Partial observation risk-seeking insider trading in continuous time*

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Partial observation risk-seeking insider trading in continuous time *

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Abstract In this paper, a continuous-time insider trading model is investigated, we require that an insider is a risk-seeking and market makers may receive partial signal on risky asset. With the help of filtering theory and dynamic programming principle, the uniqueness and existence of linear equilibrium is established. It shows that (i) as time goes by, residual information decrease, but both trading intensity and market liquidity are increasing; (ii) As partial observation precise increase, both market liquidity and residual information are decreasing, however, the trading intensity will increase. On the whole, the risk-seeking insider is eager to trade all the trading period.

Keywords Insider trading, risk-seeking, partial observation. HJB. Linear Bayesian equilibrium

JEL Classification: D81, D82, G12


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1 Introduction

In Kyle’s insightful and pioneering paper [1], he gave a dynamic model on insider trading, where a risk-neutral insider received a liquidation value of a fundamental asset. With the help of informational advantage, the insider slowly releases her private information to profit, and all the private information is incorporated into market price at the end of trading. Using the framework of Kyle [1], we will characterize the optimal equilibrium in our model, where we assume the insider is risk-seeking. That is to say, we consider that the insider’s desire of trading is becoming stronger and stronger.

In fact, Gong and Zhou [2] had considered the risk-seeking insider trading model, they firstly maximized the risky profit, and then maximized the guaranteed profit. Unlike their methods, we adopt an exponential utility to deal with this problem. This viewpoint comes from risk aversion model, Baruch [3] had used the exponential utility to study risk-averse insider trading model. Immediately afterwards, Cho [4] selected the same utility function to handle the risk-averse model. Recently, Xiao and Zhou [5] established a risk-averse insider trading model of a risky asset whose value is driven by an arithmetic Brownian motion, and gave a closed form linear Bayesian equilibrium. In a word, market liquidity is a monotonically decreasing function on time if the insider is risk aversion. Actually, there is much literature on risk-averse model, see [6–8].

Different of risk-averse conclusion, the market liquidity in our model is a monotonically increasing function.

Recently, Zhou [9] investigated a linear strategy equilibrium in continuous time, where market makers allowed to know partial information on risky asset. Moreover, he pointed out that there is no Nash equilibrium in a Cournot competition when two insiders adopt a linear strategy. Subsequently, Xiao and Zhou [10, 11] combined both partial observation and random deadline, and then expanded Zhou’s model [9].

Therefore, in this paper, we studies a continuous-time insider trading model with a risk-seeking insider together with partial observation by market makers. Especially, we will establish the uniqueness and existence of Bayesian equilibrium with risk-seeking insider.

The rest of the paper is organized as follows. In section 2, an insider trading model in continuous time will be introduced. Section 3 gives some necessary condition. The main conclusions in the paper is contained in section 4, we will give existence and uniqueness of linear Bayesian equilibrium with risk-seeking insider together with partial observation. In section 5, we get some numerical simulation for our main conclusions. Finally, section 6 is conclusions.
2 The model

We assume that all of the randomness in this paper come from a common filtered probability space \((\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P})\) and satisfy the usual conditions.

In risky market, there is a risky asset traded in a finite time interval \([0,1]\), which is normal random variable with mean 0 and variance \(\sigma_v^2\).

As in classical model, we assume that there are three representative agents in the market:

(i) **an insider**, who gets the perfect knowledge of the liquidation value \(v\) and submits his order \(x_t\), as in the form [1]:

\[
dx_t = \beta_t(v - p_t)dt, \quad x_0 = 0
\]

where \(\beta\) is a deterministic measurable function, called the intensity of insider trading [1];

(ii) **liquidity traders**, who have no information on the risky asset and submit total order \(z_t\), where \(z_t\) evolves as the dynamics [4]:

\[
z_t = \sigma_z B_{zt}
\]

where \(\sigma_z > 0\), \(B_z\) is a standard Brownian motion and independent of \(v\);

(iii) **market makers**, who collect the total trading volume

\[
y_t = x_t + z_t
\]

and observe the signal

\[
u = v + \epsilon
\]

where \(\epsilon\) is distributed \(N(0, \sigma^2_\epsilon)\) and independent of \(v\) and \(B_{zt}\). According to trading volume \(y_t\) and signal \(u\), market makers set the market price \(p_t = p(y_t, u)\).

