On Q-rung orthopair Z-Numbers with an application in Multi-Criteria Decision Making

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Research Article

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On $Q$-rung orthopair $Z$-Numbers with an application in Multi-Criteria Decision Making

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Abstract

A $Q$-rung orthopair $Z$-number $Z_Q = (C_Q, R_Q)$ defined on a non-empty finite set $X$ is a pair of two $Q$-rung orthopair fuzzy sets, where $C_Q$ is $Q$-rung orthopair fuzzy set defined on $X$ and $R_Q$ is the reliability of the $C_Q$ in the form of $Q$-rung orthopair fuzzy set. In this paper, we have proposed the notion of $Q$-rung orthopair fuzzy $Z$-numbers. Furthermore, arithmetic operations on these $Z$-numbers have been introduced. Furthermore, a series of dice similarity measures have been defined for $Q$-rung orthopair fuzzy $Z$-numbers. To solve the multi-criteria decision-making problems, a TOPSIS method based on these distance measures has been discussed. Furthermore, the application of the proposed method has been studied to solve an industrial problem. A sensitivity analysis of the involved parameters has been done to illustrate the stability of the proposed approach. Finally, to demonstrate the rationality and superiority of the proposed approach, a comparison of the proposed method with current methods has been carried out.

Keywords: Multi-criteria decision-making, TOPSIS method, $Z$-number, $Q$-rung orthopair fuzzy sets.

1 Introduction

Multi-criteria decision-making (MCDM) is the process of selecting the best alternative among a set of alternatives evaluated for some given attributes of a problem. In
solving MCDM, we often encounter uncertainties. Therefore, the classical crisp set theory is no longer effective in handling these problems. To handle these types of problems, Zadeh (1965) gave the concept of fuzzy set theory, which is an extension of the crisp set, and assigned a membership value between zero and one to each set element. The concept of the fuzzy set is further extended by Atanassov (1986, 1989) into the intuitionistic fuzzy set (IFS) and interval-valued intuitionistic fuzzy set (IVIFS), which consists of membership and the non-membership degrees for each element of the set, with the condition that the sum of the membership and non-membership degrees should not exceed one.

Yagar (2014) expanded the concept of the IFS into the Pythagorean fuzzy set (PFS). Which is more suitable for handling decision-making problems as in these sets, instead of the direct sum of membership and non-membership grades, the sum of their squares is restricted to unity. Hence, the expert can express his beliefs about the membership grades with more freedom. Yagar (2017) has further developed the idea of Pythagorean fuzzy sets into Q-rung orthopair fuzzy sets (Q-ROFS), in which the sum of the $Q^{th}$ power ($Q \geq 1$) of the membership and non-membership degrees is constrained by unity. Many researchers have explored IFS, PFS, and Q-ROFS to solve real-life MCDM problems. Wang et al. (2012) have proposed MCDM based on induced intuitionistic normal fuzzy-related aggregation operators. Recently, Thao and Chou (2022) presented a novel similarity measure and entropy of intuitionistic fuzzy sets and its application to software quality evaluation. Furthermore, the MCDM approach based on Pythagorean fuzzy sets was introduced by Mahanta and Panda (2021), Gao and Deng (2021), Pan et al. (2022), and Farhadinia (2022). Some MCDM approach based on Q-ROFS has also been studied in the Liu et al. (2022), Deveci et al. (2022), Garg et al. (2021), and Cheng et al. (2021). Recently, applications of MCDM have also been studied in Inuiguchi et al. (2022) for ill-known pairwise comparison data.

Chen and Hwang (1992) introduced the technique for order preference by similarity to an ideal solution (TOPSIS) method, which has been extensively used to solve MCDM problems in recent years. Describing the information measure between the ideal and available alternatives is an essential step of the TOPSIS method. Recently, Milosevic et al. (2017) proposed IBA (interpolative Boolean algebra) similarity measures for IFS and described their applications in machine learning algorithms. Further, Xiao (2021) defined a new distance measure based on Jensen–Shannon divergence and its applications to MCDM. Like IFS, many distance measures have also been defined for PFS and Q-ROFS. Li and Zeng (2018) define a variety of distance measures for Pythagorean fuzzy sets and Pythagorean fuzzy numbers, which account for the four parameters of Pythagorean fuzzy sets. Further, Xiao et al. (2019) presented the divergence measure for PFS and explained its applications to medical diagnosis. Recently, Zeng et al. (2021) proposed the weighted induced logarithm distance measure and its applications to multi-attribute decision-making. Furthermore, Peng et al. (2019) presented a series of distance, similarity, entropy, and inclusion measure for Q-ROFS and their application in MCDM. Recently, Pinar et al. (2022) presented a distance measure for Q-rung picture fuzzy sets and its application to MCDM.
problems. Later on, Jan et al. (2020) proposed generalized dice similarity measures for Q-rung orthopair fuzzy sets with applications to MCDM problems. Besides these, some notable information measures have been defined in (Yatsalo et al., 2020; Wang et al., 2019; Shen et al., 2018; Zhang et al., 2022; Ejegwa 2020; Du 2018).

The Q-Rung orthopair Z-Number (Q-ROZN) is the combined concept of Q-Rung orthopair fuzzy sets and Z-number. Joshi et al. (2020), Kumar and Gupta (2023), and Garg (2017) have proposed the MCDM methods in which they have considered the confidence level for Q-Rung orthopair fuzzy number, Q-rung orthopair normal fuzzy numbers and Pythagorean fuzzy numbers, respectively. However, in all these works, they do not consider the dissatisfaction level for the evaluated alternatives. Hence, they fail to capture real-life situations completely because, nowadays, the MCDM problems are complex and uncertain. Hence, to reach a higher level of accuracy, some modifications should be made to the existing methods to tackle such problems. Zadeh (2011) introduced the concept of Z-number as a pair $\langle A, B \rangle$ of two fuzzy sets where the first fuzzy set is defined on the values of the considered set and $B$ is the reliability of the fuzzy set $A$ in the form of the fuzzy set. Further, a Q-rung orthopair fuzzy Z-number is the extension of the Z-number where both the sets in the pair are of the form of Q-rung orthopair fuzzy sets.

