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Study on Creep Damage Model of Yellow Sandstone with Step Loading and Unloading under the Action of Wetting-drying Cycles

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Abstract: This study performs uniaxial compression and step loading and unloading creep tests on rock samples under different wetting-drying cycles with yellow sandstone to investigate the creep characteristics of rock bodies after excavation or excavations with step loading and unloading under external forces. Moreover, it constructs a creep damage model based on the nonlinear rheological theory and damage mechanics to describe the instantaneous elastic strain, instantaneous plastic strain, nonlinear viscoelastic strain, and nonlinear viscoplastic strain of rocks. Besides, it proposes a creep model parameter identification method based on the creep curve characteristics. The results have shown that the action of wetting-drying cycles causes a weakened decrease in elastic modulus, uniaxial compressive strength, and long-term compressive strength of yellow sandstone, in line with the exponential function change relationship. The specimen damage type evolves from shear damage of a single oblique section to conical shear damage of a double oblique section. Additionally, the creep rate is not constant in the steady-state creep stage but constantly changes with time, and the fluctuation range increases with the increase in the wetting-drying cycles. The creep damage model parameters $E_{me}$, $E_{mp}$, $E_{ve}$, and $\eta_{ve}$ do not differ much under the same stress condition, and each parameter is influenced by the creep stress $\sigma$ at a specific law. The $E_{me}$, $E_{mp}$, and $\sigma$ have a linear relationship, whereas the $E_{ve}$, $\eta_{ve}$, and $\sigma$ have an exponential one.
1. Introduction

The creep is an inherent rock mechanical property involved in various rock engineering damage problems such as tunnel support, slope engineering design, and stability of the underground chamber in hydropower stations. Slope landslides are serious geological hazards under various climatic or construction working conditions (Fang et al. 2023). In water-related high steep rock slopes, part of the water-related rock body is in a periodic cycle of wetting and drying due to the change in the water level at the early stage. On the other hand, the slope deformation may experience a long-term multi-stage (or step) loading and unloading process under external forces, including engineering acting force, hydraulic pressure, and seismic forces, once the side slope excavation and unloading disturbance or excavation are completed. Creep is involved in the long-term stability of the project and cannot be neglected (Zhang et al. 2015; Wang et al. 2018) Hence, it is important to study the creep properties of yellow sandstone step loading and unloading under wetting-drying cycles.

Many scholars have investigated sandstone's creep deformation characteristics and constitutive model, explored the stress-strain versus time variation law during creep, and constructed a rich rock creep constitutive model (Guo et al. 2022; Zhao et al. 2021; Ping et al. 2016; Wei et al. 2022). Luo et al. (2018) conducted a series of wetting-drying cycle tests to investigate the relationship between the creep deformation of sandstone and its rate, long-term compressive strength, and the number of wetting-drying cycles. Liu et al. (2020) found that the sandstone microstructure changes from dense and compact to a loose and porous state after experiencing the action of wetting-drying cycles. Zhao et al. (2020) studied the creep deformation characteristics of red sandstone under multi-stage loading and unloading conditions. Studies on rock constitutive creep models mainly include empirical model theory, element combination model theory, and damage model theory (Gray et al. 2015; Andrews et al. 2015; Shan et al. 2021; Li et al. 2021; Wu et al. 2018). Cao et al. (2020) obtained the creep rate versus time by introducing time-hardening and damage evolution theories and constructed a unified creep model that can describe rock
creep's strong and weak creep behavior with a simple form and easy-to-find parameters. Hu et al. (2021) treated rock strength parameters as damage evolution with stress and time, developed a novel damage nonlinear viscoelastic body, and tied it to the Burgers model to quantitatively describe the accelerated creep properties of rocks. Zhang et al. (2021) and Li et al. (2021) proposed a nonlinear Kelvin model with a viscoelastic damage model to develop a creep damage model describing the nonlinear creep behavior of rocks under cyclic loading and unloading conditions.

