Mechanical Buckling Analysis of Functionally Graded Plates Using an Accurate Shear Deformation Theory

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Mechanical Buckling Analysis of Functionally Graded Plates Using an Accurate Shear Deformation Theory

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Abstract: In this present study, we are interested in the use of a precise theory of shear deformation for the buckling analysis of plates with functional gradation simply supported such as the refined theory of plates with four variables, several parameters of comparison have been used, dimensional and non-dimensional, the displacement field is compatible with this study, the non-use of shear correction factors is satisfied, the choice of material is very precise in such a way are variable according to the thickness of the plate and on the other hand to make comparison with other researcher and confirms that this study gives precise results and converges, the transverse shear stresses vary through the thickness, the results found is also studied and discussed.

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INTRODUCTION

Functionally graded materials (FGM) and their excellent properties have led to a wide and rapid technological revolution for researchers and industry in various fields and that for his utilsations particular at very high temperatures. The basic concept of FGMs has been presented by Koizumi and other researchers through studies referencing in the literature[1-4].

For years several studies are based on numerous forms of structure such as plates that are widely used in many fields of engineering, tunnels, dams and buildings. Most of plated structures that can withstand tensile loads, poor resistance to compressive forces. Usually, the buckling phenomena observed in compressed plates occur quite suddenly and can lead to catastrophic structural rupture. It is therefore important to know the buckling capacities of the plates in order to avoid premature failure.

The subject of this research is the buckling behavior of structural elements in isotropic materials subjected to mechanical loads. Subsequently, many scientists developed equilibrium and stability equations for plates and shells in laminated and functionally graduated composite materials and used them to determine the buckling and the vibrational behavior of structures. [5].

In order to static and dynamic analysis of plate structures, a number of plate theories are available based on considering the transverse shear deformation of plate. The classical plate

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theory (CPT) in which the transverse shear deformation effects are neglected and the normal to the mid-plane remains straight and normal to the middle surface during the deformation. As a result, the classical plate theory usually underestimates deflection and overestimates the natural frequencies and buckling loads for thick plates [6].

The investigates the buckling responses of functionally graded material (FGM) plate subjected to uniform, linear, and non-linear in-plane loads. New nonlinear in-plane load models are proposed based on trigonometric and exponential function. Non-dimensional critical buckling loads are evaluated using non-polynomial based higher order shear deformation theory. Navier's method, which assures minimum numerical error, is employed to get an accurate explicit solution [7].

Nowadays, these types of materials are still treated as modern materials that, through varying different properties throughout their thickness, can carry loads in hard conditions, especially in high-temperature environment. The gradual changes in volume fraction of the components and non-homogenous structure allow continuous graded macroscopic properties to be obtained [8]. On the basis of a new theory of shear deformation and of the theory of modified torque stresses, the hygro-thermal buckling of microplates and porous microbeams in sandwich FGM is studied. Contrary to the classical theory of elasticity, implies a parameter of the material length scale and can thus capture the effect of small size. The theory of four variable shear deformation with a new shape function is used to derive the stability equations governing microplates and microbeams from the principle of virtual work [9].

Two new higher order transverse shear deformation theories (NHSDTs) with five variables have been proposed for the analysis of functionally graded material (FGM) plate. A governing differential equation (GDE) of the FGM plate is developed using energy principle [10].

A study of the pre- and post-buckling state of square plates built from functionally graded materials (FGMs) and pure ceramics is presented. In contrast to the theoretical approach, the structure under consideration contains a finite number of layers with a step-variable change in mechanical properties across the thickness. An influence of ceramics content on a wall and a number of finite layers of the step-variable FGM on the buckling and post-critical state was scrutinized by [10].

The functionally graded materials (FGMs) are microscopically inhomogeneous, in which the mechanical properties vary smoothly and continuously from one surface to the other. They are usually made from a mixture of ceramics and metals to attain the significant requirement of material properties. The determination of accurate behaviour of the FGMs largely depends on the theory used to model the structure because in the FGMs, material properties vary continuously as a function of position in the preferred direction. Various concepts have been developed to inculcate the appropriate analysis of the FGM plates. The classical Kirchhoff plate theory neglects transverse shear deformation and gives acceptable results for relatively thin plates. In order to circumvent this problem an earlier attempts were made by Reissner [11] and Mindlin [12]. However, a shear correction factor must be incorporated to overcome the problem of a constant transverse shear stress distribution and its value depends on various parameters, such as applied loads, boundary conditions and geometric parameters, etc. The inaccuracy occurs due to neglecting the effects of transverse shear and normal strains in the plate [12]. Due to continuous variation in material properties, the first-order shear deformation theory and higher order shear deformation theory may be conveniently used in the analysis. It is noted that the first-order shear deformation theory proposed by Mindlin [12] does not satisfy the parabolic variation of transverse shear strain in the thickness direction. Subsequently, many higher order theories were proposed, notable among them are [13–17]. The higher order theories assume the in-plane displacements as a cubic expression of the thickness coordinate and the out-of-plane displacement to be constant. Thus, the development of higher order shear deformation theory to assimilate the behaviour of FGM structures has been of high importance to the researchers.

In the literature, studies of the buckling of FGM structures are as follows, several researchers have studied different phenomena such as buckling analysis of FGM plates under uniform, linear and non-linear in-plane loading is studied by[18].

The thermal effect for FGM plates is interested in [19] under a work to determine by thermal buckling of clamped thin rectangular FGM plates resting on Pasternak elastic foundation.

Yaghoobi H and his co-authors examined the exact solution for thermal buckling of functionally graded plates resting on elastic foundations with various boundary conditions [20]. Czechowski L and his colleagues studied the study of buckling and post-buckling of a step-variable FGM box [21]. A new quasi-3D HSDT for buckling and vibration of FG plate was considered by [22].

In addition, much study has been done to the semi analytic for the circular plates as the research of [23] in a form of demonstration described under semi analytical solution for buckling analysis of variable thickness two-directional functionally graded circular plates with non uniform elastic foundations.

Shi-Rong Li et al. have study correspondence relations between deflection, buckling load, and frequencies of thin functionally graded material plates and those of corresponding homogeneous plates [24].

H. Farahmand et al. to develop with Navier's solutions the size effect that a study expressed by Navier solution for buckling analysis of size-dependent functionally graded micro-plates [25]. A theory of transverse shear deformation for homogeneous monoclinic plates has been studied by [26].

M. Karama et al. were interested in the mechanical behavior of laminated composite beams by the new model of multilayer laminated composite structures with continuity of transverse shear stresses [27]. B. Sidda Reddy et al. to use the theory of high order shear deformation to analyze the buckling of functionally graduated material plates [28]. Otherwise, the simple refined theory is used for the analysis of buckling of plates with functional gradation by [29].

The aspect of the new theory of shear deformation for several structures such as laminated composite plates is studied by [30].

In this study, a new and accurate shear deformation theory is presented for mechanical buckling analysis of functionally graded plates. Numerical examples cover the effects of the gradient index, plate aspect ratio, side-to-thickness ratio on the critical buckling load of functionally graded plates are investigated and discussed.

1 FORMULATION

Consider a functionally graded plate of thickness h, side length a in the x-direction, and b in the y-direction as shown in Figure 1.

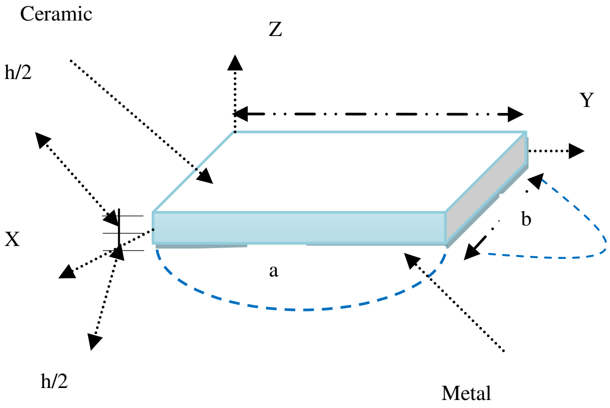

Fig 1. *Geometry for rectangular FG plate.*

The assumptions of the present theory are as follows; see Shimpi, [31] and Bouderba et al [32]:
- The transverse displacements are partitioned into bending and shear components;
- The in-plane displacement is partitioned into extension, bending, and shears components;
- The bending parts of the in-plane displacements are similar to those given by classical plate theory (CPT);
- The shear parts of the in-plane displacements give rise to the nonlinear variations of shear strains and hence to shear stresses through the thickness of the plate in such a way that the shear stresses vanish on the top and bottom surfaces of the plate.

Based on these assumptions, the following displacement field relations can be obtained

$$U(x,y,z)=u(x,y)-z\frac{\partial w_b}{\partial x}-\psi(z)\frac{\partial w_s}{\partial x}$$
$$V(x,y,z)=v(x,y)-z\frac{\partial w_b}{\partial y}-\psi(z)\frac{\partial w_s}{\partial y} \qquad (1)$$
$$W(x,y,z)=w_b(x,y)+w_s(x,y)$$

where U, V, W are displacements in the x, y, z directions, u, v and w_b , w_s are mid-plane displacements and $\psi(z)$ is a shape function that represents the distribution of the transverse shear strain and stress through the thickness, as presented in Table 1.
Table 1: Different shear shape strain functions.
Table 1. Different shear shape strain functions.