The main contributions of the paper are as follows:

- The Q-rung orthopair Z-number is defined, which has more flexibility than the previous fuzzy numbers in terms of satisfaction level for the alternative and dissatisfaction level for the evaluated alternative. The comparison of the Q-rung orthopair fuzzy Z-number with the previous fuzzy numbers has been summarized as follows:

<table>
<thead>
<tr>
<th>Fuzzy number</th>
<th>Satisfaction level for alternative</th>
<th>Dissatisfaction level for alternative</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pythagorean fuzzy number</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Q-rung orthopair fuzzy set</td>
<td>✗</td>
<td>✗</td>
</tr>
<tr>
<td>Pythagorean fuzzy number with confidence level</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>Q-rung orthopair with confidence level</td>
<td>✓</td>
<td>✗</td>
</tr>
<tr>
<td>Q-rung orthopair Z number</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Table 1 Comparison of different fuzzy numbers

- The algebraic operations for Q-rung orthopair Z-numbers have been defined.
- A TOPSIS method based on the defined Q-rung orthopair Z-numbers has been discussed.
- Sensitivity analyses on the involved parameters have been done to show the stable ranking of the proposed method.

The remaining paper is framed as follows: Section 2 contains preliminaries and the arithmetic operations on Q-ROZN. The Dice similarity measure for Q-ROZN and their properties have been presented in Section 3. Section 4 explains the TOPSIS method based on the defined dice similarity measures. Further, an industrial application of the proposed MCDM method has been discussed in Section 5. Furthermore, sensitivity
analyses have been done on the parameters $Q$ and $\alpha$ in Section 6, and a comparative analysis of the proposed approach with the existing methods is illustrated in Section 7. Finally, the concluding remark and future directions have been mentioned in Section 8.

2 Preliminaries and Definitions

This section introduces the notion of the $Q$-rung orthopair $Z$-numbers. Further, the arithmetic operations and some preliminaries have also been presented.

2.1 $Q$-rung orthopair $Z$-number

Let $X$ be a non-empty set. Then, a $Q$-rung orthopair $Z$-number ($Q$-ROZN), $Z_Q$ on $X$ can be defined as:

$$Z_Q = \{C_Q(x), R_Q(x), \, x \in X\}$$

where $C_Q$ is a $Q$-ROFS defined on the set $X$ as:

$$C_Q = \{(x, t(x), u(x)), \, x \in X\}.$$ 

The functions $t : X \rightarrow [0, 1]$ and $u : X \rightarrow [0, 1]$ are the membership and non-membership degrees for $x \in X$, respectively, with the condition that

$$0 \leq (t(x))^Q + (u(x))^Q \leq 1, \quad Q \geq 1,$$

and the reliability $R_Q$ of $C_Q$ is also defined in the form of $Q$-ROFS. Here, the reliability $R_Q(x)$ is defined for the statement “$X$ is $C_Q(x)$.” The possibility of ($X$ is $C_Q(x)$) = $t(C_Q(x))$ and the possibility of ($X$ is not $C_Q(x)$) = $u(C_Q(x))$.

Hence, a $Q$-ROZN can be expressed as

$$Z_Q = \{(t(x), u(x)), (t(C(x)), u(C(x))), \, x \in X\}.$$ 

Throughout the manuscript, for notational convenience, a $Q$-ROZN of the form

$$Z_Q = \{(t(x_i), u(x_i)), (t(C(x_i)), u(C(x_i))), \, x_i \in X\}$$

is represented by $((t_{x_i}, u_{x_i}), (t_{C(x_i)}, u_{C(x_i)}))$.

**Example 1.** According to the different institutions, the projection of Indian GDP growth is given as follows:

- According to Reserve Bank of India (RBI), “Indian economy will grow by 9.2% in the financial year 2022-23, very sure.”
- According to the International Monetary Fund (IMF), “Indian economy will grow by 8.5% in the financial year 2022-23, not so sure.”
- According to Asian Development Bank (ADB), “Indian economy will grow by 8.0% in the financial year 2022-23, very likely.”
- According to World Bank (WB), “Indian economy will grow by 8.7% in the financial year 2022-23, likely.”
Then, this economic projection can be represented in the form of Q-ROZN as in Table 2:

<table>
<thead>
<tr>
<th>Projection of GDP</th>
<th>Projection of GDP in terms of Q-ROZN</th>
</tr>
</thead>
<tbody>
<tr>
<td>RBI</td>
<td>((0.9, 0.3), (0.9, 0.2))</td>
</tr>
<tr>
<td>IMF</td>
<td>((0.7, 0.5), (0.3, 0.9))</td>
</tr>
<tr>
<td>ADB</td>
<td>((0.9, 0.4), (0.8, 0.3))</td>
</tr>
<tr>
<td>WB</td>
<td>((0.6, 0.4), (0.7, 0.6))</td>
</tr>
</tbody>
</table>

Table 2 Example of Q-ROZN

In RBI (Table 2), the first component of Q-ROZN (0.9, 0.3) represents the Q-ROFS “Indian economy will grow by 9.2% in the financial year 2022-23,” and the reliability of this Q-ROFN “very sure” is expressed by the Q-ROFN (0.9, 0.2).

2.2 Operational laws for Q-ROZN

Let \( Z_Q^1 = ((t_{x_1}, u_{x_1}), (t_{C_1}, u_{C_1})) \) and \( Z_Q^2 = ((t_{x_2}, u_{x_2}), (t_{C_2}, u_{C_2})) \) be two Q-ROZNs, and \( \lambda \) be a non-negative real number. Then,

(i) \( Z_Q^1 \oplus Z_Q^2 = (((t_{x_1} + t_{x_2} - t_{x_1} t_{x_2})^{\frac{1}{Q}}, u_{x_1} u_{x_2}), ((t_{C_1} + t_{C_2} - t_{C_1} t_{C_2})^{\frac{1}{Q}}, u_{C_1} u_{C_2})) \).

(ii) \( Z_Q^1 \otimes Z_Q^2 = (((t_{x_1}, u_{x_1}) t_{x_2}, u_{x_2} + u_{x_1} u_{x_2} t_{x_2})^{\frac{1}{Q}}, u_{C_1} t_{C_2} + u_{C_1} u_{C_2} t_{C_2})^{\frac{1}{Q}}, u_{C_1} u_{C_2})) \).

(iii) \( \lambda Z_Q^1 = (((1 - (1 - t_{x_1}^{\lambda})^{\frac{1}{Q}}, u_{x_1}^{\lambda}), (1 - (1 - t_{C_1}^{\lambda})^{\frac{1}{Q}}, u_{C_1}^{\lambda}))) \).

(iv) \( Z_Q^1 \lambda = (((t_{x_1}^{\lambda}, (1 - (1 - u_{x_1}^{\lambda})^{\frac{1}{Q}}), (1 - (1 - u_{C_1}^{\lambda})^{\frac{1}{Q}})).

