Effect of symptomatic rate on spreading of COVID-19 evaluated by a flexible compartment model

Hiroo Ohmori (ohmori@edu.k.u-tokyo.ac.jp)

Department of Natural Environmental Studies, Graduate School of Frontier Sciences, The University of Tokyo

https://orcid.org/0000-0002-5993-0996

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Abstract

For COVID-19, many of the infected individuals become symptomatic after the latent period is ended and should be isolated from the community. After the isolation period is ended, they become recovered individuals who have immunity and return to the community. Thus, they are capable of infecting susceptible individuals only during the latent period. Some infected individuals, however, are asymptomatic through the recovery period, which includes the latent period. They are infectious but not isolated. They stay in the community and continue infecting susceptible individuals through the recovery period, inducing an increase in the number of infected individuals, though they become recovered individuals who also have immunity in the community after the recovery period is ended. The number of ‘symptomatic and isolated’ infected individuals and/or of ‘asymptomatic and staying’ infected individuals is controlled by the symptomatic rate. Thus, the symptomatic rate affects the number of infectious individuals continuing to infect susceptible individuals in the community. At the same time, the symptomatic rate affects the number of isolated individuals and, as a result, the number of recovered individuals and the population excluding the individuals kept in isolation. Since the contact rate between infected individuals and susceptible individuals is affected by both the number of recovered individuals having returned to the community and the population excluding the individuals kept in isolation, the symptomatic rate also affects the contact rate. Namely, the symptomatic rate affects not only the number of infectious individuals but also the contact rate. This symptomatic rate could change commonly depending on the characteristics of virus and/or on the health conditions of infected individuals. However, the symptomatic rate could be changed/decided in the middle of infection duration by political and/or medical interventions because the symptomatic rate practically means the isolation rate, and the isolation rate and/or the number of isolated individuals could be decided by some political and/or medical interventions induced by, for example, the capacity of hospital care. Therefore, the evaluation of the effect of the symptomatic rate could provide reference materials for political and/or medical measures. The effects of vaccination and isolation by PCR were also examined for cases with different symptomatic rates. The results of evaluation by a flexible compartment model, which is a model including the symptomatic rate as a term in the calculation equation and being able to evaluate the effect of isolated/recovered individuals on the spread of COVID-19, show that a small difference in the symptomatic rate causes a large difference in the number of infected individuals and in the infection duration. Vaccination and PCR tests were effective in reducing the number of infected individuals for cases with any symptomatic rates.

1. Introduction

For COVID-19, infected individuals should be isolated from the community when they become symptomatic after the latent period is ended. Thus, the infected individuals do not infect susceptible individuals in the community after the latent period, though when the isolation period is ended, they become recovered individuals who have immunity and then return to the community. The infection has actually occurred during the latent period worldwide. However, when some infected individuals are
asymptomatic, which means the symptomatic rate is not 1.0 but less than 1.0, they are not isolated. They
stay in the community and continue infecting susceptible individuals until the recovery period is ended,
and then they become recovered individuals who also have immunity in the community. For this case, the
number of infected individuals increases compared with that of the case with a symptomatic rate of 1.0,
where all infected individuals become symptomatic and are isolated. Thus, the symptomatic rate directly
affects the frequency of occurrence of infection.

On the other hand, in a community mixed with infected individuals, susceptible ones and recovered ones,
the contact rate between the infected individuals and the susceptible ones is reduced by the contact of
the infected individuals with the recovered ones, when the number of recovered individuals has been
increased. At the same time, since the contact rate between the infected individuals and the susceptible
ones is calculated using the ratios of the numbers of infected individuals, of susceptible ones and of
recovered ones per the population excluding the individuals kept in isolation, the contact rate is affected
not only by the number of recovered individuals but also by the population excluding the individuals kept
in isolation and the dead. Since the number of isolated/recovered individuals is controlled by the
symptomatic rate, the population excluding the individuals kept in isolation is also affected by the
symptomatic rate. Therefore, the symptomatic rate affects the contact rate, indicating that the
symptomatic rate indirectly affects the frequency of occurrence of infection.

As mentioned above, the symptomatic rate affects not only the number of infectious individuals who stay
and continue infecting susceptible individuals in the community but also the contact rate between
infected individuals and susceptible individuals. Namely, the symptomatic rate has direct and indirect
effects on the spread of COVID-19.

In the calculation, the symptomatic rate practically indicates the ratio of the number of isolated
individuals. Namely, the symptomatic rate can be recognized and used as the ‘isolation rate’. The
symptomatic rate changes commonly depending on the characteristics of the virus and/or on the health
conditions of infected individuals. However, the symptomatic rate could be changed/decided in the
middle of the infection duration by political and/or medical interventions because the isolation rate
and/or the number of isolated individuals could be changed/decided by some political and/or medical
interventions, such as the capacity of hospital care. In some circumstances, political and/or medical
interventions could be taken at any time during the infection duration. For such cases, the same things
could occur as the symptomatic rate is set/changed in the middle of the infection duration. Therefore, the
evaluation of the effect of the symptomatic rate, which changes at some point during the infection
duration, must provide reference materials for political and/or medical measures. Moreover, the effects of
vaccination and isolation by PCR test, the former of which causes a decrease in the number of
susceptible individuals and a reduction in the contact rate as the recovered individuals do, and the latter
of which causes changes in the population and in the number of isolated/recovered individuals, are also
examined for cases of different symptomatic rates.
The effect of the symptomatic rate on the spread of COVID-19 is evaluated by the flexible compartment mode specific to COVID-19 proposed by Ohmori [1], which contains the symptomatic rate as an independent valuable in the calculation equations and, as the dependent valuables, the number of isolated/recovered individuals and the population excluding the individuals kept in isolation, both of which change through the course of infection. Using this model, the number of isolated/recovered individuals induced by the symptomatic rate set at any time can be calculated, and the simulation for the number of infected individuals affected by the changes in the number of isolated/recovered individuals and in the population excluding the individuals kept in isolation can be performed. Then, the effect of the symptomatic rate can be evaluated by comparing the results of the simulation among the cases with different symptomatic rates.

2. Framework of the flexible compartment model used here

The flexible compartment model proposed by Ohmori[1] consists of six categories of ‘Susceptible (Remainder): $RM$; ‘Vaccinated: $V$; ‘Recovered: $RI, RT, RAS$; ‘Infected (‘Infectious’, ‘Patient’): $P$; ‘Isolated: $I, PI$; and ‘Death: $DAS, DTI, DT$’, as shown in Fig. 1. ‘Susceptible’ is the number of susceptible individuals who are not infected but could become infected. ‘Vaccinated’ is the number of vaccinated individuals who have immunity by vaccination and live and work in the real community in Fig. 1. ‘Recovered’ is the number of recovered individuals who have been isolated from the real community to the isolation community when they have been symptomatic after the end of the latent period, have recovered from the disease and have immunity after the infectious period (the recovery period) is ended, and then have returned to the real community. ‘Infected’ is the number of infected individuals who have been infected and are capable of infecting susceptible individuals. ‘Isolated’ is the number of individuals kept in isolation, which means the number of infected individuals who have been isolated, kept in the ‘isolation community’ until the recovery period is ended, and then become recovered individuals who have immunity. ‘Death’ is the number of individuals who have died of infection after the latent period, that is, the death toll.

A compartment on the left side, containing ‘Susceptible’, ‘Vaccinated’, ‘Recovered’, ‘Infected’ and ‘Death’, is a ‘real community’. Its population, $N$, is changed by subtracting the number of isolated individuals ($I, PI$) and death ($DAS$) and by adding the number of recovered individuals ($RI, RT$). Another compartment on the right side, containing ‘Isolated’, ‘Recovered’ and ‘Death’, is the ‘isolation community’, whose population, that is, the number of individuals kept in isolation, is also changed by subtracting the number of recovered individuals ($RI, RT$) returning to the real community and of the death ($DTI, DT$). The large compartment consisting of the two compartments mentioned above is the ‘whole community’ of which the population is expressed by $TN(n)$. $TN(n)$ includes the number of individuals living in the two compartments and the toll of death ($DAS, DTI, DT$) occurring at the fatality rate in the two compartments. $TN(n)$ and $N(n)$ are changed due to the number of ‘Susceptible ($NAP(n)$)’ and/or ‘Infected ($UP(n)$)’ coming in and/or going out of the community. As shown above, each compartment contains individuals belonging to different categories, and its population changes by interacting with another compartment and with the outside.
The infected individuals in the community, \( P(n) \), are separated into three groups of those (\( I(n) \)) who are confirmed to be infected due to being test positive, those (\( PL(n) \)) who become symptomatic after the latent period and those (\( AS(n) \)) who are asymptomatic through the recovery period. The isolated individuals are composed of two groups of those (\( I(n) \)) who are confirmed to be infected due to being test positive and then isolated and those (\( PL(n) \)) who are symptomatic in the community and then isolated. Each of them needs to be calculated in a different manner according to the coefficients of the test positive rate and the symptomatic rate.

The recovered individuals are composed of two groups. One is the group of recovered individuals in the right compartment, consisting of those (\( RL(n') \)) who were isolated due to being test positive, \( I(n) \), and have been recovered and those (\( RT(n') \)) who were isolated due to being symptomatic in the community, \( PL(n) \), and have been recovered. For the date (\( n' \)) of returning to the community, each of them must be calculated in a different manner according to the isolation durations that are different from each other. The total number of recovered individuals (\( RL(n') + RT(n') \)) is not always equal to that of infected individuals (\( I(n) + PL(n) \)) because \( RL(n') + RT(n') \) is the number subtracting the total number of deaths (\( DTI(n') + DT(n) \)) from (\( I(n) + PL(n) \)). The other is the group of recovered individuals in the left compartment, \( RAS(n') \), who are the individuals who recovered from the ‘asymptomatic’ in the community. The number of recovered individuals (\( RAS(n') \)) is not always equal to that of ‘asymptomatic’ infected individuals (\( AS(n) \)) because \( RAS(n') \) is the number subtracting the number of deaths (\( DAS(n) \) from \( AS(n) \)).

The ‘death’ individuals are also composed of two groups of those (\( DAS(n) \)) who are asymptomatic and die of infection after the latent period in the community and those (\( DTI(n) \) and \( DT(n) \)) who die during the isolation period. Each of them needs to be calculated in a different manner according to different fatality rates. The vaccinated individuals, \( V(n) \), have immunity and live and work in the community. Vaccination decreases the number of susceptible individuals and reduces the contact rate between infected individuals and susceptible individuals.

### 3. Calculation process of the flexible compartment model

#### 3.1. Contact rate in the community mixed with infected, susceptible and recovered individuals

The contacts between infected individuals and susceptible individuals that cause infections actually occur in the real community mixed with infected, susceptible and recovered individuals. Thus, infected individuals contact not only susceptible individuals but also recovered individuals. From a physical point of view, the contact of the infected individuals with the recovered ones must reduce the contact rate between the infected individuals and the susceptible ones when the number of recovered individuals has increased. Accounting for the reduction effect of the recovered individuals on the contact rate, the contact rate should be given by the following equation:

\[
cr(n) = \frac{S(n)}{N(n)}(1 - \delta(\frac{R(n)}{N(n)}))
\]  

(1)
where the \( cr(n) \) is the contact rate, \( n \) is the date starting from 1 when the infection begins, \( S(n) \) is the number of susceptible individuals in the community, \( R(n) \) is the number of recovered individuals who have returned to the community, and \( N(n) \) is the population including the recovered individuals having returned to the community but excluding the individuals kept in isolation and the dead. The term \(- \delta(R(n)/N(n))\) is the reduction effect of the recovered individuals on the contact rate between the infected individuals and the susceptible ones, and the term \((1-\delta(R(n)/N(n)))\) is the ‘reduction rate’ of the contact rate. The reduction effect increases with a decreasing value of \((1-\delta(R(n)/N(n)))\), meaning that the contact rate decreases with an increase in the number of recovered individuals. ‘\( \delta \)’ is a coefficient expressing the activity level of the recovered individuals in the community. When the value of \( \delta \) is given by 1, the activity is the same level as the susceptible individuals, and when the value of \( \delta \) is given by 0, the recovered individuals are not active, meaning a similar condition as they are kept in isolation, though the number of recovered individuals is added to the population. This contact rate, \( cr(n) \), is used for the calculation of the number of infected individuals, as shown by Eq. (2).