Model	$\psi(z)$ function
Ambartsumian [33]	$\psi(z)=\frac{z}{2}\left(\frac{h^2}{4}-\frac{z^2}{3}\right)$
Kaczkowski [34], Panc [35], Reissner [15]	$\psi(z)=\frac{5z}{4}\left(1-\frac{4z^2}{3h^2}\right)$
Levinson[36], Murthy [37] and Reddy [38]	$\psi(z)=z\left(1-\frac{4z^2}{3h^2}\right)$
Touratier [39]	$\psi(z)=\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$
Soldatos [26]	$\psi(z)=h\sinh\left(\frac{z}{h}\right)-z\cosh\left(\frac{1}{2}\right)$
Karama et al [27] and Aydogdu [30]	$\psi(z)=ze^{-2\left(\frac{z}{h}\right)^2}$

Consider a FG plate made of ceramic and metal, the material properties of FGM such as Young's modulus E are assumed to vary through the plate thickness with a power law distribution of the volume fraction of the two materials [40]:
$$E(z)=E_m+(E_{cm})V_C(z)$$
$$E_{cm}=E_c-E_m \qquad (2)$$
where E_c and E_m are the corresponding properties of the ceramic and metal, respectively, and k is the volume fraction exponent which takes values greater than or equal to zero. The volume fraction V_C (z) follows a simple power law as [41, 42]:
$$V_C(z)=((z/h)+0.5)^k$$
$$V_C+V_m=1 \quad ;$$

The kinematic relations can be obtained as follows:
$$\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}+\psi(z)\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix},$$
$$\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=g(z)\begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix}, \qquad (3a)$$
where:
$$\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{Bmatrix},\quad \begin{Bmatrix}k_x^b\\ k_y^b\\ k_{xy}^b\end{Bmatrix}=\begin{Bmatrix}-\frac{\partial^2 w_b}{\partial x^2}\\ -\frac{\partial^2 w_b}{\partial y^2}\\ -2\frac{\partial^2 w_b}{\partial x\partial y}\end{Bmatrix},$$
$$\begin{Bmatrix}k_x^s\\ k_y^s\\ k_{xy}^s\end{Bmatrix}=\begin{Bmatrix}-\frac{\partial^2 w_s}{\partial x^2}\\ -\frac{\partial^2 w_s}{\partial y^2}\\ -2\frac{\partial^2 w_s}{\partial x\partial y}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{yz}^s\\ \gamma_{xz}^s\end{Bmatrix}=\begin{Bmatrix}\frac{\partial w_s}{\partial y}\\ \frac{\partial w_s}{\partial x}\end{Bmatrix} \qquad (3b)$$
And $\psi(z)$ given by [9]:
$$\psi(z)=z-h\tan^{-1}\left(\frac{z}{h}\right)+\left(\frac{16z^3}{15h^2}\right),$$
$$g(z)=1-\frac{d\psi(z)}{dz} \qquad (4)$$
The linear constitutive relations are:

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&0\\ C_{12}&C_{22}&0\\ 0&0&C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x\\ \varepsilon_y\\ \gamma_{xy}\end{Bmatrix}$$
And

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{bmatrix} C_{44} & 0 \\ 0 & C_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \quad (5)$$

where (σ_x, σ_y, τ_{xy}, τ_{yz}, τ_{yx}) and (ε_x, ε_y, γ_{xy}, γ_{yz}, γ_{yx}) are the stress and strain components, respectively.

Stiffness coefficients, C_{ij}, can be expressed as

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2},\ C_{12} = \frac{\nu\, E(z)}{1-\nu^2},$$
$$C_{44} = C_{55} = C_{66} = \frac{E(z)}{2(1+\nu)}, \quad (6)$$

The strain energy of the plate can be written

$$U = \frac{1}{2}\int_V \begin{bmatrix} \sigma_x\,\varepsilon_x + \sigma_y\,\varepsilon_y + \tau_{xy}\,\gamma_{xy} \\ +\tau_{yz}\,\gamma_{yz} + \tau_{zx}\,\gamma_{zx} \end{bmatrix} dV \quad (7)$$

Substituting Eqs. (3) and (5) into Eq.(7) and integrating through the thickness of the plate, the strain energy of the plate can be rewritten as

$$U = \frac{1}{2}\int_A \Big[N_x\,\varepsilon_x^0 + N_y\,\varepsilon_y^0 + N_{xy}\,\varepsilon_{xy}^0 + M_x^b\,k_x^b + M_y^b\,k_y^b + M_{xy}^b\,k_{xy}^b + M_x^s\,k_x^s$$
$$+ M_y^s\,k_y^s + M_{xy}^s\,k_{xy}^s + S_{yz}^s\,\gamma_{yz}^s + S_{xz}^s\,\gamma_{xz}^s \Big] dxdy \quad (8)$$

Where the resultants forces, moments and shear forces, wich are all defined by

$$\left(N_x, N_y, N_{xy}\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) dz,$$
$$\left(M_x^b, M_y^b, M_{xy}^b\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) z dz \quad (9a)$$
$$\left(M_x^s, M_y^s, M_{xy}^s\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) \psi dz$$
$$\left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right) g dz \quad (9b)$$

Substituting Eq. (5) into Eq. (9) and integrating through the thickness of the plate, the stress resultants are given as

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix},$$
$$\begin{Bmatrix} S_{yz}^s \\ S_{xz}^s \end{Bmatrix} = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \quad (10)$$

Where

$$N = \left\{N_x, N_y, N_{xy}\right\}^t$$
$$M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}^t,$$
$$M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}^t,$$
$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\},$$
$$k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\},$$
$$k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}$$
$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$$
$$B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}$$
$$D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}$$
$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}$$
$$D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix} \quad (10a)$$

where A_{ij}, B_{ij}, etc., are the plate stiffness, defined by

$$\left\{A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}\right\} = \sum_{n=1}^{3} \int_{h_n}^{h_{n+1}} \left\{1, z, z^2, z^3, z^4, z^6\right\} C_{ij} dz, \quad (i, j = 1,2,6)$$

$$B_{ij}^s = -\frac{1}{4} B_{ij} + \frac{5}{3h^2} E_{ij}, \quad (i, j = 1,2,6)$$

$$D_{ij}^{s} = -\frac{1}{4}D_{ij} + \frac{5}{3h^2}F_{ij}, \quad (i, j = 1,2,6) \quad (11)$$

$$H_{ij}^{s} = \frac{1}{16}D_{ij} - \frac{5}{6h^2}F_{ij} + \frac{25}{9h^4}H_{ij}, \quad (i, j = 1,2,6)$$

$$\{A_{ij}, D_{ij}, F_{ij}\} = \sum_{n=1}^{3}\int_{h_n}^{h_{n+1}}\{1, z^2, z^4\}C_{ij}dz, \quad (i, j = 4,5)$$

$$A_{ij}^{s} = \frac{25}{16}A_{ij} - \frac{25}{6h^2}D_{ij} + \frac{25}{h^4}F_{ij}, \quad (i, j = 4,5)$$

The work done by applied forces can be written as

$$V = \frac{1}{2}\int_A \begin{bmatrix} N_x^0\left(\frac{\partial(\mathrm{w_b}+\mathrm{w_s})}{\partial x}\right)^2 \\ +N_y^0\left(\frac{\partial(\mathrm{w_b}+\mathrm{w_s})}{\partial y}\right)^2 \\ +2N_{xy}^0\frac{\partial(\mathrm{w_b}+\mathrm{w_s})}{\partial x}\frac{\partial(\mathrm{w_b}+\mathrm{w_s})}{\partial y} \end{bmatrix} dxdy \quad (12)$$

Where N_x^0 , N_y^0 , N_{xy}^0 are in-plane pre-buckling forces.

