{yj}(\psi) \\ F_{zj}(\psi) \end{bmatrix} = \frac{N}{4\pi} \cdot a_p \cdot \begin{bmatrix} a_{xx} & a_{xy} & a_{xz} \\ a_{yx} & a_{yy} & a_{yz} \\ a_{zx} & a_{zy} & a_{zz} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}
\end{aligned}
\]  

(18)

where \([A]\) is the direction dynamic milling force coefficient matrix, and it changes with time. Eq. (18) is related to the entry angle and exit angle of the cutting tool, and it can be expressed in the time domain as:

\[
\{ F(t) \} = \frac{N}{4\pi} \cdot a_p \cdot \begin{bmatrix} A(t) \end{bmatrix} \{ \Delta t \}
\]  

(19)

\([A(t)]\) is expanded according to the Fourier series, and its higher-order term does not affect stability prediction. Therefore, only the first term of \([A(t)]\) is retained, and the following equation can be obtained:

\[
\begin{aligned}
[A_0] &= \frac{1}{T} \int_0^T [A(t)] dt = \frac{1}{\psi_a} \int_{\psi_a}^{\psi_e} \begin{bmatrix} A(\psi) \end{bmatrix} d\psi = \begin{bmatrix} a_{xx}^0 & a_{xy}^0 & a_{xz}^0 \\ a_{yx}^0 & a_{yy}^0 & a_{yz}^0 \\ a_{zx}^0 & a_{zy}^0 & a_{zz}^0 \end{bmatrix}
\end{aligned}
\]  

(20)

where:
\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \sin^2 \psi \sin \kappa \right) - \sin^2 \psi \sin \kappa - K_n(\beta) \left( \frac{-\sin 2\gamma}{2} + \psi \right) \right] \right] \]

\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \sin \psi \right) \right] \right] \]

\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \frac{\sin 2\kappa}{2} \right) \right] \right] \]

\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \frac{-\sin 2\gamma}{2} + \psi \right) \right] \right] \]

\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \sin \psi \right) \right] \right] \]

\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \sin \psi \right) \right] \right] \]

\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \sin \psi \right) \right] \right] \]

\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \sin \psi \right) \right] \right] \]

\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \sin \psi \right) \right] \right] \]

\[ a_0^p = \sum_{i=1}^{n} \left[ K_n(\beta) \left[ -K_n(\beta) \left( \sin \psi \right) \right] \right] \]

where, \( n \) represents the workpiece of fiber layering angles, and \( \kappa \) can be expressed by a helix angle.

Assuming that \( \omega_c \) is the regenerative chatter frequency, the regenerative vibration displacement produced by \( \omega_c \) in the frequency domain can be expressed as:

\[ \{ \Delta(t) \} = \{ G(i\omega_c) \} \{ F \} e^{i\omega_c t} - \{ G(i\omega_c) \} \{ F \} e^{i\omega_c (t-T)} = \left( 1 - e^{-i\omega_c T} \right) \{ G(i\omega_c) \} \{ F \} e^{i\omega_c t} \]  

(22)

where \( [G(i\omega)] \) is the transfer function matrix of the cutter workpiece contact area. In the dynamic milling force equation (Eq. (19)), the dynamic cutting force can be obtained as:

\[ \{ F(t) \} = \frac{N}{4\pi} \cdot a_p \cdot \left( 1 - e^{-i\omega_c t} \right) \cdot [A_0] \cdot \{ G(i\omega_c) \} \cdot \{ F(t) \} \]  

(23)

When its determinant is 0, there is a non-zero solution to Eq. (22):

\[ \det \left\{ [I] - \frac{1}{2} a_p \left( 1 - e^{-i\omega_c t} \right) [A_0] [G(i\omega_c)] \right\} = 0 \]  

(24)

The final characteristic equation is:

\[ \det \{ [I] + \Lambda [\Phi] \} = 0 \]  

(25)
The eigenvalues of the equation are:

\[ \Lambda = \frac{N}{4\pi} a_p \left(1 - e^{-i\omega T}\right) \]  \hspace{0.5cm} (26)

According to the Lyapunov stability criterion, when the real part of the characteristic value is negative, the system is stable; when the real part of the eigenvalue is positive, the system is unstable; when the real part of the eigenvalue is 0, it corresponds to the critical condition of system stability.

