Multi-source Directional Heat Conduction Method Based on Coordinate Transformation

Yang Li (liyangxx@xjtu.edu.cn)
Xi’an Jiaotong University

Wenlei He
Xi’an Jiaotong University

Zhe Nie
Xi’an Jiaotong University

Jingyao Tian
Xi’an Jiaotong University

Huijie Zhang
Xi’an Jiaotong University

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Multi-source Directional Heat Conduction Method Based on Coordinate Transformation

Yang Li, Wenlei He, Zhe Nie, Jingyao Tian, Huijie Zhang

(iHarbour Academy of Frontier Equipment, Xi'an Jiaotong University, Xi'an 712000)

Abstract: Based on the optical transformation theory, the formal invariance of heat conduction equation was used to realize the transformation of constitutive parameters of heat conducting materials through coordinate transformation, so as to control the distribution of heat flow in physical space. In addition, on the basis of the research on the control method of heat flow propagation path, a multi heat source directional heat conduction method with complex boundary was studied for the case that the object was affected by a variety of internal and external heat sources, and the temperature field was uneven, resulting in irregular deformation. Based on translation transformation and compression transformation, the thermal conductivity distribution of directional heat conducting structural materials was derived, and the controllable propagation of heat flow from high temperature region to low temperature region was realized. By analyzing various distributions of multiple heat sources, an applicable directional heat conduction structure with multiple sources and complex boundaries was designed to realize passive control of temperature field. The performance of directional heat conduction in transient state was analyzed by Comsol software simulation.

Key words: Multi heat source; Directional heat conduction; Translation transformation; Compression transformation

1 Introduction

When the object is affected by various internal and external heat sources, the heat flowing path is irregular, the temperature rise and temperature field distribution are complex, which make it difficult to control its thermal deformation. Recently, domestic and foreign scholars have carried out a lot of research on the impact of heat on objects. Through analytical calculation, numerical simulation and experimental testing, the temperature field distributions of the objects can be obtained. Furthermore, in order to solve the problem of machining accuracy degradation caused by thermal deformation, engineers and scholars have tried many methods in practical. For example, Chen et al. [1] improved the thermal stiffness of the structure by designing a thermal symmetric structure to reduce the thermal deformation of components. By adding additional cooling system, thermal controllers and auxiliary heat sources [2], the part with high temperature was cooled, and the part with low temperature was heated, so as to make the overall temperature
field uniform.

The traditional methods mentioned above help to balance temperature field actively. In this paper, a passive control method, called directional heat conduction method was proposed. By designing material parameters, the passive control of heat flow was realized, so as to achieve the goal of obtaining a relatively low temperature rise and an ideal distribution temperature field. In terms of heat flow regulation, it is mainly based on the transformation optics theory. In 2006, Pendry [3] and Leonhardt [4] published articles about the cloak of invisibility on Science. From then on, the transformation optics theory developed rapidly. Researchers from various countries have designed various transformation optical devices using transformation optics methods, such as invisibility cloak [5-9], concentrator [10], illusion optical devices [11-14], etc. In addition, the transformation idea in transformation optics are also applied to the fields of acoustics [15-18] and heat [19,20]. For example, in 2015, Wu et al. [21] theoretically deduced the relationship between the heat flow propagation path and the material parameters, and designed a cylindrical heat cloak. When the heat flows, the external temperature field of the object covered by the cylindrical heat cloak will not be interfered. The material parameter expression of the heat cloak stealth area for transient heat conduction was calculated by Guenneau et al. [19]. The effect of heat stealth was numerical simulated as well. In addition to the cylindrical heat cloak, the square heat cloak and the heat cloak of arbitrary shape was investigated by Yu et al. [22] and He et al. [23].

Inspired by the heat cloak, Sun et al. [24] carried out a simple directional heat transfer study. Compared with the heat cloak, the heat flow does not need to return to the original propagation path, but transmits directionally to some certain surfaces, so as to keep the target object at a low temperature, which can suppress its infrared radiation to achieve infrared stealth.

At present, there are a few researches on other heat shields except the cylindrical one. They only focuses on the effect of thermal stealth in theory in order to achieve the constant temperature in a certain area. The directional heat conduction in one object, especially with multi-heat sources draws little attention. But it is important and valuable to study the control of specific heat flow path to achieve the uniformity of temperature field distribution. In this paper, by using translation transformation and compression transformation, the method of directional heat conduction with multiple sources was studied, and the directional heat conduction structures for three typical
models were designed. The thermal conductivity distribution of directional thermal conductive structural materials was deduced theoretically. By using Comsol software, the effect of directional heat conduction was simulated.