The governing equations of equilibrium can be obtained as follows

$$\delta u: \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0, \quad \delta v: \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0$$

$$\delta w_b: \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + P(w) = 0 \quad (13)$$

$$\delta w_s: \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x\partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} + P(w) = 0$$

Where

$$P(w) = N_x^0\frac{\partial^2(\mathrm{w_b}+\mathrm{w_s})}{\partial x^2} + N_y^0\frac{\partial^2(\mathrm{w_b}+\mathrm{w_s})}{\partial y^2} + 2N_{xy}^0\frac{\partial^2(\mathrm{w_b}+\mathrm{w_s})}{\partial x\partial y} \quad (14)$$

Substituting from Eq. (6) into Eq. (13), we obtain the following equation,

$$\begin{aligned} &A_{11}d_{11}u + A_{66}d_{22}u + (A_{12}+A_{66})d_{12}v - B_{11}d_{111}w_b \\ &-(B_{12}+2B_{66})d_{122}w_b - (B_{12}^s+2B_{66}^s)d_{122}w_s \\ &-B_{11}^s d_{111}w_s = 0, \end{aligned}$$

$$\begin{aligned} &A_{22}d_{22}v + A_{66}d_{11}v + (A_{12}+A_{66})d_{12}u \\ &-B_{22}d_{222}w_b - (B_{12}+2B_{66})d_{112}w_b \\ &-(B_{12}^s+2B_{66}^s)d_{112}w_s - B_{22}^s d_{222}w_s = 0, \end{aligned} \quad (15)$$

$$\begin{aligned} &B_{11}d_{111}u + (B_{12}+2B_{66})d_{122}u \\ &+(B_{12}+2B_{66})d_{112}v + B_{22}d_{222}v - D_{11}d_{1111}w_b \\ &-2(D_{12}+2D_{66})d_{1122}w_b \\ &-D_{22}d_{2222}w_b - D_{11}^s d_{1111}w_s \\ &-2(D_{12}^s+2D_{66}^s)d_{1122}w_s - D_{22}^s d_{2222}w_s \\ &+P(w) = 0 \end{aligned}$$

$$\begin{aligned} &B_{11}^s d_{111}u + (B_{12}^s+2B_{66}^s)d_{122}u \\ &+(B_{12}^s+2B_{66}^s)d_{112}v + B_{22}^s d_{222}v \\ &-D_{11}^s d_{1111}w_b - 2(D_{12}^s+2D_{66}^s)d_{1122}w_b \\ &-D_{22}^s d_{2222}w_b - H_{11}^s d_{1111}w_s \\ &-2(H_{12}^s+2H_{66}^s)d_{1122}w_s \\ &-H_{22}^s d_{2222}w_s + A_{55}^s d_{11}w_s \\ &+A_{44}^s d_{22}w_s + P(w) = 0 \end{aligned}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators:

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l},$$

$$d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i},$$

$$(i, j, l, m = 1,2). \quad (16)$$

Consider a simply supported rectangular plate with length *a* and width *b* which is subjected to in-pline loading in two directions $(N_x^0 = \gamma_1 N_{cr}, N_y^0 = \gamma_2 N_{cr}, N_{xy}^0 = 0)$.

Based on the Navier method, the following expansions of displacements (u, v, w_b, w_s)

$$\begin{Bmatrix} u \\ v \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\begin{Bmatrix} U_{mn}\cos(\lambda x)\sin(\mu y) \\ V_{mn}\sin(\lambda x)\cos(\mu y) \\ W_{bmn}\sin(\lambda x)\sin(\mu y) \\ W_{smn}\sin(\lambda x)\sin(\mu y) \end{Bmatrix} \quad (17)$$

where $\lambda = m\pi / a$, $\mu = n\pi / b$, and (U_{mn}, V_{mn}, W_{bmn}, W_{smn}) are unknown functions to be determined. Substituting Eq. (17) into Eq. (15), the closed-form solution of buckling load N_{cr} can be obtained from

$$\begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} \\ K_{12} & K_{22} & K_{23} & K_{24} \\ K_{13} & K_{23} & K_{33}+\overline{K} & K_{34}+\overline{K} \\ K_{14} & K_{24} & K_{34}+\overline{K} & K_{44}+\overline{K} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{bmn} \\ W_{smn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (18)$$

Where

$K_{11} = A_{11}\lambda^2 + A_{66}\mu^2$,

$K_{12} = \lambda\,\mu\left(A_{12} + A_{66}\right)$,

$K_{13} = -\lambda\,[B_{11}\lambda^2 + (B_{12} + 2B_{66})\,\mu^2]$,

$K_{14} = -\lambda\,[B_{11}^s\lambda^2 + (B_{12}^s + 2B_{66}^s)\,\mu^2]$,

$K_{22} = A_{66}\lambda^2 + A_{22}\mu^2$,

$K_{23} = -\mu\,[(B_{12} + 2B_{66})\,\lambda^2 + B_{22}\mu^2]$,

$K_{24} = -\mu\,[(B_{12}^s + 2B_{66}^s)\,\lambda^2 + B_{22}^s\mu^2]$,

$K_{33} = D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4$,

$K_{34} = D_{11}^s\lambda^4 + 2(D_{12}^s + 2D_{66}^s)\lambda^2\,\mu^2 + D_{22}^s\,\mu^4$,

$K_{44} = H_{11}^s\lambda^4 + 2(H_{11}^s + 2H_{66}^s)\lambda^2\mu^2 + H_{22}^s\mu^4 + A_{55}^s\lambda^2 + A_{44}^s\mu^2$,

$$\overline{K} = N_{cr}(\gamma_1\lambda^2 + \gamma_2\mu^2). \quad (19)$$

3 NUMERICAL RESULTS AND DISCUSSIONS

For having a numerical study and investigation, it is assumed that FGM plate is made of Aluminum (*Al*) as the metal part (E_m =70 GPa) and Alumina (*Al_2O_3*) as ceramic part (E_c = 380 GPa). Also, the Poisson ratio is constant and equal to v = 0.3.

Table 1: *Comparison of nondimensionalized critical buckling load ($\overline{N}$) of simply supported Al/Al_2O_3 plate subjected to uniaxial compression ($\lambda_1 = -1$, $\lambda_2 = 0$).*

b/h	Power-law index, K=0.1			
	CPT [28]	FSDT [28]	HSDT [28]	**Present RSDT**
10	17.68	16.76	16.74	**16.776**
20	17.68	17.4	17.41	**17.452**
40	17.68	17.62	17.59	**17.629**
50	17.68	17.64	17.61	**17.650**
100	17.68	17.67	17.64	**17.679**

b/h	K=1			
	CPT [28]	CPT [28]	CPT [28]	**Present RSDT**
10	9.78	9.78	9.78	**9.339**
20	9.78	9.78	9.78	**9.667**
40	9.78	9.78	9.78	**9.753**
50	9.78	9.78	9.78	**9.763**
100	9.78	9.78	9.78	**9.777**

b/h	K=10			
	CPT [28]	CPT [28]	CPT [28]	**Present RSDT**
10	5.87	5.87	5.87	**5.452**
20	5.87	5.87	5.87	**5.767**
40	5.87	5.87	5.87	**5.851**
50	5.87	5.87	5.87	**5.861**
100	5.87	5.87	5.87	**5.875**

In table 01 the effect of the uniaxial compressive load for a functionally graded plate simply supported in the (λ_1 = -1, λ_2 = 0) is clear, the results found by this theory (RSDT) converge with others theories such as CPT, FSDT and HSDT, we take three digits after the decimal point for more precision, the values of (*b / h*) increases, the results for (CPT) remains constant for the three values of (k), the transverse shear deformation effects of plate are not considered in the CPT, the values of nondimensional critical buckling load predicted by CPT are independent of thickness ratio, which accounts for the transverse shear deformation effects, are dependent of thickness ratio. While the CPT overestimates the nondimensional critical buckling load of functionally graded plate. The amplification of the critical load compared to the thickness is very clear, in this table the values of (FSDT) and (HSDT) are closer compared to the other. For the ratio (*b / h*=20) and a value of k = 0.1 there is also a change in critical buckling mode, On the other hand for the other three theories the values of increases with the increase of (*b / h*) and the volume fraction exponent (k).

Table2: *Comparison of nondimensionalized critical buckling load ($\overline{N}$) of simply supported Al/Al$_2$ O$_3$ plate subjected to uniaxial compression along the x-axis ($\lambda_1 = -1$, $\lambda_2 = 0$).*

a/b	*a/h*	*Theory*	Power-law index, k							
			0	0.5	1	2	5	10	20	100
0.5	5	*B. Sidda Reddy et al**	6.714	4.409	3.39	2.61	2.124	1.90	1.705	1.371
		*Thai and Choi**	6.7203	4.4235	3.4164	2.6451	2.1484	1.9213	1.7115	1.3737
		Present	**6.7203**	**4.4235**	**3.4164**	**2.6451**	**2.1484**	**1.9213**	**1.7115**	**1.3737**
	10	*B. Sidda Reddy et al**	7.397	4.81	3.70	2.87	2.40	2.18	1.93	1.5231
		*Thai and Choi**	7.405	4.82	3.71	2.88	2.41	2.18	1.93	1.52
		Present	**7.405**	**4.821**	**3.711**	**2.889**	**2.416**	**2.189**	**1.939**	**1.525**
	20	*B. Sidda Reddy et al**	7.590	4.924	3.78	2.95	2.48	2.26	2.00	1.56
		*Thai and Choi**	7.599	4.93	3.79	2.95	2.49	2.26	2.00	1.56
		Present	**7.599**	**4.931**	**3.793**	**2.958**	**2.494**	**2.269**	**2.005**	**1.568**
	50	*B. Sidda Reddy et al**	7.64	4.95	3.81	2.973	2.51	2.28	2.02	1.57
		*Thai and Choi**	7.65	4.96	3.81	2.97	2.51	2.29	2.025	1.58
		Present	**7.655**	**4.963**	**3.817**	**2.978**	**2.517**	**2.292**	**2.025**	**1.581**
	100	*B. Sidda Reddy et al**	7.65	4.96	3.81	2.97	2.51	2.292	2.02	1.58
		*Thai and Choi**	7.66	4.968	3.82	2.98	2.52	2.29	2.028	1.58
		Present	**7.663**	**4.968**	**3.820**	**2.981**	**2.520**	**2.296**	**2.028**	**1.583**
1	5	*B. Sidda Reddy et al**	16.00	10.57	8.146	6.23	4.97	4.44	3.98	3.25
		*Thai and Choi**	16.02	10.62	8.22	6.34	5.05	4.48	4.00	3.25
		Present	**16.021**	**10.625**	**8.224**	**6.343**	**5.053**	**4.481**	**4.007**	**3.259**
	10	*B. Sidda*	18.54	12.08	9.299	7.21	5.99	5.42	4.82	3.81

