Adaptive Neural Consensus Tracking Control for Multi-agent Systems with Unknown State and Input Hysteresis

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Abstract

An indirect adaptive consensus control method is presented for MASs with Unknown Bouc-Wen hysteresis states and input. All states of MASs are measured by sensors subjected to hysteresis. In order to reduce the effect of hysteresis with multi-value and rate-dependent characteristics, an adaptive compensated scheme is proposed. NNs are introduced to approximate the time-varying control gain which is coupled by input hysteresis and states hysteresis. The parameters of inverse compensation model are approximated by adaptive laws. The proposed control scheme can guarantee that the consensus errors of followers converge to a predefined interval of zero asymptotically. In addition, the transient performance of MASs can be further ensured. A simulation example is included to verify the effectiveness of the presented control approach.

Keywords: Adaptive neural control, Input and states hysteresis, Inverse compensation, Consensus control

1. Introduction

Multi-agent systems (MASs) has an extensive application in military, satellite clusters and so on [1, 2, 3, 4], and therefore numerous researchers focus on

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it. Therein, there are many significant results for linear or low-order nonlinear MASs which have achieved the ideal tracking consensus [5, 6, 7]. Moreover, these interesting works were applied in the high-order MASs [8, 9, 10]. However, the dynamics of MASs in the important works need to be completely known.

Considering that unknown nonlinearities universally exist in the dynamics of MASs, some novel corresponding methods, such as neural networks (NNs) and fuzzy-logic systems (FLSs), are presented to handle the unknown dynamics. In [11], an innovative consensus tracking control scheme was proposed for MASs with unknown dynamics. In [12], by employing NNs, the consensus control method of MASs with unknown control sign and dynamic uncertainties was presented. In [13], a tracking control scheme for MASs with unknown function existing in each subsystem was presented. However, as the scale of the MAS expands, the number of the estimated parameters in [13] becomes unbearable, which requires lots of computation.Such a problem was properly solved in [14] for nonlinear system and then in [15, 16] for MASs by treating the norm of the ideal weight vector as a unknown constant. This novel scheme is widespreadly adopted in [17, 18, 19, 20] for the neural or fuzzy controller design. However, as a result of estimating the norm, control schemes in the aforementioned papers [15, 16, 17, 18, 19, 20] only achieve the system stability with unknown tracking error. These schemes introduce some additional terms and thus the derivative of Lyapunov function is difficult to be designed negatively. To produce the asymptotic tracking control of nonlinear system, in [21], based on an innovative Lyapunov function design, a performance-oriented fuzzy control approach was proposed. The method in [21] not only ensures the fixed tracking accuracy and transient performance, but also does not increase the computational burden. In [22], a neural control method is presented for uncertain MASs with predefined accuracy. However, there are few results for MASs with fixed tracking performance in the presence of hysteresis.

Adaptive control for nonlinear systems and MASs with input hysteresis has attracted lots of attention, because the hysteresis problem exists in extensive devices and actual systems, such as electromagnetism and mechanical actuators

[23, 24, 25, 26]. To eliminate the effect of input hysteresis, in [24], by constructing an inverse compensation function, an output-feedback control scheme was proposed. Even so, the perfect compensation requires that the hysteresis model is completely known. To solve the issue of unknown hysteresis, a promising adaptive method was developed in [27]. In [28], an adaptive control method was proposed for stochastic nonlinear systems with input hysteresis and unknown control gain. Moreover, a fuzzy event-triggered based control scheme was proposed for stochastic MASs with Bouc-Wen hysteresis input in [29].

Nevertheless, the mentioned works are states feedback control methods or output feedback control methods. They require accurate values of the states or the actual system output. In practical application, the system variables are usually measured by sensors with errors, which makes the above methods impractical. It is reported in [30, 31, 32, 33] that various sensors suffer from hysteresis and in such case the exact values of system states become unknown. For nonlinear system , the using of inaccurate state value may cause performance degradation and even system instability [34]. There is a fundamental difference between the nonlinearities in actuator and in sensor, because the controller design only can use the inaccurate value. It is a challenging task to relieve the impacts caused by hysteresis existing in sensors. In [23, 35], two adaptive control approach for linear system with uncertain hysteresis in both actuator and sensor were presented. It is assumed that hysteresis only exists in the sensor measuring the output. To remove this obstacle, [36] discusses an adaptive control for nonlinear systems in the presence of both input and state hysteresis with prescribed accuracy. So far, there are few researches on MASs with state hysteresis and input hysteresis. The control scheme in[36] can not be applied in MASs because the distributed consensus controller for agent in MAS is designed using information from itself and its neighbor. The states from different agent are subjected to different unknown hysteresis. How to design the controller for MASs with state hysteresis is an interesting work.

Driven by the above discussion, this article explores the predefined precision consensus control of MASs with hysteresis in actuators and measuring sensors.

The controllers are fully distributed requiring partial information of MASs. The adaptive compensation scheme is presented to mitigate the effects of state hysteresis. Since the parameters of input hysteresis are unknown, NNs are utilized to estimate the input hysteresis and adaptive laws are given to approximate the weight matrix. The contributions of this article are epitomized as follows.

- Compared with the issue that hysteresis only exists in the actuator [24, 27, 28, 29], state hysteresis causes more technical difficulties in back-stepping design. Unlike [23, 35, 36], for the MASs with state hysteresis, the state of agent's neighbor is subjected to different hysteresis. Hysteresis is related to the states of agent and its neighbors which cannot be obtained in the current step. Compared with [36], the compensation of hysteresis is more difficult because we need to handle several different hysteresis nonlinearties from agent's neighbors in the same step. To handle the unknown dynamics caused by state hysteresis, adaptive laws are designed at each step to estimate the upper and lower bounds of the time-varying term. The unavailable states from anget's neighbors are approximated by NNs. In addition, an inverse compensation is proposed for input hysteresis.

- Most control methods [12, 13, 14, 15, 16, 17, 18, 19, 20] achieved system stability with unknown error, while in this paper by using the smooth sign function to construct Lyapunov function, all followers can track the leader asymptotically. Compared with the control algorithm for nonlinear system in [21], the uncertainties from agent's neighbor are handled by an altered tuning strategy. Unlike [22], the actual system states of agent and its neighbor are unavailable and the controllers are developed using the value measured by sensors. The proposed method can ensure a predefined steady-state error and improve the transient performance of MASs. Moreover, MASs can still maintain stability and performance even in the presence of interference.

2. Problem Statement

2.1. Communication Graph

The interaction between nodes is described by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, in which $\mathcal{V} = \{0, 1, ..., K\}$ means a set of agents and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes a set of communicated edge. Generally, the node labeled 0 is treating as the leader and K is the number of followers.The adjacency matrix is defined as $\mathcal{A} = [a_{ik}]_{N\times N}$. If the ith node can receive message from the kth node, $a_{ik} > 0$, otherwise $a_{ik} = 0$. Moreover, $\mathcal{L} = \mathcal{D} - \mathcal{A}$ denotes Laplacian matrix where $\mathcal{D} = diag(d_1, ..., d_K)$ and $d_i = \sum_{p=1}^{K} a_{ik}$. b_i shows the communication from leader to the ith follower.

2.2. Radial Basis Function NNs

It has been proven that RBFNNs are very influential in dealing with unknown dynamics [37]. This article combines NNs and a novel Lyapunov function to ensure the asymptotic stability of MASs with unknown dynamics. The RBF NNs are employed to approximate the unknown function.

$$\pi(\bar{x}) = W^{*T} R(\bar{x}) + \Delta(\bar{x}) \tag{1}$$

$\Delta(\bar{x})$ is an unknown but bounded approximation error. W^* denotes the ideal weight vector defined as

$$W^{*T} = \arg \min_{W \in \mathbb{R}^l} \{\sup_{\bar{x}\in\Omega} |\pi(\bar{x}) - W^T R(\bar{x})|\}$$

$R = [\nabla_1, ..., \nabla_M]^T$ and ∇_k $(p = 1, ..., M)$ is the Gaussian function defined as

$$\nabla_p = \exp\left[-\frac{(\bar{x} - p_k)^T(\bar{x} - p_k)}{\ell_i^2}\right], \tag{2}$$

where ℓ_k and $p_p = [p_{k1}, ..., p_{kq}]^T$ respectively denote the width and the center of basis function.

The following lemma describes a useful property of neural networks which will be helpful in reducing the number of variable inputs.

Lemma 1. [15] For $\bar{x}_k = [x_1, \ldots, x_k]^T$ and $k \leq j$, we have

$$\|R(\bar{x}_j)\|^2 \leq \|R(\bar{x}_k)\|^2 \tag{3}$$

2.3. Problem Formulation

Consider a MASs with hysteresis phenomenon presenting in both actuators and sensors. The dynamics of the followers are modeled as:

$$\begin{aligned} \dot{x}_{i,p} &= x_{i,p+1} + \pi_{i,p}\left(\bar{x}_{i,p}\right) \quad 1 \leq p \leq n_i - 1, \\ \dot{x}_{i,n_i} &= u_i + \pi_{i,n_i}\left(\bar{x}_{i,n_i}\right) \quad 1 \leq i \leq N \\ u_i &= H_i(v_i), \quad y_i = x_{i,1} \end{aligned} \tag{4}$$

where $\pi_{i,p}$, y_i and $\bar{x}_{i,p} = [x_{i,1}, ..., x_{i,p}]^T$ represent unknown smooth function, system output and state vector, respectively. v_i is the control signal we actually designed and u_i is the output of actuator acting on the plant. They are not equal when hysteresis exists in actuator. In this paper, the most popular Bouc-Wen model is adopted.