2.3 Hesitancy degree for Q-ROZN

Let \( Z_Q^1 = ((t_1, u_1), (t_{C_1}, u_{C_1})) \) be a Q-ROZN. Then, the hesitancy degree for Q-ROZN is defined as

\[
I(Z_Q^1) = (1 - (t_1^{Q} + t_{C_1}^{Q} - t_1^{Q} t_{C_1}^{Q}) - u_1^{Q} u_{C_1}^{Q})^{1/Q}.
\]

2.4 Dice similarity measure (Dice, 1945)

Let \( X = (x_1, x_2, ..., x_n) \) and \( Y = (y_1, y_2, ..., y_n) \) be two vectors, where \( x_i, y_i \geq 0 \), for all \( i = 1, 2, ..., n \). Then, the dice similarity measure between \( X \) and \( Y \) is defined as follows:

\[
D = \begin{cases} 
0, & \text{if } X = Y = 0, \\
2 \sum_{j=1}^{n} x_j y_j \over \sum_{j=1}^{n} x_j^2 + \sum_{j=1}^{n} y_j^2, & \text{otherwise}.
\end{cases}
\]

Clearly, the dice similarity measure takes value in the interval \([0, 1]\) only.

3 Dice similarity measure for Q-ROZN

In this section, a series of dice similarity measures (DSM) for Q-ROZN has been introduced, and further, some necessary properties have been proved
The dice similarity measure between two Q-ROZNs for any $Q \geq 1$ can be defined as

$$D^1(Z_Q^1, Z_Q^2) = \frac{\sum_{j=1}^{m} \left( (t_{ij}^Q + t_{ij}^C_{ij} - \frac{t_{ij}^Q t_{ij}^C_{ij}}{t_{ij}^Q + t_{ij}^C_{ij}})^2 + (t_{ij}^Q + t_{ij}^C_{ij} - \frac{t_{ij}^Q t_{ij}^C_{ij}}{t_{ij}^Q + t_{ij}^C_{ij}})^2 + u_{ij1}^Q u_{ij2}^Q + u_{ij3}^Q u_{ij4}^Q \right)}{m}$$

Example 2. Let $Z_Q^1 = \{(0.8, 0.4), (0.3, 0.9), (0.6, 0.7), (0.8, 0.1), (0.7, 0.4), (0.5, 0.5)\}$ and $Z_Q^2 = \{(0.9, 0.2), (0.3, 0.8), (0.7, 0.4), (0.8, 0.2), (0.3, 0.7), (0.6, 0.7)\}$ be two Q-ROZNs. Then, the dice similarity measure between these $Q$-ROZNs for $Q = 4$ is given by

$$D^1(Z_Q^1, Z_Q^2) = \frac{\sum_{j=1}^{m} \left( (t_{ij}^Q + t_{ij}^C_{ij} - \frac{t_{ij}^Q t_{ij}^C_{ij}}{t_{ij}^Q + t_{ij}^C_{ij}})^2 + (t_{ij}^Q + t_{ij}^C_{ij} - \frac{t_{ij}^Q t_{ij}^C_{ij}}{t_{ij}^Q + t_{ij}^C_{ij}})^2 + u_{ij1}^Q u_{ij2}^Q + u_{ij3}^Q u_{ij4}^Q \right)}{m}$$

**Theorem 1.** The proposed DSM $D^1(Z_Q^1, Z_Q^2)$ satisfies the following properties:

(i) $0 \leq D^1(Z_Q^1, Z_Q^2) \leq 1$.

(ii) $D^1(Z_Q^1, Z_Q^2) = D^1(Z_Q^2, Z_Q^1)$.

(iii) $D^1(Z_Q^1, Z_Q^2) = 1$ if $Z_Q^1 = Z_Q^2$.

**Proof.** (i) Since, for any $Q \geq 1$, and for all $j = 1, 2, \ldots, m$,

$$|(t_{ij}^Q + t_{ij}^C_{ij} - \frac{t_{ij}^Q t_{ij}^C_{ij}}{t_{ij}^Q + t_{ij}^C_{ij}})^2 + (t_{ij}^Q + t_{ij}^C_{ij} - \frac{t_{ij}^Q t_{ij}^C_{ij}}{t_{ij}^Q + t_{ij}^C_{ij}})^2 + u_{ij1}^Q u_{ij2}^Q + u_{ij3}^Q u_{ij4}^Q| \geq 0.$$
The proof follows on the lines of Theorem 1.

Hence,

\[
\frac{2}{m} \sum_{j=1}^{m} \left( t_{ij}^Q + t_{ij}^C \right)^2 \geq \left( t_{ij}^Q + t_{ij}^C \right)^2 + \left( t_{ij}^Q + t_{ij}^C \right)^2 + (u_{ij} u_{Cij} u_{2j} u_{C2j})^2 \leq 1
\]

or, \(D^1(Z_Q^2, Z_Q^2) \leq 1\).

Next, to prove \(D^1(Z_Q^1, Z_Q^2) \geq 0\).

Since \(0 \leq t_{ij}, t_{Cij}, u_{ij}, u_{Cij} \leq 1\), \(0 \leq (t_{ij}^Q + t_{ij}^C - t_{ij}^Q t_{ij}^C) \leq 1\) for \(i = 1, 2, j = 1, 2, \ldots, m\) and \(Q \geq 1\), therefore, \(D^1(Z_Q^1, Z_Q^2) \leq 1\).

\((i)\) \(D^1(Z_Q^1, Z_Q^2) = \frac{2}{m} \sum_{j=1}^{m} \left( t_{ij}^Q + t_{ij}^C \right)^2 \geq \left( t_{ij}^Q + t_{ij}^C \right)^2 + \left( t_{ij}^Q + t_{ij}^C \right)^2 + (u_{ij} u_{Cij} u_{2j} u_{C2j})^2 \leq 1\).

\((ii)\) \(D^1(Z_Q^1, Z_Q^2) = \frac{2}{m} \sum_{j=1}^{m} \left( t_{ij}^Q + t_{ij}^C \right)^2 \geq \left( t_{ij}^Q + t_{ij}^C \right)^2 + \left( t_{ij}^Q + t_{ij}^C \right)^2 + (u_{ij} u_{Cij} u_{2j} u_{C2j})^2 \leq 1\).

