Path tracking optimization of tractor-trailer wheeled platforms taken into account the wheels dynamics

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Path tracking optimization of tractor-trailer wheeled platforms taken into account the wheels dynamics

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Abstract:

Tractor-trailer wheeled platforms are recognized as articulated robotic platforms contain a frontrunner active mobile platform (the tractor) pulling one or more passive ones (the trailers). These categories of mobile platforms are broadly implemented in cooperative activities thanks to their novel features like as: maximum load carrying capacity and minimum energy consumption. The presented article deals with path tracking optimization of the tractor-trailer moving platform employing indirect method of the optimal control. Firstly, the nonlinear motion relations of the modular platform are obtained pertaining to kinematic constraints and inertia characteristics of wheels. Then, the obtained dynamic equations are supposed as constraints of the problem and an appropriate objective criterion is determined for the optimal trajectory tracking. Then, the problem is solved and various simulations are presented. The obtained results reveal proficiency and applicability of the proposed procedure for the optimal tracking of the tractor-trailer wheeled mobile platforms.

Keywords: Tractor-trailer, platform, path tracking, optimization, wheel dynamics.

1. Introduction

Mobile articulated platforms have many advantages such as larger operating workspace and more maneuverability over their fixed colleagues. With respect to their novel features, they are extensively implemented in variant practical industries and commercial applications [1-3]. Therefore, the mobile robots (especially the wheeled ones) have received vast attraction and treated by many robotic researchers [4-6]. In addition, to enhance the output and effectiveness, it is appealingly endeavored to generate optimal paths for mobile robotic platforms. In other words, optimal point-to-point motion planning and optimal trajectory tracking of the mobile robotic platforms have received the most attentions by many robotic researchers: Lee et al. [7] studied optimal path planning of holonomic movable platforms by localizing vision sensors. They
assumed holonomic wheels which cannot be a proper model for the most common wheels used in robotic applications. Nazemizadeh et al. [8] derived nonlinear dynamic relations of a nonholonomic movable robot and investigated its optimal control in general tasks from an initial to final locations. Also, they investigated optimal path planning of the kinematically constrained robots among fixed [9] and moving [10] obstacles. Also, time-optimal trajectory planning of differential-wheeled moving robots was presented in [11] based on a geometric approach and the bang-bang control input. Mirzaeinejad and Shafei [12] derived kinematic and dynamic relations of a wheeled moving robot by applying the recursive Gibbs–Appell technique and studied its optimal trajectory tracking retaining the predictive control method.

On the other point of view, an enormous deal of interests is devoted to a kind of articulated moving platforms called ‘Tractor-Trailer’. This kind of mobile robots is known as efficient ones for public transportation because of their vast advantages such as maximum load carrying capacity and lowest energy consumption. The tractor-trailer wheeled moving platform (TTWMP) is an articulated mobile platform includes of prior active moving platform pulling some unactuated platforms. Due to several prominences, the TTWMP has been obtained many practical applications and studied by several robotic researchers: Ritzen et al. [13] proposed handling control of the trailer to enhance maneuverability of the moving platform through circular paths. They only considered the kinematics of the TTWMP and developed an advanced controller to track a given path. Though, the TTWMP is only modeled kinematically and dynamic behavior and motor restrictions of the moving platform were not considered. Li et al. [14] studied time-optimal path design of tractor-trailer robots optimizing simultaneous dynamic of the system. Also in [15], trajectory tracking control of TTWMPs was investigated taken into account nonholonomic constraints and kinematic model. Herein, a feedback linearization procedure was proposed to lessen tracking deviations of the system. Yue et al. [16] proposed a hierarchical control procedure including model predictive and adaptive fuzzy controllers for path tracking of tractor–trailer vehicles. Also, in [17] Trajectory tracking of a tractor-trailer robot using model predictive control was presented. Kinematic relations of the TTWMP were derived and the tracking controller was designed to minimize a predefined cost function. Yue et al. [18] proposed a control strategy based on neural network to estimate of a disturbance-like parameter of the system and control TTWMPs. Furthermore in [19], a feedback controller was developed to control an articulated TTWMP. A simple model of the TTWMP was developed by selecting proper parameters and a neural based control strategy is employed to track a reference path. Khalaji [20] derived the kinetic relations of a TTWMP taken into account nonholonomic restraints of the moving platform. However he estimated the nonlinear kinetic relations of the system, but he ignored the dynamic features of wheels. Liu et al. [21] proposed the neural adaptive control methodology to track a desired path for TTWMPs. However, they implemented the controller on the dynamic model of the robot, but of the wheels inertia were not taken into account.