The reduction effect is caused not only by the recovered individuals but also by the vaccinated individuals who have been vaccinated, have immunity and are living and working in the community. The reduction effect of vaccination should be taken into account for the simulation. The model contains the reduction effect of both the recovered individuals and the vaccinated individuals.

3.2. Calculation of the number of infected individuals, the number of individuals newly infected a day and the increment/decrement in the number of infected individuals

It should be confirmed that the number of infected individuals, \( P(n) \), is not equal to the number of individuals infected each day, \( AP(n) \). The former is the total number of individuals being infected and infectious in the community, that is, the sum of the number of infected individuals during the latent period and/or the recovered period in the community, whereas the latter is the number of individuals newly infected for one day. However, \( P(1) \), the initial number of infected individuals in the community, is arbitrarily set for simulation. The increment and/or decrement in the number of infected individuals on date \( n \) in the community, \( \Delta P(n) \), can be calculated by subtracting the number of individuals isolated on date \( n \) from the number of individuals newly infected on date \( n \).

For simulation, using an Excel file, calculation is performed based on Eq. (2). The Excel file and the meaning of individual terms/variables are explained in the supplementary files. The independent variables of twenty terms can be set arbitrarily by you, and the values of the dependent variables of fifty-five terms are uniquely determined based on the independent values given by you.

The number of individuals newly infected on date \( n \), \( AP(n) \), is given by the following equation:

\[
AP(n\text{(night)})=(pfc(n)/lp(n))*(RM(n)/N(n))*icf(n)*
\]
\[(1-(al(n)\times(CR(n)+CRT(n))+al(n)\times CRAS(n)+alV(n)\times V(n))/N(n))\times(RP(n)/N(n))\times RM(n) \quad (2)\]

where the coefficient \(pfc(n)\) is the potential (biological) infectious capacity of coronavirus, which is an approximate value indicating the number of susceptible individuals infected during the latent period, \(lp(n)\). The latent period, \(lp(n)\), is the time interval between when an individual is infected and when he/she is symptomatic. \(pfc(n)/lp(n)\) is the number of susceptible individuals infected by an infected individual a day, and its value, including decimal places, is used in the calculation. The coefficient \(icf(n)\) is the infection reduction rate by infectious control measures preventing the spread of virus, such as facemasks, partitions and disinfectants. The coefficient \(al\) is the activity level of the recovered individuals having returned to the community from the isolation, \(al\) is the activity level of the individuals recovered from the 'asymptomatic' in the community and \(alV\) is the activity level of the vaccinated individuals.

In a strict sense, as explained in section 4.1 'Symptomatic rate', since the infection occurs during the daytime from the morning to the evening for the purpose of calculation, \(AP(n)(\text{night})\), which is the value of \(AP(n)\) at night, is the correct number of individuals newly infected a day, which has increased/decreased from the \(AP(n)\) in the morning, which is equal to the \(AP(n)\) of the previous night, that is, \(AP(n-1)(\text{night})\).

\(RM(n)\) is the number of susceptible individuals in the community, \(N(n)\) is the population excluding the individuals kept in isolation and dead, \(RP(n)\) is the number of infected individuals excluding the individuals kept in isolation and the dead, \(CRI(n)\) is \(\Sigma RI(n)\) and \(RI(n)\) is the number of recovered individuals who were isolated due to being test positive and have returned to the community, \(CRT(n) = \Sigma RT(n)\) and \(RT(n)\) is the number of recovered individuals who have been isolated due to being symptomatic in the community and have returned to the community, \(CRAS(n) = \Sigma RAS\) and \(RAS(n)\) is the number of recovered individuals who were infected but did not become symptomatic, were asymptomatic, were not isolated, were staying in the community, had continued infecting until the recovery period was ended and then have become the recovered individuals, and \(V(n)\) is the number of vaccinated individuals who have immunity as the recovered individuals do.

\(RM(n)\) is the number of susceptible individuals in the community, as mentioned above. It indicates the number of susceptible individuals subtracting the number of individuals confirmed to be infected due to being test positive and isolated, \((CI)\), the cumulative number of individuals having been infected, \((CAP)\), which includes the number of individuals having recovered, and the number of vaccinated individuals \((V)\) from the total population of the community, \((TN)\), and therefore, is given by:

\[RM(n) = TN(n) - (CI(n) + CAP(n) + V(n)) \quad (3)\]

where \(TN(n)\) is the total population of the whole community, such as a city. \(TN(1)\) is the initial population of the community arbitrarily given by yourself, and \(TN(n)\) is changed by coming in-going out of the
susceptible \((NAP(n))\) and/or the infected \((UP(n))\).

\[
CI(n) = \sum I(n)
\]

is the number of individuals isolated due to being test positive. Since all the individuals confirmed to be infected due to test positivity are not always isolated and the individuals decided to be isolated are isolated on the day next to the date when they are confirmed to be infected, in the actual calculation, \(I(n)\) is given by:

\[
I(n) = CP(n-1) \times i(n-1)
\]

(4)

where the coefficient \(i(n)\) is the isolation rate for the individuals who are confirmed to be infected due to being test positive. \(CP(n)\) is the number of individuals confirmed to be infected due to being test positive. The individuals decided to be isolated are isolated on the next day for the purpose of calculation.

Conversely, the individuals confirmed to be infected on the previous day, date \((n-1)\), are isolated on date \(n\). Thus, \(I(n)\) is given by

\[
CP(n) = T(n) \times bp(n) \times ir(n)
\]

(5)

where \(T(n)\) is the number of individuals having PCR test and/or the antibody test, which you can set arbitrarily on any days when tests are performed, and the coefficient \(bp(n)\) is the magnification of incidence rate for the test to the incidence rate, \(ir(n)\), in the community. The incident rate, \(ir(n)\), is given by:

\[
ir(n) = P(n) / TN(n)
\]

(6)

where \(P(n)\) is the number of infected individuals already having existed in the community, though \(P(1)\) which is the initial number of infected individuals in the community and is arbitrarily given by yourself. \(TN(n)\) is the total population of the whole community. Since individuals who have the test are mainly close contacts, the incidence rate for the test would be biased to be higher than \(ir(n)\). The incidence rate for the test is given by the magnification with respect to \(ir(n)\). As a result, the value of \(bp(n) \times ir(n)\) indicates the positive rate for the PCR test, \(tir(n)\). The positive rate, \(tir(n)\), is given by:

\[
tir(n) = bp(n) \times ir(n)
\]

(7)

\(CAP(n)\) means \(\sum AP(n)\) and indicates the cumulative number of individuals newly infected a day up to the morning of date \(n\), which is equal to the cumulative number of infected individuals up to the night of date \((n-1)\).

\(V(n)\) is the number of vaccinated individuals who are vaccinated and have immunity and is given by:

\[
V(n) = TN(1) \times (v(n) - b(n))
\]

(8)

where \(TN(1)\) is the initial total population of the community, the coefficient \(v(n)\) is the vaccination rate, and \(b(n)\) is the breakthrough rate. The term \((v(n) - b(n))\) indicates an immunity acquisition rate for the
purpose of calculation. However, note the following: For the individuals who were vaccinated and had once immunity, some of them would suffer breakthrough infection considerably later after the date when they were vaccinated. For example, when vaccination is carried on date \( n \), breakthrough infection could occur on date \( (n+m) \), where \( m \) would indicate dozens of days. Therefore, \( b(n) \) should be applied to the individuals who were vaccinated on date \( (n-m) \) and were infected by ‘breakthrough infection’ on date \( n \). The value of \( m \) is arbitrarily supposed/decided by you for simulation. The value of \( b(n) \) should be set to 0 on the day when breakthrough infection does not occur. The vaccinated individuals who have become infected are reset to be susceptible individuals on the date when breakthrough infection occurs.

\( N(n) \) is the population excluding the individuals kept in isolation and dead, that is, the total number of individuals living and working in the community, which is explained in section 4-2 ‘Population excluding the individuals kept in isolation’.

\( RP(n) \) is the number of infected individuals excluding the individuals kept in isolation and the dead, that is, the number of infected individuals living and working in the community, for example, the infected individuals during the latent period and/or asymptomatic infected individuals even after the latent period. Thus, \( RP(n) \) could be called the ‘Spreader’ who continues to infect susceptible individuals in the community. When \( n \) is 1, meaning the first day of simulation, \( RP(1) \) is equal to \( P(1) \), which is the initial number of infected individuals in the community and is arbitrarily given by yourself. Since \( RP(n) \) is the number of infected individuals minus the number of isolated individuals (\( PI(n) \)), of the dead (\( DAS(n) \)) and of the recovered individuals living in the community (\( RAS(n) \)) from the total number of infected individuals up to date \( n \), it is given by the following equation:

\[
RP(n) = \sum (AP(n) - PI(n-1) - DAS(n-1) - RAS(n))
\]  

(9)

where \( PI(n) \) is the number of symptomatic individuals who become symptomatic on the day after the end of the latent period in the real community and are isolated on the next day. \( PI(n) \) is given by:

\[
PI(n) = AP(n-(lp+1)) \cdot syr(n-(lp+1))
\]  

(10)

where \( lp(n) \) is the latent period, and \( (n-(lp+1)) \) indicates the ‘latent period+1’ before date \( n \), meaning the day after the end of the latent period, because the infected individuals become symptomatic and are isolated on the day after the end of the latent period, as explained in 4-1 ‘Symptomatic rate’. The value of \( AP(n-(lp+1)) \) is the number of infected individuals who were newly infected on date \( (n-(lp+1)) \), which is the day after the end of the latent period. The coefficient \( syr(n) \) is the symptomatic rate on date \( n \). When \( syr(n) \) is 1, all the infected individuals become symptomatic and isolated.

Conversely, therefore, the number of asymptomatic infected individuals, \( AS(n) \), is given by:

\[
AS(n) = AP(n-(lp+1)) - PI(n)
\]  

(11)

\( DAS(n) \) is the number of individuals who are asymptomatic and die of infection after the latent period in the community, that is, the death toll in the community. It is given by:
\[ DAS(n) = AS(n - \text{trunc}((rp-lp)/2)) \times fr(n - \text{trunc}((rp-lp)/2)) \]  

(12)

where the coefficient \( rp(n) \) is the recovery period, which is the time interval between when an individual is infected and when he/she recovers from the disease and is not capable of infecting. The ‘(rp-lp)’ is equivalent to the isolation period. \( AS(n - \text{trunc}((rp-lp)/2)) \) is the number of ‘asymptomatic’ infected individuals who were not isolated and were staying in the community on the day ‘\text{trunc}((rp-lp)/2)’ days before date \( n \). \( AS(n) \) is the number of infected individuals subtracting \( PI \) from \( AP \), as shown by Eq. (11), the coefficient \( fr(n) \) is the fatality rate for the asymptomatic infected individuals in the community. Death of infected individuals occurs on the middle date in the isolation period, that is, ‘\text{trunc}((rp-lp)/2)’. Namely, some of the infected individuals who have been isolated on date \( n \) die on date \( (n+\text{trunc}((rp-lp)/2)) \). For asymptomatic individuals, the same condition is applied. When the number of asymptomatic individuals infected on date \( n \) is \( AS(n) \), the \( AS(n) \times fr(n) \) individuals also die on date \( (n+\text{trunc}((rp-lp)/2)) \). Conversely, the death toll of the asymptomatic individuals on date \( n \), \( DAS(n) \), is given by Eq. (12).

\[ RAS(n) = \sum RI(n) \], and \( RI(n) \) is the number of recovered individuals who were isolated due to being test positive. \( RI(n) \) is given by:

\[ RI(n) = I(n-rpI) - DTI(n-(1+\text{trunc}((rpI)/2))) \]  

(14)

where \( I(n) \) is the number of individuals isolated due to being test positive and given by Eq. (4). \( DTI(n) \) is the death toll of the individuals isolated due to being test positive and is given by:

\[ DTI(n) = I(n-\text{trunc}(rpI/2)) \times frI(n-\text{trunc}(rpI/2)) \]  

(15)

where \( frI(n) \) is the fatality rate for the isolated individuals and \( rpI(n) \) is the isolation period for the isolated individual due to being test positive, which is less than or equal to \( rp \). The death of isolated individuals occurs on the middle date in the isolation period. When the number of individuals isolated on date \( n \) is \( I(n) \), the \( I(n) \times frI(n) \) individuals die on day ‘\((n+rpI/2)\) days’ after date \( n \). Conversely, for the death toll on date \( n \), the date of death of \( I \) is \((n-\text{trunc}(rpI/2))\).

\[ CRT(n) = \sum RT(n) \], and \( RT(n) \) is the number of recovered individuals who have been isolated due to being symptomatic in the community. \( RT(n) \) is given by:

\[ RT(n) = PI(n-(rp-lp))-DT(n-(1+\text{trunc}((rp-lp)/2))) \]  

(16)
where \( DT(n) \) is the death toll of the individuals isolated due to being symptomatic.

\[
DT(n) = PI(n \text{trunc}((rp-lp)/2)) \times frI(n \text{trunc}((rp-lp)/2)) \tag{17}
\]

where \( PI(n) \) is the number of ‘symptomatic’ infected individuals given by Eq. (10) and \( frI(n) \) is the fatality rate for the isolated individuals. The ‘\((rp-lp)\)’ is the isolation period for the individuals isolated due to being symptomatic.