According to Euler's formula:

\[ e^{-i\omega T} = \cos \omega_T - i \sin \omega_T \]  \hspace{0.5cm} (27)

The critical axial cutting depth at the chatter frequency \( \omega_C \) can be obtained as:

\[ a_{p,lim} = -\frac{4\pi}{N} \left[ \frac{\Lambda_R \left(1 - \cos \omega_T\right) + \Lambda_I \sin \omega_T}{1 - \cos \omega_T} + i \frac{\Lambda_I \left(1 - \cos \omega_T\right) - \Lambda_R \sin \omega_T}{1 - \cos \omega_T} \right] \]  \hspace{0.5cm} (28)

Since \( a_{p,lim} \) is a real number, the imaginary part is 0. Then, \[ \Lambda_I \left(1 - \cos \omega_T\right) - \Lambda_R \sin \omega_T = 0 \]  \hspace{0.5cm} (29)

with:

\[ \kappa = \frac{\Lambda_I}{\Lambda_R} \frac{\sin \omega_T}{\left(1 - \cos \omega_T\right)} \]  \hspace{0.5cm} (30)

The final expression of axial tangent depth under chatter-free conditions can be represented as:

\[ a_{p,lim} = -\frac{2\pi \Lambda_R}{N} \left(1 + \kappa^2\right) \]  \hspace{0.5cm} (31)

The system chatter frequency is \( \omega_C \). Then, \[ \omega_C T = \pi - 2\eta + 2k\pi \]  \hspace{0.5cm} (32)

where \( k \) is the number of lobes of all the vibration patterns left on the milling surface by the cutter tooth at the chatter frequency \( \omega_C \) in the cycle \( T \). Then, the spindle speed corresponding to the ultimate cutting depth can be expressed as:
By investigating the stability limit relationship of the corresponding spindle speed, the stability lobe diagram can be obtained, and the stability cutting limit of the thin-wall CFRP multi-degree-of-freedom system can be predicted and analyzed, which provides a theoretical basis for selecting the process parameters to suppress the milling chatter phenomenon.

4. Results and discussion

4.1. Calculate the stability lobe diagram

To obtain the milling force coefficient varying with the fiber cutting angle, eight groups of CFRP one-way plate and multidirectional plate groove milling experiments with different fiber directions were conducted. The material properties are the same as those of the CFRP thin-walled curved workpiece. The milling experiments were carried out in the Guangyang five-axis machining center. All experiments were conducted with carbide double-edged ball milling cutters with a tool radius \( R \) of 5 mm and a helix angle \( \beta_0 \) of 30°. The experimental device is illustrated in Fig. 7. The milling force was measured by a three-way tool holder dynamometer. Then, the experimental results of milling force on a unidirectional plate were used to calibrate the milling force coefficient, and the results of milling force on a multidirectional plate were used to verify the accuracy of the milling force coefficient. The specific processing parameters are listed in Table 1.

When milling a unidirectional plate, the three-way milling force of the tool was measured. Meanwhile, 20 points on average were selected in each tool rotation cycle, and the milling force coefficient was calibrated following the milling force calculation method in Section 2. The milling force
force coefficient varies with the tangential angle of the fiber, and the change curve is shown in Fig. 8. The results indicate that the influence of the fiber cutting angle on the milling force is obvious. Then, polynomial fitting was performed on the milling force coefficient, and it was found that when the 6th-order polynomial fitting was used, the obtained curve was highly consistent with the experimental results. By analyzing the goodness of fit, it can be seen from the coefficient of determination $R^2$ in Table 2 that the fitting results have high reliability. The fitting results of the cutting force coefficient are as follows:

\[
\begin{align*}
K_n(\beta) &= 8.675\beta^6 - 158.3\beta^5 + 1053\beta^4 - 3061\beta^3 + 3516\beta^2 - 697.6\beta + 104.7 \\
K_a(\beta) &= 8.769\beta^6 - 155.8\beta^5 + 994.7\beta^4 - 2701\beta^3 + 2727\beta^2 - 206.4\beta + 226 \\
K_{nc}(\beta) &= 6.005\beta^6 - 110.8\beta^5 + 754\beta^4 - 2226\beta^3 + 2327\beta^2 + 252\beta - 77.03 \\
K_{ac}(\beta) &= 0.3606\beta^6 - 6.889\beta^5 + 48.67\beta^4 - 153.6\beta^3 + 201\beta^2 - 64.86\beta + 2.378 \\
K_w(\beta) &= 0.3819\beta^6 - 7.295\beta^5 + 51.13\beta^4 - 158.3\beta^3 + 199.1\beta^2 - 56.81\beta + 2.885 \\
K_{wc}(\beta) &= 0.455\beta^6 - 8.683\beta^5 + 61.81\beta^4 - 197.4\beta^3 + 256.3\beta^2 - 67.91\beta + 1.376
\end{align*}
\]