In fact, in previous researches, the inertia characteristics of wheels are neglected in deriving of dynamic governing equations of the TTWMPs. However, it cannot be an accurate presumption particularly for multi section robots with many wheels such as the TTWMPs. Therefore, in the presented article, the optimal trajectory tracking of the TTWMP is performed with regards to
nonholonomic and inertia effects of the wheels. An innovative set of generalized coordinates are defined and the nonlinear kinetic equations of the TTWMP are obtained considering mass and inertia effects of wheels. Then the optimal tracking of the mobile platform is organized as an optimal control formulation and solved. The simulated results are presented which shows the efficiency of the proposed method for optimal trajectory tracking of the robot. Moreover, the article is organized as: In next section, a comprehensive formulation for tracking optimization of the article is presented. Also, kinematics and nonlinear kinematic equations of the mobile platform is stated in section 3. Finally, some simulations are presented and discussed in section 4 and the article is ended in section 5.

2. Optimal trajectory tracking formulation

Here in, the optimal trajectory tracking of the TTWMP is organized as an optimal control formulation. To form the optimal tracking problem, governing relations of the system is generally considered in the state space form as follows:

\[
\dot{X}(t) = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} + \begin{bmatrix} S X_2 \\ N(X_1, X_2) + Z(X_1) \end{bmatrix} U
\]  

(1)

Where \( X = [X_1 \; X_2]^T \) is the state vector and \( U \) is the input vector. Assuming \( \Omega \) be the set of the permissible control inputs, the aim is to determine the optimal input \( U^*(t) \in \Omega \) which the TTWMR can track a desired path while the following cost criterion must be minimized:

\[
J_0(X, U) = \int_{t_0}^{t_f} L(X(t), U(t), t) \; dt
\]  

(2)

Where \( t_0 \) and \( t_f \) are start and stop time of movement, respectively. To track a given trajectory, the definition of a proper cost function is a critical matter. Herein, the cost function is defined as:

\[
J_0(X, U) = \int_{t_0}^{t_f} \left( \|X - X_{ref}\|_W^2 + \|U\|_R^2 \right) \; dt
\]  

(3)

Where \( \|X - X_{ref}\|_W^2 = (X - X_{ref})^T W (X - X_{ref}) \) is the generalized squared norm of the tracking errors considering a weighting matrix \( W \) and \( \|U\|_R^2 = U^T R U \) is the generalized squared norm of the input vector with respect to a weighting matrix \( R \). As the optimal trajectory tracking of the TTWMR is organized as an optimal control problem, an indirect solution is employed to obtain the optimality conditions. The proposed procedure can treat the optimal problems with underactuated and high nonlinear systems like as the TTWMP. The solution is started by forming the Hamiltonian function (Korayem et al., 2014):

\[
H(X(t), U(t), P(t), t) = L(X(t), U(t), t) + P^T(t) \; G(X(t), U(t), t)
\]  

(4)

Where \( P(t) \) is known as the co-state vector. Then, indirect solution procedure is employed to solve the problem and resulted in the following optimality conditions:

\[
\dot{X}^*(t) = \frac{\partial H}{\partial P}(X^*, U^*, P^*, t)
\]

\[
\dot{P}^*(t) = -\frac{\partial H}{\partial X}(X^*, U^*, P^*, t)
\]

\[
O = \frac{\partial H}{\partial U}(X^*, U^*, P^*, t)
\]

(5)
where $U^*$ is optimal input value. Also, Eq. (5) leads to a set of differential equations where build a two-point boundary value problem with respect to start and end positions of the moving mobile platform.