\((iii)\) If \(Z_{Q1} = Z_{Q2}\), then \(t_{ij} = t_{2j}, t_{Cij} = t_{C2j}\) and \(u_{ij} = u_{2j}, u_{Cij} = u_{C2j}\). Hence,

\[
D^1(Z_{Q1}, Z_{Q2}) = \frac{2}{m} \sum_{j=1}^{m} \left( t_{ij}^Q + t_{ij}^C \right)^2 \geq \left( t_{ij}^Q + t_{ij}^C \right)^2 + \left( t_{ij}^Q + t_{ij}^C \right)^2 + (u_{ij} u_{Cij} u_{2j} u_{C2j})^2 \leq 1.
\]

\[\square\]

**Definition 2.** The weighted dice similarity measure (WDSM) \(D_w^1\) for Q-ROZN is defined as

\[
D_w^1(Z_Q^1, Z_Q^2) = \frac{m \sum_{j=1}^{m} \left( t_{ij}^Q + t_{ij}^C \right)^2}{(t_{ij}^Q + t_{ij}^C)^2 + (t_{ij}^Q + t_{ij}^C)^2 + (u_{ij} u_{Cij} u_{2j} u_{C2j})^2}
\]

where \(w = (w_1, w_2, \ldots, w_m)\) is the weighted vector such that \(0 \leq w_j \leq 1\), \(j = 1, 2, \ldots, m\), and \(\sum_{j=1}^{m} w_j = 1\).

**Theorem 2.** The proposed WDSM \(D_w^1(Z_Q^1, Z_Q^2)\) satisfies the following properties:

\((i)\) \(0 \leq D_w(Z_Q^1, Z_Q^2) \leq 1\).

\((ii)\) \(D_w(Z_Q^1, Z_Q^2) = D_w(Z_Q^2, Z_Q^1)\).

\((iii)\) \(D_w(Z_Q^1, Z_Q^2) = 1\) if \(Z_Q^1 = Z_Q^2\).

**Proof.** The proof follows on the lines of Theorem 1. \[\square\]
Definition 3. The weighted dice similarity measure $D_w^2$ for picture $Q$-ROZN is defined as

$$D_w^2(Z_Q^1, Z_Q^2) = \sum_{j=1}^{m} w_j \left\{ \left( (t_{1j}^Q + t_{C_1j}^Q - t_{1j}^Q t_{C_1j}^Q) (t_{2j}^Q + t_{C_2j}^Q - t_{2j}^Q t_{C_2j}^Q) + (u_{1j} u_{C_1j}^Q u_{2j} u_{C_2j}^Q)^Q \right) \right\}$$

where $w = (w_1, w_2, \ldots, w_m)$ is the weighted vector such that $0 \leq w_j \leq 1$, $j = 1, 2, \ldots, m$, and $\sum_{j=1}^{m} w_j = 1$.

Theorem 3. The proposed WDSM $D_w^2(Z_Q^1, Z_Q^2)$ satisfies the following properties:

(i) $0 \leq D_w^2(Z_Q^1, Z_Q^2) \leq 1$.

(ii) $D_w^2(Z_Q^1, Z_Q^2) = D_w^2(Z_Q^2, Z_Q^1)$.

(iii) $D_w^2(Z_Q^1, Z_Q^2) = 1$ if $Z_Q^1 = Z_Q^2$.

Definition 4. The weighted dice similarity measure $D_w^3$, including hesitancy for $Q$-ROZN, is defined as

$$D_w^3(Z_Q^1, Z_Q^2) = \sum_{j=1}^{m} w_j \left\{ \left( \frac{(t_{1j}^Q + t_{C_1j}^Q - t_{1j}^Q t_{C_1j}^Q) (t_{2j}^Q + t_{C_2j}^Q - t_{2j}^Q t_{C_2j}^Q) + (u_{1j} u_{C_1j}^Q u_{2j} u_{C_2j}^Q)^Q}{(t_{1j}^Q + t_{C_1j}^Q - t_{1j}^Q t_{C_1j}^Q)^2 + (t_{2j}^Q + t_{C_2j}^Q - t_{2j}^Q t_{C_2j}^Q)^2 + u_{1j}^2 u_{C_1j}^2 + u_{2j}^2 u_{C_2j}^2} \right) \right\}$$

where $w = (w_1, w_2, \ldots, w_m)$ is the weighted vector such that $0 \leq w_j \leq 1$, $j = 1, 2, \ldots, m$, and $\sum_{j=1}^{m} w_j = 1$. 
Theorem 4. The proposed DSM $D_w^1(Z_Q^1, Z_Q^2)$ satisfies the following properties:
(i) $0 \leq D_w^0(Z_Q^1, Z_Q^2) \leq 1$.
(ii) $D_w^0(Z_Q^1, Z_Q^2) = D_w^0(Z_Q^0, Z_Q^1)$.
(iii) $D_w^0(Z_Q^1, Z_Q^2) = 1$ if $Z_Q^1 = Z_Q^2$.

Definition 5. Let $Z_Q^1 = \{(t_1(x_j), u_1(x_j)), (t_1(C_1(x_j)), u_1(C_1(x_j)))$, $j = 1, 2, \ldots, m \}$ and $Z_Q^2 = \{(t_2(x_j), u_2(x_j)), (t_2(C_2(x_j)), u_2(C_2(x_j)))$, $j = 1, 2, \ldots, m \}$ be two $Q$-ROZFS. Then, the generalized dice similarity measure between $Z_Q^1$ and $Z_Q^2$ can be defined as

$$(D_w^1)_{\alpha}(Z_Q^1, Z_Q^2) = \frac{m}{\alpha} \sum_{j=1}^{m} w_i \left[ (t_{ij}^Q + t_{ij}^Q - t_{ij}^Q t_{ij}^Q) (t_{ij}' + t_{ij}' - t_{ij}' t_{ij}'^Q) + (u_{ij} u_{ij} u_{ij} u_{ij})^{Q} \right]$$

and

$$(D_w^2)_{\alpha}(Z_Q^1, Z_Q^2) = \frac{m}{\alpha} \sum_{j=1}^{m} w_i \left[ (t_{ij}^Q + t_{ij}^Q - t_{ij}^Q t_{ij}^Q) (t_{ij}' + t_{ij}' - t_{ij}' t_{ij}'^Q) + (u_{ij} u_{ij} u_{ij} u_{ij})^{Q} \right]$$

where $0 \leq \alpha \leq 1$.