To verify the accuracy of the cutting force coefficient, the multi-directional plate milling force calculation method proposed in Section 2 was adopted, and the milling force coefficient obtained above was substituted into the formula to perform a simulation calculation of the milling force and compare it with the experimental results. The comparison between the simulated value and the measured value of the three-way milling force is shown in Fig. 9. It can be seen from the comparison results that the simulated value and the measured value are relatively consistent, and the maximum error is less than 18.43%, indicating the high accuracy of the obtained cutting force coefficient curve.

According to the calculation methods of dynamic milling force and milling stability region proposed in the previous two sections, a typical CFRP thin-walled curved circular workpiece is adopted to verify the validity of the proposed method. The thin-wall CFRP curved circular component and its
clamping method are shown in Fig. 10. The material properties of CFRP are listed in Table 3.

To analyze the milling stability of a CFRP thin-walled curved workpiece, the modal parameters of the workpiece must be determined. The hammering modal test is a simple and fast method to obtain the modal parameters of the workpiece. Therefore, following the hammering method a modal parameter acquisition test was performed on the circular CFRP thin-walled curved surface workpiece. The Kdl-02l force hammer was selected as the pulse excitation source to measure the frequency response function, and a three-way acceleration sensor (sensitivity: 9.81mv /g, 9.73 mv /g, 10.18 mv /g) was adopted to measure the vibration response of the workpiece. The average experimental results for each experiment were obtained by 10 times knocking. Then, the Mi-7008 data acquisition and analyzer was used to analyze the experimental data. The principle of the modal test is shown in Fig. 11. Since the high-order mode has little influence on the chatter of the workpiece, only the first-order modes of the workpiece were considered. The modal parameter results are listed in Table 4.

According to the above calibrated milling force coefficients and the identified structural modal parameters, based on the dynamic solution method introduced in Section 3, the stability domain lobe diagram in the surface milling of the circular CFRP thin-walled surface workpiece was predicted. The results are presented in Fig. 12. The spindle speed range is 0-10000 rpm, and the axial tangential depth range is 0-4 mm. The part below the red line represents the absolutely stable region, where the machining system is stable regardless of the spindle speed. The curve represents the critical axial depth of cutting, where the part below the curve represents the machining stability region, and the part above the curve represents the chatter region.

4.2. Experimental verification
To verify the reliability of the predicted stability lobe diagram in the stability domain (Fig. 13), several groups of processing parameters were selected from the diagram, and surface milling experiments were conducted for a CFRP thin-walled curved workpiece. The feed speed was 300 mm/min, and the reverse milling method was adopted. Other processing parameters were the same as those used in the simulation calculation. The amplitude of the workpiece was measured by a single-point laser vibrometer during machining. Meanwhile, to determine whether chatter occurs in the milling process, the processing surface quality, milling force frequency domain signal, and workpiece amplitude were judged, respectively. The analysis results of A (4800 rpm, 0.7 mm) and B (4300 rpm, 0.7 mm) in Fig. 13 are presented in Fig. 14.

In the process of machining the CFRP thin-walled curved workpiece, the milling force was measured, and the frequency spectrum was obtained by FFT transformation of the time-domain signal of the milling force. As shown in Fig. 14 (a) and (b), when the process parameters corresponding to point A of the lobe diagram were adopted, the spindle speed was 4800 rpm, and the cutter tooth passing frequency was 160 Hz. On spectrum (a), only the frequency doubling of the cutter tooth frequency appeared, and there was no chatter frequency, indicating that the machining is in stable cutting. When the process parameters of point B were adopted, in addition to the frequency doubling of cutter tooth frequency, the chatter frequency of 372 Hz also appeared, which was close to the first-order natural frequency of the workpiece, indicating that point B was the chatter point. By observing the amplitude of the workpiece during the processing of the two groups of process parameters, it was found that the amplitude of Fig. 14 (d) was significantly larger than that of Fig. 14 (c), which also proved that point A was the stable cutting point and point B was the chatter point. The processing parameters corresponding to points A and B on the lobe diagram respectively show the surfaces obtained after
processing, as shown in Fig. 14 (e) and (f). Besides, it can be seen from Fig. 14 (e) that using the processing parameters corresponding to point A obtained a better processing surface quality. Fig. 14 (f) shows that there are obvious vibration marks on the workpiece surface with relatively high roughness, which further proves that chatter occurred in the milling process of point B.