Briefly speaking, the insider’s profit from time \(t\) to 1 is given by

\[
\pi_t = \int_t^1 \beta_s (v - p_s)^2 ds.
\]

As a risk-seeking insider, she/he has an exponential utility function from time \(t\) to 1 in the form

\[
U(\pi_t) = \exp^{\pi_t}.
\]

Next, we will give the concept of an equilibrium in our model.
Definition 1. A risk-seeking linear Bayesian equilibrium is a pair of \((\beta, \lambda)\) such that

(i) maximization of utility: for the given \(\lambda\), function \(\beta\) maximizes
\[
E[U(\pi_0)|\mathcal{F}_t^I] = E[\exp^\int_0^t \beta_t(v-p_t)^2dt | \mathcal{F}_t^I]
\] (2.7)
where the insider’s information field \(\mathcal{F}_t^I = \sigma\{v\} \vee \sigma\{p_s, 0 \leq s \leq t\}\) at time \(t\), and

(ii) market efficiency: for the given \(p_t\), the function \(\lambda\) satisfies
\[
p_t = E[v|\mathcal{F}_t^M]
\] (2.8)
where \(\mathcal{F}_t^M = \sigma\{y_s \vee u, 0 \leq s \leq t\}\) at time \(t\).

3 Necessary Condition in Market

To ensure the well-posedness, the following three technical conditions must be guaranteed, \(\int_0^1 \beta_t^2 dt < \infty\), \(\int_0^1 \lambda_t^2 dt < \infty\), and \(E(\exp^\int_0^1 \beta_t(v-p_t)^2 dt) < \infty\). Moreover, before obtaining the existence of linear Bayesian equilibria, the following filtering equation and their conclusions are important, so we can get the following analysis. Obviously, the signal-observation system on a probability space \((\Omega, \mathcal{F}, P)\) is given by

\[
\begin{align*}
\text{Signal } v: & \quad dv = 0; \\
\text{Observation } y: & \quad d\xi_t = (A_0 + A_1 v)dt + A_2 dB_t
\end{align*}
\] (3.1)
where \(\xi_t = \left(\begin{array}{c} y_t \\ u \end{array}\right)\), \(\xi_0 = \left(\begin{array}{c} 0 \\ v + \epsilon \end{array}\right)\), \(A_0 = \left(\begin{array}{c} -\beta_t \\ 0 \end{array}\right)\), \(A_1 = \left(\begin{array}{c} \beta_t \\ 0 \end{array}\right)\), \(A_2 = \left(\begin{array}{cc} \sigma_z & 0 \\ 0 & 0 \end{array}\right)\), \(B_t = \left(\begin{array}{c} B_{zt} \\ 0 \end{array}\right)\).

By the Theorem 12.1 in [12] and Lemma 3.1 in [10], we obtain
\[
dp_t = \lambda_t dy_t
\] (3.2)
where \(p_0 = 0\), \(\lambda_t = \frac{\Sigma_t \beta_t}{\sigma_z^2}\) is a deterministic measurable function, which is called price pressure [3], and
\[
\frac{d\Sigma_t}{dt} = -\frac{\Sigma_t^2 \beta_t^2}{\sigma_z^2}
\] (3.3)
where \(\Sigma_t = E[(v-p_t)^2]\), and \(\Sigma_0 = \frac{\sigma_z^2 \epsilon^2}{\sigma_z^2 + \sigma_e^2}\).