The dynamics of the leader are described as:

$$\begin{aligned} \dot{x}_0 &= f_0(x_0) \\ y_0 &= x_0 \end{aligned} \tag{5}$$

Assumption 1. *[38] The leader's output y_0 and its derivative $\dot{x}_0$ are bounded. The reference signal of the leader is known to some agents.*

In this paper, the exact system states are not available and only the hysteresis states received by sensors $\hat{x}_{i,1}, \hat{x}_{i,2}, ..., \hat{x}_{i,n_i}$ can be used in our control design. According to [24, 39], the relationships between the received signals and the genuine system states are given as the Bouc-Wen model:

$$\hat{x}_{i,p} = H_{i,p}(x_{i,p}) = \mu_{i,p1} x_{i,p} + \mu_{i,p2} \xi_{i,p} \tag{6}$$

where $\mu_{i,p1}$ and $\mu_{i,p2}$ are unknown and $\text{sign}\left(\mu_{i,p1}\right) = \text{sign}\left(\mu_{i,p2}\right)$. Without loss of generality, the parameters $\mu_{i,p1}$ and $\mu_{i,p2}$ are assumed to be positive. $\xi_{i,p}$ is given by

$$\dot{\xi}_{i,p} = \dot{x}_{i,p} \hbar_{i,p}(\xi_{i,p}, \dot{x}_{i,p}) \tag{7}$$

$$\hbar_{i,p}(\xi_{i,p}, \dot{x}_{i,p}) = 1 - \text{sign}(\dot{x}_{i,p}) h_{i,p} |\xi_{i,p}|^{l_{i,p}-1} \xi_{i,p} - \chi_{i,p} |\xi_{i,p}|^{l_{i,p}} \tag{8}$$

where $h_{i,p} > |\chi_{i,p}|$ and $l_{i,p} \geq 1$.

The following lemma gives an important property which will help our method development.

Lemma 2. $\hbar_{i,p}$ is nonnegative along with time.

Proof. From [24], we have

$$\xi_{i,p} \leq \sqrt[l_{i,p}]{\frac{1}{h_{i,p} + \chi_{i,p}}}.$$

Substituting into the definition of $\hbar_{i,p}$ (8), one has that $\hbar_{i,p} \geq 0$ is bounded. Similarly, $\hbar_i \geq 0$ holds, where $\hbar_i$ is defined in (67) □

Remark 1. It is a huge technical challenge that all exact states of agents are unavailable in this article. Different from [24, 27, 28, 29], hysteresis affects each measuring sensor, and in the meanwhile, the characteristics of hysteresis are totally unknown. Compared with [36], the hysteresis information from agent's neighbor are more difficult to handle because it is related to another different unknown hysteresis nonlinearity.

Using the measured states instead of actual system states, the consensus tracking error of the ith agent is defined as:

$$\begin{aligned} z_{i,1} &= b_i\left(\hat{x}_{i,1} - y_r\right) + \sum_{j=1}^{N} a_{i,j}\left(\hat{x}_{i,1} - \hat{x}_{j,1}\right) \\ z_{i,p} &= \hat{x}_{i,p} - \alpha_{i,p-1}, \quad p = 2, ..., n_i \end{aligned} \tag{9}$$

The following lemma is needed.

Lemma 3. [11] Chose $\mathcal{B} = diag\{b_i\}$, $\mathcal{Y} = [y_r, ..., y_r]^T$, $y = [y_1, ..., y_N]^T$ $z_1 = [z_{1,1}, ..., z_{N,1}]^T$ and $P = \mathcal{L} + \mathcal{B}$, and then one has:

$$\|y - \mathcal{Y}\| \leq \frac{\|z_1\|}{\Theta(P)} \tag{10}$$

where Θ denotes the minimum singularvalue value of P.

3. consensus control

In this section, the consensus control schemes are presented for MASs with states hysteresis and input hysteresis. RBFNNs are utilized to approximate unknown function at each step. Before our beginning, for the sake of convenience the unknown function $\pi_{i,p}$ can be expressed as

$$\pi_{i,p} = \kappa_i^T \psi_{i,p} \tag{11}$$

where

$$\psi_{i,p} = [\underbrace{0}_{M_{i,1}+\cdots+M_{i,p-1}}, R_{i,p}^T, \underbrace{0}_{M_{i,p+1}+\cdots+M_{i,n_i}+p-1}, 1, \underbrace{0}_{n_i-k}]^T \tag{12}$$

and

$$\kappa_i = \left[W_{i,1}^T, \ldots, W_{i,n_i}^T, \Delta_{i,1}, \ldots, \Delta_{i,n_i}\right]^T \tag{13}$$

Define

$$\theta_i = \sup\left\{\left(\kappa_i^T \kappa_i\right)^{\frac{1}{2}}\right\} \tag{14}$$

To continue our work, two smooth functions are introduced [40].

$$sg_{i,k}\left(z_{i,k}\right) = \begin{cases} \frac{z_{i,k}}{|z_{i,k}|}, & |z_{i,k}| \geq \epsilon_{i,k} \\ \frac{z_{i,k}}{\left(\epsilon_{i,k}^2 - z_{i,k}^2\right)^{n_i-p+2} + |z_{i,k}|}, & |z_{i,k}| < \epsilon_{i,k} \end{cases} \tag{15}$$

$$f_{i,k}\left(z_{i,k}\right) = \begin{cases} 1, & |z_{i,k}| \geq \epsilon_{i,k} \\ 0, & |z_{i,k}| < \epsilon_{i,k} \end{cases} \tag{16}$$

The relationship (6) between actual system state and sensor output is rewritten as

$$x_{i,p} = \frac{\hat{x}_{i,p} - \mu_{i,p2}\xi_{i,p}}{\mu_{i,p1}} \tag{17}$$

Now we can develop the control scheme and the details are presented in the following two cases.

3.1. case 1: only states hysteresis

In this case, $H_i(v_i) = v_i$, that is , the actuator is not affected by hysteresis.

Step 1: From the definition of consensus errors (9) and Eq.(17), $\dot{z}_{i1}$ can be derived as

$$
\begin{aligned}
\dot{z}_{i,1} &= b_i(\mu_{i,11}\dot{x}_{i,1} + \mu_{i,12}\dot{x}_{i,1}\hbar_{i,1}(\xi_{i,1}, \dot{x}_{i,1}) - \dot{y}_r) \\
&\quad + \sum_{j=1}^{N} a_{ij}(\mu_{i,11}\dot{x}_{i,1} + \mu_{i,12}\dot{x}_{i,1}\hbar_{i,1}(\xi_{i,1}, \dot{x}_{i,1}) \\
&\quad - \mu_{j,11}\dot{x}_{j,1} - \mu_{j,12}\dot{x}_{j,1}\hbar_{j,1}(\xi_{j,1}, \dot{x}_{j,1})) \\
&= (d_i + b_i)(\mu_{i,11} + \mu_{i,12}\hbar_{i,1})(x_{i,2} + \pi_{i,1}(\bar{x}_{i,1})) - b_i\dot{y}_r \\
&\quad - \sum_{j=1}^{N} a_{ij}(\mu_{j,11} + \mu_{j,12}\hbar_{j,1})(x_{j,2} + \pi_{j,1}(\bar{x}_{j,1})) \\
&= r_{i,1}(\alpha_{i,1} - \mu_{i,22}\xi_{i,2} + z_{i,2}) + r_{i,1}\mu_{i,21}\pi_{i,1} - b_i\dot{y}_r \\
&\quad - \sum_{j=1}^{N} a_{ij} r_{j,1}(\hat{x}_{j,2} - \mu_{j,22}\xi_{j,2} + \mu_{j,21}\pi_{j,1}(\bar{x}_{j,1}))
\end{aligned} \tag{18}
$$

where $r_{i,1} = (d_i + b_i)(\mu_{i,11} + \mu_{i,12}\hbar_{i,1})/\mu_{i,21}$ and $r_{j,1} = (\mu_{j,11} + \mu_{j,12}\hbar_{j,1})/\mu_{j,21}$. From the definition of $r_{i,1}$, we can directly conclude that $r_{i,1}$ is positive and bounded. Define $\eta_{i,p}$ representing the upper bound of $\frac{1}{r_{i,p}}$. $\hat{\eta}_{i,p}$ is the estimation of $\eta_{i,p}$ and the error $\tilde{\eta}_{i,p} = \eta_{i,p} - \hat{\eta}_{i,p}$.

Choose a Lyapunov function candidate as

$$
V_{i,1} = \frac{1}{n_i + 1}\left(|z_{i,1}| - \epsilon_{i,1}\right)^{n_i+1} f_{i,1} + \frac{1}{2\gamma_i\eta_{i,1}}\tilde{\eta}_{i,1}^2 \tag{19}
$$

where $\epsilon_{i,1}$ is a predefined tracking accuracy. Subsequently, we have

$$
\begin{aligned}
\dot{V}_{i,1} &= \left(|z_{i,1}| - \epsilon_{i,1}\right)^{n} f_{i,1} sg_{i,1}(z_{i,1})\,\dot{z}_{i,1} - \frac{\tilde{\eta}_{i,1}}{\gamma_i\eta_{i,1}}\dot{\eta}_{i,1} \\
&= \left(|z_{i,1}| - \epsilon_{i,1}\right)^{n_i} f_{i,1} sg_{i,1}(z_{i,1})\left(r_{i,1}(\alpha_{i,1} + z_{i,2}) + \bar{g}_{i,1}\right) \\
&\quad - \frac{\tilde{\eta}_{i,1}}{\gamma_i\eta_{i,1}}\dot{\eta}_{i,1}
\end{aligned} \tag{20}
$$

where the unknown continuous function $\bar{g}_{i,1}(\bar{Z}_{i,1})$ satisfies

$$
\begin{aligned}
\bar{g}_{i,1} &= [-r_{i,1}\mu_{i,22}\xi_{i,2} + r_{i,1}\mu_{i,21}\pi_{i,1} - b_i\dot{y}_r \\
&\quad - \sum_{j=1}^{N} a_{ij} r_{j,1}(\hat{x}_{j,2} - \mu_{j,22}\xi_{j,2} + \mu_{j,21}\pi_{j,1}(\bar{x}_{j,1}))]
\end{aligned} \tag{21}
$$