For calculation, Eq. (2) is transformed to the following equation:

\[
AP(n\text{night}) = p(n) \times RM(n) \tag{18}
\]

where \( p(n) \) is the infection coefficient, which is the infectious capacity, including the contact rate changing with the numbers of susceptible individuals, infected individuals, recovered individuals and vaccinated individuals in the (real) community:

\[
p(n) = \frac{pfc(n)}{lp(n)} \times \frac{RM(n)}{N(n)} \times icf(n) \times (1-\frac{AL(n)}{N(n)}) \times \frac{RP(n)}{N(n)} \tag{19}
\]

where \( AL(n) \) is the sum of the activity levels of the recovered individuals and vaccinated individuals:

\[
AL(n) = alI(n) \times (CRI(n)+CRT(n)) + al(n) \times CRAS(n) + alV(n) \times V(n) \tag{20}
\]

The term \( AL(n)/N(n) \) is equivalent to the term \( \delta(R(n)/N(n)) \), and the term \( (1-(AL(n)/N(n))) \) is equivalent to the term \( (1-\delta(R(n)/N(n))) \) of Eq. (1). When the value of \( AL(n) \) is given by 1, the activity of the recovered individuals is the same level as the susceptible individuals, and when the value of \( AL(n) \) is given by 0, the recovered individuals are not active, meaning a similar condition as they are kept in isolation, though the number of recovered individuals who have returned to the community is added to the population.

The value \( pfc(n)/lp(n) \) is the infection rate (persons/person/day) for an infected individual, and \( (RP(n)/N(n)) \) indicates the ratio of the number of infected individuals living in the community. Thus, the value of \( (pfc(n)/lp(n)) \times (RP(n)/N(n)) \) indicates the probability of occurrence of infection by the total number of infected individuals in the community. The value of \( (RM(n)/N(n)) \times (1-(AL(n)/N(n))) \) indicates the probability of occurrence of contact for a susceptible individual. Therefore, the coefficient \( p(n) \) indicates the possibility of infection per susceptible individual per day in the community with mixed infected, susceptible and recovered individuals.

The infected individuals, \( P(n) \), are given by the following equation:

\[
P(n) = RP(n) + AP(n\text{night}) = RP(n) + p(n) \times RM(n) \tag{21}
\]
Taking the number of isolated individuals into consideration, the rate of change in the number of infected individuals, $\Delta P(n)$, that is, the increment and/or decrement in the number of infected individuals a day in the community, can be calculated by subtracting the number of individuals isolated on date $n$ due to being symptomatic from the number of individuals newly infected on date $n$ and is given by the following difference equation:

$$\Delta P(n) = AP(n) - P(n) = AP(n) - syr(n')*AP(n')$$  \hspace{1cm} (22)$$

where $n'$ means the date ‘the latent period’ before date $n$ and indicates that $P(n)$ is the number of infected individuals who have been newly infected on date $n'$ and have been symptomatic on the day after the end of the latent period and then have been isolated on date $n$, as explained in section 4-1 ‘Symptomatic rate’.

For the simulation using Eq. (2) and/or Eq. (18), when $n$ is 1, meaning the first day of simulation, $N(1)$ is equal to $TN(1)$, which is the initial total population of the community. $TN(1)$ is arbitrarily set by yourself. $RP(1)$ is equal to $P(1)$, which is the initial number of infected individuals. $P(1)$ is also arbitrarily set by yourself. $RM(1)$ is the initial number of susceptible individuals in the community and is inevitably decided by subtracting $P(1)$ from $N(1)$.

4. Method

4.1. Symptomatic rate

The symptomatic rate, $syr$, is the ratio of the number of individuals who become symptomatic per all the number of individuals newly infected a day. However, note the following: Symptoms occur on the day after the latent period. Namely, the individuals who were infected on date $n$ become symptomatic on date $(n+lp)$, and $syr(n)$ indicates the ratio of the number of individuals who were infected on date $n$ and become symptomatic on date $(n+lp)$ to the total number of individuals newly infected on date $n$. Thus, for the purpose of calculation, date ‘$n$’ of $syr(n)$ indicates date $n$ when the symptomatic individuals are infected, though they become symptomatic on date $(n+lp)$. The latent period includes the day when they are infected. They are isolated due to being symptomatic on the next day, that is, on date $(n+(lp+1))$. Namely, the infected individuals who were infected on date $n$ become symptomatic on date $(n+lp)$ and are isolated on the next day, $(n+(lp+1))$, that is, the day after the end of the latent period. Therefore, again, the symptomatic rate expressed as ‘$syr(n)$’ means the symptomatic rate for the individuals infected on date $n$ and the individuals infected on date $n$ are isolated on date $(n+(lp+1))$.

As all infected individuals are not always symptomatic, all infected individuals are not always isolated. The number of isolated individuals, $P(n)$, which indicates the number of ‘symptomatic’ infected
individuals isolated a day on date \( n \), is given by the following equation:

\[
P(n) = AP(n-(lp+1)) \times syr(n-(lp+1))
\]  \hfill (23) (=10)

where \( n \) indicates the day after the end of the latent period, and date \( (n-(lp+1)) \) indicates the date when the individuals who are isolated on date \( n \) were infected. Thus, the value of \( AP(n-(lp+1)) \) is the number of infected individuals who were newly infected on date \( (n-(lp+1)) \), and some of them should be isolated on date \( n \). Date \( (n-(lp+1)) \) is equivalent to \( n' \) of Eq. (22).

For example, when the latent period, \( lp \), is 5 and \( n \) is 157, \( lp+1=6 \), then \( (n-(lp+1)) = (157-6) =151 \). Thus, the number of individuals isolated on the 157\(^{th} \) is \( AP(151) \times syr(151) \). When the value of the symptomatic rate is set to 0.8 on the 151\(^{st} \), 80% of the individuals infected on the 151\(^{st} \) become symptomatic on the 156\(^{th} \), which is the day after the end of the latent period and are isolated on the 157\(^{th} \) (Figs. 2 and 3). However, 20% of the individuals infected on the 151\(^{st} \) do not become symptomatic on the 156\(^{th} \), being asymptomatic and then not isolated, staying in the community. The number of asymptomatically infected individuals, \( AS(n) \), is given by Eq. (11):

\[
AS(n) = AP(n-(lp+1)) - P(n)
\]  \hfill (24) (=11)

Asymptomatic individuals continue to infect susceptible individuals until the recovery period ends and then become recovered individuals in the community.

In the actual calculation in the Excel file, \( AP(n) \) in the morning on date \( n \) is given by:

\[
AP(n) = RPM(n) - RP(n-1)
\]  \hfill (25)

where \( RP(n-1) \) is the number of infected individuals existing on the previous day, excluding the individuals kept in isolation and the dead, and \( RP(n) \) is given by Eq. (9), and \( RPM(n) \) is the remaining number of infected individuals in the community excluding the number of individuals isolated due to being test positive but including the number of individuals who are test positive but are not isolated:

\[
RPM(n) = P(n) + UP(n) - I(n-1)
\]  \hfill (26)

where \( P(n) \) is the number of infected individuals in the morning on date \( n \), meaning the number of infected individuals before any isolated individuals and/or the dead have been taken away, \( UP(n) \) is the number of infected individuals coming in/going out of the community and \( I(n) \) is the number of individuals isolated due to being test positive. The value of \( AP(n) \) includes the symptomatic infected individuals, the asymptomatic infected individuals and the individuals who test positive but are not isolated, staying in the community.
The value of $AP(n)$ in the morning on date $n$ is equal to that of the previous night, that is, $AP(n-1(night))$. After isolating the individuals who became symptomatic on the previous day (on date $(n-1)$) and removing the dead who died on the previous day, the calculation of the number of infected individuals is performed. For the purpose of calculation, the infection occurs during the daytime from the morning to the evening, and thus, the number of individuals newly infected on that day (on date $n$) is given as $AP(n(night))$ by Eq. (2), and the number of infected individuals existing at night on that day is given by $P(n(night))$ by Eq. (21).

### 4.2. Population excluding the individuals kept in isolation

The population excluding the individuals kept in isolation and dead in the real community, $N(n)$, is changed through the infection duration, as shown in Figs. 2 and 3, and is given by the following equation:

$$N(n) = TN(n-1) - (CI(n-1) + CPI(n-1) + CDAS(n-1) + CDT(n-1)) + CRI(n-1) + CRT(n-1)$$  \hspace{1cm} (27)

where $TN(n)$ is the total population of the community. When $n$ is 1, meaning the first day of simulation, $N(1)$ is $TN(1)$ and $TN(1)$ is the initial population of the community, which is arbitrarily set by yourself, and other terms of Eq. (27) are all 0. Using ‘$(n-1)$’ means that the population on the previous night becomes the population in the morning of date $n$. $T(n)$ is changed by the number of susceptible individuals ($NAP(n)$) and/or infected individuals ($UP(n)$) who come in and/or leave the community.

$CI(n)$ is $\sum I(n)$. $I(n)$ is the number of individuals isolated due to being positive for the PCR test and given by Eq. (4).

$CPI(n)$ is $\sum PI(n)$. $PI(n)$ is the number of individuals isolated due to being symptomatic and, as previously explained, it is given by Eqs. (10) and (23).

$CDAS(n)$ is $\sum DAS(n)$, and $DAS(n)$ is given by Eq. (12).

$CDT(n)$ is $\sum DT(n)$, and $DT(n)$ is the death toll of the individuals isolated due to being symptomatic. $DT(n)$ is given by:

$$DT(n) = PI(n \cdot \text{trunc}((rp-lp)/2)) \cdot frI(n \cdot \text{trunc}((rp-lp)/2))$$  \hspace{1cm} (28) (=17)

where $PI(n)$ is given by Eqs. (10) and (23), and $frI(n)$ is the fatality rate for the isolated individuals.

$CRI(n)$ is $\sum RI(n)$, and $RI(n)$ is the number of recovered individuals who were isolated due to being test positive. $RI(n)$ is given by:
where $DTI(n)$ is the death toll of the individuals isolated due to being test positive. $DTI(n)$ is given by:

$$DTI(n) = \frac{I(n-rpI)}{1 + \text{trunc}(rpI/2)}$$

(30) (=15)

$CRT(n) = \sum RT(n)$, and $RT(n)$ is the number of recovered individuals who have been isolated due to being symptomatic. $RT(n)$ is given by:

$$RT(n) = PI(n-(rp-lp)) - DTI(n-(1 + \text{trunc}((rp-lp)/2)))$$

(31) (=16)

$N(n)$ is changed by the occurrence of isolation and death. When the $syr$ is set to 1.0 through the infection duration, ‘$syr1$’, meaning that all the infected individuals become symptomatic and are isolated, $N(n)$ decreases down to 981,124 on the 206th and then recovers to 1,000,000 on the 366th. On the other hand, when $syr(151)$ is set to 0.8 on the 151st, ‘$sry (1-150)1.0; (151-)0.8$’, meaning that 80% of the individuals newly infected a day become symptomatic and are isolated and 20% of the individuals newly infected a day are asymptomatic and stay in the community on and after the 151st, $N(n)$ decreases down to 947,350 on the 206th and then recovers to 1,000,000 on the 323rd (Figs. 2 and 3).