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		Reddy et al								
		Thai and Choi	18.57	12.12	9.33	7.26	6.03	5.45	4.83	3.81
		Present	**18.578**	**12.123**	**9.339**	**7.263**	**6.035**	**5.453**	**4.835**	**3.819**
	20	*B. Sidda Reddy et al**	19.31	12.53	9.649	7.51	6.32	5.75	5.08	3.98
		*Thai and Choi**	19.35	12.56	9.66	7.53	6.34	5.76	5.09	3.99
		Present	**19.353**	**12.567**	**9.667**	**7.537**	**6.345**	**5.767**	**5.099**	**3.992**
	50	*B. Sidda Reddy et al**	19.54	12.67	9.743	7.601	6.42	5.84	5.16	4.03
		*Thai and Choi**	19.58	12.69	9.763	7.61	6.43	5.8	5.17	4.04
		Present	**19.581**	**12.697**	**9.763**	**7.617**	**6.437**	**5.861**	**5.178**	**4.044**
	100	*B. Sidda Reddy et al**	19.57	12.69	9.75	7.61	6.43	5.86	5.17	4.04
		*Thai and Choi**	19.61	12.71	9.77	7.62	6.45	5.87	5.18	4.05
		Present	**19.614**	**12.716**	**9.777**	**7.629**	**6.451**	**5.875**	**5.189**	**4.051**
1.5	*5*	*B. Sidda Reddy et al**	28.15[a]	19.09[a]	14.76[a]	11.06[a]	8.25[a]	7.20[a]	6.56[a]	5.612[a]
		*Thai and Choi**	28.19[a]	19.25[a]	15.03[a]	11.42[a]	8.47[a]	7.29[a]	6.61[a]	5.63[a]
		Present	***28.199[a]***	***19.251[a]***	***15.034[a]***	***11.423[a]***	***8.473[a]***	***7.295[a]***	***6.610[a]***	***5.632[a]***
	10	*B. Sidda Reddy et al**	40.58[a]	26.72[a]	20.57[a]	15.81[a]	12.74[a]	11.42[a]	10.22[a]	8.27[a]
		*Thai and Choi**	40.74[a]	26.90[a]	20.80[a]	16.07[a]	12.95[a]	11.53[a]	10.29[a]	8.31[a]
		Present	**40.747[a]**	**26.909[a]**	**20.802[a]**	**16.079[a]**	**12.950[a]**	**11.538[a]**	**10.296[a]**	**8.311[a]**
	20	*B. Sidda Reddy et al**	45.64[a]	29.71[a]	22.85[a]	17.75[a]	14.81[a]	13.425[a]	11.90[a]	9.39[a]
		*Thai and Choi**	45.89[a]	29.90[a]	23.02[a]	17.92[a]	14.94[a]	13.52[a]	11.98[a]	9.44[a]
		Present	**45.893[a]**	**29.905[a]**	**23.029[a]**	**17.922[a]**	**14.947[a]**	**13.527[a]**	**11.984[a]**	**9.445[a]**
	50	*B. Sidda Reddy et al**	47.29[a]	30.67[a]	23.59[a]	18.39[a]	15.51[a]	14.12[a]	12.48[a]	9.76[a]
		*Thai and Choi**	47.57[a]	30.86[a]	23.74[a]	18.51[a]	15.628[a]	14.21[a]	12.56[a]	9.82[a]
		Present	**47.579[a]**	**30.869[a]**	**23.741[a]**	**18.518[a]**	**15.624[a]**	**14.215[a]**	**12.563[a]**	**9.821[a]**

	100	*B. Sidda Reddy et al**	47.53[a]	30.82[a]	23.69[a]	18.48[a]	15.62[a]	14.23[a]	12.57[a]	9.81[a]
		*Thai and Choi**	47.82[a]	31.01[a]	23.84[a]	18.60[a]	15.72[a]	14.31[a]	12.65[a]	9.87[a]
		Present	**47.829**[a]	**31.012**[a]	**23.847**[a]	**18.606**[a]	**15.725**[a]	**14.319**[a]	**12.650**[a]	**9.877**[a]
2	*5*	*B. Sidda Reddy et al**	37.67[b]	26.11[b]	20.29[b]	14.99[b]	10.65[b]	9.04[c]	8.317[c]	7.40[b]
		*Thai and Choi**	37.74[b]	26.36[b]	20.74[b]	15.58[b]	10.95[b]	9.15[c]	8.39[c]	7.44[b]
		Present	**37.740**[b]	**26.364**[b]	**20.749**[b]	**15.582**[b]	**10.955**[b]	**9.150**[c]	**8.399**[c]	**7.440**[b]
	10	*B. Sidda Reddy et al**	63.78[a]	42.14[a]	32.46[a]	24.86[a]	19.84[a]	17.72[a]	15.90[a]	12.96[a]
		*Thai and Choi**	64.08[a]	42.50[a]	32.89[a]	25.37[a]	20.21[a]	17.92[a]	16.02[a]	13.03[a]
		Present	**64.084**[a]	**42.501**[a]	**32.898**[a]	**25.373**[a]	**20.212**[a]	**17.923**[a]	**16.028**[a]	**13.034**[a]
	20	*B. Sidda Reddy et al**	73.80[a]	48.10[a]	37.00[a]	28.71[a]	23.86[a]	21.61[a]	19.18[a]	15.17[a]
		*Thai and Choi **	74.3[a]	48.49[a]	37.35[a]	29.05[a]	24.14[a]	21.81[a]	19.33[a]	15.27[a]
		Present	**74.314**[a]	**48.492**[a]	**37.356**[a]	**29.052**[a]	**24.141**[a]	**21.811**[a]	**19.338**[a]	**15.279**[a]
	50	*B. Sidda Reddy et al**	77.20[a]	50.09[a]	38.51[a]	30.02[a]	25.32[a]	23.04[a]	20.36[a]	15.93[a]
		*Thai and Choi **	77.80[a]	50.48[a]	38.83[a]	30.28[a]	25.53[a]	23.27[a]	20.53[a]	16.05[a]
		Present	**77.800**[a]	**50.489**[a]	**38.834**[a]	**30.286**[a]	**25.536**[a]	**23.228**[a]	**20.530**[a]	**16.056**[a]
	100	*B. Sidda Reddy et al**	77.71[a]	50.38[a]	38.74[a]	30.22[a]	25.54[a]	23.26[a]	20.55[a]	16.04[a]
		*Thai and Choi **	78.32[a]	50.78[a]	39.05[a]	30.47[a]	25.74[a]	23.45[a]	20.71[a]	16.17[a]
		Present	**78.326**[a]	**50.788**[a]	**39.054**[a]	**30.471**[a]	**25.749**[a]	**23.445**[a]	**20.713**[a]	**16.174**[a]

[a]Mode for plate is $(m,n) = (2, 1)$.

[b]Mode for plate is $(m,n) = (3, 1)$.

[c]Mode for plate is $(m,n) = (4, 1)$.

*Taken from *B. Sidda Reddy et a l*(2013)[28].

$$\overline{N} = N_{cr}\frac{a^2}{E_m h^3} \qquad (20)$$

For table 2 the plate FG subjected to a uniaxial compression along the axis x ($\lambda_1 = -1$, $\lambda_2 = 0$). The fixing mode for this plate is always simply supported, the ratio (*a/b*) varies with a regular step of 0.5 and for each step of (*a/b*) the ratio *(a/h)* varies from 5 to 100 , for fixed values of (*a/b*) and k, nondimensionalized critical buckling load increases with the ratio (*a/h*), for fixed values of (*a/b*) and (*a/h*), nondimensionalized critical buckling load decreases with the increase in k value, from a value of 1 for (*a/b*) the results in the table are almost doubled, For the ratio (*a/b*=2), (*a/h*=5) and all the values of k there is also a change in critical buckling mode, for a fixed value of k and increases in the values of the ratios (*a/b*) and (*a/h*) on an overestimation of nondimensionalized critical buckling load, on the other hand for fixed values of the ratios (*a/b*) and (*a/h*) and ascending values of k we have an underestimation of nondimensionalized critical buckling load.