Since $\pi_{i,1}$ and π_{j1} is unknown and contain the exact value of system sates, $\bar{g}_{i,1}(\bar{Z}_{i,1})$ is unknown with $\bar{Z}_{i,1} = [x_{i,1}, x_{j1}, x_{j2}, \hat{x}_{i,1}, \hat{x}_{j1}, \hat{x}_{j2}, \dot{y}_r]^T$. We employ the RBFNN $\kappa_i^T \psi_{i,1}$ to approximate $\bar{g}_{i,1}(\bar{Z}_{i,1})$, and according to Lemma 1, with $Z_{i,1} = [\hat{x}_{i,1}, \hat{x}_{j,1}]^T$, it can be obtained that

$$\begin{aligned} \dot{V}_{i,1} &= (|z_{i,1}| - \epsilon_{i,1})^{n_i} f_{i,1} sg_{i,1}(r_{i,1}(\alpha_{i,1} + z_{i,2}) + sg_{i,1}\kappa_i^T \psi_i(\bar{Z}_{i,1})) - \frac{\tilde{\eta}_{i,1}}{\gamma_i \eta_{i,1}} \dot{\eta}_{i,1} \\ &\le (|z_{i,1}| - \epsilon_{i,1})^{n_i} f_{i,1} sg_{i,1}(r_{i,1}(\alpha_{i,1} + z_{i,2}) + \theta_i \omega_{\theta_{i,1}}) - \frac{\tilde{\eta}_{i,1}}{\gamma_i \eta_{i,1}} \dot{\hat{\eta}}_{i,1} \end{aligned} \tag{22}$$

where

$$\omega_{\theta_{i,1}} = sg_{i,1} \sqrt{\psi_{i,1}(Z_{i,1})^T \psi_{i,1}(Z_{i,1}) + d^2} \tag{23}$$

with d representing a design parameter.

Define $\varpi_{i,p}$ as the upper bound of $r_{i,p}$, $\hat{\varpi}_{i,p}$ is a estimate of $\varpi_{i,p}$ and $\tilde{\varpi}_{i,p} = \varpi_{i,p} - \hat{\varpi}_{i,p}$. To ensure that $\dot{V}_{i,1}$ is negative, the tuning function, virtual control law and adaptive law are given as follows:

$$\tau_{\theta_{i,1}} = (|z_{i,1}| - \epsilon_{i,1})^{n_i} f_{i,1} sg_{i,1} \omega_{\theta_{i,1}} \tag{24}$$

$$\dot{\eta}_{i,1} = -\gamma_i (|z_{i,1}| - \epsilon_{i,1})^{n_i} f_{i,1} sg_{i,1} \bar{\alpha}_{i,1} \tag{25}$$

$$\bar{\alpha}_{i,1} = -sg_{i,1}(z_{i,1}) \left[\left(c_{i,1} + \frac{\hat{\varpi}_{i,1}^2}{4} \right) (|z_{i,1}| - \epsilon_{i,1})^{n_i} + \hat{\varpi}_{i,1}(\epsilon_{i,2} + 1) \right] - \omega_{\theta_{i,1}} \hat{\theta}_i \tag{26}$$

$$\alpha_{i,1} = \hat{\eta}_{i,1} \bar{\alpha}_{i,1} \tag{27}$$

where ζ_i and $c_{i,1}$ are design parameters.

Lemma 4. If $\hat{\theta}_i(0) > 0$ is chosen, then $\hat{\theta}_i > 0$ holds along with time.

Proof. From (56) one has $\dot{\hat{\theta}}_i(0) > 0$, and therefore $\hat{\theta}_i > \hat{\theta}_i(0) > 0$. □

Similarly, we can prove that $\hat{\varpi}_{i,p} > 0$ and thus we have

$$(|z_{i,1}| - \epsilon_{i,1})^{n_i} f_{i,1} sg_{i,1} r_{i,1} \alpha_{i,1} \le 0 \tag{28}$$

Hence

$$
\begin{aligned}
(|z_{i,1}|-\epsilon_{i,1})^{n_i} f_{i,1} sg_{i,1} r_{i,1}\alpha_{i,1} &\le (|z_{i,1}|-\epsilon_{i,1})^{n_i} f_{i,1} sg_{i,1}\frac{1}{\eta_{i,p}}(\eta_{i,p}-\tilde{\eta}_{i,p})\bar{\alpha}_{i,1} \\
&\le (|z_{i,1}|-\epsilon_{i,1})^{n_i} f_{i,1} sg_{i,1}\bar{\alpha}_{i,1} - (|z_{i,1}|-\epsilon_{i,1})^{n_i} f_{i,1} sg_{i,1}\frac{\tilde{\eta}_{i,p}}{\eta_{i,p}}\bar{\alpha}_{i,1}
\end{aligned} \tag{29}
$$

By combing (24)-(29), $\dot{V}_{i,1}$ (22) can be calculated as

$$
\begin{aligned}
\dot{V}_{i,1} \le &-\left(c_{i,1}+\frac{\hat{\varpi}_{i,1}^2}{4}\right)(|z_{i,1}|-\epsilon_{i,1})^{2n_i} f_{i,1} \\
&+(|z_{i,1}|-\epsilon_{i,1})^{n_i} f_{i,1}(\varpi_{i,1}|z_{i,2}|-\hat{\varpi}_{i,1}(\epsilon_{i,2}+1))+\tau_{\theta_{i,1}}\tilde{\theta}_i
\end{aligned} \tag{30}
$$

Remark 2. If the quadratic Lyapunov function $V=\frac{1}{2}z^2$ is chosen and the NN $W^tR+\Delta$ are utilized to approximate the unknown function, we can only achieve that $\dot{V}\le cV+\sigma$ where $c>0$ is a constant and σ is positive and unknown [12, 13, 14, 15, 16, 17, 18, 19, 20]. This method cannot guarantee that the derivative of Lyapunov function is negative, because the using of Young's inequality introduces the unknown term into the stability analysis. In this paper, the smooth function sg is utilized and thus the using of Young's inequality is avoided. With function sg, we can achieve that $\dot{V}\le 0$. The control algorithm for nonlinear system in [21] can not be directly extended to MASs, because of the uncertainties from agent's neighbor.

Remark 3. The consensus tracking error is defined in (9) only using the available information. Moreover, due to the existence of state hysteresis, the design of virtual controller $\alpha_{i,p}$ at step k may require the states $x_{i,p+1}$ and $x_{j,p+1}$. The rate-dependent term $\dot{\xi}_{i,p}$ introduces the state $x_{i,p+1}$ into the virtual controller design. However, according to [41, 42], $x_{i,p+1}$ can not be obtained in step k. Thus $r_{i,p}$ is unavailable. According to Lemma 2, it is easy to prove that $r_{i,p}$ is bounded and non-negative. Then the upper and lower bound of the time-varying term $r_{i,p}$ are approximated by $\hat{\varpi}_{i,p}$ and $\hat{\eta}_{i,p}$ respectively. Moreover, the Lemma 1 and NNs are introduced to handle the unaccessible state from its neighbor $x_{j,p+1}$. With these adjustments, we can continue the controller design.

Step 2: Select the second Lyapunov function as:

$$V_{i,2} = V_{i,1} + \frac{1}{n_i}\left(|z_{i,2}| - \epsilon_{i,2}\right)^{n_i} f_{i,2} + \frac{1}{2\gamma_i \eta_{i,2}}\tilde{\eta}_{i,2}^2 + \frac{1}{2\zeta_i}\tilde{\varpi}_{i,1}^2 \tag{31}$$

From (9) and (17), one can directly derive that

$$\begin{aligned}
\dot{z}_{i,2} &= \dot{\hat{x}}_{i,2} - \dot{\alpha}_{i,1} \\
&= r_{i,2}(z_{i,3} + \alpha_{i,2} - \mu_{i,32}\xi_{i3}) + r_{i,2}\mu_{i,31}\pi_{i,2} \\
&\quad - \frac{\partial \alpha_{i,1}}{\partial \hat{x}_{i,1}}(\mu_{i,11} + \mu_{i,12}\hbar_{i,1})(x_{i,2} + \pi_{i,1}(\bar{x}_{i,1})) \\
&\quad - \sum_{j \in N_i} \frac{\partial \alpha_{i,1}}{\partial \hat{x}_{j,1}} r_{j,1}(\hat{x}_{j,2} - \mu_{j,22}\xi_{j,2} + \mu_{j,21}\pi_{j,1}(\bar{x}_{j,1})) \\
&\quad - \frac{\partial \alpha_{i,1}}{\partial \hat{\varpi}_{i,1}}\dot{\hat{\varpi}}_{i,1} - \frac{\partial \alpha_{i,1}}{\partial \hat{\eta}_{i,1}}\dot{\hat{\eta}}_{i,1} - \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i}\dot{\hat{\theta}}_i - \frac{\partial \alpha_{i,1}}{\partial y_r}\dot{y}_r
\end{aligned} \tag{32}$$

where $r_{i,2} = (\mu_{i,21} + \mu_{i,22}\hbar_{i,2})/\mu_{i,31}$. Define $\eta_{i,2}$ representing the upper bound of $\frac{1}{r_{i,2}}$. $\hat{\eta}_{i,2}$ is a estimate of $\eta_{i,2}$ and $\tilde{\eta}_{i,2} = \eta_{i,2} - \hat{\eta}_{i,2}$. There exists an unknown function $\bar{g}_{i,2}(\bar{Z}_{i,2})$, such that

$$\begin{aligned}
\bar{g}_{i,2}(\bar{Z}_{i,2}) = &-r_{i,2}\mu_{i,32}\xi_{i,3} + r_{i,2}\mu_{i,31}\pi_{i,2} - \frac{\partial \alpha_{i,1}}{\partial \hat{x}_{i,1}}(\mu_{i,11} + \mu_{i,12}\hbar_{i,1})(x_{i,2} + \pi_{i,1}(\bar{x}_{i,1})) \\
&- \sum_{j \in N_i} \frac{\partial \alpha_{i,1}}{\partial \hat{x}_{j,1}} r_{j,1}(\hat{x}_{j,2} - \mu_{j,22}\xi_{j,2} + \mu_{j,21}\pi_{j,1}(\bar{x}_{j,1}) - \frac{\partial \alpha_{i,1}}{\partial y_r}\dot{y}_r
\end{aligned} \tag{33}$$

where $\bar{Z}_{i,2} = [\bar{x}_{i,2}, \bar{x}_{j,2}, \bar{\hat{x}}_{i,2}, \bar{\hat{x}}_{j,2}, \dot{y}_r, \hat{\eta}_{i,1}, \hat{\theta}_i, \hat{\varpi}_{i,1}]^T$.