In this paper, the nonlinear creep properties of yellow sandstone under different wetting-drying cycle conditions are obtained through the uniaxial step loading and unloading creep tests. Besides, a nonlinear creep damage model is developed based on damage mechanics and nonlinear rheological theory. The experimental data are analyzed, and the values of each creep parameter in the inverse performance model are fitted. The model is expected to effectively characterize the creep characteristics of yellow sandstone's step loading and unloading creep under wetting-drying cycles and reflect the nonlinear characteristics of the accelerated creep of yellow sandstone.

2. The test process and results analysis

2.1 Test system and process

Standard rock samples of φ50 × 100 mm were collected from the field, and their longitudinal wave velocity was tested. Thereafter, the samples with similar wave velocities were strictly selected for subsequent tests. Based on the changes in porosity of the rock samples measured by the weighing method during the wetting-drying cycles process, it was found that soaking yellow sandstone for 24 h and then drying it for 24 h provides a complete cycle of the wetting-drying process. Accordingly, 0, 1, 5, and 10 cycles of wetting-drying cycles tests were completed. The average compressive strength of the raw yellow sandstone sample was determined using the uniaxial compression test as 62 MPa, and the cyclic stresses at each level were set based on this strength value. In order to facilitate later data comparison and analysis, the test loading and unloading rates were set to 0.5 MPa/s, and each cycle was loaded to the set stress level and then held for 6 h. After that, the test was unloaded to 5 MPa
and held for 2 h, and this cycle was continued until the sample was destroyed. The uniaxial compression and creep tests were conducted using CDT1305-2 microcomputer-controlled electronic pressure tester and YAW-3000A electro-hydraulic pressure tester, respectively, and the test procedure is shown in Fig. 1.

![Diagram of test setup and flowchart](image)

2.2 Creep test results of the yellow sandstone grading and unloading

Fig. 2(a) shows the creep deformation curve of yellow sandstone after a different number of wetting-drying cycles. It can be seen that the creep deformation becomes large with the increase in the number of wetting-drying cycles and axial stress level. When the axial stress level is low ($\sigma \leq 0.8\sigma_c$), the specimen only experiences decay creep and steady-state creep. With the increase in time, the axial creep increases nonlinearly and then tends to a specific value. As shown in Fig. 2(b), (c), and (d), when the axial stress level is high ($\sigma = 0.9\sigma_c$), the specimen undergoes decay creep, steady-state creep, and accelerated creep until the final damage, and the overall creep rate shows a fast decreasing-roughly constant-fast increasing trend. The creep increases approximately linearly with time at creep time ($32.78 \leq t \leq 36.49$, $32.9 \leq t \leq 34.79$, and $32.58 \leq t \leq 34.07$) for the original, cyclic intrusion 5 times, and cyclic intrusion 10 times rock samples, respectively, with linear correlation coefficients of 0.9939, 0.9832, and 0.9963, respectively. Thus, they can be treated as stable creep stages. The creep rates of all specimens showed small fluctuations with increasing time, and the fluctuation range increased with the number of wetting-drying cycles,
i.e., there was no strict steady-state creep in the yellow sandstone under this test condition.

Fig. 2 Axial creep curves of yellow sandstone: (a) total creep curve; (b) original rock sample; (c) 5 times of wetting-drying cycles; (d) 10 times of wetting-drying cycles

Fig. 3 shows the long-term strength of yellow sandstone determined based on the steady-state creep rate under different test conditions. The stress-steady-state creep rate curve is plotted, where the horizontal coordinate of the tangent point p in this curve intersection is the long-term compressive strength value. The long-term compressive strength of yellow sandstone decreased by 2.32% and 3.19% after 5 and 10 cycles of wetting-drying, respectively.
Fig. 3 Long-term strength determined based on steady-state creep rate: (a) Raw rock samples; (b) 5 cycles of wetting-drying; (c) 10 cycles of wetting-drying

Fig. 3 also presents the yellow sandstone's creep damage pattern after different wetting-drying cycles. It can be seen that the original rock sample and 5 cycles of wetting-drying specimens contain only one penetrating overall oblique shear surface after damage, which is relatively complete and has less secondary crack development. In contrast, 10 cycles of wetting-drying specimens contain two groups of penetrating main cracks on the surface and are accompanied by the production of many secondary cracks, and large areas of rock pieces are detached near the main cracks. The damage type of the specimen gradually evolves from shear damage of a single oblique section to conical shear damage of a double oblique section.