Table 3: *Comparison of nondimensionalized critical buckling load ($\overline{N}$) of simply supported Al/Al_2O_3 plate subjected to biaxial compression ($\lambda_1 = -1$, $\lambda_2 = -1$).*

a/b	a/h	Theory	Power-law index, k							
			0	0.5	1	2	5	10	20	100
0.5	5	*B. Sidda Reddy et al**	5.371	3.527	2.715	2.092	1.700	1.527	1.364	1.097
		*Thai and Choi**	5.376	3.539	2.733	2.116	1.719	1.537	1.369	1.099
		Present	**5.3762**	**3.5388**	**2.7331**	**2.1161**	**1.7187**	**1.5370**	**1.3692**	**1.0989**
	10	*B. Sidda Reddy et al**	5.918	3.850	2.961	2.302	1.925	1.747	1.548	1.218
		*Thai and Choi**	5.926	3.857	2.969	2.312	1.933	1.752	1.551	1.220
		Present	**5.9243**	**3.8565**	**2.9689**	**2.3117**	**1.9332**	**1.7517**	**1.5509**	**1.2200**
	20	*B. Sidda Reddy et al**	6.072	3.940	3.029	2.362	1.991	1.812	1.602	1.253
		*Thai and Choi**	6.079	3.857	3.034	2.367	1.996	1.815	1.604	1.255
		Present	**6.0794**	**3.9452**	**3.0344**	**2.3665**	**1.9955**	**1.8152**	**1.6043**	**1.2546**
	50	*B. Sidda Reddy et al**	6.117	3.966	3.049	2.379	2.011	1.831	1.618	1.263
		*Thai and Choi**	6.124	3.971	3.053	2.382	2.014	1.834	1.620	1.265
		Present	**6.1244**	**3.9707**	**3.0533**	**2.3823**	**2.0137**	**1.8338**	**1.6199**	**1.2647**
	100	*B. Sidda Reddy et al**	6.123	3.970	3.052	2.382	2.014	1.834	1.620	1.265
		*Thai and Choi**	6.131	3.974	3.056	2.385	2.016	1.837	1.622	1.266
		Present	**6.1308**	**3.9744**	**3.0560**	**2.3846**	**2.0164**	**1.8365**	**1.6222**	**1.2662**
1	5	*B. Sidda Reddy et al**	8.001	5.288	4.073	3.120	2.487	2.221	1.994	1.626
		*Thai and Choi**	8.011	5.313	4.112	3.172	2.527	2.240	2.004	1.629
		Present	**8.0105**	**5.3127**	**4.1122**	**3.1713**	**2.5264**	**2.2404**	**2.0036**	**1.6294**
	10	*B. Sidda Reddy et al**	9.273	6.045	4.650	3.608	2.998	2.715	2.410	1.906
		*Thai and Choi**	9.289	6.062	4.670	3.632	3.018	2.726	2.417	1.910
		Present	**9.2897**	**6.0615**	**4.6694**	**3.6316**	**3.0176**	**2.7264**	**2.4173**	**1.9100**
	20	*B. Sidda Reddy et al**	9.658	6.270	4.821	3.757	3.162	2.876	2.544	1.992
		*Thai and Choi**	9.676	6.283	4.834	3.769	3.172	2.883	2.549	1.996
		Present	**9.6765**	**6.2833**	**4.8337**	**3.7685**	**3.1725**	**2.8834**	**2.5495**	**1.9961**
	50	*B. Sidda Reddy et al**	9.772	6.336	4.872	3.801	3.212	2.925	2.584	2.018
		*Thai and Choi**	9.791	6.349	4.882	3.809	3.219	2.931	2.589	2.0217
		Present	**9.7912**	**6.3485**	**4.8817**	**3.8085**	**3.2184**	**2.9305**	**2.5892**	**2.0217**
	100	*B. Sidda Reddy et al**	9.788	6.345	4.879	3.807	3.219	2.932	2.590	2.021
		*Thai and Choi**	9.807	6.358	4.889	3.815	3.225	2.938	2.595	2.025
		Present	**9.8072**	**6.3578**	**4.8886**	**3.8148**	**3.2254**	**2.9376**	**2.5951**	**2.0254**
1.5	5	*B. Sidda Reddy et al**	11.665	7.782	6.000	4.559	3.544	3.138	2.833	2.354
		*Thai and Choi**	11.682	7.830	6.080	4.664	3.618	3.172	2.851	2.360
		Present	**11.6819**	**7.829856,**	**6.0799**	**4.6637**	**3.6176**	**3.1718**	**2.8510**	**2.3599**
	10	*B. Sidda Reddy et al**	14.571	9.528	7.331	5.671	4.666	4.212	3.749	2.987
		*Thai and Choi**	14.608	9.569	7.379	5.728	4.712	4.238	3.766	2.996
		Present	**14.6084**	**9.5685**	**7.3793**	**5.7279**	**4.7124**	**4.2384**	**3.7657**	**2.9959**

	20	*B. Sidda Reddy et al**	15.542	10.098	7.766	6.046	5.075	4.611	4.082	3.203
		*Thai and Choi**	15.589	10.133	7.798	6.076	5.101	4.630	4.096	3.214
		Present	**15.5887**	**10.1332**	**7.7977**	**6.0761**	**5.1006**	**4.6299**	**4.0961**	**3.2135**
	50	*B. Sidda Reddy et al**	15.837	10.270	7.897	6.160	5.203	4.737	4.186	3.270
		*Thai and Choi**	15.888	10.336	7.924	6.182	5.221	4.753	4.200	3.280
		Present	**15.8875**	**10.3036**	**7.9236**	**6.1815**	**5.2212**	**4.7531**	**4.1995**	**3.2803**
	100	*B. Sidda Reddy et al**	15.880	10.295	7.916	6.177	5.222	4.756	4.201	3.280
		*Thai and Choi**	15.931	10.328	7.942	6.197	5.239	4.771	4.215	3.290
		Present	**15.9312**	**10.3284**	**7.9419**	**6.1969**	**5.2389**	**4.7712**	**4.2147**	**3.2900**
2	5	*B. Sidda Reddy et al**	15.698	10.581	8.172	6.156	4.661	4.088	3.712	3.143
		*Thai and Choi**	15.724	10.662	8.309	6.335	4.775	4.138	3.739	3.153
		Present	**15.7234**	**10.6622**	**8.3092**	**6.3353**	**4.7754**	**4.1382**	**3.7392**	**3.1534**
	10	*B. Sidda Reddy et al**	21.429	14.071	10.830	8.345	6.782	6.095	5.444	4.378
		*Thai and Choi**	21.505	14.155	10.932	8.464	6.875	6.148	5.477	4.396
		Present	**21.5049**	**14.1552**	**10.9323**	**8.4644**	**6.8749**	**6.1481**	**5.4769**	**4.3958**
	20	*B. Sidda Reddy et al**	23.590	15.346	11.802	9.177	7.674	6.964	6.171	4.858
		*Thai and Choi**	23.697	15.426	11.875	9.247	7.737	7.007	6.204	4.888
		Present	**23.6970**	**15.4260**	**11.8755**	**9.2469**	**7.7326**	**7.0067**	**6.2039**	**4.8802**
	50	*B. Sidda Reddy et al**	24.276	15.746	12.108	9.442	7.970	7.255	6.412	5.011
		*Thai and Choi**	24.394	15.824	12.170	9.493	8.013	7.293	6.444	5.036
		Present	**24.3944**	**15.8244**	**12.1699**	**9.4931**	**8.0132**	**7.2926**	**6.4440**	**5.0358**
	100	*B. Sidda Reddy et al**	24.378	15.805	12.153	9.482	8.015	7.299	6.448	5.034
		*Thai and Choi**	24.497	15.883	12.213	9.529	8.055	7.335	6.480	5.059
		Present	**24.4974**	**15.8830**	**12.2132**	**9.5294**	**8.0549**	**7.3353**	**6.4799**	**5.0589**

*Taken from *B. Sidda Reddy et al* (2013) [28].

Table 3 presents the results obtained by our theory and that of B. Sidda Reddy et al. and Thai and Choi. Pour nondimensionalized critical buckling load ($\overline{N}$) of simply supported Al/Al_2O_3 plate subjected to biaxial compression ($\lambda_1 = -1$, $\lambda_2 = -1$). for fixed values of (*a/b*) and k, nondimensionalized critical buckling load increases with the ratio (*a/h*), for fixed values of (*a/b*) and (*a/h*), nondimensionalized critical buckling load decreases with the increase in k value, from a value of 1 for (*a/b*) the results in the table are almost doubled. For a fixed value of k and increases in the values of the ratios (*a/b*) and (*a/h*) on an overestimation of nondimensionalized critical buckling load, on the other hand for fixed values of the ratios (*a/b*) and (*a/h*) and ascending values of k we have an underestimation of nondimensionalized critical buckling load. For the ratio (*a/b*=2), (*a/h*=5) and all the values of k there is also a change in critical buckling mode.