2 Directional heat conduction method based on transformation of coordinates for multi heat sources

2.1 Multi heat source analysis

The thermal deformation of an object is determined by its temperature distribution. Based on the second law of thermodynamics, heat always flow from the high-temperature object/region to the low-temperature object/region. As shown in Fig. 1, when there is only a single heat source in an object with simple, a regular circle for example, the heat flow path is also simple if its material is isotropic, which is indicated by the straight arrow. The formed temperature field is uniform. The arc lines is isotherm line, and its thermal deformation is simple expansion in the diameter direction. However, when there are multi heat sources or objects with complex structures, the heat flow path is no longer a simple and regular straight line, and the resulting temperature field is unevenly distributed, as shown in Fig. 2. In Fig. 2, A and B are two heat sources, and the heat flow path is a curve. At this time, the isotherm lines are also distributed in an irregular curve.

![Fig. 1. Single heat source](image1)

![Fig. 2. Two heat sources](image2)
When an object is affected by multi internal and external heat sources, its internal heat flow path is complex, the temperature rise and temperature field distribution are uncontrollable, which will lead to the irregular deformation. As shown in Fig. 3, from one side of the object, two streams of heat namely \( q_1 \) and \( q_2 \) \((q_1 > q_2)\) flows into it. According to the heat conduction law, the temperature of the upper part is warming up faster than the lower part, so the temperature is distributed unevenly. It will cause the upper part of the object to be hot and the lower part to be cold, and it will bend, which leads to the decline of its accuracy in application, as shown in Fig.3.

![Fig. 3 Object bending diagram](image)

2.2 Directional heat conduction method for multi heat sources

Based on the invariance of space coordinate transformation, the electromagnetic wave propagation in the transformation space could be accurately controlled in transformation optics field. Because the heat conduction equation is invariant to space coordinate transformation, the optical transformation theory and coordinate transformation were used to study the directional heat conduction method, which can regulate the heat in the area with high temperature to the area with low temperature, so as to control heat flow path and equalize the temperature distribution.

For example, there are two different heat sources, namely \( q_1 \) and \( q_2 \) \((q_1 > q_2)\) on one side of the object as shown in Fig.4. Commonly, the temperature above the object is high and the temperature below is low. But by using coordinate transformation, the direction of heat flow could be controlled. The heat in the upper area with high temperature of the object can be forced to transfer to the lower area with low temperature along a certain direction (along Aa), thus the regional temperature gap is reduced.
The process of heat diffusion in the structure can be described by the heat conduction equation. The general passive heat conduction equation can be written as follows:

$$
\rho c \frac{\partial T}{\partial t} - \nabla \cdot (\kappa \nabla T) = 0
$$

where $\nabla$ represents the gradient; $\kappa$ is the thermal conductivity and $T$ is the temperature.

Due to the invariance in transformation space, Equation (1) can also be expressed as:

$$
\rho^' c^' \frac{\partial T^'}{\partial t} - \nabla^' \cdot (\kappa^' \nabla T^') = 0
$$

where $\nabla^'$ represents the gradient in transformation space; $\kappa^'$ and $T^'$ are respectively the thermal conductivity and temperature of the transformation space medium; $\rho^'$ and $c^'$ are the density and heat capacity of the transformation space.

The parameters of the transformation space medium are calculated as follows:

$$
\rho^' c^' = \frac{\rho c}{\det(J)}
$$

$$
\kappa^' = \frac{J \kappa J^T}{\det(J)}
$$

where $J$ is a Jacobian transformation matrix, $J^T$ is a transpose of $J$, $\det(J)$ is the determinant of the matrix.

When the transformation space is two-dimensional, the Jacobian matrix can be expressed as:

$$
J = \frac{\partial (x^', y^')}{\partial (x, y)} = \begin{bmatrix}
\frac{\partial x^'}{\partial x} & \frac{\partial x^'}{\partial y} \\
\frac{\partial y^'}{\partial x} & \frac{\partial y^'}{\partial y}
\end{bmatrix}
$$
Where, \((x', y')\) represents transformation space and \((x, y)\) represents physical space.