4 Existence and uniqueness of linear Bayesian equilibrium

We now turn to investigate linear Bayesian equilibrium with risk-seeking insider.
Let \((\beta, \lambda)\) is a linear Bayesian equilibrium. From Definition 1, the insider’s value function is given by for any time \(t \in [0, 1]\)

\[
V(t, v - p_t) = \max_{\beta \in \mathcal{U}(t, \beta)} E[\exp \int_t^1 \tilde{\beta}_s(v - p_s)^2 ds | \mathcal{F}_t] = E[\exp \int_t^1 \beta_s(v - p_s)^2 ds | \mathcal{F}_t]
\]

(4.1)

where the two conditions for (4.1) are needed:

\[
\lim_{t \to 1^-} V(t, (v - p_t)) = 1, \quad \Sigma_0 = \int_0^1 \lambda_t^2 \sigma_z^2 dt.
\]

(4.2)

Of course, the first is obvious, and the second can be inferred by the equation (3.2), (3.3) and \(\lim_{t \to 1^-} \Sigma_t = 0\).

The following conclusion is the stochastic version of Bellman’s optimality principle for above insider trading problem.

**Proposition 4.1.** Given \(t \in [0, 1]\), for \(\hat{t} \in [t, 1]\)

\[
V(t, v - p_t) = \max_{\beta \in \mathcal{U}(t, \beta)} E[\exp \int_t^1 \tilde{\beta}_s(v - p_s)^2 ds (1 - \exp - \int_{\hat{t}}^t \tilde{\beta}_s(v - p_s)^2 ds) + V(\hat{t}, v - p_{\hat{t}}) | \mathcal{F}_t].
\]

**Proof** From (4.1), we have

\[
V(t, v - p_t) = \max_{\beta \in \mathcal{U}(t, \beta)} E[\exp \int_t^1 \tilde{\beta}_s(v - p_s)^2 ds | \mathcal{F}_t]
\]

\[
= \max_{\beta \in \mathcal{U}(t, \beta)} E[\exp \int_t^1 \tilde{\beta}_s(v - p_s)^2 ds (1 - \exp - \int_{\hat{t}}^t \tilde{\beta}_s(v - p_s)^2 ds) + \exp \int_t^1 \tilde{\beta}_s(v - p_s)^2 ds | \mathcal{F}_t]
\]

\[
= \max_{\beta \in \mathcal{U}(t, \beta)} E[\exp \int_t^1 \tilde{\beta}_s(v - p_s)^2 ds (1 - \exp - \int_{\hat{t}}^t \tilde{\beta}_s(v - p_s)^2 ds) + V(\hat{t}, v - p_{\hat{t}}) | \mathcal{F}_t].
\]

By the dynamic programming principle, the insider’s optimality condition is portrayed as follows.

**Proposition 4.2.** If the value function of insider is determined by Proposition 4.1, then the HJB equation will be given by

\[
\frac{\partial V}{\partial t} + \frac{1}{2} \lambda_t^2 \sigma_z^2 \frac{\partial^2 V}{\partial p^2} + \max_{\beta \in \mathcal{R}} [\beta (\lambda_t(v - p_t) \frac{\partial V}{\partial p} + (v - p_t)^2 V)] = 0.
\]

(4.3)

**Proof** Applying Itô’s formula to the difference \(V(\hat{t}, v - p_{\hat{t}}) - V(t, v - p_t)\), and as \(\hat{t} - t \to o\), the limit of

\[
1 - \exp - \int_{\hat{t}}^t \beta_s(v - p_s)^2 ds
\]

\[
\hat{t} - t
\]

is equivalent to \(\beta_t(v - p_s)^2\). And then using HJB equation in [13], the conclusion is hold.
Proposition 4.3. If the value function satisfy the HJB equation (4.3), then it will be given by

\[ V(t, m_t) = \frac{c_2}{\sqrt{\lambda_t}} \exp^{\frac{\sigma_t^2}{4} t} \exp^{-\frac{\sigma_t^2}{2} m_t^2} \]

where

\[ \lambda_t = \frac{1}{c_1 - \sigma_t^2 t} \]  

(4.4)

and the two constants \( c_1 \) and \( c_2 \) satisfy the following two equations respectively

\[ c_1 = \sqrt{\frac{\sigma_t^4 + 4 \sigma_t^2 + \sigma_t^2}{2}} \]

and

\[ c_2 = \sqrt{\frac{c_1 (1 + \sigma_t^2 (c_1 - \sigma_t^2))}{c_1 - \sigma_t^2}} \exp^{\frac{\sigma_t^2}{2} m_t^2}. \]