Table 4: *Comparison of nondimensionalized critical buckling load ($\overline{N}$) of simply supported Al/Al_2O_3 plate subjected to biaxial compression and tension ($\lambda_1 = -1$, $\lambda_2 = 1$).*

a/b	a/h	Theory	Power-law index, k							
			0	0.5	1	2	5	10	20	100
0.5	5	*B. Sidda Reddy et al**	8.953	5.879	4.525	3.487	2.833	2.545	2.274	1.829
		*Thai and Choi**	8.960	5.898	4.555	3.527	2.865	2.562	2.282	1.832
		Present	**8.9604**	**5.8980**	**4.5551**	**3.5268**	**2.8646**	**2.5617**	**2.2820**	**1.8316**
	10	*B. Sidda Reddy et al**	9.863	6.416	4.934	3.837	3.208	2.911	2.580	2.031
		*Thai and Choi**	9.874	6.428	4.948	3.853	3.222	2.200	2.585	2.033
		Present	**9.8738**	**6.4275**	**4.9481**	**3.8529**	**3.2219**	**2.9195**	**2.5849**	**2.0334**
	20	*B. Sidda Reddy et al**	10.120	6.566	5.049	3.936	3.319	3.020	2.670	2.089

		*Thai and Choi**	10.132	6.575	5.057	3.944	3.326	3.025	2.674	2.091
		Present	**10.1324**	**6.5753**	**5.0574**	**3.9442**	**3.3259**	**3.0253**	**2.6739**	**2.0911**
		*B. Sidda Reddy et al**	10.195	6.610	5.082	3.965	3.352	3.052	2.697	2.105
	50	*Thai and Choi**	10.207	6.618	5.089	3.971	3.356	3.056	2.700	2.108
		Present	**10.2073**	**6.6179**	**5.0889**	**3.9706**	**3.3562**	**3.0564**	**2.6999**	**2.1079**
		*B. Sidda Reddy et al**	10.206	6.616	5.087	3.969	3.356	3.057	2.700	2.108
	100	*Thai and Choi**	10.218	6.624	5.093	3.974	3.361	3.061	2.704	2.110
		Present	**10.2181**	**6.6240**	**5.0934**	**3.9744**	**3.3606**	**3.0609**	**2.7037**	**2.1103**
		*B. Sidda Reddy et al**	26.16[a]	17.63[a]	13.62[a]	10.26[a]	7.76[a]	6.81[a]	6.18[a]	5.23[a]
	5	*Thai and Choi**	26.20[a]	17.77[a]	13.84[a]	10.55[a]	7.95[a]	6.89[a]	6.23[a]	5.25[a]
		Present	**26.205**[a]	**17.771**[a]	**13.849**[a]	**10.558**[a]	**7.959**[a]	**6.897**[a]	**6.232**[a]	**5.255**[a]
		*B. Sidda Reddy et al**	35.71[a]	23.45[b]	18.04[a]	13.90a	11.30[a]	10.15[a]	9.07[a]	7.297[a]
	10	*Thai and Choi**	35.84[a]	23.59[b]	18.22[a]	14.10a	11.45[a]	10.24[a]	9.12[a]	7.32[a]
		Present	**35.842**[a]	**23.592**[b]	**18.222**[a]	**14.107**[a]	**11.459**[a]	**10.247**[a]	**9.128**[a]	**7.326**[a]
		*B. Sidda Reddy et al**	39.31[a]	25.57[a]	19.67[a]	15.29[a]	12.79[a]	11.60[a]	10.28[a]	8.09[a]
1	20	*Thai and Choi**	39.49[a]	25.71[a]	19.79[a]	15.41[a]	12.88[a]	11.67[a]	10.34[a]	8.13[a]
		Present	**39.497**[a]	**25.709**[a]	**19.793**[a]	**15.411**[a]	**12.888**[a]	**11.678**[a]	**10.340**[a]	**8.134**[a]
		*B. Sidda Reddy et al**	40.46[a]	26.24[a]	20.179[a]	15.73[a]	13.28[a]	12.09[a]	10.68[a]	8.35[b]
	50	*Thai and Choi**	40.65[a]	26.37[a]	20.283[a]	15.82[a]	13.35[a]	12.15[a]	10.74[a]	8.39[b]
		Present	**40.658**[a]	**26.375**[a]	**20.285**[a]	**15.821**[a]	**13.356**[a]	**12.154**[a]	**10.740**[a]	**8.393**[b]
		*B. Sidda Reddy et al**	40.62[a]	26.34[a]	20.25a	15.80[a]	13.35[a]	12.16[a]	10.74[a]	8.39[b]
	100	*Thai and Choi**	40.82[a]	26.47[a]	20.35[a]	15.88[a]	13.42[a]	12.22[a]	10.79[a]	8.43[b]
		Present	**40.831**[a]	**26.472**[a]	**20.355**[a]	**15.882**[a]	**13.425**[a]	**12.225**[a]	**10.800**[a]	**8.431**[b]
		*B. Sidda Reddy et al**	28.97[b]	19.92[b]	15.45[b]	11.47[b]	8.29[b]	7.15[b]	6.54[b]	5.72[b]
	5	*Thai and Choi**	29.02[b]	20.11[b]	15.78[b]	11.90[b]	8.52[b]	7.24[b]	6.60[b]	5.74[b]
		Present	**29.025**[b]	**20.110**[b]	**15.782**[b]	**11.901**[b]	**8.525**[b]	**7.242**[b]	**6.601**[b]	**5.748**[b]
		*B. Sidda Reddy et al**	37.884	24.773	19.060	14.745	12.133	10.950	9.748	7.767
	10	*Thai and Choi**	37.982	24.878	19.186	14.893	12.252	11.020	9.791	7.789
		Present	**37.9819**	**24.8781**	**19.1863**	**14.8925**	**12.2523**	**11.0199**	**9.7909**	**7.7894**
		*B. Sidda Reddy et al**	40.408	26.255	20.190	15.718	13.194	11.988	10.612	8.329
1.5	20	*Thai and Choi**	40.531	26.346	20.244	15.798	13.262	12.038	10.650	8.355
		Present	**40.5307**	**26.3463**	**20.2739**	**15.7979**	**13.2616**	**12.0379**	**10.6499**	**8.3550**
		*B. Sidda Reddy et al**	41.177	26.702	20.532	16.016	13.528	12.317	10.883	8.502
	50	*Thai and Choi**	41.308	26.789	20.601	16.072	13.575	12.358	10.919	8.529
		Present	**41.3076**	**26.7894**	**20.6013**	**16.0719**	**13.5752**	**12.3580**	**10.9186**	**8.5287**
		*B. Sidda Reddy et al**	41.289	26.768	20.582	16.059	13.577	12.365	10.923	8.527
	100	*Thai and Choi**	41.421	26.854	20.649	16.112	13.624	12.405	10.958	8.554
		Present	**41.4211**	**26.8539**	**20.6489**	**16.1118**	**13.6212**	**12.4052**	**10.9581**	**8.5541**
		*B. Sidda Reddy et al**	26.164	17.636	13.620	10.261	7.768	6.814	6.187	5.239
	5	*Thai and Choi**	26.206	17.770	13.849	10.559	7.959	6.897	6.232	5.256
		Present	**26.2057**	**17.7704**	**13.8486**	**10.5589**	**7.9589**	**6.8970**	**6.2320**	**5.2556**
2		*B. Sidda Reddy et al**	35.715	23.451	18.050	13.909	11.303	10.159	9.073	7.297
	10	*Thai and Choi**	35.842	23.592	18.221	14.107	11.458	10.247	9.128	7.326
		Present	**35.8416**	**23.5920**	**18.2206**	**14.1073**	**11.4583**	**10.2469**	**9.1281**	**7.3263**
	20	*B. Sidda Reddy et al**	39.317	25.576	19.670	15.295	12.791	11.607	10.286	8.096
		*Thai and Choi**	39.495	25.710	19.793	15.412	12.888	11.678	10.340	8.134

		Present	**39.4951**	**25.7100**	**19.7925**	**15.4115**	**12.8878**	**11.6779**	**10.3399**	**8.1336**
	50	*B. Sidda Reddy et al**	40.460	26.243	20.179	15.737	13.284	12.092	10.687	8.352
		*Thai and Choi**	40.657	26.374	20.283	15.822	13.355	12.154	10.740	8.393
		Present	**40.6573**	**26.3739**	**20.2833**	**15.8219**	**13.3554**	**12.1543**	**10.7401**	**8.3931**
	100	*B. Sidda Reddy et al**	40.629	26.341	20.254	15.803	13.358	12.165	10.747	8.390
		*Thai and Choi**	40.829	26.472	20.355	15.882	13.425	12.226	10.800	8.432
		Present	**40.8291**	**26.4717**	**20.3554**	**15.8823**	**13.4249**	**12.2255**	**10.7998**	**8.4315**

[a]Mode for plate is $(m,n) = (2, 1)$.

[b]Mode for plate is $(m,n) = (1, 2)$.

*Taken from *B. Sidda Reddy et al* (2013) [28].