Based on the form invariance of the heat conduction equation under the coordinate transformation, the space transformation is equalized as the transformation of heat conduction parameters by using the optical transformation theory, and the artificial control of the heat flow is realized.

3 Directional heat conduction structure based on coordinate transformation for multi heat sources

3.1 Directional heat conduction structure of case 1

According to optical transformation theory, space transformation can be realized by the structure parameter transformation. In this paper, directional heat conduction structure for the multi heat sources was designed. In the structure, coordinate transformation was used to change the parameters of the transformation space medium material, so as to change the heat transmission path and achieve balanced temperature field. In this section, structures with three different kinds of distribution of multi heat sources were designed. In the first case, there was a high heat source at the top of the subject and the heat would flow along the horizontal direction resulting in the situation of "hot top and cold bottom" of the structure. In the second case, there was a high heat source at the bottom of the subject. And it would result in the situation of "cold top and hot bottom" of the structure. In the third case, there was a high heat source in the middle of the side of the subject which would cause the situation of "hot in the middle and cold on both sides".

In this paper, the coordinate transformation was divided into translation transformation and compression transformation. When the high heat source is located at the top of the subject, more heat is supposed to flow along the path as shown in dotted line in Fig.5. By using the translation transformation, the heat is flowing along the new path as shown in black full line, in which \(\theta\) represents the angle of translation.
Then the coordinates after translation are:

\[ x_{11} = x \] (6)

\[ y_{11} = y - x \tan(\theta) \] (7)

The Jacobian matrix after translation transformation is:

\[
J_{11} = \begin{bmatrix} 1 & 0 \\ -\tan \theta & 1 \end{bmatrix}
\] (8)

According to (4), the thermal conductivity of transformation space medium is:

\[ \kappa_{11xx}' = \kappa \] (9)

\[ \kappa_{11xy}' = -\tan(\theta)\kappa \] (10)

\[ \kappa_{11yy}' = (\tan^2(\theta) + 1)\kappa \] (11)

By using the compression transformation, the heat in width is compressed in a certain area. As shown in Fig. 6, the heat above the oA side of the model can be compressed in area \( \Omega' \).

The coordinate relationship after compression transformation is:
\[
x_{c1} = x
\]
\[
y_{c1} = \frac{k_1 x + b}{b_1} y
\]

Where \( k_i = \frac{b_1 - b}{c}, b_1 \) is the compression width, \( c \) is the compression distance. And \( a \) and \( b \) are the length and width of the rectangle.

The Jacobian matrix after compression transformation is:
\[
J_{c1} = \begin{bmatrix} 1 & B \\ B & B^2 + A^2 \end{bmatrix}
\]

where \( B = \frac{k_1}{b_1} y, A = \frac{k_1 x + b}{b_1} \).

The thermal conductivity after compression transformation is changed to:
\[
\kappa_{c1xx} = \frac{k}{A}
\]
\[
\kappa_{c1yy} = \frac{B}{A}
\]
\[
\kappa_{c1yy} = \frac{B^2}{A} k + Ak
\]

For case 1, supposing the structural parameters \( a = 0.351 \text{m} \) and \( b = 1.307 \text{m} \), the material is cast iron (\( \kappa = 76.2 \text{W/(m·K)} \)), the thermal conductivity in area \( \Omega \) after space transformation, can be calculated by (9) - (11) in which the translation angle is \( \theta = 60^\circ \):
\[
k_{1xx} = 76.2 \text{W/(m·K)}
\]
\[
k_{1xx} = -131.98 \text{W/(m·K)}
\]
\[
k_{1yy} = 304.8 \text{W/(m·K)}
\]

For case 1, if \( b_1 = 0.6 \text{ m}, c = 0.351 \text{ m} \), then the thermal conductivity of the area \( \Omega \) after compression transformation can be calculated by (15) - (17).

For transient simulation, Comsol multiphysics software was used. The simulation conditions were as follows: the material outside \( \Omega \) is cast iron and the material parameters of iron are shown in Table 1. The material parameters in the area \( \Omega \) are given by the formula. The initial temperature of all areas is 293.15K. All sides are convection heat transfer boundaries, and the heat
transfer coefficient is \( h = 9.8W/(m^2 \cdot K) \). The oA side has two boundary heat sources, the upper heat source value is \( 10000W/m^3 \), and the lower heat source value is \( 2000W/m^3 \).