It is to be noted that

$$(D_w^1)_{\alpha=0.5}(Z_Q^1, Z_Q^2) = D_w^1(Z_Q^1, Z_Q^2)$$

$$(D_w^2)_{\alpha=0.5}(Z_Q^1, Z_Q^2) = D_w^2(Z_Q^1, Z_Q^2)$$

$$(D_w^3)_{\alpha=0.5}(Z_Q^1, Z_Q^2) = D_w^3(Z_Q^1, Z_Q^2)$$

Similarly, one can find the reduced similarity measure for other values of $\alpha$ from the generalized dice similarity measure $(D_w^1)_{\alpha}$, $(D_w^2)_{\alpha}$, and $(D_w^3)_{\alpha}$. Also, it can be seen from the above that the generalized dice similarity measure is more powerful compared to the previously defined DSMs for $Q$-ROZN because one can easily reduce the generalized dice similarity measure to the standard dice similarity measures by giving a specific value to the generalization parameter $\alpha$. Further, the standard DSMs failed to differentiate the alternatives in some cases. However, the generalized DSM worked well for those cases also.
4 TOPSIS method based on Q-ROZN

In this section, a TOPSIS method based on the proposed distance measures has been presented.

In an MCDM problem, suppose we have \( n \) criteria \( \{C_1, C_2, \ldots, C_n\} \), along with \( m \) alternatives \( \{A_1, A_2, \ldots, A_m\} \). The weighted vector for these criteria is given as \( \{w_1, w_2, \ldots, w_n\} \), where \( \sum_{i=1}^{n} w_i = 1 \). Then, in the following steps, one can solve the MCDM problem:

**Step 1.** Compose the initial decision matrix (Table 3) according to the expert evaluation of each alternative in the form of Q-ROZN.

<table>
<thead>
<tr>
<th></th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( \ldots )</th>
<th>( C_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_1 )</td>
<td>((t_{11}, u_{11}), (t_{C11}, u_{C11}))</td>
<td>((t_{12}, u_{12}), (t_{C12}, u_{C12}))</td>
<td>( \ldots )</td>
<td>((t_{1n}, u_{1n}), (t_{C1n}, u_{C1n}))</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>((t_{21}, u_{21}), (t_{C21}, u_{C21}))</td>
<td>((t_{22}, u_{22}), (t_{C22}, u_{C22}))</td>
<td>( \ldots )</td>
<td>((t_{2n}, u_{2n}), (t_{C2n}, u_{C2n}))</td>
</tr>
<tr>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>( A_m )</td>
<td>((t_{m1}, u_{m1}), (t_{Cm1}, u_{Cm1}))</td>
<td>((t_{m2}, u_{m2}), (t_{Cm2}, u_{Cm2}))</td>
<td>( \ldots )</td>
<td>((t_{mn}, u_{mn}), (t_{Cmn}, u_{Cmn}))</td>
</tr>
</tbody>
</table>

Table 3 Initial decision matrix

Step 2. Identify the different criteria, whether it is a criterion of cost nature or benefit in nature. Let \( Z_{Qij} = (t_{ij}, u_{ij}), (t_{Cij}, u_{Cij}) \) be a Q-ROFS for \( i \)th alternative, considering \( j \)th criteria. Then, normalize the decision matrix according to the following rules:

For benefit-oriented attribute:

\[
\bar{Z}_{Qij} = (t_{ij}, u_{ij}), (t_{Cij}, u_{Cij})
\]

For cost-oriented attribute:

\[
\bar{Z}_{Qij} = (u_{ij}, t_{ij}), (u_{Cij}, t_{Cij}).
\]

Step 3. Define the positive and negative ideal solutions as follows:

\[
Z_{Q+}^i = (\max_{1 \leq j \leq n} t_{ij}, \min_{1 \leq j \leq n} u_{ij}), (\max_{1 \leq j \leq n} t_{Cij}, \min_{1 \leq j \leq n} u_{Cij}), \quad i = 1, 2, \ldots, m
\]

\[
Z_{Q-}^i = (\min_{1 \leq j \leq n} t_{ij}, \max_{1 \leq j \leq n} u_{ij}), (\min_{1 \leq j \leq n} t_{Cij}, \max_{1 \leq j \leq n} u_{Cij}), \quad i = 1, 2, \ldots, m
\]

Step 4. Find the distance measure of each alternative from positive and negative ideal solutions as follows

\[
d_i^+ = \left( \sum_{j=1}^{n} (D(Z_{Qij}, Z_{Q+}^i))^2 \right)^{\frac{1}{2}}, \quad i = 1, 2, \ldots, m,
\]
The normalized decision matrix is composed in Table 4. Then, the relative closeness coefficient for each alternative can be computed using the relation:
\[
R_i = \frac{d_i^-}{d_i^- + d_i^+}, \quad i = 1, 2, \ldots, m.
\]

Step 5. Rank the different alternatives based on the relative closeness coefficient. The higher relative closeness coefficient implies a better alternative.

5 An industrial application

Suppose an entrepreneur wants to set up a startup company. Let the person has the following five options in choosing a suitable company:
1. Textile industry \((A_1)\).
2. Semiconductor industry \((A_2)\).
3. Iron & steel industry \((A_3)\).
4. Crude oil products industry \((A_4)\).
5. Pharmaceutical industry \((A_5)\).

Along with these alternatives, suppose the entrepreneur has decided to consider the following five criteria:
1. Initial investment required to start the company \((C_1)\).
2. Availability of raw materials from different sources \((C_2)\).
3. Competition exists in the market already \((C_3)\).
4. Expected profit according to initial assessment \((C_4)\).
5. Requirement of technical human resources \((C_5)\).

According to the concerned field expert, the initial decision matrix has been composed in Table 4. Then, in the following steps, the MCDM problem can be solved:

Table 4 Initial decision matrix

<table>
<thead>
<tr>
<th></th>
<th>(C_1)</th>
<th>(C_2)</th>
<th>(C_3)</th>
<th>(C_4)</th>
<th>(C_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A_1)</td>
<td>[(0.9, 0.2), (0.5, 0.2)]</td>
<td>[(0.8, 0.03), (0.6, 0.4)]</td>
<td>[(0.5, 0.04), (0.4, 0.7)]</td>
<td>[(0.5, 0.8), (0.9, 0.2)]</td>
<td>[(0.2, 0.9), (0.6, 0.4)]</td>
</tr>
<tr>
<td>(A_2)</td>
<td>[(0.7, 0.9), (0.3, 0.8)]</td>
<td>[(0.4, 0.9), (0.7, 0.3)]</td>
<td>[(0.6, 0.5), (0.3, 0.8)]</td>
<td>[(0.5, 0.6), (0.9, 0.2)]</td>
<td>[(0.9, 0.1), (0.6, 0.4)]</td>
</tr>
<tr>
<td>(A_3)</td>
<td>[(0.5, 0.7), (0.4, 0.7)]</td>
<td>[(0.6, 0.8), (0.3, 0.8)]</td>
<td>[(0.8, 0.4), (0.9, 0.2)]</td>
<td>[(0.7, 0.5), (0.6, 0.4)]</td>
<td>[(0.9, 0.8), (0.6, 0.4)]</td>
</tr>
<tr>
<td>(A_4)</td>
<td>[(0.3, 0.9), (0.7, 0.3)]</td>
<td>[(0.5, 0.8), (0.4, 0.7)]</td>
<td>[(0.3, 0.9), (0.6, 0.4)]</td>
<td>[(0.7, 0.6), (0.9, 0.2)]</td>
<td>[(0.5, 0.6), (0.6, 0.4)]</td>
</tr>
<tr>
<td>(A_5)</td>
<td>[(0.4, 0.7), (0.9, 0.2)]</td>
<td>[(0.7, 0.3), (0.7, 0.3)]</td>
<td>[(0.6, 0.5), (0.3, 0.8)]</td>
<td>[(0.4, 0.4), (0.4, 0.7)]</td>
<td>[(0.5, 0.6), (0.6, 0.4)]</td>
</tr>
</tbody>
</table>