For the difference between them, the change in the difference $\Delta N = N(sry(1-150)1.0; (151-)0.8) - N(syr1.0)$ is shown in Fig. 4. $N(n)$ of ‘$syr(151-)0.8$’ is larger than that of ‘$syr1.0$’ for the early duration from the 158th to 173rd, with a maximum difference of 1,504 on the 166th. During from the 174th to the 240th, however, $N(n)$ becomes considerably less, with the maximum difference of -33,774 on the 206th, because the increment of the number of ‘isolated’ individuals newly infected a day overcomes that of the number of the newly infected individuals staying in the community. From the 241st to the 335th, $N(n)$ is larger again, with a maximum difference of 1,028 on 252nd. After 336th, there is no difference between them.

Since, as shown by Eqs. (1) and (2), where $N(n)$ is included as a denominator of the fraction, the contact rate between infected individuals and susceptible individuals is controlled by the population as well as the number of infected individuals and susceptible individuals, and the change in the number of infected individuals should be affected by the change in the population. It would be examined in section 5-3 ‘Changes in the number of the population excluding the number of individuals kept in isolation and its relation to the change in the number of individuals newly infected a day by different symptomatic rate’.

5. Results

5.1. Change in the number of infected individuals affected by the occurrence of asymptomatic staying infected individuals

The number of infected individuals is calculated by
\[ AP(n_{night}) = \left( \frac{pfc(n)}{lp(n)} \right) \left( \frac{RM(n)}{N(n)} \right) \cdot \text{icf}(n) \times \]
\[ (1 - (aI(n)\cdot CRT(n) + aI(n)\cdot CRAS(n) + aI\cdot V(n)\cdot V(n)) / N(n)) \times \left( \frac{RP(n)}{N(n)} \right) \times RM(n) \quad (32) (= 2) \]

and the number of isolated individuals, \( Pl(n) \), which indicates the number of 'symptomatic' infected individuals isolated a day on date \( n \), is given by the following equation, which was previously shown in Eqs. (10) and (23):

\[ Pl(n) = AP(n - (lp+1)) \times syr(n - (lp+1)) \quad (33)(=23, 10) \]

When the symptomatic rate, \( syr \), is set to 1.0 on and after the first day of simulation, since all the infected individuals become symptomatic and are isolated, the number of individuals isolated on date \( (n + (lp+1)) \) is equal to that of individuals infected on date \( n \), as explained in section 4-1 ‘Symptomatic rate’ (Fig. 5).

‘\( P \)’ is the number of infected individuals, which is the sum of the infected individuals existing in the community during the latent period, and ‘\( I2 \)’ is the number of individuals kept in isolation, which is the sum of the isolated individuals existing during the isolation period, which is the period from the date when they are isolated to the date when they recover from the disease and return to the community. Since the isolation period is relatively longer than the latent period, the number of individuals kept in isolation, \( I2 \), is larger than that of infected individuals, \( P \).

Now, the number of individuals newly infected a day, \( AP \), reaches 2,135 at the peak on the 191\(^{st}\) and 192\(^{nd}\) and then decreases to 0 on the 326\(^{th}\). The number of infected individuals, \( P \), reaches 14,895 at the peak on the 194\(^{th}\) and then decreases to 0 on the 354\(^{th}\), with a total number of infected individuals of 141,788 (Table 1).

When the symptomatic rate is 0.8, 80% of individuals newly infected on a day become symptomatic and are isolated, and the remaining 20% are asymptomatic and continue staying in the community. Changes in the number of individuals newly infected a day, \( AP \), the number of individuals who become symptomatic and isolated a day, \( Pl \), the number of individuals who are asymptomatic and staying in the community, \( AS \), the number of infected individuals in the community, \( P \), and the number of individuals kept in isolation, \( I2 \), are shown in Fig. 6.

Since the asymptomatic infected individuals staying in the community continue infecting the susceptible individuals in the community until the recovery period is ended, the number of individuals newly infected a day, \( AP \), reaches 8,670 at the peak on the 125\(^{th}\) and then decreases to 0 on the 217\(^{th}\). It is approximately 4 times larger than that of the case with a \( syr \) of 1.0. Out of 8,670 infected individuals, 1,374 individuals were asymptomatic and staying in the community (\( AS \)), and 6,936 individuals were isolated due to being symptomatic (\( Pl \)). Namely, the number of isolated individuals, \( Pl \), which indicates the number of individuals who need to get treatment, also rapidly increases up to 6,936 on the 131\(^{st}\), though it is 2,135 on the 197\(^{th}\) for the case with a \( syr \) of 1.0 (Table 1).
Since the sum of the ‘asymptomatic’ infected individuals who are staying in the community during the recovered period becomes markedly large, even though the ‘asymptomatic rate’ is not so large, the number of infected individuals, \( P \), is larger than that of the individuals kept in isolation, \( I_2 \).

On the other hand, as shown in Fig. 7, the relation between the number of ‘asymptomatic staying’ infected individuals, \( AS \), and the number of individuals newly infected a day, \( AP \), that is, the relation of \( AS - AP \), shows a convex curve in the first half, meaning \( \Delta AP / \Delta AS > 1 \). This indicates that a small increase in the number of ‘asymptomatic staying’ infected individuals induces a large increase in the number of individuals newly infected a day, and due to feedback, the increase in the number of infected individuals induces the increase in ‘asymptomatic staying’ infected individuals in turn. In the second half, the relation shows a concave curve. Considering the direction of the arrow, it could be understood to be \( \Delta AS / \Delta AP > 1 \). This indicates that a small decrease in the number of individuals newly infected a day induces a large decrease in the number of ‘asymptomatic staying’ infected individuals. For the relation of \( AS - PI \), since the number of isolated individuals is simply apportioned by 80% of the number of individuals newly infected a day, it shows a linear relation to the number of ‘asymptomatic staying’ infected individuals.

As a result, the number of infected individuals, \( P \), reaches 73,678 at the peak on the 129\(^{th} \) and then decreases to 0 on the 240\(^{th} \), with a total number of infected individuals of 336,096. Thus, when the symptomatic rate is set to 0.8, the date of the peak is considerably advanced, and the infection duration becomes considerably shorter. However, the number of infected individuals at the peak markedly increases, and the total number is also markedly larger. This indicates that the occurrence of ‘asymptomatic staying’ infected individuals, which is induced by a symptomatic rate less than 1.0, could cause a rapid and large increase in the number of infected individuals, though the infection duration becomes short.

**5.2. Changes in the number of infected individuals by different symptomatic rates**

Changes in the number of \( AP(n) \) (the number of individuals newly infected on date \( n \)) and \( CAP \) (the cumulative number of infected individuals up to date \( n = \) the cumulative number of individuals newly infected a day up to date \( n \)) by different symptomatic rates are shown in Table 2 and Fig. 8. As mentioned previously in section 5.1, when the symptomatic rate is 1.0, meaning that all the infected individuals become symptomatic and are isolated, the number of individuals newly infected a day, \( AP \), reaches 2,135 at the peak on the 191\(^{st} \) and 192\(^{nd} \) and then decreases to 0 on the 326\(^{th} \). The number of infected individuals, \( P \), reaches 14,895 at the peak on the 194\(^{th} \) and then decreases to 0 on the 354\(^{th} \), with a total number of infected individuals of 141,788. However, when the symptomatic rate is 0.9, meaning that 90% of individuals newly infected on a day become symptomatic and are isolated, the number of infected individuals a day reaches 5,411 at the peak on the 145\(^{th} \). Out of 5,411 infected individuals, 541
individuals are asymptomatic and staying in the community, and 4,870 individuals become symptomatic on the 150\textsuperscript{th} and are isolated on the 151\textsuperscript{st}. The number of infected individuals reaches 42,001 at the peak on the 149\textsuperscript{th} and then decreases to 0 on the 274\textsuperscript{th}, with a total number of infected individuals of 251,341. The number of infected individuals at the peak becomes markedly larger, though the date of the peak is considerably brought forward. The total number of infected individuals also becomes markedly larger, although the infection duration becomes considerably shorter (Fig. 8 and Table 2).

When the symptomatic rate is 0.5, meaning that half of the infected individuals become symptomatic and are isolated, the number of newly infected individuals a day reaches 16,342 at the peak on the 97\textsuperscript{th}. Half of it, 8,171, is asymptomatic and stays in the community, and the remaining half becomes symptomatic on the 102\textsuperscript{nd} and is isolated on the 103\textsuperscript{rd}. The number of infected individuals reaches 173,623 at the peak on the 103\textsuperscript{rd} and then decreases to 0 on the 200\textsuperscript{th}, with a total number of infected individuals of 490,235.

When the symptomatic rate is 0.3, meaning that 30\% of the infected individuals become symptomatic and are isolated, the number of newly infected individuals a day reaches 19,879 at the peak on the 88\textsuperscript{th}. Out of 19,879 infected individuals, 5,964 individuals become symptomatic on the 93\textsuperscript{rd} and are isolated on the 94\textsuperscript{th}, and the rest, 13,915, are asymptomatic and staying in the community. The number of infected individuals reaches 239,318 at the peak on the 95\textsuperscript{th} and then decreases to 0 on the 188\textsuperscript{th}, with a total number of infected individuals of 546,614.

As examined above, with a decrease in the symptomatic rate, the number of infected individuals markedly increases, though the infection duration becomes markedly short (Table 2). The relations between the symptomatic rate, \( syr \), and the number of individuals newly infected a day, \( AP \), at the peak, the number of infected individuals, \( P \), at the peak and the total number of infected individuals, \( CAP \), are shown in Fig. 9. The number of infected individuals markedly increases with a decrease in the symptomatic rate. Although they show non-linear relations, when \( syr \) decreases by 0.1, the peak number of \( AP \) increases by approximately 4,000, the peak number of \( P \) increases by approximately 35,000, and the total number of infected individuals increases by approximately 100,000 individuals in the first half from 1 to 0.5 of \( syr \).

5.3. Changes in the number of the population excluding the number of individuals kept in isolation and its relation to the change in the number of individuals newly infected a day by different symptomatic rates

As previously explained in section 4-2 ('Population excluding the individuals kept in isolation'), the change in the number of isolated individuals induces the change in the population. Since the symptomatic rate
controls the change in the number of isolated individuals, the symptomatic rate controls the change in the population, as shown in Fig. 10.

When the syr is set to 1.0 through the simulation, \( N(n) \) decreases down to 981,124 on the 206\(^{th} \) and then recovers to 1,000,000 on the 366\(^{th} \). When syr is set to 0.9, the minimum value of the population is 956,681 at the bottom on the 156\(^{th} \) and then recovers to 1,000,000 on the 281\(^{st} \). When syr is set to 0.8, the minimum value of the population is 938,613 at the bottom on the 136\(^{th} \) and then recovers to 1,000,000 on the 244\(^{th} \). When syr is set to 0.5, the minimum value of the population is 928,450 at the bottom on the 108\(^{th} \) and then recovers to 1,000,000 on the 197\(^{th} \). It shows the smallest value among all cases with different symptomatic rates (Fig. 10).

When syr is set to 0.3, the minimum value of the population is 948,051 at the bottom on the 99\(^{th} \) and then recovers to 1,000,000 on the 181\(^{st} \). The minimum value has increased, though the date of the bottom became brought forward. When syr is set to 0.2, the minimum value of the population is 962,992 at the bottom on the 96\(^{th} \) and then recovers to 1,000,000 on the 174\(^{th} \). Therefore, the minimum value of the population for the individual symptomatic rate decreases with a decrease in the symptomatic rate, reaches 928,450 for the symptomatic rate of 0.5, and then increases.

The correlation between the symptomatic rate, syr, and the minimum population, \( N \), is shown in Fig. 11. The correlation shows a concave curve with a bottom. The value of \( N \) decreases with a decrease in the value of syr, reaching 928,450 at the bottom when the value of syr is 0.5. After the bottom, the value of \( N \) increases with a decrease in the value of syr. When the symptomatic rate is set to 0, since any infected individuals are not symptomatic, are asymptomatic and are not isolated, the population does not decline, keeping the initial population at 1,000,000.