With $Z_{i,2} = [\bar{\hat{x}}_{i,2}, \bar{\hat{x}}_{j,2}, \hat{\eta}_{i,1}, \hat{\theta}_i, \hat{\varpi}_{i,1}]^T$, according to Lemma 1, the virtual control law is designed as:

$$\begin{aligned}
\bar{\alpha}_{i,2} = -sg_{i,2}(z_{i,2}) \Bigg[&\left(c_{i,2} + 1 + \frac{\hat{\varpi}_{i,2}^2}{4}\right)\left(|z_{i,2}| - \epsilon_{i,2}\right)^{n_i - 1} \\
&+ \hat{\varpi}_{i,2}(\epsilon_{i,3} + 1) + \sqrt{\left(\frac{\partial \alpha_1}{\partial \hat{\eta}_{i,1}}\dot{\hat{\eta}}_{i,1}\right)^2 + \left(\frac{\partial \alpha_1}{\partial \hat{\varpi}_{i,1}}\dot{\hat{\varpi}}_{i,1}\right)^2 + d^2} \\
&+ \sqrt{\left(\frac{\partial \alpha_1}{\partial \hat{\theta}_i}\right)^2 + d^2} \cdot \Gamma_i \tau_{\theta_{i,2}} \Bigg] - \omega_{\theta_{i,2}}\hat{\theta}_i
\end{aligned} \tag{34}$$

$$\omega_{\theta_{i,2}} = sg_{i,2}\sqrt{\psi_{i,2}(Z_{i,2})^T \psi_{i,2}(Z_{i,2}) + d^2} \tag{35}$$

$$\alpha_{i,2} = \hat{\eta}_{i,2}\bar{\alpha}_{i,2} \tag{36}$$

$$\dot{\hat{\eta}}_{i,2} = -\gamma_i \left(|z_{i,2}| - \epsilon_{i,2}\right)^{n_i-1} f_{i,2} sg_{i,2}\bar{\alpha}_{i,2} \tag{37}$$

$$\tau_{\theta_{i,2}} = \tau_{\theta_{i,1}} + \left(|z_{i,2}| - \epsilon_{i,2}\right)^{n_i-1} f_{i,2} sg_{i,2}\omega_{\theta_{i,2}} \tag{38}$$

$$\dot{\hat{\varpi}}_{i,1} = \zeta_i \left(|z_{i,1}| - \epsilon_{i,1}\right)^{n_i} f_{i,1} \left|z_{i,2}\right| \tag{39}$$

where γ_i, Γ_i and $c_{i,2}$ are positive constants to be selected.

In the review of (31)-(39), the derivative of $V_{i,2}$ can be obtained

$$\begin{aligned} \dot{V}_{i,2} \leq & -c_{i,1} \left(|z_{i,1}| - \epsilon_{i,1}\right)^{2n_i} f_{i,1} - \left(c_{i,2} + \frac{\hat{\varpi}_{i,2}^2}{4}\right) \left(|z_{i,2}| - \epsilon_{i,2}\right)^{2(n_i-1)} f_{i,2} \\ & + \left(|z_{i,2}| - \epsilon_{i,2}\right)^{n_i-1} f_{i,2} \left[\varpi_{i,2} \left|z_{i3}\right| - \hat{\varpi}_{i,2} \left(\epsilon_{i3} + 1\right)\right] \\ & - \left(|z_{i,2}| - \epsilon_{i,2}\right)^{n_i-1} f_{i,2} \sqrt{\left(\frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i}\right)^2 + d^2} (\Gamma_i \tau_{\theta_{i,2}} - \dot{\hat{\theta}}_i) + M_{i,2} + \tilde{\theta}_i \tau_{\theta_{i,2}} \end{aligned} \tag{40}$$

where

$$\begin{aligned} M_{i,2} = & -\frac{\hat{\varpi}_{i,1}^2}{4} \left(|z_{i,1}| - \epsilon_{i,1}\right)^{2n_i} f_{i,1} + \left(|z_{i,1}| - \epsilon_{i,1}\right)^{n_i} f_{i,1} [\hat{\varpi}_{i,1} \left|z_{i,2}\right| \\ & - \hat{\varpi}_{i,1} \left(\epsilon_{i,2} + 1\right) - \left(|z_{i,2}| - \epsilon_{i,2}\right)^{2(n_i-1)} f_{i,2}] \end{aligned} \tag{41}$$

According to Lemma 4, $\hat{\varpi}_{i,p} > 0$ holds if we choose $\hat{\varpi}_{i,p}(0) > 0$. Then with the work in [40], $M_{i,2} \leq 0$ is always satisfied.

Remark 4. The proof of $M_{i,2} \leq 0$ needs that $\hat{\varpi}_{i,1} > 0$, such that the updated law $\dot{\hat{\varpi}}_{i,1}$ (39) are given is step 2. Accordingly, the updated law $\dot{\hat{\varpi}}_{i,p}$ will be designed in step $p+1$.

Step p: Similarly, we choose a Lyapunov function

$$\begin{aligned} V_{i,p} = V_{i,p-1} & + \frac{1}{n_i - p + 2} \left(|z_{i,p}| - \epsilon_{i,p}\right)^{n_i-p+2} f_{i,p} \\ & + \frac{1}{2\gamma_i \eta_{i,p}} \tilde{\eta}_{i,p}^2 + \frac{1}{2\zeta_i} \tilde{\varpi}_{i,p-1}^2 \end{aligned} \tag{42}$$

Calculating the time derivative of 9, we have

$$\begin{aligned}\dot{z}_{i,p} &= \dot{\hat{x}}_{i,p} - \dot{\alpha}_{i,p-1} = r_{i,p}(z_{i,p+1} + \alpha_{i,p}) + \bar{g}_{i,p} \\ &- \sum_{p=1}^{p-1} \frac{\partial \alpha_{i,p-1}}{\partial \hat{\varpi}_{i,p}} \dot{\hat{\varpi}}_{i,p} - \sum_{p=1}^{p-1} \frac{\partial \alpha_{i,p-1}}{\partial \hat{\eta}_{i,p}} \dot{\hat{\eta}}_{i,p} - \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i\end{aligned} \tag{43}$$

$$\bar{g}_{i,p} = -r_{i,p}\mu_{i,(p+1)2}\xi_{i,p+1} + r_{i,p}\mu_{i,(p+1)1}\pi_{i,p} - \sum_{p=1}^{p-1} \frac{\partial \alpha_{i,p-1}}{\partial \hat{x}_{i,p}} \dot{\hat{x}}_{i,p} \tag{44}$$

$$- \sum_{p=1}^{p-1} \sum_{j \in N_i} \frac{\partial \alpha_{i,p-1}}{\partial \hat{x}_{j,p}} \dot{\hat{x}}_{j,p-1} - \frac{\partial \alpha_{j,p-1}}{\partial y_r} \dot{y}_r \tag{45}$$

The control law and adaption laws are directly given as

$$\alpha_{i,p} = \hat{\eta}_{i,p} \bar{\alpha}_{i,p} \tag{46}$$

$$\begin{aligned}\bar{\alpha}_{i,p} &= -sg_{i,p}(z_{i,p})[(c_{i,p} + 1 + \frac{\hat{\varpi}_{i,p}^2}{4})(|z_{i,p}| - \epsilon_{i,p})^{n_i - p + 1} + \hat{\varpi}_{i,p}(\epsilon_{i,p+1} + 1) \\ &+ \sqrt{\sum_{p=1}^{p-1} (\frac{\partial \alpha_{p-1}}{\partial \hat{\eta}_{i,p}} \dot{\hat{\eta}}_{i,p})^2 + \sum_{j=1}^{p-1} (\frac{\partial \alpha_{i,p-1}}{\partial \hat{\varpi}_{i,p}} \dot{\hat{\varpi}}_{i,p})^2 + d^2} + \sqrt{(\frac{\partial \alpha_{i,p-1}}{\partial \hat{\theta}_i})^2 + d^2} \cdot \Gamma \tau_{\theta_{i,p}} \\ &+ \sum_{j=2}^{p-1} (|z_{i,j}| - \epsilon_{i,j})^{n_i - j + 1} f_{i,j} \sqrt{(\frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_i})^2 + d^2} \cdot \Gamma \omega_{\theta_{i,p}}] - \omega_{\theta_{i,p}} \hat{\theta}_i\end{aligned} \tag{47}$$

$$\dot{\hat{\eta}}_{i,p} = -\gamma_i \left(|z_{i,p}| - \epsilon_{i,p}\right)^{n_i - p + 1} f_{i,p} sg_{i,p} \bar{\alpha}_{i,p} \tag{48}$$

$$\tau_{\theta_{i,p}} = \tau_{\theta_{i,p-1}} + \left(|z_{i,p}| - \epsilon_{i,p}\right)^{n_i - p + 1} f_{i,p} sg_{i,p} \omega_{\theta_{i,p}} \tag{49}$$

$$\omega_{\theta_{i,p}} = sg_{i,p} \sqrt{\psi_{i,p}\left(Z_{i,p}\right)^T \psi_{i,p}\left(Z_{i,p}\right) + d^2} \tag{50}$$

$$\dot{\hat{\varpi}}_{i,p-1} = \zeta_i \left(|z_{i,p-1}| - \epsilon_{i,p-1}\right)^{n_i - p + 2} f_{i,p-1} \left|z_{i,p}\right| \tag{51}$$

where $c_{i,p}$ is a positive design parameter.