Proof. Obviously, the HJB equation (4.3) is equivalent to the following two equations:

\[
\begin{cases}
\frac{\partial V}{\partial t} + \frac{1}{2} \lambda_t^2 \sigma_t^2 \frac{\partial^2 V}{\partial p^2} = 0; \\
\lambda_t \frac{\partial V}{\partial p} + (v - p_t)V = 0.
\end{cases}
\]

(4.5)

The second equation of (4.5) can be viewed as an ordinary differential equation with respect to \( p_t \), which has a solution of the form

\[ V(t, p_t) = g(t) \exp^{\frac{p_t^2}{2}} \exp^{-\frac{v p_t}{\lambda_t}} \]

(4.6)

where \( g(t) \) is some deterministic function on \([0,1]\).

Substituting (4.6) into the first equation of (4.5), we can obtain the equation

\[ g'(t) - g(t) (v p_t - \frac{p_t^2}{2}) \left( \frac{1}{\lambda_t} \right)' + \frac{g(t)}{2} \sigma_t^2 (v - p_t)^2 + \frac{1}{2} \lambda_t g(t) \sigma_t^2 = 0, \]

it is equivalent to

\[ g'(t) + g(t) \frac{\sigma_t^2}{2} (v^2 + \lambda_t) - \frac{g(t) p_t^2}{2} \left( \frac{1}{\lambda_t} \right)' + \frac{g(t) \sigma_t^2}{2} (v - p_t) + g(t) v p_t \left( \frac{1}{\lambda_t} \right)' + \sigma_t^2 = 0, \]

Taking expectation for the above equation, the following system can be hold

\[
\begin{cases}
(\frac{1}{\lambda_t})' = -\sigma_t^2; \\
g'(t) + g(t) \frac{\sigma_t^2}{2} (\sigma_t^2 + \lambda_t) = 0.
\end{cases}
\]

(4.7)
Then by the first equation of (4.7),
\[ \lambda_t = \frac{1}{c_1 - \sigma^2 z t} \]  
where \( \frac{1}{\lambda_0} = c_1 \) is some constant real number. We now bring (4.8) into the second equation of (4.7)
\[ g(t) = \frac{c_2}{\sqrt{c_1}} \sqrt{-\sigma^2 z t + c_1 \exp^{-\frac{\sigma^2 z t}{2}}} \]  
where \( g_0 = c_2 \) is some constant real number. According to the boundary conditions in (4.2), the constant \( c_1 \) can be solved by the following system:
\[
\begin{cases}
\frac{c_2}{\sqrt{c_1(1 + \sigma^2(c_1 - \sigma^2 z))}} \sqrt{c_1 - \sigma^2 z} \exp^{-\frac{\sigma^2 z t}{2}} = 1; \\
\frac{c_2^2 \Sigma_0 - c_1 \Sigma_0 \sigma^2}{c_1 - \sigma^2 z} = 0.
\end{cases}
\]  
Namely
\[ c_1 = \frac{\sigma^2 z \pm \sqrt{\sigma^4 z + 4 \sigma^2 z \Sigma_0}}{2} \]
by (4.8), we assert
\[ c_1 = \frac{\sigma^2 z + \sqrt{\sigma^4 z + 4 \sigma^2 z \Sigma_0}}{2} \]
and
\[ c_2 = \frac{\sqrt{c_1(1 + \sigma^2(c_1 - \sigma^2 z))}}{c_1 - \sigma^2 z} \exp^{-\frac{\sigma^2 z t}{2}}. \]

**Theorem 1.** There is an unique linear equilibrium \((\beta, \lambda)\) satisfying
\[ \beta_t = \frac{c_1 - \sigma^2 z}{1 - t}, \quad \lambda_t = \frac{1}{c_1 - \sigma^2 z t}. \]