Table 1 Material parameters of iron

<table>
<thead>
<tr>
<th>Density/(kg/m^3)</th>
<th>Thermal conductivity/(W/(m \cdot K))</th>
<th>Specific heat capacity/(J/(kg \cdot K))</th>
</tr>
</thead>
<tbody>
<tr>
<td>7870</td>
<td>76.2</td>
<td>440</td>
</tr>
</tbody>
</table>

In order to analyze the performance of directional heat conduction in transient state, temperature fields of different cases are simulated based on the Comsol multiphysics software. After the translation transformation, the temperature distribution of case 1 at 60 minutes is shown in Fig. 8. After the compression transformation, the temperature distribution of case 1 at 60 minutes is shown in Fig. 9. The corresponding thermal diffusion diagram is shown in Fig. 10. Compared with the temperature distribution without heat flow control as shown in Fig. 7, it can be seen that the heat is directionally transferred in a certain path and the temperature difference in the whole region is reduced significantly, which means after transformation the direction of heat is changed significantly. Thermal diffusion diagram of the translation transformation of case 1 is shown in Fig. 10. Thermal diffusion diagram of compression transformation of case 1 is shown in Fig. 11. It can be seen from the figures that the heat flows from the upper left area of high temperature to the lower temperature area in a specific direction.
3.2 Directional heat conduction structure of case 2

As shown in Fig. 12, the heat is commonly flowing along the horizontal direction, resulting in the situation of "hot bottom and cold top". But by using the translation transformation, the heat under the oA of the model can be controlled and translated for a distance along a certain angle. That is to say, the heat flow is moved up to the upper area with low temperature, so as to achieve the uniformity of the temperature field.
Fig. 12. Schematic diagram of translation transformation of heat flow for case 2

If the translation angle is $\theta$, the coordinate relationship before and after translation is:

$$x_{t2} = x$$  \hspace{1cm} (18)

$$y_{t2} = y + x \tan(\theta)$$  \hspace{1cm} (19)

After translation transformation, the Jacobian matrix is:

$$J_{t2} = \begin{bmatrix} 1 & 0 \\ \tan \theta & 1 \end{bmatrix}$$  \hspace{1cm} (20)

According to (4), the thermal conductivity of transformation space medium is:

$$\kappa_{t2xx} = \kappa$$  \hspace{1cm} (21)

$$\kappa_{t2xy} = \tan(\theta) \kappa$$  \hspace{1cm} (22)

$$\kappa_{t2yy} = (\tan^2(\theta) + 1) \kappa$$  \hspace{1cm} (23)

As shown in Fig. 13, by using the compression transformation, the heat at the bottom of oA is compressed to area $\Omega'$ to achieve the uniformity of the temperature field.

The coordinate relationship after and before compression transformation is:

$$x_{c2} = x$$  \hspace{1cm} (24)

$$y_{c2} = \frac{-k_2 x + b}{b_1} y$$  \hspace{1cm} (25)

where $k_2 = \frac{b - b_1}{c}$.

The Jacobian matrix after compression transformation is:

$$J_{c2} = \begin{bmatrix} 1 & D \\ D & D^2 + C^2 \end{bmatrix}$$  \hspace{1cm} (26)
where \( C = \frac{-k_2 x + b}{b_i} \) and \( D = \frac{-k_2}{b_i} y \).

For case 2, the values of \( b_1 \) and \( c \) are the same as those for case 1, then the thermal conductivity of heat conduction structure area \( \Omega \) after compression transformation can be calculated by (27) - (29).

Temperature fields of different case were simulated based on the Comsol multiphysics software. The simulation conditions, boundary conditions and heat source settings were consistent with case 1. The temperature distribution diagram of the translation transformation of the heat conduction structure of case 2 after 60 minutes is shown in Fig. 15. The temperature distribution...
diagram of the compression of the heat conduction structure of case 2 after 60 minutes is shown in Fig. 16. Compared with Fig. 14, it can be seen that after the translation and compression transformation, the direction of the heat is significantly changed, and the temperature difference of the entire region is significantly reduced. Thermal diffusion diagram of the translation transformation of case 2 is shown in Fig.17. Thermal diffusion diagram of compression transformation of bit case 2 is shown in Fig. 18. It can be seen from the figures that the heat flows to the low temperature area in the direction of translation or compression.
3.3 Directional heat conduction structure of case 3

For case 3, there is a high heat source in the middle of the structure, as shown in Fig. 19. The model can be divided into two rectangles of the same shape along the centerline DF. For the rectangle of the upper part, the heat is adjusted to the upper boundary, and for the rectangle of the lower part, the heat is adjusted to the lower boundary, so as to reduce the temperature difference of the entire region.