The population excluding the isolated individuals and the dead affects the contact rate between infected individuals and susceptible individuals because the contact rate is used as the ratio such as \( S(n)/N(n) \) and \( R(n)/N(n) \), where \( N(n) \) is the population excluding the isolated individuals and the dead but including the recovered individuals having returned to the community, \( S(n) \) is the number of susceptible individuals in the community and \( R(n) \) is the number of recovered individuals having returned to the community. Thus, the decrease in \( N \) is expected to increase the contact rate, resulting in an increase in the number of infected individuals.

The relation between the population excluding the individuals kept in isolation, \( N \), and the number of individuals newly infected a day, \( AP \), shows a non-linear relation for every symptomatic rate (Fig. 12). For each symptomatic rate, in the first half, the relation shows a convex curve, meaning \( \Delta AP/(-\Delta N)>1 \). This indicates that a small decrease in \( N \) induces a large increase in \( AP \), and due to feedback, the increase in \( AP \) induces a decrease in \( N \) in turn. Namely, for a symptomatic rate, the decrease in \( N \) increases the contact rate, resulting in an increase in \( AP \). In the second half, the relation shows a concave curve for
every symptomatic rate. Considering their curvature, however, since each curve shows an almost straight line for a large part, it could be understood that the relation is in negative proportion, indicating that $AP$ decreases simply with an increase in $N$, though the slopes of the curves are different from one another.

On the other hand, when the symptomatic rate is reduced, the number of infected individuals markedly increases due to the increase in the number of ‘staying’ infected individuals themselves. The increase in the number of individuals infected by the ‘staying’ infected individuals causes a large number of individuals to be isolated due to being symptomatic, and this large number of isolated individuals causes a marked decrease in the population, though the population slightly increases for the period just after the symptomatic rate is set, as shown Fig. 4.

5.4. Herd immunity

When the value of the symptomatic rate is set to 0.0, all infected individuals are not symptomatic and are not isolated without any intervention. They stay in the community and continue infecting susceptible individuals until the recovery period is ended, and then they become recovered individuals who have immunity in the community. For such a case, the number of infected individuals increases to a peak and then decreases. This phenomenon is explained by ‘herd immunity’, which is indirect protection against the spread of infection caused by the immunity of a large proportion of the population. The contact rate between infected individuals and susceptible individuals should be reduced by increasing the number of recovered individuals who have immunity. As a result, although the decreases in the number of susceptible individuals surely induce the decrease in the number of infected individuals, the number of infected individuals must be considerably decreased due to the increase in the number of recovered individuals. This is scientific proof of the idea of herd immunity. The cumulative number of infected individuals at the peak, which is the turning point of the infected individuals from increasing to decreasing, is one of the ‘herd immunity thresholds’, being a target value for vaccination. Although the herd immunity threshold was examined by Ohmori[1], since there were wrong numbers in the values of threshold pointed out, the herd immunity is reexamined here, correcting the wrong numbers. The results of simulation by different values of $pfc$ are shown in Fig. 13 and Table 3.

When the value of $pfc$ is 1.0, the number of individuals newly infected a day, $AP$, reaches 23,655 (28,912) at the peak on the 79th (77th), with a cumulative number of 300,807 (383,974), and then decreases to 0 on the 153rd (155th). The number in parentheses with a strikethrough line is the wrong number in the article by Ohmori[1]. The number of infected individuals, $P$, reaches 336,268 (517,318) at the peak on the 87th (89th), with a cumulative number of 466,620 (629,455). After the peak, the number of infected individuals decreases to 0 on the 178th (189th), with a total number of infected individuals of 598,287 (726,431).
However, as previously examined, when \( pfc \) is 1.0 and \( syr \) is 1.0, meaning that all the infected individuals become symptomatic and are isolated, the number of individuals newly infected a day, \( AP \), reaches 2,135 at the peak on the 191\( \text{st} \) and the 192\( \text{nd} \) with a cumulative number of 76,961 on the 191\( \text{st} \) and 79,096 on the 192\( \text{nd} \), and then decreases to 0 on the 326\( \text{th} \). The number of infected individuals, \( P \), reached 14,895 at the peak on the 194\( \text{th} \) with a cumulative number of 83,343. After the peak, the number of infected individuals decreases to 0 on the 354\( \text{th} \), with a total number of infected individuals of 141,788 (Table 3). This indicates that when the symptomatic rate is set to 0, the numbers of infected individuals become markedly large.

On the other hand, when the value of \( pfc \) is 2.0, the number of individuals newly infected a day reaches 58,221 (59,029) at the peak on the 42\( \text{nd} \) (42\( \text{nd} \)) with a cumulative number of 398,087 (464,859) and then decreases to 0 on the 88\( \text{th} \) (98\( \text{th} \)). The number of infected individuals reached 655,265 (775,424) at the peak on the 50\( \text{th} \) (56\( \text{th} \)), with a cumulative number of 714,026 (829,003). After the peak, the number of infected individuals decreases to 0 on the 108\( \text{th} \) (126\( \text{th} \)) with a total number of infected individuals of 809,959 (873,160).

Each of the numbers 466,620 (629,455), approximately 47\% (63\%) of the population for a \( syr \) of 1, and 714,026 (829,003), approximately 71\% (83\%) of the population for a \( syr \) of 2, is a ‘potential’ herd immunity threshold without any intervention. Although potential herd immunity could be achieved sooner than expected, it is surely achieved only at the cost of so many infected individuals with so much death.

Now, when the symptomatic rate is set to 0, since any infected individuals are not isolated and stay in the community, the population of the community does not change, indicating that the effect of the change in population on the spread of COVID-19 can be ignored. The change in the number of newly infected individuals, \( AP \), and the change in the number of recovered individuals having been in the community, \( RAS \), by different \( pfc \) are shown in Fig. 14. The change in the number of recovered individuals follows the change in the number of newly infected individuals 15 days later. The time lag of 15 days means the time interval between when the newly infected individuals were infected and when they have recovered in the community, that is, the time interval of ‘the recovery period +1 (= rp+1)’ days, for the purpose of calculation. The peak of \( AP(n) \) is located just behind the date when the number of recovered individuals starts to increase, suggesting that the rapid decrease in the number of infected individuals is induced by the increase in the number of recovered individuals.

The relation between the number of recovered individuals in the community, \( RAS \), and the number of newly infected individuals, \( AP \), for the cases with a \( syr \) of 0 by different \( pfc \) is shown in Fig. 15.
Since the number of recovered individuals is 0 or a few during the early stage of infection, the number of newly infected individuals rapidly increases and reaches a peak. After the peak, with an increase in the number of recovered individuals, the number of newly infected individuals decreases. The number of newly infected individuals has become considerably small on the date when the number of recovered individuals reaches the peak. After the peak of the number of recovered individuals, the effect of the recovered individuals weakens; then, the number of newly infected individuals slowly decreases to 0.

From another point of view, as explained by Eq. (20), when the value of $AL(n)$ is given by 1, the activity of the recovered individuals is the same level as that of the susceptible individuals, indicating that the reduction effect of the recovered individuals on the contact rate between infected individuals and susceptible individuals is counted in the calculation for simulation. However, when the value of $AL(n)$ is set to 0, the recovered individuals are not active, meaning that the reduction effect of the recovered individuals on the contact rate is left out of the calculation, and the case such as this with an $AL(n)$ of 0 is called the ‘Modified SIR model’ by Ohmori [1]. For the SIR model created by Kermack and McKendrick [2, 3], the infected individuals continue infecting in the community throughout the recovered period, and the population used in calculation does not change throughout the infection duration, while for the ‘Modified SIR model’, the infected individuals continue infecting in the community not throughout the recovered period but during the latent period, and the population used in calculation is changed even for the case when $AL(n)$ is set to 0. The results of the calculation for the cases with an $AL$ of 0 are also shown in Table 3 and Fig. 16.

When the $AL$ is set to 0, the number of newly infected individuals is larger than that of the case with an $AL$ of 1 throughout the infection duration. The difference in the number of individuals newly infected a day between the case with an $AL$ of 0 and the case with an $AL$ of 1, $\Delta \text{CAP}$, rapidly increases with an increase in the number of recovered individuals (Fig. 16).

The relation between the number of recovered individuals, $RAS$, and the difference, $\Delta \text{CAP}$, is shown in Fig. 17. The cumulative number of infected individuals, $\text{CAP}$, rapidly increases with an increase in the number of recovered individuals in the first half and slowly increases with decreases in the number of recovered individuals in the second half. The difference in the number of individuals newly infected a day between the case without the reduction effect of the recovered individuals, $AL$ of 0, and the case with the reduction effect of the recovered individual, $AL$ of 1, increases rapidly in proportion to the number of recovered individuals with a slope of approximately 1.7 in the first half and increases slowly in negative proportion to the number of recovered individuals with a slope of approximately 1.1 in the second half. This indicates that the reduction in the incidence of infection is considerably induced by the recovered individual.

The correlations between the number of susceptible individuals remaining in the community, $RM$, and the difference in the number of individuals newly infected a day between the case without the reduction effect of the recovered individuals ($AL$ of 0) and the case with the reduction effect of the recovered
individual (\(AL\) of 1), \(\Delta CAP\), and between the number of susceptible individuals and the number of recovered individuals, \(RAS\), are shown in Fig. 18. The number of recovered individuals, \(RAS\), increases with a decrease in the number of susceptible individuals. At the same time, the difference, \(\Delta CAP\), also increases with a decrease in the number of susceptible individuals, indicating that the increase in the number of recovered individuals induces a decrease in the number of individuals newly infected a day.

In addition, for verification, the total number of infected individuals shows a linear line for each case with an \(AL\) of 1 or 0, indicating being in proportion to the number of susceptible individuals. The difference in the total number of infected individuals between the two cases is 76,606 (=654,893-598,387), indicating that when the reduction effect by the recovered individuals is ignored, the increment in the infected individuals reaches approximately 13%.

Furthermore, when the value of the symptomatic rate is set to 1.0, the number of individuals newly infected a day, \(AP\), the number of recovered individuals who were isolated, \(RT\), the cumulative number of infected individuals, \(CAP\), and the difference in the number of individuals newly infected a day between the case with an \(AL\) of 0 and the case with an \(AL\) of 1, \(\Delta CAP\), are considerably smaller than those of the cases with a symptomatic rate of 0 examined above (Table 3 and Fig. 19). However, the difference in the total number of infected individuals between the case with an \(AL\) of 0 and the case with an \(AL\) of 1 is 54,850 (=196,638-141,788), indicating that when the reduction effect by the recovered individuals is ignored, the increment in the infected individuals reaches approximately 39%.

5.5. Changes in the number of infected individuals caused by changes in the number of isolated individuals due to political and/or medical interventions

For the flexible model used here, the ‘symptomatic’ infected individuals among the infected individuals in the community are isolated. Thus, the symptomatic rate indicates the ratio of the number of isolated individuals. Namely, the symptomatic rate is practically used as the isolation rate for the individuals infected a day. The appearance of symptoms commonly depends on the characteristics of the virus and/or on the health conditions of infected individuals. Therefore, the isolation rate also depends on the characteristics of the virus and/or on the health conditions of infected individuals. However, sometimes, the number of isolated individuals is controlled/decided by political and/or medical interventions for some reasons induced by the capacity of hospital care and others. Namely, in some circumstances, some political and/or medical interventions could be taken to control the number of isolated individuals at any time throughout the infection duration. For such cases, the same things could occur as the symptomatic rate is set/changed in the middle of the infection duration.
As previously noted, when the symptomatic rate is set to 1, the number of individuals newly infected a day reaches 2,135 at the peak on the 191\textsuperscript{st} and 192\textsuperscript{nd} and then decreases to 0 on the 326\textsuperscript{th}, with a total number of infected individuals of 141,788 (Fig. 20, Table 4). When the symptomatic rate is changed from 1 to 0.8 on the 201\textsuperscript{st}, the number of individuals newly infected a day decreases from 2,135 to 1,995 on the 200\textsuperscript{th} with a cumulative number of infected individuals of 95,696, and further to 1,706 on the 207\textsuperscript{th} with a cumulative number of infected individuals of 108,574, then increases again to 2,210 at the second peak on the 228\textsuperscript{th} with a cumulative number of infected individuals of 151,034. After the second peak, the number of individuals newly infected a day decreases to 0 on the 397\textsuperscript{th} with a total number of infected individuals of 228,640, approximately 1.6 times larger than that of the case without any change in the symptomatic rate (Fig. 20, Table 4). The total number of infected individuals of the first term to the 200\textsuperscript{th} is 95,696 and that of the second term is 132,950.