Step 1. Compose the initial decision matrix as given in Table 4.
Step 2. Identify the different criteria for cost and benefit-oriented attributes. Further, normalize the different attributes according to the formulas given in the algorithm. The normalized decision matrix is composed in Table 5.
Step 3. Design the positive and negative ideal solutions for each criterion according to the given formulas, given in Table 6.
and negative ideal solutions is given in Tables 5-8.

### Table 5 Normalized decision matrix

<table>
<thead>
<tr>
<th>Alternative/Criterion</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$0.9209$</td>
<td>$0.9070$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9020$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
</tr>
<tr>
<td>$A_3$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
</tr>
<tr>
<td>$A_4$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
</tr>
<tr>
<td>$A_5$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
<td>$0.9050$</td>
</tr>
</tbody>
</table>

### Table 6 Positive & negative ideal solutions

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Positive ideal solution</th>
<th>Negative ideal solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$(0.9, 0.4), (0.8, 0.3)$</td>
<td>$(0.2, 0.9), (0.2, 0.9)$</td>
</tr>
<tr>
<td>$C_2$</td>
<td>$(0.9, 0.4), (0.4, 0.6)$</td>
<td>$(0.3, 0.8), (0.3, 0.8)$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>$(0.8, 0.4), (0.9, 0.2)$</td>
<td>$(0.3, 0.9), (0.6, 0.4)$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>$(0.8, 0.4), (0.9, 0.2)$</td>
<td>$(0.6, 0.6), (0.2, 0.9)$</td>
</tr>
<tr>
<td>$C_5$</td>
<td>$(0.9, 0.2), (0.6, 0.4)$</td>
<td>$(0.1, 0.9), (0.4, 0.6)$</td>
</tr>
</tbody>
</table>

### Step 4. Determine the distance of each alternative from the corresponding positive and negative ideal solutions. The computed distance of each alternative from positive and negative ideal solutions is given in Tables 7-8.

### Table 7 Distances from positive ideal solution

<table>
<thead>
<tr>
<th>Alternative/Criterion</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.0019</td>
<td>0.0302</td>
<td>0.064</td>
<td>0.2961</td>
<td>0.2966</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.3</td>
<td>0.3</td>
<td>0.2554</td>
<td>0.2975</td>
<td>0.0217</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.2483</td>
<td>0.2999</td>
<td>0.3</td>
<td>0.2159</td>
<td>0.266</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.2946</td>
<td>0.2948</td>
<td>0.0999</td>
<td>0.2991</td>
<td>0.1242</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.1664</td>
<td>0.0147</td>
<td>0.0998</td>
<td>0.0379</td>
<td>0.1242</td>
</tr>
</tbody>
</table>

### Table 8 Distances from negative ideal solution

<table>
<thead>
<tr>
<th>Alternative/Criterion</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>0.9536</td>
<td>0.853</td>
<td>0.801</td>
<td>0.0741</td>
<td>0.0707</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.0082</td>
<td>0.048</td>
<td>0.4217</td>
<td>0.0715</td>
<td>0.9327</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.0134</td>
<td>0.0451</td>
<td>0.2498</td>
<td>0.1746</td>
<td>0.1165</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.0274</td>
<td>0.0547</td>
<td>0.7488</td>
<td>0.0678</td>
<td>0.3838</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.1058</td>
<td>0.9269</td>
<td>0.7499</td>
<td>0.7397</td>
<td>0.4169</td>
</tr>
</tbody>
</table>

Then, the distances $d_i^+$, $d_i^-$, $i = 1, 2, \ldots, 5$ of alternatives from positive and negative ideal solutions are $d_1^+ = 0.4271$, $d_2^+ = 0.5738$, $d_3^+ = 0.5992$, $d_4^+ = 0.5372$, $d_5^+ = 0.2339$, and $d_1^- = 1.5385$, $d_2^- = 1.0273$, $d_3^- = 1.5385$, $d_4^- = 0.8464$, $d_5^- = 1.4675$. Further, the relative closeness coefficient $R_i$ of each alternative are $R_1 = 0.782695948$, $R_2 = 0.641627126$, $R_3 = 0.719711805$, $R_4 = 0.611722178$, and $R_5 = 0.862511863$.

### Step 5. Hence, the final ranking of the alternatives for the given problem is $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$. Therefore, the pharmaceutical industry is the best one to invest in, and the crude oil product industry is the worst one.
6 Sensitivity Analyses

6.1 Sensitivity analysis on the generalization parameter $\alpha$

In this section, a sensitivity analysis on the generalization parameter $\alpha$ has been done considering the dice similarity measure $(D_{\alpha}^1)$ defined in Section 3 and for the application discussed in Section 5. The ranking of alternatives for different values of $\alpha$ has been summarized in Table 9.

<table>
<thead>
<tr>
<th>Values of $\alpha$</th>
<th>Closeness Coefficient</th>
<th>Alternative Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$A_2$</td>
<td>$A_3$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.7271</td>
<td>0.6965</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5113</td>
<td>0.5286</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5003</td>
<td>0.4619</td>
</tr>
<tr>
<td>0.3</td>
<td>0.4991</td>
<td>0.4233</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5006</td>
<td>0.3979</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5039</td>
<td>0.3791</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5092</td>
<td>0.3633</td>
</tr>
<tr>
<td>0.7</td>
<td>0.5174</td>
<td>0.348</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5274</td>
<td>0.3313</td>
</tr>
<tr>
<td>0.9</td>
<td>0.5295</td>
<td>0.31</td>
</tr>
<tr>
<td>1.0</td>
<td>0.2471</td>
<td>0.2319</td>
</tr>
</tbody>
</table>

Table 9: Ranking of alternatives varying $\alpha$

From Table 9 and Figure 1, it can be seen that $A_5$ is the best alternative for all the values of $\alpha \leq 0.7$. While $A_1$ becomes the best alternative for values of $\alpha$ in between 0.8 $\leq \alpha \leq$ 1.0. Therefore, $A_5$ remains the best alternative for almost all values of the generalization parameter $\alpha$. The decision maker is free to choose any value of the generalization parameter $\alpha$ according to their suitability.