Table 4 shows a comparison between the theory cited in the latter, Pour nondimensionalized critical buckling load ($\overline{N}$) of simply supported Al/Al_2O_3 plate subjected to biaxial compression and tension ($\lambda_1 = -1$, $\lambda_2 = 1$). for fixed values of (*a/b*) and k, nondimensionalized critical buckling load increases with the ratio (*a/h*), for fixed values of (*a/b*) and (*a/h*), nondimensionalized critical buckling load decreases with the increase in k value. For a fixed value of k and increases in the values of the ratios (*a/b*) and (*a/h*) on an overestimation of nondimensionalized critical buckling load, on the other hand for fixed values of the ratios (*a/b*) and (*a/h*) and ascending values of k we have an underestimation of nondimensionalized critical buckling load. For the ratio (*a/b*=1.5), (*a/h*=5) and all the values of k there is also a change in critical buckling mode.

Fig 2: *Effect of side-to-thickness ratios (a/h) on nondimensionalized critical buckling load ($\overline{N}$) under uniaxial compression for a simply supported FG plate for various material variation parameters (k).*

Fig 2: *Effect of side-to-thickness ratios (a/h) on nondimensionalized critical buckling load ($\overline{N}$) under uniaxial compression for a simply supported FG plate for various material variation parameters (k).*

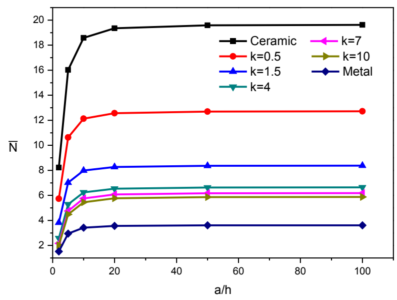

Fig 2: *Effect of side-to-thickness ratios (a/h) on nondimensionalized critical buckling load ($\overline{N}$) under uniaxial compression for a simply supported FG plate for various material variation parameters (k).*

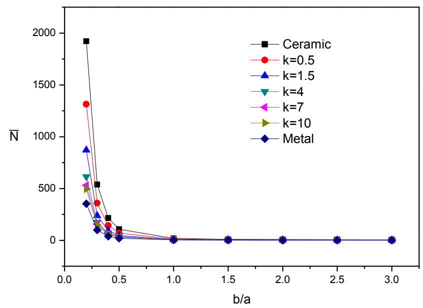

Fig 3: *Effect of aspect ratios (b/a) on nondimensionalized critical buckling load $\overline{N}$ under uniaxial compression for a simply supported FG plate for various material variation parameters (k) , (a = 10h).*

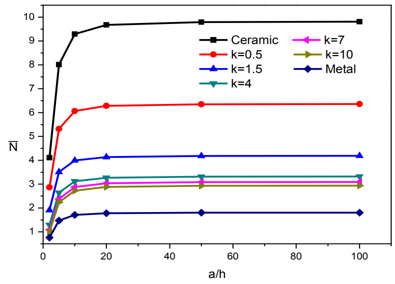

Fig 4: *Effect of side-to-thickness ratios (a/h) on nondimensionalized critical buckling load ($\overline{N}$) under biaxial compression for a simply supported FG plate for various material variation parameters (k).*

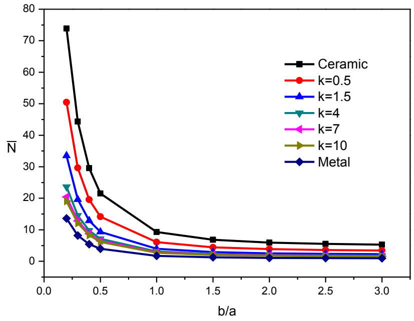

Fig 5: *Effect of aspect ratios (b/a) on nondimensionalized critical buckling load ($\overline{N}$) under biaxial compression for a simply supported FG plate for various material variation parameters (k), (a = 10h).*

For Figures 2 and 3, we studied Effect of side-to-thickness ratios (*a/h*) on nondimensionalized critical buckling load ($\overline{N}$) under uniaxial compression for a simply supported FG plate for various material variation parameters (k). The variation of this load is quite clear as a function of these ratios, these figures show that the critical dimensionless buckling loads can go up to a value of 20 for the plates rich in ceramic on the other hand for the plates rich in metal does not exceed not the value of 4 that is to say it is lower. The critical buckling loads of FG plates are intermediate to those of ceramic and metal. For values of k varies between 0.5 and 10, Figure 2 shows that the effect of shear deformation is significant for a ratio (*a/h*) less than 20 and decreases with values greater than 20. For Figure 3 The increase in the ratio (*b/a*) decreases the critical buckling load in such a way that it is almost convergent, which explains the decrease in the rigidity of the plate.

For Figures 4 and 5, The effect of side-to-thickness ratio (a / h) for values varying from 0 to 100, aspect ratio (b / a) for values varying from 0 to 30 and power law index values (k) for values varying from 0.5 to 10 on nondiensionalized critical buckling load for a simply supported FG plate under biaxial compression is shown in Figures 4 and 5. For the biaxial nondiensionalized critical buckling load is better converged for values greater than 30.

4CONCLUSIONS

In this study the plates with functional gradation were developed by the refined theory of four variables or one to study the behavior of mechanical buckling. For this theory the number of primary variables is still lower than that of the theories of shear deformation plates of first order and of higher level that implies the non-use of the shear correction factor. We can conclude that the present theory is precise and effective for predicting the mechanical buckling load of functionally graduated plates simply supported, For the certain ratio values (*a/b*), (*a/h*) and all the values of the volume fraction exponent (k), there is also a change in critical buckling mode, in certain cases the increase in the ratio (*b/a*) implies the reduction in the rigidity of the plate, in addition, the results found present a convergence with other semilaries and especially that of B. Sidda Reddy, this study remains useful to evaluate other future plate theories, Hence, it can be said that the proposed refined plate theory is accurate and simple in solving the buckling behaviour of FG plates.