3. Definition of damage variables for wetting-drying cycles and load action

The relationship between elastic modulus and uniaxial compressive strength of yellow sandstone after wetting-drying cycles and the number of wetting-drying cycles was obtained based on the uniaxial compressive strength tests of yellow sandstone after wetting-drying cycles for 0, 1, 5, and 10 times, as shown in Fig. 4. The modulus
of elasticity decreased by 28.94%, 39.81%, and 43.07%, and the compressive strength decreased by 2.98%, 9.37%, and 11.31%, respectively, after the wetting-drying cycles. After a certain number of wetting-drying cycles, the damage to the rock caused changes in the microstructure and deterioration in the material's mechanical properties, mainly in the elastic modulus. As a result, the relative change in the elastic modulus was used to define the damage caused by wetting-drying cycles and was combined with the fitted relationship between the elastic modulus and the number of wetting-drying cycles in Fig. 4.

\[
D_i = 1 - \frac{E_N}{E_0} = 0.4152 - 0.4152 \times \exp \left(-\frac{N}{0.8427}\right)
\]

where \(E_0\) is the value of the elastic modulus of yellow sandstone before wetting-drying cycles, and \(E_N\) is the value of the elastic modulus of yellow sandstone after \(N\) cycles of wetting-drying.

**Fig. 4** Modulus of elasticity and compressive strength versus the number of wetting-drying cycles

The simultaneous stress-strain curves of the yellow sandstone at the stable creep stage were obtained by processing the experimental data shown in Fig. 2(a) as shown in Fig. 5. The stress-strain curve modulus of elasticity was found to decrease with the increase in load holding time and gradually converge to a constant value. At low-stress levels, the simultaneous stress-strain curve was kept as a straight line, and the average value of the elastic modulus of the corresponding curve for the straight part (stress level less than 43.4 MPa) was obtained, as shown in Fig. 6.
Fig. 5 Time-stress-strain curves for different rock samples: (a) Original rock sample; (b) Wetting-drying cycles 5 times; (c) Wetting-drying cycles 10 times

The damage produced by rock creep conditions is reflected in the transient loading and aging creep damages. Additionally, since the former accounts for a smaller proportion, the aging creep damage was mainly considered in this paper. Ma et al. (2022) proposed the damage evolution model in Eq. 2.

Fig. 6 Modulus of elasticity versus time
\[ D'_t = D_t = \frac{E_0 - E'_\infty}{E_0} [k - \exp(-at)] = A[k - \exp(-at)] \]  

(2)

where \( k \), \( a \) is the coefficient related to the degree of damage. \( a \) mainly reflects the speed of the damage variable tends to zero and \( k \) mainly reflects the initial damage variable size. \( E'_\infty \) is the long-term modulus of elasticity of the specimen in the stable creep stage.

Combined with the relationship between damage factor and elastic modulus time at different moments of Eq. 2, the elastic modulus at different ages was fitted to the original rock sample to obtain the following relation:

\[ D'_t = 0.09 [1 - \exp(-1.5t)] \]  

(3)

Comparing Eqs. 2 and 3, the correlation coefficients for the degradation effect of age-related creep damage in the steady-state creep phase of yellow sandstone under different test conditions can be obtained as shown in Table 1:

<table>
<thead>
<tr>
<th>Type</th>
<th>( E'_\infty ) (GPa)</th>
<th>( a )</th>
<th>( k )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original rock sample</td>
<td>5.228</td>
<td>1.50</td>
<td>1.00</td>
<td>0.981</td>
</tr>
<tr>
<td>5 cycles of wetting-drying</td>
<td>4.338</td>
<td>1.27</td>
<td>1.00</td>
<td>0.986</td>
</tr>
<tr>
<td>10 cycles of wetting-drying</td>
<td>4.327</td>
<td>0.85</td>
<td>1.00</td>
<td>0.992</td>
</tr>
</tbody>
</table>