Then we can derive the following equation in the same way as in step 2.

$$
\begin{aligned}
\dot{V}_{i,p} \leq \sum_{j=1}^{k} -c_{i,p} \left(|z_{i,1}| - \epsilon_{i,1}\right)^{2(n_i-j+1)} f_{i,j} \\
-\frac{\hat{\varpi}_{i,p}^2}{4} \left(|z_{i,p}| - \epsilon_{i,p}\right)^{2(n_i-p+1)} f_{i,p} \\
+ \left(|z_{i,p}| - \epsilon_{i,p}\right)^{n_i-p+1} f_{i,p} \left[\varpi_{i,p} \left|z_{i,p+1}\right| - \hat{\varpi}_{i,p} \left(\epsilon_{i,p+1} + 1\right)\right] \\
+\tilde{\theta}_i \tau_{\theta_{i,p}} \\
-\sum_{j=2}^{k} \left(|z_{i,j}| - \epsilon_{i,j}\right)^{n_i-j+1} f_{i,j} \sqrt{\left(\frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_i}\right)^2 + d^2} \cdot (\Gamma_i \tau_{\theta_{i,p}} - \dot{\hat{\theta}}_i)
\end{aligned} \tag{52}
$$

Step n In this case, we consider the MASs without input hysteresis. The plant can directly receive control signal, i.e., $H_i(v_i) = v_i$. The distributed consensus controller and the updated laws are given to achieve our control goal:

$$
v_i = \hat{\eta}_{i,n_i} \bar{\alpha}_{i,n_i} \tag{53}
$$

$$
\begin{aligned}
\bar{\alpha}_{i,n_i} = & - sg_{i,n_i}(z_{i,n_i}) \Big[(c_{i,n_i} + 1)(|z_{i,n_i}| - \epsilon_{i,n_i}) \\
& + \sqrt{\sum_{j=1}^{n_i-1} \left(\frac{\partial \alpha_{n_i-1}}{\partial \hat{\eta}_{i,j}} \dot{\hat{\eta}}_{i,j}\right)^2 + \sum_{j=1}^{n_i-1} \left(\frac{\partial \alpha_{i,n_i-1}}{\partial \hat{\varpi}_{i,j}} \dot{\hat{\varpi}}_{i,j}\right)^2 + d^2} \\
& + \sqrt{\left(\frac{\partial \alpha_{i,n_i-1}}{\partial \hat{\theta}_i}\right)^2 + d^2} \cdot \Gamma_i \tau_{\theta_{i,n_i}} \\
& + \sum_{j=2}^{n_i-1} (|z_{i,j}| - \epsilon_{i,j})^{n_i-j+1} f_{i,j} \sqrt{\left(\frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_i}\right)^2 + d^2} \cdot \Gamma_i \omega_{\theta_{i,n_i}} \Big] \\
& - \omega_{\theta_{i,n_i}} \hat{\theta}_i
\end{aligned} \tag{54}
$$

$$
\dot{\hat{\eta}}_{i,n_i} = -\gamma_i \left(|z_{i,n_i}| - \epsilon_{i,n_i}\right) f_{i,n_i} sg_{i,n_i} \bar{\alpha}_{i,n_i} \tag{55}
$$

$$
\dot{\hat{\theta}}_i = \Gamma_i \tau_{\theta_{i,n_i}} \tag{56}
$$

$$
\dot{\hat{\varpi}}_{i,n_i-1} = \zeta_i \left(|z_{i,n_i-1}| - \epsilon_{i,n_i-1}\right)^2 f_{i,n_i-1} \left|z_{i,n_i}\right| \tag{57}
$$

where $\tau_{\theta_{i,n_i}}$ and $\omega_{\theta_{i,n_i}}$ are defined in (49) and (50). Define the Lyapunov function for this step.

$$\begin{aligned} V_{i,n_i} &= V_{i,n_i-1} + \frac{1}{2}\left(|z_{i,n_i}| - \epsilon_{i,n_i}\right)^2 f_{i,n_i} \\ &+ \frac{1}{2\gamma_i \eta_{i,n_i}} \tilde{\eta}_{i,n_i}^2 + \frac{1}{2\zeta_i} \tilde{\varpi}_{i,n_i-1}^2 + \frac{1}{2\Gamma_i} \tilde{\theta}_i^2 \end{aligned} \tag{58}$$

In view of (53)-(57), we can get the derivative of V_{i,n_i}

$$\dot{V}_{i,n_i} \leq -\sum_{p=1}^{n_i} c_{i,p} \left(|z_{i,p}| - \epsilon_{i,p}\right)^{2(n_i-p+1)} f_{i,k} \tag{59}$$

With the design of control signal and updated laws, the following theorem can be concluded.

Theorem 1. Considering the MASs (4) with states hysteresis, if the controller (53) and adaptation laws (55)-(57) are implemented, we can obtain that:

- all variables of the MASs (4) are SGUUB.
- the consensus tracking error asymptotically converges to a predefined bound along with time. i.e., $\lim_{t\to\infty} |z_{i,1}| \leq \epsilon_{i,1}$. Furthermore, $\|y - \mathcal{Y}\| \leq \frac{\epsilon_1}{\underline{\Lambda}}$, where $\epsilon_1 = \max\{\epsilon_{i,1}\}$.

Proof. Define the Lyapunov function

$$V = \sum_{i=1}^{N} V_{i,n_i} \tag{60}$$

From (59), one has

$$\dot{V} \leq -\sum_{i=1}^{N}\sum_{p=1}^{n_i} c_{i,p} \left(|z_{i,p}| - \epsilon_{i,p}\right)^{2(n_i-p+1)} f_{i,k} \leq 0 \tag{61}$$

Thus $z_{i,p}$, $\hat{\theta}_i$, $\hat{\varpi}_{i,p-1}$ and $\hat{\eta}_{i,p}$ are bounded. Moreover, through the definition of $z_{i,p}$ (9), we can get that $\hat{x}_{i,1}$ is bounded because the output of leader y_0 is bounded. $\alpha_{i,1}$ is consisting of variables $\hat{\theta}_i$, $\varpi_{i,1}$, $\eta_{i,1}$, $\hat{x}_{i,1}$ and $\hat{x}_{j,1}$, such that $\alpha_{i,1}$ is also bounded. Subsequently, we can further get that $\hat{x}_{i,2}$ is bounded from (9). In the same way, the boundedness of $\alpha_{i,p}$ and $\hat{x}_{i,p}$ can be proved. Then the control signal v_i is also bounded. Noticing Eq.(6), the real system states $x_{i,p}$ are ensured to be bounded. Next it is proved that the consensus errors converge to a predefined neighborhood of zero. For convenience we define $s_i = (|z_{i,1}| - \epsilon_{i,1})^{n_i+1} f_{i,1}$. Then one has

$$\dot{V}_{i,n_i} \leq -c_{i,1} s_i^2 \tag{62}$$

Integrate both sides of the inequality

$$\int_0^\infty s_i(t)^2 dt \leq \frac{1}{c_{i,1}} V_{i,n_i}(0) \tag{63}$$

According to Barbalat's lemma, it is proved that $\lim_{t\to\infty} s_i = 0$, i.e., $\lim_{t\to\infty} |z_{i,1}| \leq \epsilon_{i,1}$. Along with Lemma 3, $\|y - \mathcal{Y}\| \leq \frac{\epsilon_1}{\underline{\Delta}}$ is achieved. □

The $\mathbb{L}_2$-norm of the consensus error can be further proved

$$\begin{aligned} \|s_i\|_2 &= \sqrt{\int_0^\infty s_i(t)^2 dt} \\ &\leq \frac{1}{\sqrt{2c_{i,1}}} \sum_{p=1}^{n_i} [\frac{1}{n_i - p + 2} (|z_{i,p}(0)| - \epsilon_{i,p})^{n_i - p + 2} f_{i,p}(z_{i,p}(0)) \\ &\quad + \frac{1}{2\gamma_i \eta_{i,p}} \tilde{\eta}_{i,p}^2(0) + \frac{1}{2\zeta_i} \tilde{\varpi}_{i,p-1}^2(0)] + \frac{1}{2\Gamma_i} \tilde{\theta}_i^2(0) \end{aligned} \tag{64}$$

Remark 5. It is proved that the consensus tracking errors can be designed in advance by using the Lyapunov function with smooth sign function. The $\mathbb{L}_2$-norm performance is obviously adjustable. By increasing the design parameters ζ_i, Γ_i and γ_i, the transient performance can be improved. The consensus tracking errors can be reduced by selecting a suitable $\epsilon_{i,1}$.