3. Nonlinear dynamic equation of the TTWMP

Herein, the nonholonomic constraints and nonlinear dynamics of the tractor-trailer movable platform are developed. Respecting to Fig. 1, the TTWMP includes a frontrunner active tractor platform and a pulling trailer moving platform.

![Figure 1. Schematics of a tractor-trailer moving platform](image)

according to Fig. 1, parameters of the TTWMR are given as: $c_1$ and $c_2$ are the gravity points of the frontrunner and the pulling platform, respectively; $L_1$ and $L_2$ are the distance among the wheels of the tractor and the trailer platform, respectively; $d_1$ and $d_2$ are the distance between the wheels of frontrunner and the pulling platform to their center of gravity, respectively; $b$ is the distance among points $o_1$ and $o_2$; $\phi_1$ and $\phi_2$ are the orientation of the frontrunner and pulling platforms, respectively; $\dot{\theta}_{1r}$ and $\dot{\theta}_{1l}$ are the rotational speeds of the right and left wheels of the frontrunner platform and $\dot{\theta}_{2r}$ and $\dot{\theta}_{2l}$ are the rotational speeds of the right and left wheel of the pulling platform, respectively.

To obtain equations of the nonholonomic restraints of the TTWMP, no lateral and longitudinal slippage of each wheel of the tractor-trailer robot must occur. To obtain these nonholonomic constraints, the position and velocity vectors of the wheels should be written and finally the nonholonomic kinematic equations of the robot are calculated as:
\[\begin{align*}
-x\sin\varphi_2 + y\cos\varphi_2 - b\dot{\theta}_1 &= 0 \\
x\cos\varphi_2 + y\sin\varphi_2 + L_2\dot{\varphi}_2 - r\dot{\theta}_2 &= 0 \\
x\cos\varphi_2 + y\sin\varphi_2 - L_2\dot{\varphi}_2 + r\dot{\theta}_2 &= 0 \\
-x\sin\varphi_1 + y\cos\varphi_1 &= 0 \\
x\cos\varphi_1 + y\sin\varphi_1 + L_1\dot{\varphi}_1 - r\dot{\theta}_1 &= 0 \\
x\cos\varphi_1 + y\sin\varphi_1 - L_1\dot{\varphi}_1 - r\dot{\theta}_1 &= 0
\end{align*}\]  
(6)

By defining the generalized coordinates of the articulated moving platform as
\(\xi = [x \ y \ \varphi_1 \ \varphi_2 \ \theta_2 \ \theta_1 \ \theta_1]^T\) the kinematic constraints of the TTWMP can be stated in the matrix form as:
\[\Lambda(\xi) \ \ddot{\xi} = 0\]  
(7)

Where the jacobian matrix \(\Lambda\) is:
\[
\Lambda = 
\begin{bmatrix}
-sin\varphi_2 & cos\varphi_2 & 0 & -b & 0 & 0 & 0 & 0 \\
-cos\varphi_2 & sin\varphi_2 & 0 & L_2 & -r & 0 & 0 & 0 \\
-cos\varphi_2 & sin\varphi_2 & 0 & -L_2 & 0 & -r & 0 & 0 \\
-sin\varphi_1 & cos\varphi_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
cos\varphi_1 & sin\varphi_1 & L_1 & 0 & 0 & 0 & -r & 0 \\
cos\varphi_1 & sin\varphi_1 & -L_1 & 0 & 0 & 0 & -r & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]  
(8)

In the next step, the nonlinear kinematic relations of the TTWMP are obtained taking into account the kinematic restrictions. Now, employing the Lagrange’s principle, the governing formulation of the system can be stated in the close form as:
\[M \dddot{\xi} + \ddot{V}(\xi, \dot{\xi}) = E \ddot{U} + \dot{\lambda}^T \ddot{\lambda}\]  
(9)