Table 1 Parameters related to the effect of aging creep damage

Considering the rock damage caused by wetting-drying cycles as microscopic damage and the damage caused by creep loading as macroscopic damage, the coupled damage variables superposition is shown in Fig. 7. Under Lemaitre strain equivalence hypothesis (2019), combined with the definition of damage variables previously mentioned, the rock coupled damage variables were obtained as:
Fig. 7 Coupled damage variables: (a) macro-micro damage; (b) microdamage; (c) macro damage; (d) no damage

\[
D'_{12} = 1 - (1-D_1)(1-D_2) = 1 - \frac{(E_N - A \frac{E_N}{E_0})e^w + A \frac{E_N}{E_0}}{1 - A + A \frac{E_N}{E_0}} = 1 - \frac{f \cdot e^w + h}{m \cdot e^w + n}
\] (4)

where \((E_N/E_0) - A(E_N/E_0) = f\), \(A(E_N/E_0) = h\), \(1 - A + A(E_N/E_0) = m\), and \(A(E_N/E_0) = n\).

When \(D_1 = 0\) and \(D'_{12} = D'_1\), the rock exhibits macroscopic damage under load only. In addition, when \(D_2 = 0\) and \(D'_{12} = D'_1\), only microscopic damage by wetting-drying cycles is presented, where \(D'_{12}\) reflects the evolution of time-dependent creep damage under the coupled wetting-drying cycles-load effect.

4. Creep damage model

The total deformation \(\varepsilon\) of yellow sandstone under creep stress can be obtained by combining Fig. 8 with the analysis of the test results in Section 2.3. It can be seen that the total deformation \(\varepsilon\) is divided into time-independent instantaneous strain \(\varepsilon_m\) and time-dependent creep strain \(\varepsilon_v\). Among them, instantaneous strain can be divided into instantaneous elastic strain \(\varepsilon_{me}\) and instantaneous plastic strain \(\varepsilon_{mp}\), and creep strain can be divided into viscoelastic strain \(\varepsilon_{ve}\) which can be gradually recovered with time and non-recoverable viscoplastic strain \(\varepsilon_{vp}\).
\[ \begin{align*}
\varepsilon &= \varepsilon_m + \varepsilon_v = \varepsilon_{\text{mes I}} + \varepsilon_{\text{mes II}} + \varepsilon_{\text{mp}} + \varepsilon_{\text{ve}} + \varepsilon_{\text{vp}} \\
\varepsilon_m &= \varepsilon_{\text{mes}} + \varepsilon_{\text{mp}} \\
\varepsilon_v &= \varepsilon_{\text{ve}} + \varepsilon_{\text{vp}} \\
\varepsilon_{\text{rp}} &= \varepsilon_{\text{mp}} + \varepsilon_{\text{mes I}} + \varepsilon_{\text{vp}}
\end{align*} \] (5)

where \( \varepsilon_{\text{rp}}, \varepsilon_{\text{mes I}}, \) and \( \varepsilon_{\text{mes II}} \) are the residual strain, partially recovered elastic strain I, and unrecovered elastic strain II, respectively.

**Fig. 8** Typical creep test curve with and without unloading

4.1. Consider transient elastic and transient plastic bodies after damage

Based on the creep deformation of yellow sandstone at loading and strain recovery characteristics at unloading, the creep test curve can be decomposed by Eq. 5, and each instantaneous strain at different stress levels can be obtained after processing, as shown in Fig. 9.

**Fig. 9** Relationship between instantaneous elastic and plastic deformation and axial stress level

Since the stress loading phase is very short compared to the creep phase, the
instantaneous deformation can be considered completed instantaneously and is not related to time but to stress only. Hence, only the damage caused by wetting-drying cycles must be considered. The transient deformation and stress are approximately linear. Thus, the properties of the transient elastomer can be described in series with the transient plastic body, and the mechanical model, upon considering the damage, can be obtained, as shown in Fig. 10.

\[
\begin{align*}
\sigma & = E_{me}(D_1) \varepsilon_{me} + E_{mp}(D_1) \varepsilon_{mp} \\
\varepsilon_{me} & = \frac{\sigma}{E_{me}(1 - D_1)} \\
\varepsilon_{mp} & = \frac{\sigma}{E_{mp}(1 - D_1)}
\end{align*}
\]

where \(\sigma\) is the stress and \(E_{me}\) and \(E_{mp}\) are the instantaneous modulus of elasticity of the elastomer and plastic body.