On the other hand, when the symptomatic rate is changed on the 251\textsuperscript{st} from 1 to 0.8, the number of individuals newly infected a day decreases from 2,135 to 149 on the 250\textsuperscript{th} with a cumulative number of infected individuals of 139,858, and further to 83 on the 262\textsuperscript{nd} with a cumulative number of infected individuals of 141,063, then increases slightly to 88 at the second peak on the 266\textsuperscript{th} with a cumulative number of infected individuals of 141,401. After the second peak, the number of newly infected individuals decreased to 0 on the 827\textsuperscript{th} with a total number of infected individuals of 154,454, approximately 1.1 times larger than that of the case without any change in the symptomatic rate (Fig. 20, Table 4). The infection duration, which ends when the infected individual and/or the isolated individual disappears for the purpose of calculation, is 1,029 days, indicating that the duration becomes extremely long (Table 4). The total number of infected individuals of the first term to the 250\textsuperscript{th} is 139,858 and that of the second term is 14,596, indicating that the effect of reduction of the symptomatic rate becomes less. When the symptomatic rate is changed on the 300\textsuperscript{th} from 1 to 0.8, the number of individuals newly infected a day decreases from 2,135 to 3 on the 300\textsuperscript{th} with a cumulative number of infected individuals of 141,745 and further to 0 on the 784\textsuperscript{th} without any peak. The total number of infected individuals is 142,430 (Table 4). Although the infection duration, 1,500 days, becomes markedly longer, the total number of infected individuals only slightly increases from 141,788 to 142,430. As examined above, the earlier the reduced symptomatic rate is set, the more the number of infected individuals increases.

When the symptomatic rate is reduced from 1 to 0.5 on the 301\textsuperscript{st}, the number of individuals newly infected a day surely decreases from 2,135 at the peak to 3 on the 300\textsuperscript{th} with a cumulative number of infected individuals of 141,745, suggesting that the infection has almost subsided (Fig. 21, Table 4). The number of individuals newly infected a day decreases further to 2 for the period from the 305\textsuperscript{th} to the 311\textsuperscript{th}. After the bottom, however, the number of individuals newly infected a day rapidly increases up to 1,866 at the second peak on the 488\textsuperscript{th}. Then, the number of individuals newly infected a day decreases down to 0 on the 690\textsuperscript{th} with a total number of infected individuals of 303,846, indicating that the total number of infected individuals for the second term after the 301\textsuperscript{st} is 162,101. It is more than that of the first term, 141,745 (Fig. 21, Table 4). The infection duration was as long as 748 days.
Furthermore, when the symptomatic rate is reduced from 1 to 0.0 on the 301\textsuperscript{st}, meaning that any newly infected individuals are not isolated and are staying in the community, the number of individuals newly infected a day decreases from 2,135 at the peak to 3 on the 300\textsuperscript{th} with a cumulative number of infected individuals of 141,745, suggesting that the infection has almost subsided, as mentioned previously. The number of individuals newly infected a day decreases further to 2 for the period from the 305\textsuperscript{th} to the 308\textsuperscript{th}. After the bottom, the number of individuals newly infected a day rapidly increases up to 7,238 at the second peak on the 413\textsuperscript{th} (Fig. 22, Table 4). Then, the number of individuals newly infected a day decreases down to 0 on the 534\textsuperscript{th} with a total number of infected individuals of 471,006, indicating that the total number of infected individuals for the second term after the 301\textsuperscript{st} is 329,261. It is approximately 3 times larger than that of the first term, 141,745. The infection duration becomes as long as 573 days.

Thus, a large reduction in the symptomatic rate, meaning a large decrease in the number of isolated individuals and/or a large increase in the number of infected individuals staying in the community, must cause a serious spread of infection with a longer infection duration, even if the reduced symptomatic rate is set at a late time in the infection duration.

5.6. Effect of vaccination on spreading of infection for cases with different symptomatic rates

The number of individuals newly infected on date \(n\), \(AP(n)\), is given by the following equation:

\[
AP(n) = p(n) \times RM(n) \tag{34} \quad (=18)
\]

where \(RM(n)\) is the number of susceptible individuals in the community and given by:

\[
RM(n) = TN(n) - (CI(n) + CAP(n) + V(n)) \quad \tag{35}(=3)
\]

\(p(n)\) is the infection coefficient and given by:

\[
p(n) = \frac{pfc(n)}{lp(n)} \times (RM(n) / N(n)) \times icf(n) \times (1 - (AL(n) / N(n))) \times (RP(n) / N(n)) \quad \tag{36} (=19)
\]

and \(AL(n)\) is the sum of the activity levels of the recovered individuals and vaccinated ones, given by:

\[
AL(n) = aII(n) \times (CRI(n) + CRT(n)) + aI(n) \times CRAS(n) + aV(n) \times V(n) \quad \tag{37} (=20)
\]

where the coefficient \(aV(n)\) is the activity level of the vaccinated individuals and \(V(n)\) is the number of vaccinated individuals who have immunity as the recovered individuals do.

When the number of vaccinated individuals, \(V\), increases, the number of susceptible individuals, \(RM\), decreases, as calculated by Eq. (35), and thus, Eq. (34) indicates that the increase in the number of vaccinated individuals directly decreases the number of individuals newly infected a day, \(AP\).
On the other hand, the contact rate \((cr(n))\) between the infected individuals and susceptible ones is given by:

\[
cr(n) = \frac{S(n)}{N(n)} \left(1 - \delta \left(\frac{R(n)}{N(n)}\right)\right)
\]

(38)

where \(\left(1 - \delta \left(\frac{R(n)}{N(n)}\right)\right)\) is the reduction rate, and as previously explained,

\[
(1 - \delta \left(\frac{R(n)}{N(n)}\right)) = (1 - \frac{AL(n)}{N(n)})
\]

(39)

The reduction effect on the contact rate increases with a decreasing value of \(1 - \delta \left(\frac{R(n)}{N(n)}\right)\). Namely, Eq. (39) indicates that when the number of vaccinated individuals increases, the reduction rate decreases, inducing a reduction in the contact rate. Consequently, the increase in vaccinated individuals decreases not only the number of susceptible individuals but also the value of the contact rate.

Under the condition that the latent period is 5 days, the recovery period is 14 days, the initial population of the community is 1,000,000 and the initial number of infected individuals is 1, when the symptomatic rate, \(syr\), is set to 0.8, and the vaccination rate is 0, meaning that the number of vaccinated individuals is 0, the number of individuals newly infected a day reaches 8,670 at the peak on the 125th and then decreases to 0 on the 217th with a total number of infected individuals of 336,096 (Fig. 23, Table 5).

However, when the vaccination rate, \(v\), is 0.01, not on and after the first day but on and after the 101st, meaning that the vaccinated individuals are just 10,000 on and after the 101st, the number of individuals newly infected a day reached 7,574 at the peak on the 125th and then decreased to 0 on the 223rd with a total number of infected individuals of 310,984, indicating 25,112 less than that of the case with no vaccination. For other cases with different symptomatic rates, each case shows that the number of individuals newly infected a day and the total number of infected individuals are both markedly smaller than those of the case without any vaccinated individuals. As examined above, vaccination is considerably effective in decreasing the number of infected individuals even for cases with a symptomatic rate less than 1, even though a ‘symptomatic rate less than 1’ causes a marked increase in the number of infected individuals.

**5.7. Effect of PCR test on spreading of infection for the case with different symptomatic rates**

The number of individuals confirmed to be infected due to being test positive, \(CP(n)\), is given by:

\[
CP(n) = T(n) * bp(n) * ir(n) = T(n) * tir(n)
\]

(40)
where \( T(n) \) is the number of individuals having PCR test and/or the antibody test, which you can set arbitrarily on any days when tests are performed, and the coefficient \( bp(n) \) is the magnification of incidence rate for the test to the incidence rate, \( ir(n) \), in the community. The incident rate, \( ir(n) \), is given by:

\[
ir(n) = \frac{P(n)}{TN(n)} \tag{41} (=6)
\]

where \( P(n) \) is the number of infected individuals already having existed in the community, though \( P(1) \) is the initial number of infected individuals in the community and is arbitrarily given by yourself. \( TN(n) \) is the population of the whole community.

The coefficient \( tir(n) \) is the positive rate for the PCR test and is given by:

\[
tir(n) = bp(n) \cdot ir(n) \tag{42} (=7)
\]

As previously explained, since all the individuals confirmed to be infected due to test positivity are not always isolated and the individuals decided to be isolated, \( I(n) \), are isolated on the following day of the date when they are confirmed to be infected, in the actual calculation, \( I(n) \) is given by:

\[
I(n) = CP(n-1) \cdot i(n-1) \tag{43} (=4)
\]

where the coefficient \( i(n) \) is the isolation rate for the individuals who are confirmed to be infected due to being test positive. \( i(n) \) indicates the ratio of the number of isolated individuals to the total number of infected individuals confirmed. When all the individuals confirmed to be infected due to a positive test are isolated, the value of \( i(n) \) should be set to 1.

The population excluding the individuals kept in isolation and dead in the real community, \( N(n) \), is given by:

\[
N(n) = TN(n-1) - (CI(n-1) + CPI(n-1) + CDAS(n-1) + CDT(n-1)) + CR(n-1) + CRT(n-1) \tag{44} (=27)
\]

where \( CI(n-1) \) is \( \Sigma I(n-1) \), meaning the cumulative number of individuals isolated due to being test positive up to the date \( (n-1) \). Since most of the individuals who are confirmed to be infected due to test positivity are isolated from the community, the PCR test causes changes in the population and in the number of isolated/recovered individuals.

When the test is started on the 101\textsuperscript{st} under the condition that the magnification \( (bp(n)) \) is 5, the isolation rate \( (i(n)) \) is 1, meaning that all the infected individuals confirmed are isolated, the latent period is 5 days, the recovery period is 14 days, the population of the community is 1,000,000 and the initial number of infected individuals is 1, the changes in the number of infected individuals and isolated individuals are shown in Fig. 24 and Table 6.

When the symptomatic rate, \( syr \), is set to 0.8, the number of individuals newly infected a day reaches 8,670 at the peak on the 125\textsuperscript{th} and then decreases to 0 on the 217\textsuperscript{th}, with a total number of infected individuals of 336,096 (Fig. 24, Table 6). However, when the test with just 1,000 tested individuals is
started not on the first day but on the 101\textsuperscript{st}, meaning that 1,000 individuals are tested every day on and after the 101\textsuperscript{st}, the number of individuals newly infected a day reached 7,534 at the peak on the 124\textsuperscript{th} and then decreased to 0 on the 216\textsuperscript{th} with a total number of infected individuals of 311,819, indicating 24,277 less than that of the case with no test. For other cases with different symptomatic rates, each case shows that the number of individuals newly infected a day and the total number of infected individuals are both markedly smaller than those of the case without any tested individuals. Therefore, it can be said that the ‘PCR test and isolation treatment’ is considerably effective in decreasing the number of infected individuals even for cases with a symptomatic rate less than 1, even though a ‘symptomatic rate less than 1’ causes a marked increase in the number of infected individuals.