3.2. case 2 states hysteresis and input hysteresis

In the just described work, the control method for MASs with states hysteresis was developed. Now the case that actuators are affected by hysteresis are further considered. The Bouc-Wen model can be expressed as

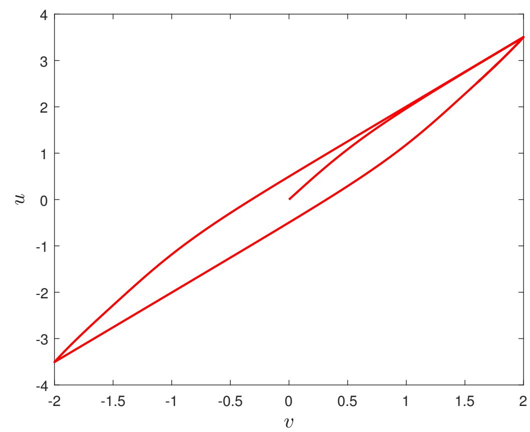
$$H_i(v_i) = \mu_{i1} v_i + \mu_{i2} \xi_i \tag{65}$$

where μ_{i1} and μ_{i2} are unknown constants and $\text{sign}(\mu_{i1}) = \text{sign}(\mu_{i2})$. ξ_i is given by

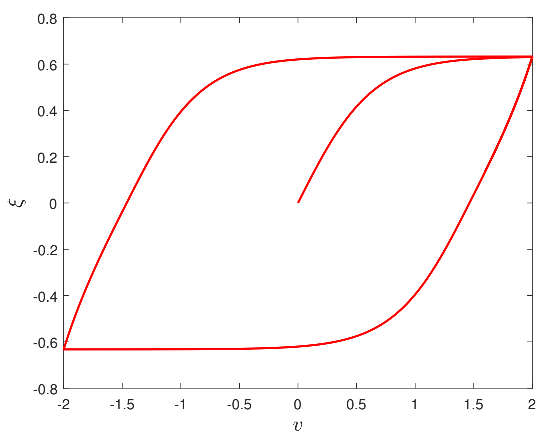
$$\dot{\xi}_i = \dot{v}_i \hbar_i(\xi_i, \dot{v}_i) \tag{66}$$

$$\hbar_i(\xi_i, \dot{v}_i) = 1 - \text{sign}(\dot{v}_i) h_i |\xi_i|^{l_i - 1} \xi_i - \chi_i |\xi_i|^{l_i} \tag{67}$$

where $h_i > |\chi_i|$ and $l_i \geq 1$. h_i, ξ_i and l_i are the unknown parameters representing the shape, the amplitude and the smoothness respectively.The Bouc-Wen model is shown in Fig.1. The inverse compensation strategy are given as follows



(a) Hysteresis



(b) Hysteresis variable ξ

Figure 1: Bouc-Wen model

$$v_i = \widehat{H}_i\left(u_{id}\right) = \frac{1}{\hat{\mu}_{i1}} u_{id} - \frac{\hat{\mu}_{i2}}{\hat{\mu}_{i1}} \xi_{id} \tag{68}$$

$$\dot{\xi}_{id} = \dot{u}_{id} \hbar_i(\xi_{id}, \dot{u}_{id}) \tag{69}$$

$$\hbar_i(\xi_{id}, \dot{u}_{id}) = 1 - \text{sign}(\dot{u}_{id}) h_{id} |\xi_{id}|^{l_{i,d}-1} \xi_{id} - \chi_{id} |\xi_{id}|^{l_{id}} \tag{70}$$

where h_{id}, l_{id} and χ_{id} are positive constants to be chosen. In addition, $h_{id} \geq |\chi_{id}|$. The error between actual control signal and desired control signal is given by

$$\begin{aligned} u_i - u_{id} &= \mu_i^T \vartheta_i - \hat{\mu}_i^T \vartheta_{id} \\ &= \mu_i^T \vartheta_{id} + \mu_i^T \vartheta_i - \mu_i^T \vartheta_{id} - \hat{\mu}_i^T \vartheta_{id} \\ &\leq \tilde{\mu}_i^T \vartheta_{id} + \Lambda_i \end{aligned} \tag{71}$$

where $\hat{\mu}_i = [\hat{\mu}_{i1}, \hat{\mu}_{i2}]^T$, $\vartheta_i = [v_i, \xi_i]^T$, $\vartheta_{id} = [v_i, \xi_{id}]^T$, and $\Lambda_i = \mu_i^T (\vartheta - \vartheta_d) = \mu_{i2} (\xi_i - \xi_{id})$ Since ξ_i and ξ_{id} are bounded, Λ_i has an unknown upper bound such that $|\Lambda_i(t)| \leq \bar{\Lambda}_i$.

Only the actual control law in step n_i needs to be modified.

$$\begin{aligned} \dot{z}_{i,n_i} &= \dot{x}_{i,n_i} - \dot{\alpha}_{i,n_i-1} \\ &= r_{i,n_i}(u_i - u_{id} + u_{id} + \pi_{in_i}) - \dot{\alpha}_{i,n-1} \end{aligned} \tag{72}$$

Remark 6. It is obvious that the input hysteresis and state hysteresis are coupled together. The control method in [24] can not be applied for the parameters of hysteresis model are unknown. The adaptive scheme in [27, 29] with input hysteresis cannot be directly extended to MASs in this paper because they do not consider the effect of state hysteresis. In this paper, the adaptive inverse compensation scheme is presented for MASs with input hysteresis. NNs are utilized to approximate the unknown control gain and the ideal constants μ_{i2} and μ_{i1} are estimated.

The RBFNN is employed to approximate the unknown control gain. One has

$$r_{i,n_i} \vartheta_{id} = W_i^T R_i\left(Z_i\right) + \Delta_i\left(Z_i\right) \tag{73}$$

where $Z_i = [\bar{x}_{i,n_i}, v_i, \xi_{id}]^T$.

$$\begin{aligned} r_{i,n_i}(u_i - u_{id}) =& r_{i,n_i}\left(\tilde{\mu}_i^T \vartheta_{id} + \Lambda_i\right) \\ =& \tilde{\mu}_i^T \left[W_i^T R_i + \Delta_i\right] + r_{i,n_i}\Lambda_i \\ =& \tilde{\mu}_i^T \left[\hat{W}_i^T R_i + \tilde{W}_i^T R_i + \Delta_i\right] + r_{i,n_i}\Lambda_i \\ =& \tilde{\mu}_i^T \hat{W}^T R_i - \hat{\mu}_i^T \tilde{W}_i^T R_i \\ &+ \mu_i^T \tilde{W}_i^T R_i + \tilde{\mu}_i^T \Delta_i + r_{i,n_i}\Lambda_i \end{aligned} \tag{74}$$

Then

$$\begin{aligned} (|z_{i,n_i}| - \epsilon_{i,n_i}) f_{i,n_i} sg_{i,n_i} r_{i,n_i}(u_i - u_{id}) \leq \\ (|z_{i,n_i}| - \epsilon_{i,n_i}) f_{i,n_i} sg_{i,n_i} [\tilde{\mu}^T \hat{W}_i^T R_i - \hat{\mu}_i^T \tilde{W}_i^T R_i + sg_{i,n_i}\lambda_i] \end{aligned} \tag{75}$$

where $\lambda_i = \|\mu_i\|\|\bar{W}_i\|\|R_i\| + \|\bar{\mu}_i\|\bar{\Delta}_i + \varpi_{i,n_i}\bar{\Lambda}_i$. This calculation requires an important property that $\tilde{W}_i$ and $\tilde{\mu}$ always have the upper bound $\bar{W}_i$ and $\bar{\mu}_i$. The property will be guaranteed by designing the updated laws Eq.(79) and Eq.(80). The control law and updated laws are designed as follows:

$$u_{id} = \hat{\eta}_i(\bar{\alpha}_{i,n_i} - sg_{i,n_i}\hat{\lambda}_i) \tag{76}$$

$$\dot{\hat{\eta}}_i = -\gamma_i (|z_{i,n_i}| - \epsilon_{i,n_i}) f_{i,n_i} sg_{i,n_i}(\bar{\alpha}_{i,n_i} - sg_{i,n_i}\hat{\lambda}_i) \tag{77}$$

$$\dot{\hat{\lambda}}_i = \beta_i(|z_{i,n_i}| - \epsilon_{i,n_i}) f_{i,n_i} \tag{78}$$

$$\dot{\hat{\mu}}_i = \text{Proj}[\Gamma_{i\mu}(|z_{i,n_i}| - \epsilon_{i,n_i}) f_{i,n_i} \hat{W}_i^T R_i] \tag{79}$$

$$\dot{\hat{W}}_i = -\text{Proj}[\Gamma_{iW}(|z_{i,n_i}| - \epsilon_{i,n_i}) f_{i,n_i} R_i \hat{\mu}_i^T] \tag{80}$$

where β_i is a parameter to be designed. $\Gamma_{i\mu}$ and $\Gamma_{i\hat{W}}$ are well-chosen positive definite matrices. Proj$(\cdot)$ represents the projection operator developed in [41].

Theorem 2. Considering the MASs (4) with states hysteresis and input hysteresis, if the controller (76) and adaptation laws (77)-(80) are implemented, we can conclude that:

- all variables of the MASs (4) are SGUUB.
- the consensus tracking error asymptotically converges to a predefined bound along with time. i.e., $\lim_{t\to\infty}|z_{i,1}| \le \epsilon_{i,1}$. Furthermore, $\|y-\mathcal{Y}\| \le \frac{\epsilon_1}{\underline{\Delta}}$, where $\epsilon_1 = \max\{\epsilon_{i,1}\}$.