4.2. Consider nonlinear viscous elastomer after damage

The loading and unloading creep curves exhibit prominent nonlinear characteristics, which cannot be described using the traditional basic rheological model. Therefore, it is necessary to establish a nonlinear Kelvin model to describe the creep deformation characteristics after considering the damage. The essential rheological model elements that can realize the parameter nonlinearization process include elastic and viscous elements, and their mechanical models are shown in 11:

\[
\begin{align*}
\sigma & = E_{ve}(D'_{12}) \varepsilon_{ve} + \eta_{ve}(D'_{12}) \varepsilon_{ve} \\
\varepsilon_{ve} & = \frac{\sigma}{E_{ve}(1 - D'_{12})}
\end{align*}
\]

where \(E_{ve}\) and \(\eta_{ve}\) are the instantaneous modulus of elasticity and viscosity of the elastomer, respectively.
According to Eq. 7 with the Kelvin model, the damage instantiation equation of the spring element and Newtonian viscous pot can be obtained as:

\[
\begin{aligned}
E_{ve}(t) &= E_{ve}(1-D_{ve}') = E \frac{f \cdot \exp(at) + h}{m \cdot \exp(at) + n} \\
\eta_{ve}(t) &= \eta_{ve}(1-D_{ve}') = \eta \frac{f \cdot \exp(at) + h}{m \cdot \exp(at) + n}
\end{aligned}
\] (7)

Substituting Eq. 7 into Eq. 8, the creep state model of the elastic damage element and viscous damage element can be obtained by combining the solution of the first-order non-simultaneous linear differential equation as:

\[
\begin{aligned}
\varepsilon_1(t) &= \frac{\sigma}{E_{ve}(t)} \\
\varepsilon_2(t) &= \frac{\sigma t}{\eta_{ve}(t)}
\end{aligned}
\] (8)

The damage degradation produced by the viscous elastic elements in the Kelvin body damage model is not correlated with each other but only with time, so the principal equation of the Kelvin body damage model can be obtained as follows:

\[
\sigma = \sigma_1 + \sigma_2 = \frac{\sigma_0 (m \cdot \exp(at) + n)}{E_{ve}(f \cdot \exp(at) + h)} + \frac{\sigma_0 n \ln(f \cdot \exp(at) + h) + m \ln(f \cdot \exp(at) + h) + nt}{ah} + \frac{\sigma_0 n \ln(f \cdot \exp(at) + h) + m \ln(f \cdot \exp(at) + h) + nt}{ah} + \frac{\sigma_0 n \ln(f \cdot \exp(at) + h) + m \ln(f \cdot \exp(at) + h) + nt}{ah}
\] (9)

Assuming unloading at \( t = t_0 \), the left side of Eq. 10 is 0. Integrating it and combining it with Eq. 11 yields the creep deformation during the unloading phase as:

\[
\varepsilon_{ve} = \frac{\sigma}{E_{ve}} \left( \frac{m \cdot \exp(at) + n}{f \cdot \exp(at) + h} \right) \left( \frac{E_{ve} t}{\eta_{ve}} \right)
\] (11)

where \( E_{ve} \) and \( \eta_{ve} \) are the modulus of elasticity and viscosity coefficient of the linear viscous elastomer, respectively, and \( t \) is the creep time.
4.3. Consider nonlinear viscoplastic body after damage

When the creep test applied pressure level exceeds its long-term compressive strength, the rock will go through a short period of decay, steady-state creep soon into an accelerated creep, creep deformation, and the deformation rate increases rapidly with time, showing obvious nonlinear characteristics.