where \(M\) is the mass matrix, the vector \(\ddot{V}\) defines nonlinear terms of the tractor-trailer moving platform, \(\ddot{U}\) describes the input moments applied to the frontunner wheels, \(E\) is the coefficient matrix of the wheels moments, and \(\ddot{\lambda}\) is the Lagrangian multipliers vectors associated with the kinematic restrictions of the TTWMP. By implementation of the Lagrange’s principle, the matrices can be summarized as:
\[
M(\xi) = 
\begin{bmatrix}
\frac{m_1 + m_0 + 2m_w + 2m_n}{m_0} & 0 & -d_m\sin\varphi_1 & (bm_1 + 2bm_w)\sin\varphi_1 & 0 & 0 & 0 \\
0 & m_0 + m_1 + m_0 + 2m_w + 2m_n & -d_m\cos\varphi_1 & -d_m\cos\varphi_1 & 0 & 0 & 0 \\
0 & d_m\cos\varphi_1 & -d_m\cos\varphi_1 & (bm_1 + 2bm_w)\cos\varphi_1 & 0 & 0 & 0 \\
1 & d_m\sin\varphi_1 & -d_m\sin\varphi_1 & -d_m\sin\varphi_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & I_{w_1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I_{w_1} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
\[
V(q, \dot{q}) = 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
\[
E = 
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]  
(10)

Moreover, to remove the unknown Lagrangian multipliers from Eq. (9), the matrix \(\Gamma\) is determined in the null space of the matrix \(\Lambda\):
\[\Lambda(\xi) \ \Gamma(\xi) = 0\]  
(11)

Therefore, the matrix \(\Gamma\) is expressed as:
Moreover, the speed vector is determined as \( \hat{\eta} = \begin{bmatrix} \dot{\theta}_1 \\ r_1 \dot{\theta}_1 \\ l \end{bmatrix} \), and the matrix \( \Gamma \) must satisfy the following equation, either:

\[
\dot{\xi} = \Gamma \hat{\eta}
\]

(13)

Now, differentiating Eq. (13) leads to:

\[
\ddot{\xi} = \Gamma \ddot{\eta} + \dot{\Gamma} \hat{\eta}
\]

(14)

And multiplying the both sides of Eq. (9) by \( \Gamma^T \) and submitting Eq. (14) into it, the relation is achieved as:

\[
\Gamma^T \left( M(\dot{\eta} + \dot{\Gamma} \hat{\eta}) + \dot{\Gamma}^T \tilde{V} \right) = \Gamma^T E \tilde{U}
\]

(15)

Remind that in Eq. (15) \( \Gamma^T \Lambda^T = 0 \) therefore, the state vector of the TTWMP is defined as, and the nonlinear governing relations of the tractor-trailer robot can be represented in the state space form as:

\[
\dot{X} = \begin{bmatrix} \Gamma \hat{\eta} \\ (\Gamma^T M \Gamma)^{-1}(-\Gamma^T M \Gamma \hat{\eta} - \Gamma^T \tilde{V}) \end{bmatrix} + \begin{bmatrix} 0 \\ (\Gamma^T M \Gamma)^{-1}(\Gamma^T E) \end{bmatrix} \tilde{U}
\]

(16)

The nonlinear formulation of the tractor-trailer mobile platform is obtained taken into account the inertia and dynamics of the wheels. Eq. (16) can be rearranged in the form of Eq. (1) and is presumed as the restrictions of the optimal control problem discussed in the previous section.

4. Simulation results

Sad In this section the trajectory tracking of the tractor-trailer wheeled moving robots are simulated. The parameters of the articulated mobile robot is determined in table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of tractor and trailer [kg]</td>
<td>( m_t = 0.9 ) \quad m_i = 0.33</td>
</tr>
<tr>
<td>Distance from wheels to mass point of the robot [m]</td>
<td>( d_i = 0.029 ) \quad d_2 = ( 0 )</td>
</tr>
<tr>
<td>Moment of inertia of frontrunner and pulling platforms [kg.m^2]</td>
<td>( I_a = 0.0035 ) \quad I_i = ( 0.00078 )</td>
</tr>
<tr>
<td>Wheel radius [m]</td>
<td>( r = 0.026 )</td>
</tr>
<tr>
<td>Mass of the wheel [kg]</td>
<td>( m_w = 0.0005 )</td>
</tr>
<tr>
<td>Moment of inertia of the wheel [kg.m^2]</td>
<td>( I_{m1} = 0.00005 ) \quad I_{m2} = 0.00005</td>
</tr>
</tbody>
</table>
Firstly, it is assumed that the TTWMP travels alongside a circular path with the radius 1m and the obtained path are compared with desired ones [23]. Figure 2 indicates the path of the robot:

![Figure 2. Path of the TTWMP in comparison with Ref. [23]](image)

For another simulation, the wheel mass influences on the optimal motion of the robot are studied. Fig. 3 shows the optimal path of the TTWMP:

![Figure 3. Optimal path of the TTWMP for different wheel mass](image)

As it is seen in Fig. 3, the mass of wheels plays a key role in movement and optimal path design of the mobile platform and cannot be overlooked in deriving the governing kinetic
equations of the robot.

In another simulation, it is desired to track a sinusoidal trajectory. The optimal path tracking of the tractor-trailer moving platform is shown in Fig. 4:

![Figure 4. optimal tracking of a sinusoidal trajectory by the TTWMP](image)

According to Fig. 4, the modular moving platform can properly track the sinusoidal trajectory. After a few traveled paths, the TTWMR detects the trajectory accurately and it is shown the applicability of the proposed optimal tracking controller. Also, the input toques are depicted in Fig. 5:

![Figure 5. optimal input torques of the tractor to track a sinusoidal trajectory](image)
According to Fig. 5, also, the input torques have maximum values at the beginning of the motion, again. Also, the inserted torques are enough smooth and can be implemented in practice. Furthermore, the rotational speeds of the wheels of the tractor are shown in Fig. 6:

![Angular velocity of the tractor to track a sinusoidal trajectory](image)

**Figure 6.** Angular velocity of the tractor to track a sinusoidal trajectory

As it is seen in figure 6, the rotational speeds of the tractor wheels are enough smooth and feasible. For the last simulation, it is desired that the TTWMR robot track a straight line. The initial condition of the modular mobile robot is given as $\overset{\rightarrow}{x}_{\text{start}} = [0 \ 0.25 \ \frac{\pi}{2} \ \frac{\pi}{2} \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^{T}$ and the robot optimally tracks the desired trajectory. The path traveled by the TTWMP robot is showed in figure 7:

![Optimal tracking of a straight line by the TTWMP](image)

**Figure 7.** Optimal tracking of a straight line by the TTWMP
As it is seen in Fig. 7, the TTWMR can track the straight trajectory using the optimal tracking controller. Moreover, in the beginning of the path, the mobile robot maneuvers more to track the trajectory. The optimal torque exerted to the tractor wheels is depicted in Fig. 8:

![Figure 8](image)

**Figure 8.** Optimal input moments of the frontrunner platform to track a straight line

According to Fig. 8, at the beginning of the robot movement, the inserted torques have maximum values which can be related to more maneuverability to track the path and overcoming the initial inertia of the robot. Also, the rotational speeds of the wheels of the tractor are shown in Fig. 9:

![Figure 9](image)

**Figure 9.** Angular velocity of the wheels of the tractor

According to fig. 9, at the beginning of the path, the rotational speed of the right and left wheels are different and then coincide.

5. Conclusion
This article has investigated the tracking optimization of the TTWMP employing indirect procedure of the optimal control. A novel set of generalized coordinates have been defined and the nonlinear governing relations of the TTWMP derived taken into account inertia effects of the wheels. Then, the obtained governing equations have been presumed as restraints of the optimal problem and a suitable objective criterion has been defined for the optimal trajectory tracking. An agile procedure has been developed to resolve the problem and some simulations presented. The simulated results indicate the presented methodology can appealingly be applied to the tractor-trailer robot to optimal track various trajectories such as straight and sinusoidal ones. Also, the obtained optimal input torques are enough feasible and smooth which can be implemented in practice.

**Statements & Declarations:**

**Availability of data and materials**

Data supporting this study cannot be made available due to ethical/legal considerations.

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**Authors' contributions**

Mostafa Nazemizadeh: Conceptualization, Methodology, Writing – original draft, Investigation. Pouya Mallahi kolahi: Data curation, Software, Investigation. Mohammadreza Gandomkar: Writing – review & editing.

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[24]