Proof. Consider the following Lyapunov function

$$\begin{aligned}\bar{V}_{i,n_i} &= V_{i,n_i-1} + \frac{1}{2}\left(|z_{i,n_i}| - \epsilon_{i,n_i}\right)^2 f_{i,n_i} \\ &+ \frac{1}{2\gamma_i \eta_{i,n_i}}\tilde{\eta}_{i,n_i}^2 + \frac{1}{2\zeta_i}\tilde{\varpi}_{i,n_i-1}^2 + \frac{1}{2\Gamma_i}\tilde{\theta}_i^2 \\ &+ \frac{1}{2\beta_i}\tilde{\lambda}_i^2 + \frac{1}{2}\tilde{\mu}_i^T \Gamma_{i\mu}^{-1}\tilde{\mu}_i + \frac{1}{2}\operatorname{tr}(\tilde{W}_i^T \Gamma_{iW}^{-1}\tilde{W}_i)\end{aligned} \tag{81}$$

and define $\bar{V} = \sum_{i=1}^{N}\bar{V}_{i,n_i}$. From (81), one has

$$\dot{\bar{V}} \le -\sum_{i=1}^{N}\sum_{p=1}^{n_i} c_{i,p}\left(|z_{i,p}| - \epsilon_{i,p}\right)^{2(n_i-p+1)} f_{i,k} \le 0 \tag{82}$$

Thus $z_{i,p}$, $\hat{\theta}_i$, $\hat{\varpi}_{i,p-1}$, $\hat{\eta}_{i,p}$, $\hat{\lambda}_i$, $\hat{\mu}_i$ and $\hat{W}_i$ are bounded. Moreover, through the definition of $z_{i,p}$ (9), we can get that $\hat{x}_{i,1}$ is bounded because the output of leader y_0 is bounded. $\alpha_{i,1}$ is consisting of variables $\hat{\theta}_i$, $\varpi_{i,1}$, $\eta_{i,1}$, $\hat{x}_{i,1}$ and $\hat{x}_{j,1}$, such that $\alpha_{i,1}$ is also bounded. Subsequently, we can further get that $\hat{x}_{i,2}$ is bounded from (9). In the same way, the boundedness of $\alpha_{i,p}$ and $\hat{x}_{i,p}$ can be proved. Then the control signal v_i is also bounded. Noticing Eq.(6), the real system states $x_{i,p}$ are ensured to be bounded.

The property that the consensus errors converge to a predefined neighborhood of zero is also given.

$$\dot{V}_{i,n_i} \le -c_{i,1}s_i^2 \tag{83}$$

According to Barbalat's lemma, it is proved that $\lim_{t\to\infty} s_i = 0$, i.e., $\lim_{t\to\infty}|z_{i,1}| \le \epsilon_{i,1}$. Along with Lemma 3, $\|y-\mathcal{Y}\| \le \frac{\epsilon_1}{\underline{\Delta}}$ is achieved. Since the measured errors are always bounded, the boundedness of the actual system states can be obtained by the boundedness of the measured value. □

The $\mathbb{L}_2$-norm of the consensus errors can be further proved

$$
\begin{aligned}
&\|s_i\|_2 \\
\leq & \frac{1}{\sqrt{2c_{i,1}}}\sum_{p=1}^{n_i}[\frac{1}{n_i-p+2}\left(|z_{i,p}(0)|-\epsilon_{i,p}\right)^{n_i-p+2} f_{i,p}(z_{i,p}(0)) \\
& +\frac{1}{2\gamma_i\eta_{i,p}}\tilde{\eta}_{i,p}^2(0)+\frac{1}{2\zeta_i}\tilde{\varpi}_{i,p-1}^2(0)]+\frac{1}{2\Gamma_i}\tilde{\theta}_i^2(0) \\
& +\frac{1}{2\beta_i}\tilde{\tilde{\lambda}}_i^2(0)+\frac{1}{2}\tilde{\mu}_i^T(0)\Gamma_{i\mu}^{-1}\tilde{\mu}_i(0)+\frac{1}{2}\operatorname{tr}(\tilde{W}_i^T(0)\Gamma_{i\hat{W}}^{-1}\tilde{W}_i)(0)
\end{aligned} \tag{84}
$$

Remark 7. Unlike [24], the model of input hysteresis is unknown, such that the perfect inverse compensation can be achieved. In this article we design the updated law Eq.(79) to estimate the model's parameters instead of parameter identifications, which is easier to be implemented in practice .Compared with [27, 28, 29], the control input of MASs are not only affected by the unknown hysteresis in actuator, but also by the unknown control gain caused by states hysteresis, as shown in Eq.(72). The control coefficient is time-varying such that we can not directly design an inverse compensation for input hysteresis. A NN are utilized to offset the effects of time-varying term and updated law Eq.(79) are given to estimate the parameters of hysteresis model.

Remark 8. The projection operator are introduced to ensure that $\tilde{W}_i$ and $\tilde{\mu}_i$ are always bounded. The boundedness of λ_i must be ensured in advance such that we can design an adaptive law. However, it contains $\tilde{\mu}_i$ and $\tilde{W}_i$. Then the projection is introduced. It is guaranteed that $\tilde{W}_i \in \Omega_{W_i}$ and $\tilde{\mu}_i \in \Omega_{\mu_i}$ if $\tilde{W}_i(0) \in \Omega_{W_i}$ and $\tilde{\mu}_i(0) \in \Omega_{\mu_i}$. With this property, we can continue our work and design the distributed controllers.

Corollary 1. In both two cases, the MASs 4 still remain stable even if there is bounded disturbances in the system dynamics. Moreover, the conclusion in Theorem 1 and Theorem 2 still can be achieved.

Proof. Let $\Upsilon_{i,p}$ represent the disturbance. When the disturbances exist in system dynamics, according to Eq.(43), one has

$$\begin{aligned}\dot{z}_{i,p} &= \dot{\hat{x}}_{i,p} - \dot{\alpha}_{i,p-1} \\ &= r_{i,p}(z_{i,p+1} + \alpha_{i,p} + \Upsilon_{i,p}) + \bar{g}_{i,p} \\ -\sum_{p=1}^{p-1} \frac{\partial \alpha_{i,p-1}}{\partial \hat{\varpi}_{i,p}} \dot{\hat{\varpi}}_{i,p} - \sum_{p=1}^{p-1} &\frac{\partial \alpha_{i,p-1}}{\partial \hat{\eta}_{i,p}} \dot{\hat{\eta}}_{i,p} - \frac{\partial \alpha_{i,1}}{\partial \hat{\theta}_i} \dot{\hat{\theta}}_i\end{aligned} \tag{85}$$

where $r_{i,p}$ and $\Upsilon_{i,p}$ are bounded. Rewriting Eq.(13), one has

$$\kappa_i = \left[W_{i,1}^T, \ldots, W_{i,n_i}^T, \Delta_{i,1} + \bar{\Upsilon}_{i,1}, \ldots, \Delta_{i,n_i} + \bar{\Upsilon}_{i,n_i}\right]^T \tag{86}$$

where $\bar{\Upsilon}_{i,p} = r_{i,p}\Upsilon_{i,p}$. Then the unknown parameter need to be estimated is still θ_i. Thus according to the same process of controller design and stability analysis, Theorem 1 and Theorem 2 can be obtained. □

4. Illustrative example

In this section, an example is provided to illustrate the effectiveness of the aforementioned distributed consensus control method. Fig.2 denotes the com-

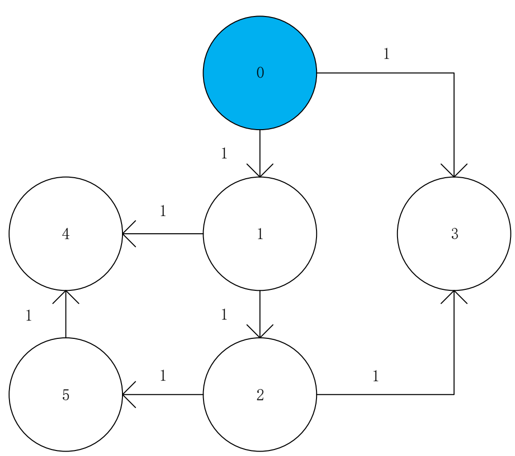


Figure 2: Directed graph of the MASs.

munication graph of the MAS which is consisting of a leader and 5 followers.

The dynamics of follower agents are given as

$$\begin{aligned}\dot{x}_{i,1} &= x_{i,2} + 0.3exp(-x_{i,1}^2) - 0.3sin(x_{i,1}^2) + 0.1sin(t)\\ \dot{x}_{i,2} &= u_i - \sin(x_{i,1}x_{i,2})\\ u_i &= H_i(v_i), \quad y_i = x_{i,1}\end{aligned} \tag{87}$$

where $\bar{x}_{i,2}$ is the exact system state vector which can not be used in our controller. The measuring sensors are subject to the hysteresis which is model by

$$\hat{x}_{i,p} = H_{i,p}(x_{i,p}) = 0.55x_{i,p} + \xi_{i,p} \tag{88}$$

$$\dot{\xi}_{i,p} = \dot{x}_{i,p}\hbar_{i,p}(\xi_{i,p}, \dot{x}_{i,p}) \tag{89}$$

$$\hbar_{i,p}(\xi_{i,p}, \dot{x}_{i,p}) = 1 - \text{sign}(\dot{x}_{i,p})|\xi_{i,p}|\xi_{i,p} - 0.5|\xi_{i,p}|^2 \tag{90}$$

It is more challenging when the input and state are both hysteretic. The control scheme in Case 2 is implemented for the example. The hysteresis in actuator is modeled by

$$H_i(v_i) = 1.2v_i + 0.5\xi_i \tag{91}$$

$$\dot{\xi}_i = \dot{v}_i\hbar_i(\xi_i, \dot{v}_i) \tag{92}$$

$$\hbar_i(\xi_i, \dot{v}_i) = 1 - 1.2\,\text{sign}(\dot{v}_i)|\xi_i|\xi_i - 0.5|\xi_i|^2 \tag{93}$$

The leader's dynamics are given as follows:

$$\begin{aligned}\dot{x}_0 = -x_0 + t^3\exp(-2t) + 1.5\cos(0.2t)\\ y_0 = x_0\end{aligned} \tag{94}$$

Remark 9. It is more common in practical and more difficult in technology that the MAS's dynamics contain unknown time-varying disturbance. Furthermore, the leader's output is also dynamic, which is more flexible in applications. Only the inaccurate values of system states can be obtained, which make the consensus control of such a MAS more difficult.