The damage variable was proposed by Kachanvo (1992), and its evolution equation is expressed as:

\[
D_c = A\sigma^V (1 - D_c)^{-V}
\]  

(13)

where \( A, V \) are the material constants, and \( D_c \) is the age damage parameter of rock creep. The integration of the above equation yields the time \( t_F \) at which creep damage occurs as:

\[
t_F = \left[ A(1+V)\sigma^V \right]^{-1}
\]  

(14)

Combining Eqs. (13) and (14) yield the following equation for the evolution of the damage variable with time:

\[
D_c = 1 - \left( 1 - \frac{t}{t_F} \right)^{\frac{1}{1+V}}
\]  

(15)

According to the Lemaitre strain equivalence principle (Song et al. 2015; Hao et al. 2013), the creep damage \( D_c \) is introduced into the viscous body with the principal differential equation in the one-dimensional stress state as Eq. 16. Moreover, the wetting-drying cycles damage \( D_1 \) (initial damage) is introduced into the creep damage stress \( \sigma_s \) using Eq. 17 (Hou et al. 2019). The mechanical model of the viscoplastic body after considering the damage is shown in Fig. 13.

\[
\varepsilon_{vp} = \frac{\sigma}{\eta_{vp}} = \frac{\sigma'}{\eta_{vp}} = \frac{\sigma}{\eta_{vp}(1-D_c)}
\]  

(16)

where \( \sigma, \sigma' \), and \( \sigma_s \) are the nominal effective stress, effective stress, and yield stress,
respectively, and $\eta_{vp}$ is the viscosity coefficient of the damaged viscous body.

$$\sigma_s = \sigma_c + KD + B \quad (17)$$

where $\sigma_c$ is the uniaxial compressive strength, and $K$ and $B$ material constants.

The relationship between yield stress and wetting-drying cycles damage can be obtained by combining Fig. 3 with Eq. 1, as shown in Fig. 12. Combining the fitted equations in the figure with Eq. 17 yields $K$ and $B$ equaling 12.397 and -9.914, respectively.

![Fig. 12 Relationship between yield stress and wetting-drying cycles damage](image)

![Fig. 13 Nonlinear viscoplastic body](image)

Substituting Eq. 15 into Eq. 16 yields the principle differential equation for the damaged viscoplastic body:

$$\dot{\varepsilon}_{vp} = \frac{\sigma - \sigma_s}{\eta_{vp}} \left(1 - \frac{t}{t_f}\right)^{\frac{1}{4+1}} \quad (18)$$

Integrating Eq. 18 and bringing in the starting condition $t = 0$ at $\varepsilon_{vp} = 0$ and Eq. 17 yields the following:
The creep damage model, after considering the damage, is obtained by connecting the above creep model elements in series, as shown in Fig. 14:

![Creep mechanical damage model](image)

**Fig. 14** Creep mechanical damage model

Eq. 20 can be obtained according to the series-parallel stress-strain relationship of the creep model in Fig. 14.

\[
\begin{align*}
\sigma &= \sigma_{me} = \sigma_{mp} = \sigma_{ve} = \sigma_{vp} \\
\varepsilon &= \varepsilon_{me} + \varepsilon_{mp} + \varepsilon_{ve} + \varepsilon_{vp} \\
\sigma_{me} &= \varepsilon_{me} E_{me} (1 - D_1) \\
\sigma_{mp} &= \varepsilon_{mp} E_{mp} (1 - D_1) \\
\sigma_{ve} &= E_{ve} (1 - D_{12}) \varepsilon_{ve} + \eta_{ve} (1 - D_{12}) \varepsilon_{ve} \\
\sigma_{vp} &= \eta_{vp} (1 - D_s) \varepsilon_{vp} + \sigma_s
\end{align*}
\]

(20)

The principal differential equation of the creep damage model can be obtained according to Fig. 14 and Eq. 20 as follows:

1. When \( \sigma < \sigma_s \)

\[
\left( \frac{E_{me} E_{mp} (1 - D_1) + E_{ve} (E_{me} + E_{mp}) (1 - D_{12})}{(E_{me} + E_{mp}) (1 - D_1) \eta_{ve} (1 - D_{12})} \right) \sigma + \frac{E_{me} + E_{mp}}{E_{me} E_{mp} (1 - D_1)} \varepsilon = \frac{E_{me} + E_{mp}}{\eta_{ve}} \varepsilon
\]

(21)