The experimental results of other process parameters selected from the lobe diagram are listed in Table 5. The surface of the No. 5 process parameter in the stable region exhibits slight vibration marks after machining, which may be caused by the dynamic milling force error, modal parameter error, and calculation error of solving the stable region. The results of other processing parameters are highly consistent with those in the stability region. Therefore, based on the stability verification experiment of multiple groups of different parameters, the accuracy of the predicted machining stability region was verified by analyzing the machining frequency and workpiece amplitude and observing the surface quality of the workpiece.

To explore the effect of chatter on the surface quality of CFRP, the machined surface morphology of CFRP during chatter was observed by electron microscope, and compared with the machined surface after metal material chatter was milling by ball-end milling cutter in reference [29], as shown in Fig. 15. When metal materials are milling with chatter, vibration marks will be left on the workpiece surface due to the dynamic cutting process between the tool and the workpiece. When milling CFRP chatter, vibration marks are still the main damage form on the workpiece surface. Different from metal, because of the characteristics of the multi-material system of CFRP, at the vibration marks fracture, due to the action of vibration, the surface fiber will be lifted by the cutting edge of the tool, resulting in burrs. In addition, some fibers will be pulled out at the vibration marks fracture, leaving deep pitting damage on the surface. Therefore, chatter will cause a variety of damage to the CFRP surface, mainly
vibration marks, and seriously reduce the milling quality of the CFRP surface.

Fig. 16 shows the surface quality after machining with selected process parameters both inside and outside the stability region. When the process parameter is selected in the stable region, the workpiece surface quality has a high quality, in Fig. 16 (I), no vibration marks appear, and the surface roughness reaches 1.552μm. When the process parameter above the limit cutting depth was selected, in Fig. 16 (II), strong vibrato appeared on the machined surface, accompanied by various damages, such as burrs and pits, and the roughness reached 6.049μm. This further proves that the prediction of the stable region has high precision.

To further verify the accuracy of the proposed method, CFRP was equivalent to a homogeneous material, and the average milling force coefficient was obtained through calibration. The machining stability region was predicted, as shown in method 2 in Fig. 17. This method was compared with the proposed dynamic milling force calculation method by considering the cutting angle and stacking characteristics of CFRP fibers, solving the dynamic equation, and predicting the milling stability region (Method 1). It can be seen from Fig. 17 that the predicted machining stability region was significantly smaller than that of Method 1 without considering the characteristics of the CFRP material. Meanwhile, in the spindle speed range of 1000-8000 rpm, the maximum cutting depth of Method 1 and 2 was 2.38 mm and 1.59 mm, respectively. Considering the material anisotropy and stacking characteristics, the predicted limit cutting depth in the stability region was increased by 49.7%.

Three groups of machining parameters A (3800 rpm, 1.3 mm), B (4900 rpm, 0.8 mm), and C (7500 rpm, 0.7 mm) were selected between the two critical axial cutting depth surfaces of machining stability domains. Then, CFRP thin-walled surface milling experiments were conducted to monitor the amplitude of the workpiece during machining and observe the machined surface quality, and based on
this, the occurrence of chatter was determined.

Three groups of process parameters were used to process CFRP thin-walled curved workpieces, respectively. The amplitude and surface quality of the processing process are illustrated in Fig. 18. When the process parameters of group A were used, the processing amplitude reached 25μm, and slight vibration marks appeared on the machined surface. When the process parameters of groups B and C were used, the amplitude was less than 25μm, the surface quality was also good, and the processing process was stable. Therefore, when CFRP was processed, the cutting edges acted on different cutting angles instantaneously, and the cutting forces of multi-directional CFRP on the same cutting edge were discontinuous. The average milling force coefficients were calculated by using the method of equivalent homogeneous materials, and the prediction accuracy of the stability region was low. Thus, it is necessary to consider different fiber directions and overlapping characteristics when predicting the stability zone of CFRP thin-walled surface milling. In the actual processing process, this can improve processing efficiency while ensuring the quality of the processed surface.