At the equilibrium, the remained information \(\Sigma_t\) at time \(t\) is
\[ \Sigma_t = \frac{1}{c_1 - \sigma^2 z} - \frac{1}{c_1 - \sigma^2 z t} \]
and the insiders total ex ante utility
\[ E[V(0, v)] = \sqrt{\frac{c_1}{(1 + 2c_1\sigma^2 v)(c_1 - \sigma^2 z)}} \exp^{-\frac{\sigma^2 z t}{2}} \]
where
\[ c_1 = \frac{\sigma^2 z + \sqrt{\sigma^4 z + 4 \sigma^2 z \Sigma_0}}{2} \]
and
\[ c_2 = \frac{\sqrt{c_1(1 + \sigma^2(c_1 - \sigma^2 z))}}{c_1 - \sigma^2 z} \exp^{-\frac{\sigma^2 z t}{2}}. \]
**Proof**  By the optimal filtering theorem [12], we have

\[ \Sigma_t = \int_t^1 \lambda_s^2 \sigma_z^2 ds, \quad (4.11) \]

\[ \lambda_t = \frac{\beta_t \Sigma_t}{\sigma_z^2}. \quad (4.12) \]

Take (4.8) into (4.12)

\[ \Sigma_t = \frac{1}{c_1 - \sigma_z^2} - \frac{1}{c_1 - \sigma_z^2 t}, \]

together this with (4.12) we obtain

\[ \beta_t = \frac{c_1 - \sigma_z^2}{1 - t}. \]

By the expected property and expression of \( c_2 \)

\[ E[V(0, v)] = \sqrt{\frac{c_1}{(1 + 2c_1 \sigma_z^2)(c_1 - \sigma_z^2)}} \exp \frac{\sigma_z^4 v^2}{2}, \]

where

\[ c_1 = \frac{\sigma_z^2 + \sqrt{\sigma_z^4 + 4\sigma_z^2 \Sigma_0^2}}{2}. \]

The proof is complete.

**Corollary 4.4.** At the equilibrium \((\beta, \lambda)\) in Theorem 1, the following results hold:

(i) As the time goes by, both optimal trading intensity and market liquidity are increasing, while residual information is decreasing.

(ii) Given a fixed time, the less partial observation accuracy, the weaker trading intensity is, however, the stronger both market liquidity and remained information are.

**Proof**  Omitted.

5  Numerical simulation

In Corollary 4.4, we give some theoretical characteristics of the equilibrium \((\beta, \lambda)\). Next, we will give numerical simulations on the equilibrium. First of all, we assume \( \sigma_v^2 = 4. \)
In Figure 1, as time $t \in (0, 1)$ goes by, insider’s private information slowly releases even close to zero. From Figure 1, we find that the private information release slowly at the beginning, while rapidly at the final, which explains the trading intensity will be strong at last. Different with risk aversion, the market liquidity is increasing with time $t$ when insider is risk-seeking.

Next, we will give a numerical simulations on partial observation accuracy as below.

In Figure 2, these simulation conclusions correspond to (ii) in Corollary 4.4. Therefore, we do not repeat it.

6 Conclusions

In this paper, we analyse a continuous-version insider trading model, in which the market makers can observe partial signal on risky asset and an insider is required to be risk-seeking, and then establish the uniqueness and existence of linear equilibrium.

It shows that (i) with the time goes by, both market liquidity and trading intensity are increasing quickly in the later trading, and residual information is slowly decreasing at the beginning.
In fact, the characteristic of market liquidity indicate that the insider is willing to trade, and the insider has a strong desire trading when there is less residual information. (ii) As the accuracy of partial observations decreases, it is true that the residual information of market will increase. Therefore, the more residual information for insider, the weaker trading intensity such that they can obtain high profits.

We remark that our model extends Kyle’s version (1985) [1] from risk seeking perspective. In Kyle’s model, the market liquidity is always a constant, and the market liquidity is a decreasing function in the model of risk aversion [3–6]. Different of models [1, 3–6], the market liquidity is a increasing function in our model. This indicates that the insider is more eager to trade in the later. Of course, our conclusion can not cover the result of other preferences, due to inconsistent preferences.

References