2. When \( \sigma_s \leq \sigma \)

\[
\left( \frac{E_{me} (1 - D_1) + E_{ve} (E_{me} + E_{mp}) (1 - D_{12})}{E_{me} E_{mp} (1 - D_1)} \right) \sigma + \frac{E_{ve} + E_{mp}}{E_{me} E_{mp} \eta_{ve} (1 - D_1) (1 - D_{12})} \varepsilon = \frac{E_{me} + E_{mp}}{\eta_{ve}} \varepsilon
\]

(22)

The creep damage loading creep equation of the rock can be obtained according
to the superposition principle as shown in Eq. 23:

\[
\varepsilon = \frac{\sigma}{E_{me}(1-D_1)} + \frac{\sigma}{E_{mp}(1-D_1)} + \frac{\sigma}{E_{ve}} \left(\frac{(m \cdot \exp(at) + n)}{(f \cdot \exp(at) + h)} \exp\left(-\frac{E_{me}}{\eta_{ve}} t\right)\right) + \frac{\sigma}{E_{ve}} \left(\frac{(m \cdot \exp(at) + n)}{(f \cdot \exp(at) + h)} \exp\left(-\frac{E_{me}}{\eta_{ve}} t\right)\right) + \frac{(\sigma - \sigma_c - KD_c - B)}{\eta_{vp}} \frac{t_f}{V} (V + 1) \left(1 - (1 - D_c)^{\gamma}\right) \quad \sigma < \sigma_c
\]

\[
\varepsilon = \frac{\sigma}{E_{me}(1-D_1)} + \frac{\sigma}{E_{mp}(1-D_1)} + \frac{\sigma}{E_{ve}} \left(\frac{(m \cdot \exp(at) + n)}{(f \cdot \exp(at) + h)} \exp\left(-\frac{E_{me}}{\eta_{ve}} t\right)\right) \quad \sigma_c \leq \sigma
\]

Indeed, some of the instantaneous elastic strain, instantaneous plastic strain, and viscoelastic strain were not recoverable under the unloading conditions in this study. Based on Eqs. 6 and 12, the creep damage equation under the unloading conditions of this paper can be obtained using Eq. 24:

\[
e^{\varepsilon} = \frac{\sigma}{E_{me}(1-D_1)} + \frac{\sigma}{E_{mp}(1-D_1)} + \frac{\sigma}{E_{ve}} \left(\frac{(m \cdot \exp(at) + n)}{(f \cdot \exp(at) + h)} \exp\left(-\frac{E_{me}}{\eta_{ve}} t\right)\right) \exp\left[-\frac{E_{me}}{\eta_{ve}} (t_f - t)\right]
\]

5. Model parameter identification and validation analysis

The parameters of the creep damage constitutive model proposed in this paper include the instantaneous elastic modulus \(E_{me}\) and the instantaneous plastic modulus \(E_{mp}\), which can be determined based on the experimental data in Eq. 6 and Fig. 9. The remaining parameters \(E_{ve}, \eta_{ve}, V, t_f, V\) and \(\eta_{vp}\) are identified using the widely used Levenberg-Marquardt nonlinear least squares method, and the results of the parameter identification are shown in Table 2.

<table>
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<tr>
<th>Type</th>
<th>Stress/MPa</th>
<th>(E_{me}/\text{GPa})</th>
<th>(E_{mp}/\text{GPa})</th>
<th>(E_{ve}/\text{GPa})</th>
<th>(\eta_{ve}/\text{GPa-h})</th>
<th>(t_f/\text{h})</th>
<th>(V)</th>
<th>(\eta_{vp}/\text{GPa-h})</th>
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<tr>
<td>Original rock sample</td>
<td>31</td>
<td>57.08</td>
<td>32.42</td>
<td>49.57</td>
<td>0.86</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td></td>
<td>37.2</td>
<td>57.20</td>
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wetting-drying

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10 cycles of wetting-drying

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The theoretical curve of the creep damage model was obtained by bringing the model parameters into Eqs. 23 and 24 are shown in Fig. 15. It is found that the experimental values agree well with the theoretical values, especially in the accelerated creep phase, i.e., the creep damage constitutive model developed in this paper can not only reflect the creep of yellow sand under different stress levels with unloading, but also characterize the nonlinear properties of its decay, steady state, and accelerated creep phases. It indicates that the creep damage constitutive model can better describe the step unloading creep deformation of yellow sandstone under the action of wetting-drying cycles.
Fig. 15 Comparison of model curves and experimental data: (a) Original rock sample; (b) Wetting-drying cycles 5 times; (c) Wetting-drying cycles 10 times