6. Conclusions

For COVID-19, when infected individuals become symptomatic after the latent period ends, they should be isolated from the community. Thus, the number of infected individuals ‘isolated’ is controlled by the symptomatic rate. However, when the infected individuals do not become symptomatic even after the latent period, they are not isolated. The number of infected individuals ‘staying in the community’ is also controlled by the symptomatic rate because the ‘asymptomatic rate’ is given by the ‘1- symptomatic rate’. Therefore, the symptomatic rate is used not only for recognizing the number of ‘symptomatic’ infected individuals but also for recognizing the number of ‘isolated’ and/or ‘staying in the community’ infected individuals. Namely, the symptomatic rate is practically used as the ‘isolation rate’. For the ‘isolated’ infected individuals, after the latent period, they do not infect susceptible individuals in the community. For the ‘staying in the community’ infected individuals, however, they continue to infect susceptible individuals during the recovery period, inducing an increase in the number of infected individuals. Although the appearance of symptoms commonly depends on the characteristics of the virus and/or on the health conditions of infected individuals, there are cases where the number of isolated individuals is decided by some political and/or medical interventions, such as the capacity of hospital care. For such cases, the symptomatic rate, that is, the isolation rate, is artificially controlled. Therefore, the evaluation of the effect of the symptomatic rate could provide reference materials for political and/or medical measures. Then, the cases with different symptomatic rates were examined under the condition that the initial population, $TN(1)$, is 1,000,000, the initial number of infected individuals, $P(1)$, is 1, the potential (biological) infectious capacity of coronavirus, $pfc(n)$, is 1.0, the latent period, $lp(n)$, is 5, and the recovery period, $rp(n)$, is 14. At the same time, herd immunity, the effects of vaccination and the PCR test on the spread of COVID-19 were evaluated based on the changes in the infected individuals simulated by the flexible compartment model. They are summarized as follows:

1. When the symptomatic rate is less than 1.0, some infected individuals stay and continue infecting susceptible individuals in the community. Thus, a ‘symptomatic rate less than 1’ causes an increase in the number of infected individuals. When the symptomatic rate is 0.8, 80% of individuals newly infected on a day become symptomatic and are isolated, and the remaining 20% are asymptomatic and stay in
the community. Since the ‘asymptomatic’ infected individuals continue infecting susceptible individuals in the community until the recovered period is ended, the number of individuals newly infected a day reaches 8,670 at the peak on the 125th, approximately 4 times larger than that, 2,135 at the peak on the 191st of the case with a $syr$ of 1. Then, the number of individuals infected a day decreases to 0 on the 217th, with a total number of infected individuals of 336,096. It is approximately 2.4 times larger than that, 141,788, of the case with a $syr$ of 1. Namely, the occurrence of ‘asymptomatic staying’ infected individuals caused by a symptomatic rate less than 1 could induce a rapid and large increase in the number of infected individuals, though the infection duration becomes short.

2. When the symptomatic rate is 1.0, meaning that all the infected individuals become symptomatic and are isolated, the infection duration is 354 days, and the total number of infected individuals is 141,788. However, when the symptomatic rate is 0.9, the infection duration becomes 274 days, and the total number of infected individuals reaches 251,341. The total number markedly increases, though the infection duration becomes considerably short. When the symptomatic rate is 0.5, meaning that half of the infected individuals become symptomatic and are isolated, the infection duration becomes 200 days, and the total number of infected individuals reaches 490,235, indicating that approximately half of the population has been infected. When the symptomatic rate is 0.3, the infection duration is shortened to 188 days, with a total number of infected individuals of 546,614, over half of the population. With a decrease in the symptomatic rate, the number of infected individuals markedly increases in a negative proportion to the change in the symptomatic rate, although the infection duration becomes markedly short.

3. The number of ‘isolated’ infected individuals and/or ‘staying in the community’ infected individuals control the change in the population of the community. The population excluding the isolated individuals and the dead affects the contact rate between infected individuals and susceptible individuals, and the decrease in the population increases the contact rate, resulting in an increase in the number of infected individuals. This indicates that a small decrease in the population induces a large increase in the number of individuals newly infected a day in the first half of the infection duration. However, in the second half, with an increase in population, the number of individuals newly infected a day decreases simply in negative proportion. On the other hand, it is notable that when the symptomatic rate is reduced less than 1, the number of infected individuals markedly increases due to the increase in the number of ‘staying’ infected individuals themselves. The large number of individuals infected by ‘staying’ infected individuals causes a large number of the individuals to be isolated due to being symptomatic, inducing a marked decrease in the population, resulting in a marked increase in the number of infected individuals.
4. When the symptomatic rate is set to 0, all the infected individuals do not become symptomatic and are not isolated, stay in the community, infect susceptible individuals until the recovery period is ended, and then become recovered individuals who have immunity in the community. For such a case, the number of infected individuals increases to a peak and then decreases. This phenomenon is explained by ‘herd immunity’. An accelerated deduction in the contact rate between the infected individuals and the susceptible ones is induced by an increase in the number of recovered individuals. As a result, the number of infected individuals, which has been increasing, reaches a peak and then decreases, though the decrease in the number of infected individuals is surely induced by the decrease in the number of susceptible individuals. The cumulative number of infected individuals at the peak is one of the ‘herd immunity thresholds’. The herd immunity threshold depends on the value of the potential infectious capacity, \(pfc\). For the case in which the initial population of the community is 1,000,000 and the initial number of infected individuals is 1, the latent period is 5 days and the recovery period is 14 days, when \(pfc\) is 1.0, the cumulative number of infected individuals reaches 466,620 (approximately 47% of the population) at the peak on the 87th, though the total number of infected individuals becomes 598,287 (approximately 60% of the population). When \(pfc\) is 2.0, the cumulative number of infected individuals reaches 714,026 (approximately 71% of the population) at the peak on the 50th, though the total number of infected individuals reaches 809,959 (approximately 81% of the population). Herd immunity could be achieved sooner than expected, but it surely is achieved only at the cost of so many infected individuals with so much death.

5. For the flexible model used here, the symptomatic rate can be practically used as the isolation rate. The isolation rate depends on the characteristics of the virus and/or on the health conditions of infected individuals as the symptomatic rate does. However, sometimes, the number of isolated individuals is controlled/decided by political and/or medical interventions for some reasons induced by the capacity of hospital care. Namely, in some circumstances, the isolation late and/or the symptomatic rate is changed by some political and/or medical interventions.

When the symptomatic rate is set to 1, the number of newly infected individuals reaches 2,135 at the (first) peak on the 191st and 192nd and then decreases to 0 on the 326th, with a total number of infected individuals of 141,788. The infection duration, which ends when the infected individual disappears for the purpose of calculation, is 354 days. However, when the symptomatic rate is changed on the 201st from 1 to 0.8, the number of newly infected individuals has a second peak of 2,210 on the 228th and then decreases to 0 on the 397th, with a total number of infected individuals of 228,640, approximately 1.6 times larger than that of the case without any change in symptomatic rate. The infection duration was 439 days. When the symptomatic rate is changed from 1 to 0.8 on the 251st, the number of newly infected individuals also has a second peak of 88 on the 266th and then decreases to 0 on the 827th, with a total number of infected individuals of 154,454, approximately 1.1 times larger than that of the case without any change in the symptomatic rate. The infection duration is 1,029 days, indicating that the
duration becomes extremely long. When the symptomatic rate is changed on the 300th from 1 to 0.8, the number of newly infected individuals decreases from the first peak of 2,135 to 3 on the 300th and further to 0 on the 784th, with a total number of infected individuals of 142,430. Although the infection duration becomes markedly longer, reaching 1,500 days, the total number of infected individuals only slightly increases. Therefore, for the same value of the symptomatic rate less than 1, the earlier the date of setting the symptomatic rate is, the larger the total number of infected individuals becomes. Notably, for the same setting date, the smaller the value of the symptomatic rate, the larger the total number of infected individuals becomes.

However, when the symptomatic rate is reduced from 1 to 0.0 on 301st, meaning that any newly infected individuals are not isolated and are staying in the community on and after the 301st, the number of newly infected individuals decreases from 2,135 at the first peak to 3 on the 300th with a total number of infected individuals of 141,745, suggesting that the infection has almost subsided. Surely, the number of newly infected individuals decreases further to 2 for the period from the 305th to the 308th. After the bottom, however, the number of newly infected individuals rapidly increases to 7,238 at the second peak on the 413th and then decreases to 0 on the 534th, with a total number of infected individuals of 471,006, approximately 3.3 times larger than that of the case without any change in the symptomatic rate. The infection duration becomes as long as 573 days. The total number of infected individuals for the second term after the 301st is 329,261. It is approximately 2.3 times larger than that of the first term, 141,745. Thus, a large reduction in the symptomatic rate, meaning a large decrease in the number of isolated individuals and/or a large increase in the number of infected individuals staying in the community, must cause a serious spread of infection with a longer infection duration, even if the reduced symptomatic rate is set at a late time in the infection duration.

6. Vaccination decreases not only the number of susceptible individuals but also the contact rate. Thus, vaccination is expected to reduce the number of infected individuals even for cases with a symptomatic rate less than 1. When the symptomatic rate is set to 0.8 and the vaccination rate is 0, meaning that the number of vaccinated individuals is 0, the number of individuals newly infected a day reaches 8,670 at the peak on the 125th and then decreases to 0 on the 217th. The total number of infected individuals was 336,096. It is approximately 2.4 times larger than that of the case with a symptomatic rate of 1, that is, 141,788. The infection duration is 240 days. However, when the vaccination rate is set to 0.01 on and after the 101st, meaning that the vaccinated individuals are just 10,000 on and after the 101st, the number of newly infected individuals becomes 7,574 at the peak on the 125th and then decreases to 0 on the 223rd with a total number of infected individuals of 310,984, indicating 25,112 less than that of the case without any vaccination. The infection duration was 247 days. For other cases with different symptomatic rates, each case shows that the number of newly infected individuals and the total number of infected individuals are both markedly smaller than those of the case without any vaccinated individuals. Even though a ‘symptomatic rate less than 1’ causes a marked increase in the
number of infected individuals, vaccination is considerably effective in decreasing the number of infected individuals, even for cases with a symptomatic rate less than 1.

7. Individuals who are confirmed to be infected due to a positive test are isolated from the community. Thus, the PCR test causes changes in the population and in the number of isolated/recovered individuals, which is expected to induce a decrease in the number of infected individuals. When the symptomatic rate is set to 0.8 and the test is not performed, indicating that the tested individuals are 0, the number of newly infected individuals reaches 8,670 at the peak on the 125th and then decreases to 0 on the 217th. The total number of infected individuals becomes 336,096. As previously noted, it is approximately 2.4 times larger than that of the case without any change in the symptomatic rate, 141,788. The infection duration is 240 days. However, when the test with 1,000 tested individuals is started on and after the 101st, meaning that 1,000 individuals are tested every day on and after the 101st, the number of newly infected individuals becomes 7,534 at the peak on the 124th and then decreases to 0 on the 216th. The total number of infected individuals was 311,819, which was 24,277 less than that of the case with no test. The infection duration is 239 days. For other cases with different symptomatic rates, each case shows that the number of newly infected individuals and the total number of infected individuals are both markedly smaller than that of the case without any tested individuals. Therefore, it can be said that the ‘PCR test and isolation treatment’ is considerably effective in decreasing the number of infected individuals even for cases with a symptomatic rate less than 1, even though a ‘symptomatic rate less than 1’ causes a marked increase in the number of infected individuals.

**Declarations**

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**Author Information**

ohmori@edu.k.u-tokyo.ac.jp
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Data availability

Data will be made available on reasonable request.

References


Tables

Tables 1-6 are available in the supplementary files section.

Figures

Figure 1
Flow of ‘Susceptible; RM, NAP, ‘Vaccinated; V, ‘Recovered; RI, RT, RAS; ‘Infected; P(I, PI, AS), UP, ‘Isolated; I, PI and ‘Death; DAS, DTI, DT’. (after Ohmori, 2022)

Figure 2

Change in the number of PI (isolated individuals/day), AS (asymptomatic staying in the community/day), N (population excluding individuals kept in isolation) and AP (infected individuals/day) by different symptomatic rates (syr); syr 1 and syr (1-150)1.0; (151-)0.8. The “syr 1; 2,135” means the syr is 1 through the infection duration with the AP and/or PI of 2,135 at the peak, and "syr (1-150)1.0; (151-)0.8; 7,420" means that the syr is 1.0 to the 150th and 0.8 on and after the 151st. with an AP of 7,420 at the peak. The initial population, TN(1), is 1,000,000, the initial number of infected individuals, P(1), is 1, the potential (biological) infectious capacity of coronavirus, pfcc(n), is 1.0, the latent period, lp(n), is 5, and the recovery period, rp(n), is 14.
Figure 3

A part of Fig. 2 from the 140th to the 210th. The value of the symptomatic rate is set to 0.8 on the 151st, and separation of infected individuals into ‘symptomatic and isolated’ and ‘asymptomatic and staying’ occurs on the 157th.