6.2 Sensitivity analysis on the parameter $Q$

In this section, a sensitivity analysis on the parameter $Q$ has been done to observe the effect on the ranking of alternatives for different values of $Q$. The results have been summarized in Table 10 and Fig. 2.

<table>
<thead>
<tr>
<th>Value of $Q$</th>
<th>$\varphi_1$</th>
<th>$\varphi_2$</th>
<th>$\varphi_3$</th>
<th>$\varphi_4$</th>
<th>$\varphi_5$</th>
<th>Alternative ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5092</td>
<td>0.4749</td>
<td>0.4379</td>
<td>0.4509</td>
<td>0.5027</td>
<td>$\varphi_1 &gt; \varphi_5 &gt; \varphi_3 &gt; \varphi_4 &gt; \varphi_2$</td>
</tr>
<tr>
<td>2</td>
<td>0.513</td>
<td>0.4233</td>
<td>0.3246</td>
<td>0.3798</td>
<td>0.5228</td>
<td>$\varphi_5 &gt; \varphi_1 &gt; \varphi_2 &gt; \varphi_4 &gt; \varphi_3$</td>
</tr>
<tr>
<td>3</td>
<td>0.514</td>
<td>0.3953</td>
<td>0.2307</td>
<td>0.3624</td>
<td>0.5681</td>
<td>$\varphi_5 &gt; \varphi_1 &gt; \varphi_2 &gt; \varphi_4 &gt; \varphi_3$</td>
</tr>
<tr>
<td>4</td>
<td>0.5039</td>
<td>0.3792</td>
<td>0.1625</td>
<td>0.3653</td>
<td>0.6188</td>
<td>$\varphi_5 &gt; \varphi_1 &gt; \varphi_2 &gt; \varphi_4 &gt; \varphi_3$</td>
</tr>
<tr>
<td>5</td>
<td>0.4888</td>
<td>0.369</td>
<td>0.1144</td>
<td>0.3699</td>
<td>0.6709</td>
<td>$\varphi_5 &gt; \varphi_1 &gt; \varphi_2 &gt; \varphi_4 &gt; \varphi_3$</td>
</tr>
</tbody>
</table>

Table 10: Ranking of alternatives varying $Q$

The sensitivity analysis shows that the ranking of alternatives for all the values of $Q \geq 2$ is nearly the same. This proves that the proposed method gives a stable ranking.
Fig. 1  Ranking of alternatives for different values of $\alpha$

Fig. 2  Ranking of alternatives for different values of $Q$
for all the values of $Q$. Hence, the decision maker is free to use any value of $Q$ for the method’s applicability. Although a higher value is more suggested for stable ranking.

6.3 Sensitivity analysis on the parameters $Q$ and $\alpha$

Effect on the ranking of the alternatives has been analyzed in the present section for varying values of both the parameter $Q$ and $\alpha$ simultaneously. The relative closeness of the alternatives in the industrial application has been summarized in Tables 11-13.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$Q = 1$</th>
<th>$Q = 2$</th>
<th>$Q = 3$</th>
<th>$Q = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_1$</td>
<td>$A_2$</td>
<td>$A_3$</td>
<td>$A_4$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.5445</td>
<td>0.4947</td>
<td>0.4616</td>
<td>0.5522</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5364</td>
<td>0.4883</td>
<td>0.4535</td>
<td>0.5706</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5261</td>
<td>0.4818</td>
<td>0.4455</td>
<td>0.4619</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5169</td>
<td>0.4752</td>
<td>0.4376</td>
<td>0.4529</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5107</td>
<td>0.4683</td>
<td>0.4297</td>
<td>0.4434</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5017</td>
<td>0.4611</td>
<td>0.4218</td>
<td>0.4335</td>
</tr>
<tr>
<td>0.6</td>
<td>0.4922</td>
<td>0.4536</td>
<td>0.414</td>
<td>0.423</td>
</tr>
<tr>
<td>0.7</td>
<td>0.4823</td>
<td>0.4456</td>
<td>0.4062</td>
<td>0.4118</td>
</tr>
<tr>
<td>0.8</td>
<td>0.4720</td>
<td>0.4372</td>
<td>0.3985</td>
<td>0.3999</td>
</tr>
<tr>
<td>0.9</td>
<td>0.4610</td>
<td>0.4280</td>
<td>0.3898</td>
<td>0.3872</td>
</tr>
<tr>
<td>1.0</td>
<td>0.4494</td>
<td>0.4180</td>
<td>0.3832</td>
<td>0.3735</td>
</tr>
</tbody>
</table>

Table 11 Relative Closeness coefficient of alternatives for $Q = 1, 2$, varying $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$Q = 3$</th>
<th>$Q = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A_1$</td>
<td>$A_2$</td>
</tr>
<tr>
<td>0.0</td>
<td>0.6904</td>
<td>0.5743</td>
</tr>
<tr>
<td>0.1</td>
<td>0.6638</td>
<td>0.5380</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6502</td>
<td>0.5154</td>
</tr>
<tr>
<td>0.3</td>
<td>0.5913</td>
<td>0.4967</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6291</td>
<td>0.4790</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6174</td>
<td>0.4608</td>
</tr>
<tr>
<td>0.6</td>
<td>0.5971</td>
<td>0.4408</td>
</tr>
<tr>
<td>0.7</td>
<td>0.5820</td>
<td>0.4166</td>
</tr>
<tr>
<td>0.8</td>
<td>0.5478</td>
<td>0.3842</td>
</tr>
<tr>
<td>0.9</td>
<td>0.4754</td>
<td>0.3315</td>
</tr>
<tr>
<td>1.0</td>
<td>0.2567</td>
<td>0.1952</td>
</tr>
</tbody>
</table>

Table 12 Relative Closeness coefficient of alternatives for $Q = 3, 4$, varying $\alpha$

Figures 3-6 represents simultaneous surface plots of the closeness coefficient, taking two alternatives at a time: $A_5$ and $A_1$, $A_1$ and $A_2$, $A_2$ and $A_4$, $A_4$ and $A_3$, respectively. From this analysis, the following conclusions can be drawn:

- Figure 6 shows that for almost all the values of the parameters, the relative closeness coefficient of alternative $A_1$ is greater than the relative closeness coefficient for the alternative $A_3$. This means that the alternative $A_3$ is worse than the alternative $A_4$ for almost all the values of $Q$ and $\alpha$. 