Table 2 shows that the creep damage model parameters do not differ much under the same creep load, such as \( E_{me} \) range from 50.39 to 57.08 GPa, \( E_{mp} \) range from 26.66 to 32.42 GPa, \( E_{ve} \) range from 49.57 to 69.50 GPa, and \( \eta_{ve} \) range from 0.56 to 0.86 GPa·h for different cycles of wetting-drying at 31 MPa stress level. The creep damage model parameters under different creep loads were statistically calculated within a reasonable range considering the non-homogeneity and discrete nature of the rock material, and the laws are shown in the box plot Fig. 16. The mean values of the parameters were also counted and fitted. It was found that \( E_{me} \) and \( E_{mp} \) show a linear and increasing and decreasing trend with creep stress, while \( E_{ve} \) and \( \eta_{ve} \) show a positive exponential increase and negative exponential decrease with creep stress, respectively.
6. Conclusions and Discussion

This paper proposes a new creep damage constitutive model based on yellow sandstone uniaxial graded loading and uniaxial compression test. This model can describe the transient deformation and creep three-stage process of yellow sandstone in a unified manner. The main conclusions are as follows:

(1) The elastic modulus and compressive strength of yellow sandstone decreased by 39.81%, 43.07% and 9.37%, 11.31% after 5 and 10 cycles of wetting-drying, respectively, and the long-term compressive strength decreased by 2.32% and 3.19%. The damage type of the specimen gradually evolves from shear damage of a single oblique section to conical shear damage of a double oblique section with the increase in the number of wetting-drying cycles.

(2) Even in the steady-state creep stage of yellow sandstone, the creep rate is not constant but constantly changes with time, and the fluctuation range increases with the increase in the number of wetting-drying cycles. Hence, the yellow sandstone used in this test does not have strict steady-state creep, but only a relatively small range of creep rate changes.

(3) The wetting-drying cycles and creep loading were considered microscopic and macroscopic damage based on the nonlinear rheological theory and continuum damage mechanics, respectively. Besides, the coupled damage variables were constructed based on the Lemaitre strain equivalence assumption $D_{12}$. Based on the
transient elastomer, transient plastic body, Kelvin model, and viscoplastic damage model, the damage variables $D_1$, $D_{12}^t$, and $D_c$ were introduced to develop a creep damage constitutive model considering wetting-drying cycles and aging creep damage. This model was verified experimentally and was found to characterize the creep deformation properties of yellow sandstone loading and unloading well.

(4) The relationship between creep damage model parameters and yellow sandstone's stress under different wetting-drying based on statistical laws was obtained, and the percentage difference of each parameter in the same creep stress state was found as 31.76%. In addition, $E_{me}$ and $E_{mp}$ showed a linear, increasing, and decreasing trend with creep stress, while $E_{ve}$ and $\eta_{ve}$ showed a positive exponential increase and a negative exponential decrease with creep stress, respectively. Therefore, the creep law of yellow sandstone can be predicted according to the number of different cycles of wetting-drying and the stress state in field yellow sandstone to avoid creep damage.

The contribution of this work is that this is provides a reference for slope engineering stability problems. Subsequent studies will carry out a large number of circulating water intrusion tests. On the one hand, based on the transient plastic body in this paper, the transient plastic deformation during rock loading is further described. On the other hand, it is still important to study whether there is a certain threshold value that can reduce or accelerate the damage caused by water-rock action.

**Author Contributions:** Conceptualization, Zhe Qin; methodology, Jihuan Han; validation, Wenlong Pang. and Wenlong Liu; investigation, Qiang Feng; writing original draft preparation, Yongde Liu.; All authors have read and agreed to the published version of the manuscript.

**Data Availability Statement:** The data presented in this study are available in article and supplementary material.

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**Conflicts of Interest:** The authors declare no conflict of interest.
7. References