The design parameters are selected as follows: $c_{i,1} = 110$, $c_{i,2} = 3$, $\zeta_{i,1} = 3$, $\gamma_i = 0.98$, $\beta_i = 2$, $d = 0.78$, $\Gamma_i = 1.63$. The tracking errors are bounded by the accuracy $\epsilon_{i,1} = 0.75$ and $\epsilon_{i,2} = 0.8$. The matrices Γ_{iW} and $\Gamma_{i\mu}$ are set to the identity matrices. h_{id}, l_{id} and χ_{id} in (70) are set as 1.1, 2 and 0.5, respectively. The initial MAS's states are given as : $x_1(0) = [0.21, 0]^T$, $x_2(0) = [0.13, 0]^T$, $x_3(0) = [0.19, 0]^T$, $x_4(0) = [0.24, 0]^T$, $x_5(0) = [0.18, 0]^T$. The leader's output are initialized as 0.21. The initial values of the adaptive parameters are listed below: $\hat{\theta}_i(0) = 0.03$, $\hat{\eta}_{i,1}(0) = \hat{\eta}_{i,2}(0) = \hat{\varpi}_{i,1}(0) = 0.1$, $\hat{\eta}_i(0) = 0.07$, $\hat{\lambda}_i(0) = 0.5$, $\hat{\mu}_i(0) = [1, 0.6]^T$ and $\hat{W}_i(0) = \begin{bmatrix} 0.8 & 0.8 \\ 0.8 & 0.8 \end{bmatrix}$. The RBFNNs used in this example contain 3 nodes. Their centers evenly spaced in $[-0.7, 0, 0.7]^T$ and $\ell_i = 2$. Define the tracking errors $e_i = |\hat{x}_{i,1} - y_0|$. The simulation results are shown in Fig.3 - Fig.8.

We can conclude that adaptive parameters are bounded as shown in Fig.6. The consensus errors converge to a small domain of zero rapidly as shown in Fig.5. It can be easy obtained from Fig.3 and Fig.4 that the genuine output and the output measured by sensors are very different. Thus the control for this case are challenging.

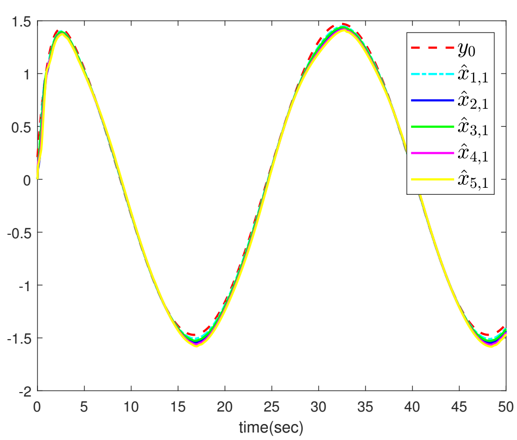


Figure 3: The outputs measured by sensors and the leader's output

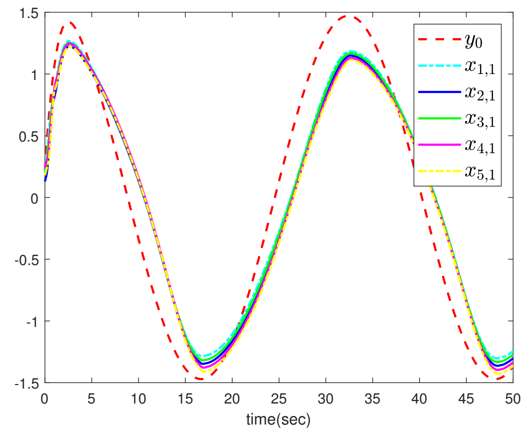


Figure 4: The genuine outputs of agents

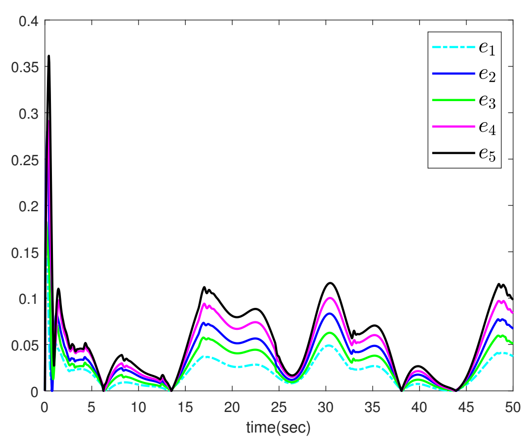
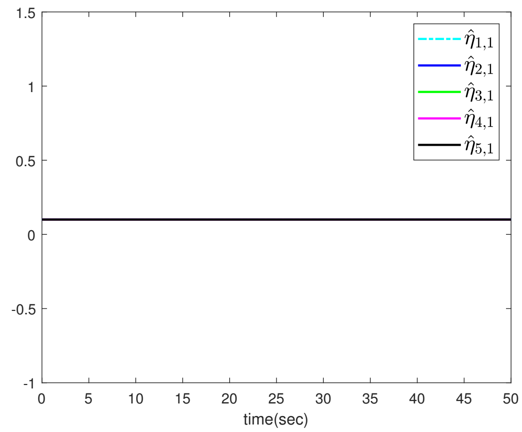
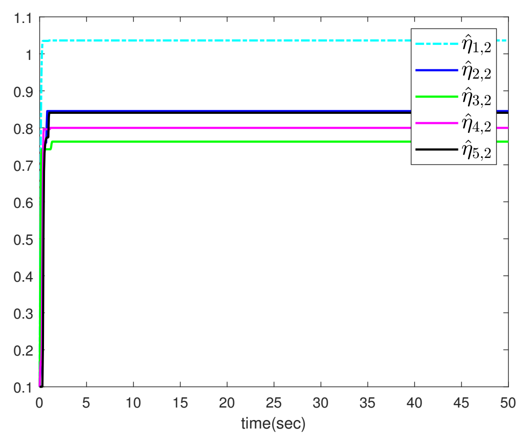


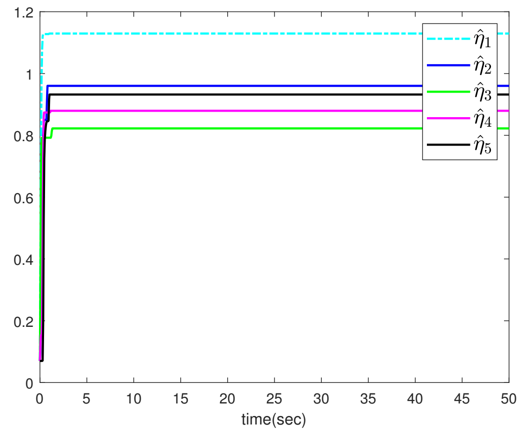
Figure 5: Consensus tracking errors



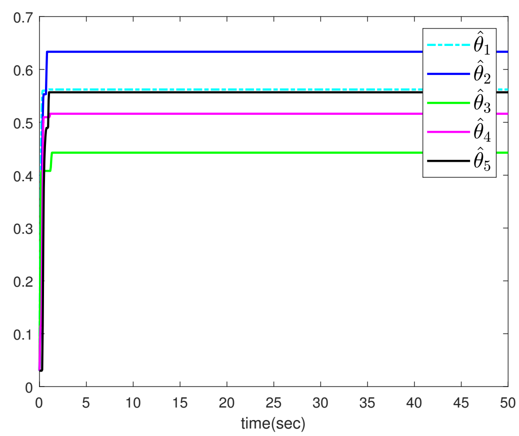
(a) Adaptive parameters $\hat{\eta}_{i,1}$



(b) Adaptive parameters $\hat{\eta}_{i,2}$



(c) Adaptive parameters $\hat{\eta}_i$



(d) Adaptive parameters $\hat{\theta}_i$

Figure 6: Adaptive parameters

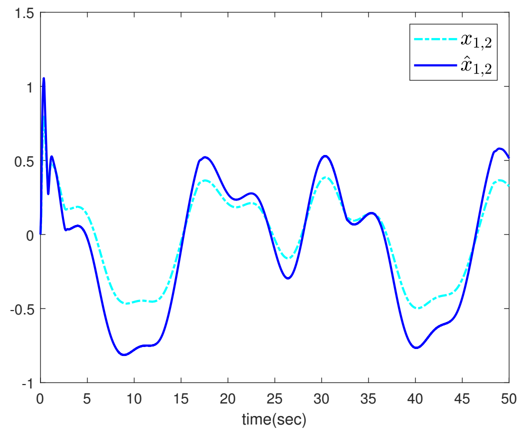


Figure 7: States of MASs $x_{1,2}$ and $\hat{x}_{1,2}$

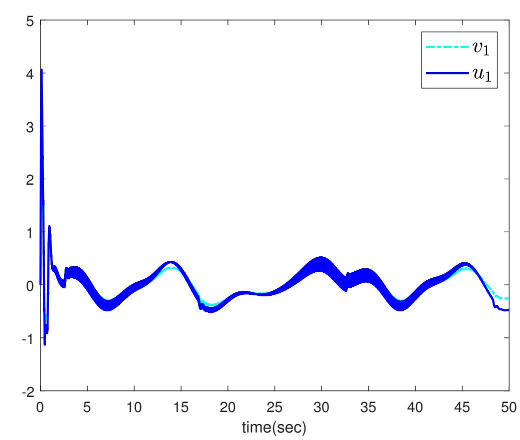


Figure 8: Designed control signal and actual control signal

5. Conclusion

The consensus control strategy for MASs with unknown states hysteresis and input hysteresis is researched. Based on back-stepping technique, we employ 2 adaptive laws to approximate the upper bound and lower bound of the unknown term introduced by states hysteresis. To handle the input hysteresis, NNs are utilized to approximate the unknown control gain which is coupled by input hysteresis and states hysteresis. Moreover, a series of differentiable functions are introduced in the design of Lyapunov function. The proposed scheme guarantees the boundedness of all signal and the consensus errors are ensured to converge to a predefined neighborhoods of zero asymptotically. In addition, the $\mathbb{L}_2$-norm of the consensus error can be further ensured. A simulation example is provided to illustrate the effectiveness of the proposed control approach.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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