15
\[
\begin{array}{cccccc}
\alpha & A_1 & A_2 & A_3 & A_4 & A_5 \\
0 & 0.7195 & 0.6923 & 0.6026 & 0.6211 & 0.9078 \\
0.1 & 0.6793 & 0.5464 & 0.4441 & 0.5592 & 0.8972 \\
0.2 & 0.6733 & 0.5267 & 0.3432 & 0.538 & 0.8843 \\
0.3 & 0.6679 & 0.5099 & 0.2702 & 0.5166 & 0.8683 \\
0.4 & 0.6623 & 0.4944 & 0.2145 & 0.4937 & 0.848 \\
0.5 & 0.6562 & 0.4794 & 0.1706 & 0.4684 & 0.8211 \\
0.6 & 0.6492 & 0.4636 & 0.1351 & 0.4392 & 0.7843 \\
0.7 & 0.4606 & 0.445 & 0.1057 & 0.4027 & 0.7303 \\
0.8 & 0.6284 & 0.4199 & 0.0805 & 0.3503 & 0.6441 \\
0.9 & 0.601 & 0.3772 & 0.0581 & 0.2506 & 0.4852 \\
1.0 & 0.1018 & 0.1019 & 0.0359 & 0.0143 & 0.0227 \\
\end{array}
\]

Table 13 Relative Closeness coefficient of alternatives for \( Q = 5 \), varying \( \alpha \)

- From the Figure 5, we can conclude that the alternative \( A_2 \) is better than alternative \( A_4 \) for nearly all the values of \( Q \) and \( \alpha \).
- Figure 4 shows that the alternative \( A_1 \) is better than the alternative \( A_2 \) for almost all the values of \( Q \) and \( \alpha \).
- Figure 3 concludes that \( 3 \), the alternative \( A_5 \) is better than \( A_1 \) for almost all the values of parameters and \( Q \) and \( \alpha \).

Hence, it is evident that the same ranking \( A_5 \succ A_1 \succ A_2 \succ A_4 \succ A_3 \) of alternatives persists for all values of the parameters \( Q \) and \( \alpha \). This shows the stability and the generalization ability of the proposed approach.
7 Comparative analysis

To show the rationality and superiority of the proposed method over the existing methods, a comparative analysis has been done of the proposed approach with some existing methods.

Suppose the Department of Mathematics, IIT Roorkee wants to select a suitable candidate for the post of Teaching Assistant. Assume that five candidates $\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4,$ and $\vartheta_5$ have applied for this post. The expert panel has to consider four criteria for selecting suitable candidates for this post, namely,

1. The candidate should have a good knowledge of applied mathematics ($\psi_1$).
2. The candidate should also have sufficient knowledge of pure mathematics ($\psi_2$).
3. The candidate with knowledge of computer programming will be preferable ($\psi_3$).
4. The candidate should have good communication skills ($\psi_4$).

The weight vectors for the above criterion are given as $(0.35, 0.2, 0.2, 0.25)$. The initial decision matrix for the problem is given in Table 14.

The alternative rankings obtained by different methods are summarised in Table 15. Assuming all the Q-ROFN are fully reliable for other methods. Also, since all the attributes are of the same type, i.e., benefit type, there is no need for normalization for the initial decision matrix methods.

The comparative analysis of the proposed method and the sensitivity analysis on the parameter has been done in Sections 6 & 7. One can see that the proposed method gives a different alternative ranking because the reliability of the alternatives, i.e.,
Fig. 5 Surface plot for the relative closeness coefficient of alternative $A_2, A_4$

Fig. 6 Surface plot for the relative closeness coefficient of alternative $A_4, A_3$
Joshi et al. (2020) have introduced the MCDM method in which results. Furthermore, a series of generalized dice similarity measures for entirely. Hence, one cannot ignore that dimension of decision-making to get rational.

dissatisfaction level, but they have completely ignored the dissatisfaction level for an evaluated alternative. However, the results discussed in the article shows that the dissatisfaction level, but they have taken the satisfaction level for the evaluated alternative, namely the confidence level for the previously defined fuzzy sets. Hence, \( Q \)-ROZN is a more flexible tool to solve TOPSIS problems. Joshi et al. (2020) have introduced the MCDM method in which they have taken the satisfaction level for the evaluated alternative, namely the confidence level, but they have completely ignored the dissatisfaction level for an evaluated alternative. However, the results discussed in the article shows that the dissatisfaction level for the evaluated alternative can change the ranking of the alternatives entirely. Hence, one cannot ignore that dimension of decision-making to get rational.

### Conclusion

\( Q \)-ROZN serves better in the fuzzy environment in comparison to \( Q \)-rung orthopair fuzzy numbers because it has greater freedom to tackle the uncertainty in comparison to the previously defined fuzzy sets. Hence, \( Q \)-ROZN is a more flexible tool to solve TOPSIS problems. Joshi et al. (2020) have introduced the MCDM method in which they have taken the satisfaction level for the evaluated alternative, namely the confidence level, but they have completely ignored the dissatisfaction level for an evaluated alternative. However, the results discussed in the article shows that the dissatisfaction level for the evaluated alternative can change the ranking of the alternatives entirely. Hence, one cannot ignore that dimension of decision-making to get rational results. Furthermore, a series of generalized dice similarity measures for \( Q \)-ROFZN has been introduced. Finally, a TOPSIS method has been introduced based on the generalized dice similarity measures, followed by a detailed sensitivity analysis of the variable involved. It can also be noted that for a stable ranking, the higher values of the parameters are more suitable. In the future, one can develop the aggregation operators for the \( Q \)-ROZN and can use these aggregation operators in solving different decision-making problems.
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References
https://doi.org/10.1007/978-3-642-46768-4_5
https://doi.org/10.1007/s40815-020-01024-3
https://doi: 10.1109/TITS.2022.3186294
https://doi.org/10.1016/j.engappai.2021.104403

https://doi.org/10.1007/s10588-017-9242-8

https://doi.org/10.1111/exsy.12609


https://doi.org/10.1007/s40747-020-00145-4


https://doi.org/10.1007/s41066-022-00314-5

https://doi.org/10.1002/int.21934


https://doi.org/10.1007/s40747-021-00551-2