Supposing the translation angle of the upper and lower rectangle are both $\theta$, then the coordinates after translation transformation can be obtained according to Eq. (18), (19), (6) and (7). The thermal conductivity of the upper rectangular area $\Omega_U$ can be calculated by (21) - (23), and the heat conductivity of the lower rectangular area $\Omega_L$ can be calculated by (9) - (11).
upward and downward by using the compression transformation in order to reduce the temperature difference of the entire region, as shown in Fig. 20.

![Compression transformation diagram of heat flow bilateral regulation](image)

Fig. 20 Compression transformation diagram of heat flow bilateral regulation

Supposing \( b_1 = 0.3 \text{ m} \), \( c = 0.351 \text{ m} \), the coordinates after compression transformation can be obtained according to Eq. (24), (25), (12) and (13). The heat conductivity of the upper rectangular area \( \Omega_U \) can be calculated by (27) - (29), and the heat conductivity of the lower rectangular area \( \Omega_L \) can be calculated by (15) - (17).

Temperature fields of different case were simulated based on the Comsol multiphysics software. The simulation conditions, boundary conditions and boundary heat source settings were consistent with case 1. The translational transformation temperature distribution diagram of the heat conduction structure of case 3 after 60 minutes is shown in Fig. 22. And the compression temperature distribution diagram of the heat conduction structure of case 3 after 60 minutes is shown in Fig. 23. Compared with the temperature distribution diagram without heat flow regulation (Fig. 21), after the translational transformation and compression transformation, the direction of heat also is changed significantly, and the temperature field in the whole area is changed evenly. Thermal diffusion diagram of the translation transformation of case 3 is shown in Fig.24. Thermal diffusion diagram of compression transformation of bit case 3 is shown in Fig. 25. It can be seen from the figures that the heat flows from the middle area to the low temperature area on both sides in a specific direction.
Fig. 21. Temperature distribution diagram without heat flow control of case 3 after 60 min

Fig. 22. Temperature distribution diagram of translation transformation of case 3 after 60 min

Fig. 23. Temperature distribution diagram of compression transformation of case 3 after 60 min
Furthermore, the differences between the average temperature of the top (AC) and the bottom edge (oB) in case 1 and 2 are computed and shown in Fig. 26-27. Then difference of average temperature between DF and oB in case 3 is shown in Fig. 28. Comparing with the structure without using the directional heat conduction method, we can see that after transformation the temperature difference between AC and oB in case 1 and 2 is reduced from about 35 °C to almost zero (Fig. 26 and Fig. 27). It can also be seen from Fig. 28 that the difference between the average temperature of DF and oA is significantly reduced from about 12 °C to close to zero after translation and compression transformation. For directional heat conduction structure with multi-sources, it could be concluded that after the translation transformation and compression transformation the heat flow in the high temperature area can be regulated to the low temperature area directionally. The temperature difference in the entire area of the thermal conduction structure was gradually reduced with the increase of time. After about six hours, the temperature difference of case 1 and case 2 were decreased by 94.3%. And the temperature difference of case 3 was decreased by 94.3%.
Fig. 26. Comparison diagram of the average temperature difference between oB and AC of case 1

Fig. 27. Comparison diagram of the average temperature difference between oB and AC of case 2

Fig. 28. Comparison diagram of the average temperature difference between oB and DF of case 3
4 Conclusion

Based on the theory of transformation optics, the multi-source directional heat conduction method was studied in this paper in combination with the fact that the object was affected by multiple heat sources and the temperature field was uneven. The thermal conductivity distribution of the directional heat conduction structure was derived by using the coordinate translation transformation and compression transformation. The performance of directional heat conduction in transient state was analyzed by Comsol software. The results showed that the directional heat conduction can be achieved through translation transformation and compression transformation, and the heat can flow along the specified direction. By comparing the average temperature difference of the three cases with high heat source at the top, high heat source at the bottom and high heat source in the middle, it could be concluded that after the translation transformation and compression transformation, the heat flow in the area with high temperature was adjusted to the area with low temperature, the temperature difference in the whole area of the heat conduction structure was reduced, and the temperature field became uniform.

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Conflict of Interest

No potential conflict of interest was reported by the authors.

Reference