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Figures

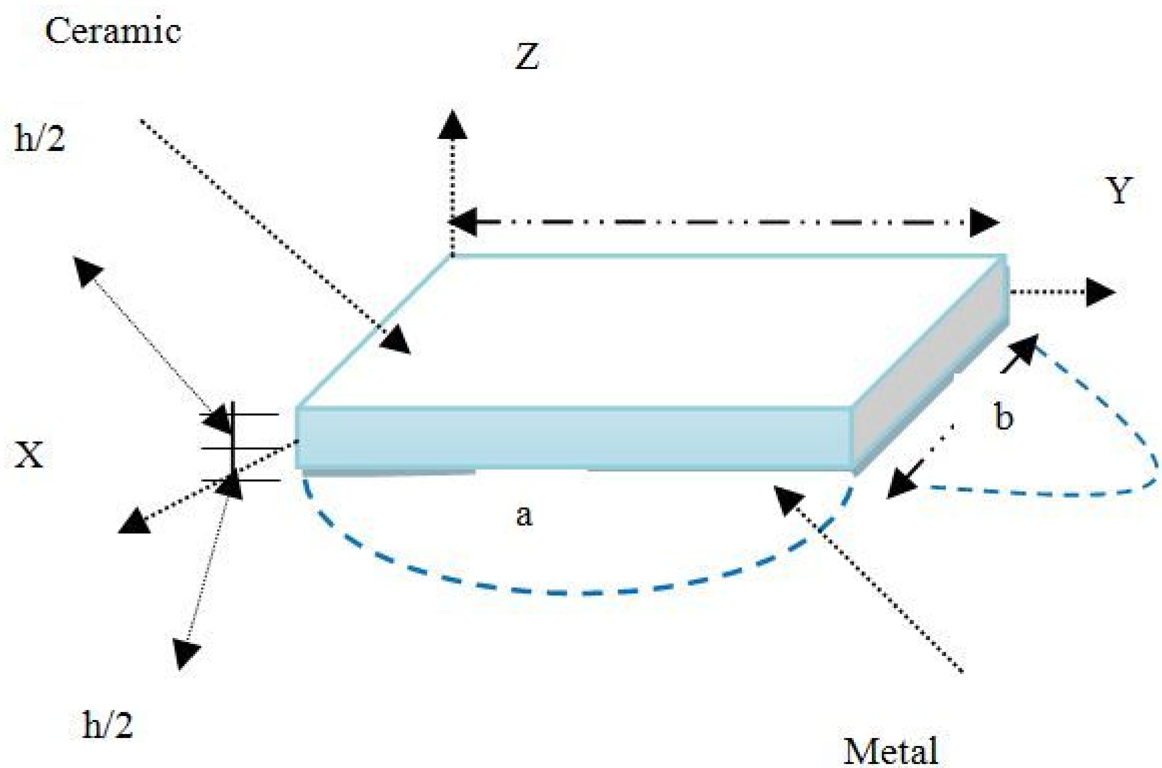

Figure 1

Geometry for rectangular FG plate

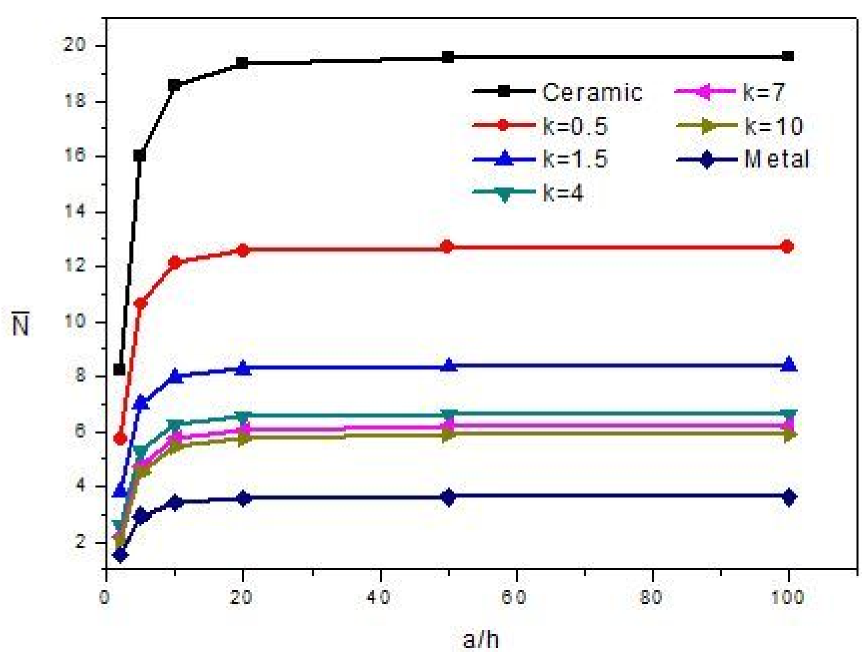

Figure 2

Effect of side-to-thickness ratios (a/h) on nondimensionalized critical buckling load ( ) under uniaxial compression for a simply supported FG plate for various material variation parameters (k).

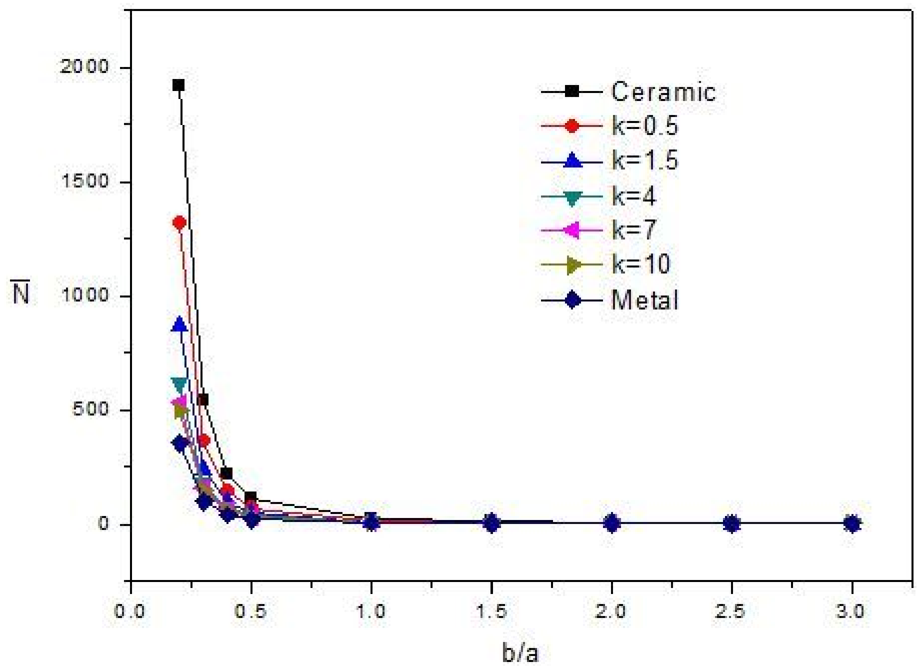

Figure 3

Effect of aspect ratios (b/a) on nondimensionalized critical buckling load under uniaxial compression for a simply supported FG plate for various material variation parameters (k) , (a = 10h).

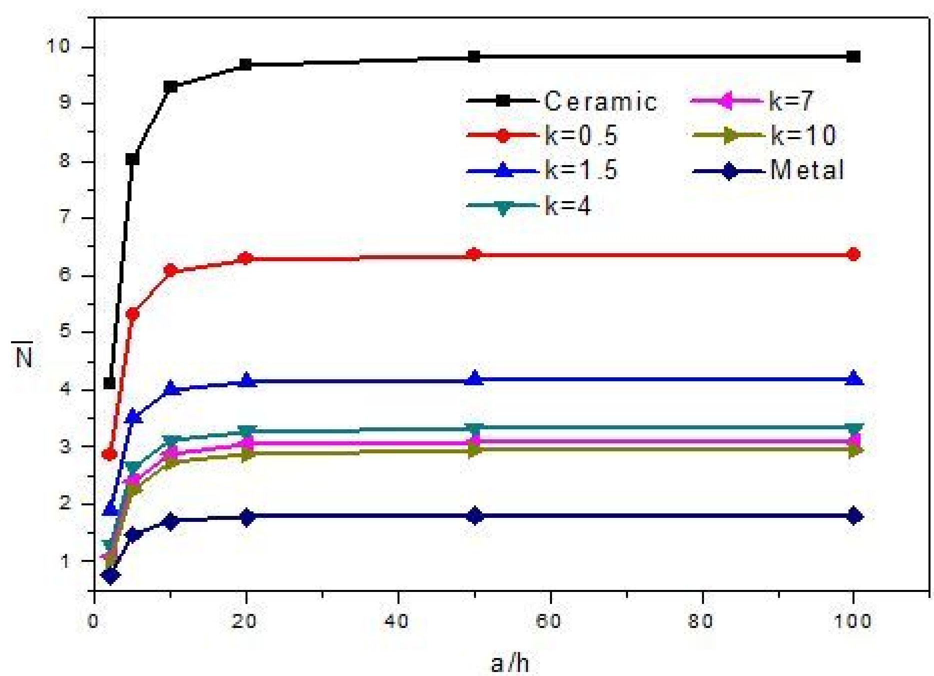

Figure 4

Effect of side-to-thickness ratios (a/h) on nondimensionalized critical buckling load ( ) under biaxial compression for a simply supported FG plate for various material variation parameters (k).

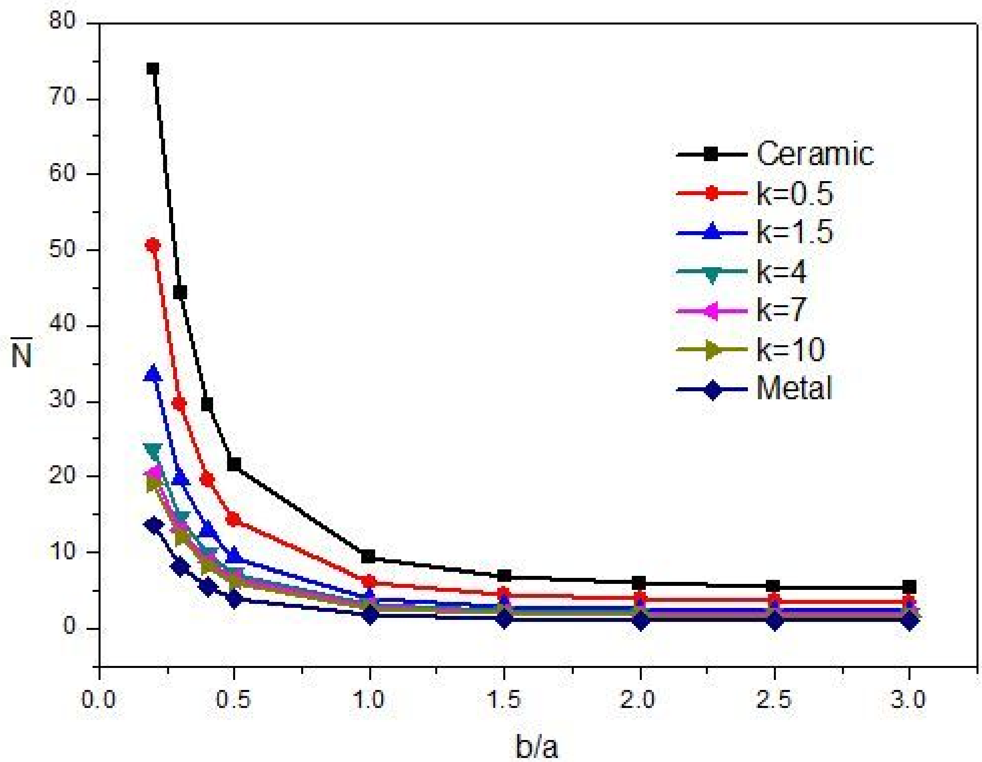

Figure 5

Effect of aspect ratios (b/a) on nondimensionalized critical buckling load ( ) under biaxial compression for a simply supported FG plate for various material variation parameters (k), (a = 10h).