Figure 4

Change in the difference in $N$ (population excluding individuals kept in isolation) between different symptomatic rates ($syr$); $\Delta N = N(syr\ (1-150)1.0;\ (151-)0.8) - N(syr1.0)$. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1.0, $lp(n)$ is 5, and $rp(n)$ is 14.
Figure 5

Change in the number of individuals infected a day ($AP$), the number of individuals isolated a day ($PI$), the number of infected individuals ($P$) and the number of isolated individuals ($I2$) for the case with the symptomatic of 1.0 from the beginning of simulation. ‘$AP; syr1; 2,135$’ means that the $syr$ is 1 throughout the infection duration with an $AP$ of 2,135 at the peak. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1.0, $lp(n)$ is 5, and $rp(n)$ is 14.

Figure 6

Change in the number of individuals infected a day ($AP$), the number of individuals isolated a day ($PI$), the number of individuals being asymptomatic and staying in the community, ($AS$), for the case with the
symptomatic of 0.8 from the beginning of simulation. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1.0, $lp(n)$ is 5, and $rp(n)$ is 14.

Figure 7

Correlation between the number of ‘asymptomatic staying’ infected individuals, $AS$, and the number of individuals newly infected a day, $AP$, and the number of isolated individuals, $PI$, for the case with the symptomatic of 0.8 from the beginning of simulation. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1.0, $lp(n)$ is 5, and $rp(n)$ is 14.

Figure 8
Change in the number of \( AP \) (infected individuals/day) and \( CAP \) (the cumulative number of infected individuals = the cumulative number of individuals newly infected a day) by different symptomatic rates (\( syr \)). \( AP, syr0.8; 8,670 \) means the number of individuals newly infected a day for the case with a symptomatic rate of 0.8 from the 1\(^{\text{st}}\) simulation, and the maximum number is 8,670. \( CAP, syr0.8; 336,096 \) means the cumulative number of individuals newly infected a day from the first day of simulation, that is, the sum of the number of infected individuals, including the recovered individuals, for the case with a symptomatic rate of 0.8 from the first day of simulation, and the total number of infected individuals is 336,096. \( TN(1) \) is 1,000,000, \( P(1) \) is 1, \( pfc(n) \) is 1.0, \( lp(n) \) is 5, and \( rp(n) \) is 14.

**Figure 9**

Correlation between the symptomatic rate, \( syr \), and the number of individuals newly infected a day, \( AP \), at the peak, the number of infected individuals, \( P \), at the peak and the total number of infected individuals, \( CAP \), by different values of the symptomatic rate, showing that the number of infected individuals markedly increases with a decrease in the symptomatic rate. \( TN(1) \) is 1,000,000, \( P(1) \) is 1, \( pfc(n) \) is 1.0, \( lp(n) \) is 5 and \( rp(n) \) is 14.
Figure 10

Change in the number of $N$ (population excluding the number of individuals kept in isolation by different symptomatic rates ($syr$). The ‘$N; syr0.8; 938,612$’ means the number of the population excluding the number kept in isolation for the case with the symptomatic rate of 0.8 from the first day of simulation, and the minimum number is 938,612. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1.0, $lp(n)$ is 5, and $rp(n)$ is 14.

Figure 11

Correlation between the symptomatic rate, $syr$, and the minimum population, $N$. When $syr$ is set to 0.5, the minimum value of the population becomes 928,450 at the bottom on the 108th, showing that it is the smallest value among all cases with different symptomatic rates. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1.0, $lp(n)$ is 5, and $rp(n)$ is 14.
Figure 12

Correlation between the population excluding the number of individuals kept in isolation, \( N \), and the number of individuals newly infected a day, \( AP \), by different values of the symptomatic rate. \( TN(1) \) is 1,000,000, \( P(1) \) is 1, \( pfc(n) \) is 1.0, \( lp(n) \) is 5 and \( rp(n) \) is 14.

Figure 13

Changes in the number of infected individuals, the number of individuals newly infected a day, the cumulative number of infected individuals and that of susceptible individuals for different \( pfc \)s when the value of \( syr \) is 0.0. \( AP \) indicates the number of individuals newly infected a day, \( P \) indicates the number of infected individuals in the community, \( CAP \) is the cumulative number of infected individuals, and \( RM \)
is the number of susceptible individuals in the community. $TN(1)$ is 1,000,000, $P(1)$ is 1, $lp(n)$ is 5 and $rp(n)$ is 14.

**Figure 14**

Change in the number of individuals newly infected a day, $AP$, and the change in the number of recovered individuals having been in the community, $RAS$, for the cases with a $syr$ of 0, by different $pfc$ of 1 and 2. The ‘$AP(79); 23,655; RAS(79); 6,888$’ indicates that the number of individuals newly infected a day is 23,655 and the number of recovered individuals who have been in the community is 6,888 on the 79th. $TN(1)$ is 1,000,000, $P(1)$ is 1, $lp(n)$ is 5 and $rp(n)$ is 14.
Figure 15

Relation between the number of recovered individuals, $RAS$, and the number of individuals newly infected a day, $AP$, for the cases with a $syr$ of 0, by different $pfc$, 1 or 2. The increase in the number of recovered individuals induces a marked decrease in the number of individuals newly infected a day. $TN(1)$ is 1,000,000, $P(1)$ is 1, $lp(n)$ is 5 and $rp(n)$ is 14.

![Figure 15](image)

Figure 16

Change in the number of individuals newly infected a day, $AP$, in the number of recovered individuals having returned to the community, $RAS$, in the cumulative number of infected individuals, $CAP$, and in the number of susceptible individuals in the community, $RM$, by different $AL$ of 1 and 0. $\Delta AP$, $AL0$' indicates the change in the number of individuals newly infected a day for the case with an $AL$ of 0, meaning that the reduction effect of the recovered individuals ($RAS$) on contact rate is left out of the calculation. $\Delta CAP$ means the difference in the number of individuals newly infected a day between the case with an $AL$ of 0 and the case with an $AL$ of 1, showing that the difference rapidly increases with an increase in the number of recovered individuals. $\Delta RM$ means the difference in the number of susceptible individuals in the community between the case with an $AL$ of 0 and the case with an $AL$ of 1, and it is completely equal to $\Delta CAP$. $TN(1)$ is 1,000,000, $P(1)$ is 1, $syr(n)$ is 0, $pfc(n)$ is 1, $lp(n)$ is 5 and $rp(n)$ is 14.
Correlation between the number of recovered individuals, $RAS(\text{AL1})$, and the ‘$\Delta CAP$’, which is the difference in the number of individuals newly infected a day between the case with an $AL$ of 0 and the case with an $AL$ of 1, together with a cumulative number of infected individuals, $CAP$, by different $AL$, showing that the difference changes in proportion to the number of recovered individuals. ‘$CAP, AL0$’ means the cumulative number of infected individuals for the case with an $AL$ of 0. ‘$RAS(AL1)$’ indicates the number of recovered individuals in the case with an $AL$ of 1. $TN(1)$ is 1,000,000, $P(1)$ is 1, $syr(n)$ is 0, $pfc(n)$ is 1, $lp(n)$ is 5 and $rp(n)$ is 14.
Correlation between the number of susceptible individuals in the community, $RM$, and the number of individuals newly infected a day, $AP$, together with a cumulative number of infected individuals, $CAP$, and the difference in the number of individuals newly infected a day between the case with an $AL$ of 0 and the case with an $AL$ of 1, $\Delta CAP$. ‘$AP; AL0$’ means the number of individuals newly infected a day for the case with an $AL$ of 0. The symptomatic rate is 0, indicating that all infected individuals are asymptomatic, are not isolated and are staying in the community. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1, $lp(n)$ is 5 and $rp(n)$ is 14.

Correlation between the number of susceptible individuals in the community, $RM$, and the number of individuals newly infected a day, $AP$, together with a cumulative number of infected individuals, $CAP$, and the difference in the number of individuals newly infected a day between the case with an $AL$ of 0 and the case with an $AL$ of 1, $\Delta CAP$. ‘$AP; AL0$’ means the number of individuals newly infected a day for the case with an $AL$ of 0. The symptomatic rate is 1, indicating that all infected individuals are symptomatic and isolated. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1, $lp(n)$ is 5 and $rp(n)$ is 14.
Figure 20

Change in the number of individuals newly infected a day, \( AP \), and in the cumulative number of infected individuals, \( CAP \), by different ‘syr setting date’. ‘\( AP; syr(1-200)1; 2,135 \) (201-0.8; 2,210)’ means the change in the number of individuals newly infected a day for the case with a \( syr \) of 1 for the period from the 1\(^{st} \) to the 200\(^{th} \) with the number of individuals newly infected a day of 2,135 at the first peak and with a \( syr \) of 0.8 for the period from the 201\(^{st} \) to the end of infection duration with the number of individuals newly infected a day of 2,210 at the second peak. \( TN(1) \) is 1,000,000, \( P(1) \) is 1, \( pfc(n) \) is 1, \( lp(n) \) is 5 and \( rp(n) \) is 14.

Figure 21
Change in the number of individuals newly infected a day, \( AP \), and in the cumulative number of infected individuals, \( CAP \), by different ‘syr setting date’ with different value of \( syr \). ‘\( AP; syr(1-300)1; 2,135; (301-)0.5; 1,866 \)’ means the change in the number of individuals newly infected a day for the case with a \( syr \) of 1 for the period from the 1\(^{\text{st}}\) to the 300\(^{\text{th}}\) with the number of individuals newly infected a day of 2,135 at the first peak and with a \( syr \) of 0.5 for the period from the 201\(^{\text{st}}\) to the end of infection duration with the number of individuals newly infected a day of 1,866 at the second peak. \( TN(1) \) is 1,000,000, \( P(1) \) is 1, \( pfc(n) \) is 1, \( lp(n) \) is 5 and \( rp(n) \) is 14.

Figure 22

Change in the number of individuals newly infected a day, \( AP \), and in the cumulative number of infected individuals, \( CAP \), by different ‘syr setting date’ with different value of \( syr \). ‘\( AP; syr(1-300)1; 2,135; (301-)0; 7,238 \)’ means the change in the number of individuals newly infected a day for the case with a \( syr \) of 1 for the period from the 1\(^{\text{st}}\) to the 300\(^{\text{th}}\) with the number of individuals newly infected a day of 2,135 at the first peak and with a \( syr \) of 0 for the period from the 301\(^{\text{st}}\) to the end of infection duration with the number of individuals newly infected a day of 7,238 at the second peak. \( TN(1) \) is 1,000,000, \( P(1) \) is 1, \( pfc(n) \) is 1, \( lp(n) \) is 5 and \( rp(n) \) is 14.
Change in the number of individuals newly infected a day, $AP$, and in the cumulative number of infected individuals, $CAP$, by different ‘syr setting date’ with different vaccination rates, 0 or 0.01. ‘$AP; syr0.8; (101~) v0.01; 7,574$’ means the change in the number of individuals newly infected a day for the case with a $syr$ of 0.8 throughout the infection duration, and $v$ is 0.01 for the period from the 101$^{st}$ to the end of the infection duration with the number of individuals newly infected a day of 7,574 at the peak. ‘$AP; syr1; (101~) 0.8; (101~) v0.01; 7,248$’ means the change in the number of individuals newly infected a day for the case where $syr$ is 1 for the period from the 1$^{st}$ to the 100$^{th}$, and $syr$ is 0.8 and $v$ is 0.01 for the period from the 101$^{st}$ to the end of infection duration with the number of individuals newly infected a day of 7,248 at the peak. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1, $lp(n)$ is 5 and $rp(n)$ is 14.
**Figure 24**

Change in the number of individuals newly infected a day, $AP$, and in the cumulative number of infected individuals, $CAP$, by different ‘syr setting date’ with different numbers in tested individuals, 0 or 1,000. ‘$AP; syr0.8; (101-) T1,000; 7,534$’ means the change in the number of individuals newly infected a day for the case with a $syr$ of 0.8 through the infection duration with the number of individuals newly infected a day of 7,534 at the peak. ‘$AP; syr1; (101-) 0.8; (101-) T1,000; 7,126$’ means the change in the number of individuals newly infected a day for the case where $syr$ is 1 for the period from the $1^{st}$ to the $100^{th}$, and $syr$ is 0.8 and $T(n)$ is 1,000 for the period from the $101^{st}$ to the end of infection duration with the number of individuals newly infected a day of 7,126 at the peak. $TN(1)$ is 1,000,000, $P(1)$ is 1, $pfc(n)$ is 1.0, $lp(n)$ is 5 and $rp(n)$ is 14.

**Supplementary Files**

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