5. Conclusion

In this study, a complete set of dynamic modeling and machining stability region prediction schemes is proposed for thin-wall CFRP surface milling. First, the dynamic milling force model was established by considering the fiber cutting angle and laminated characteristics of CFRP. Then, a dynamic model was established for surface milling of thin-wall CFRP curved workpieces by considering the dynamic characteristics of three directions. Finally, the machining stability lobe diagram was solved. The specific conclusions are as follows:

(1) The milling force coefficients of ball-end milling cutters varying with fiber cutting angles
were fitted by a sixth-order polynomial. Meanwhile, the milling force was predicted based on the milling force coefficient, and the maximum error compared with the experimental value was less than 18.43%, indicating the high accuracy of the model.

(2) The zero-order frequency domain method was used to predict the processing stability region of typical CFRP thin-walled circular curved workpieces, and the stability lobe diagram was obtained. The maximum amplitude of the processing parameters selected in the stable region is less than 48.5 μm, which proves that the prediction of machining stability is reliable.

(3) Compared with the equivalent homogeneous material method, the stability region predicted by the traditional method is too conservative when the spindle speed is within 1000-8000 rpm. The maximum axial limit cutting depth predicted by the proposed method is increased by 49.7%, which is beneficial to improving the machining efficiency.
Ethical Approval

Not applicable.

Consent to Participate

Not applicable.

Consent to Publish

The author confirms:

that the work has not been published before; that it is not under consideration for publication elsewhere; that its publication has been approved by all co-authors; that its publication has been approved by the responsible authorities at the institution where the work is carried out.

Authors Contributions

Jun Deng and Fuji Wang conceived and designed the study. Jun Deng, Yongquan Lin and Qingsong He performed the experiments. Jun Deng wrote the paper. Rao Fu reviewed and edited the manuscript. All authors read and approved the manuscript.

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Competing Interests
None.

Availability of data and materials

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also forms part of an ongoing study.

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Conflict of Interest

Dear Editors:

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

Sincerely yours,

Rao Fu.

E-mail: r.fu@dlut.edu.cn
Fig. 1. Geometric parameters of ball-end milling cutter

**Figure 1**

See image above for figure legend
Figure 2

See image above for figure legend

Fig. 2. The variation of fiber cutting angle with the tool rotation angle
Figure 3

See image above for figure legend
Fig. 4. Integral milling force of multidirectional CFRP laminates

Figure 4

See image above for figure legend
Fig. 5. Instantaneous dynamic milling thickness of the ball-end milling cutter
Figure 6

See image above for figure legend
Figure 7

See image above for figure legend
Fig. 8. The fitting results of force coefficient functions: (a) tangential cutting force coefficient $K_p$ ($\beta$) (b) tangential edge force coefficient $K_{pe}$ ($\beta$) (c) radial cutting force coefficient function $K_n$ ($\beta$) (d) radial edge force coefficient $K_{ne}$ ($\beta$) (e) axial tangential force coefficient $K_{a}$ ($\beta$), and (f) the axial edge force coefficient $K_{ae}$ ($\beta$)
Fig. 9. Comparisons of experimental and theoretical cutting forces: (a) verification experiment 1 ($n=2000$ rpm, $F=200$ mm/min, $ap=1.5$ mm, and $l=10^\circ$) (b) verification experiment 2 ($n=3000$ rpm, $F=300$ mm/min, $ap=2$ mm, and $l=15^\circ$) (c) verification experiment 3 ($n=4000$ rpm, $F=400$ mm/min, $ap=2.5$ mm, and $l=10^\circ$) and (d) verification experiment 4 ($n=5000$ rpm, $F=500$ mm/min, $ap=3$ mm, and $l=15^\circ$)
Fig. 10. The CFRP thin-walled curved workpiece and the clamping method

Figure 10

See image above for figure legend
Figure 11

See image above for figure legend
Figure 12

See image above for figure legend
Figure 13

See image above for figure legend
Fig. 14. The spectrum of the milling forces, the amplitude of the workpiece, and the surface morphology.
Figure 15

See image above for figure legend
Figure 16

See image above for figure legend

Fig. 16. Comparison of machining surface quality
Figure 17. The comparison of stability regions between the two methods

Figure 17

See image above for figure legend
Figure 18

The amplitude of the workpiece and surface morphology

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.
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- Table4.docx
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