Representation of prime numbers on the complex plane

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Short Report

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Representation of prime numbers on the complex plane

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Abstract

A novel representation of a quasi-periodic prime numbers on the complex plan has been investigated. We focus on some quantities characterizing the sum of centres of complex way represented prime numbers. The results indicate that prime numbers can be generated by rooting of real numbers and progression can be decomposed into periodic sequences. The main approach is to root-decompose it into prime numbers. Basically, it is applied to extract hidden periodicities in quasi periodic prime series. The periodicities were arranged in parabolic spirals to show their harmonic pattern. Numerical evidence was provided to confirm that primes can be generated by rooting.

Introduction

Prime numbers have a definite starting point and form an infinitely long series. Those are positive integers having 2 factors, one and itself. It can be seen that all primes have a last digit of 1, 3, 5 and 7 or 9 [1], so the primes billow over the set of natural numbers and able to show some arithmetical progression, pattern of prime numbers [2,3]. All sorts of prime patterns established through number of conjectures. In many admissible set of prime numbers, certain pattern can be found in like \( n + a_1, ..., n + a_k \) form. Finding short and long arithmetic progressions in the primes attracted interest in the last decades. At the time of writing the longest known arithmetic progression of primes is of length 23, and was found in 2004 by Markus Frind, Paul Underwood and Paul Jobling. Earlier Moran, Pritchard and Thyssen [4] found arithmetic progression length of 22. Other investigated the difference pattern between primes or distribution of gaps. We can deduct from these that the arithmetic progressions use only admissible set of primes and there is not any arithmetic form, which could use all successive primes. In this paper a series of successive prime numbers is created up to \( 10^6 \) only, due to numerical difficulties in computing and periodic patterns presented including all prime number up to that bound. The series was represented with unit values for the location of prime numbers and 0 otherwise. The Fourier transform successfully applied in mathematics and engineering [5]. The discrete Fourier transform
responses explicitly the equally distanced input series with well-defined periodicities. The aim of this
paper was to show a pattern among the prime numbers on the complex plane.

**Theorem I.**: All prime numbers can be generated using rooting on the complex plane. $\mathbb{R}^+ \to \text{prime set}$, and this function is called rooting.

**Definition**: $p \text{ modulo residuum}$, where residuum $\in ]0,2\pi[$ and $p$ represents prime numbers $\in \mathbb{Z}$ in radian (arc length), means that any $p$ prime number can be place on the unit length radius complex plane by $p = r(cos(p) + i \cdot sin(p))$, where $r = 1$ and $i$ is the imaginary unit.

**Definition**: The de Moivre’s identity states that any real number and consequently any prime number which is a subset of real numbers and integer power $n$ holds $(cos(p) + i \cdot sin(p))^n = cos(np) + i \cdot sin(np)$ and it is related to Euler’s identity as $(e^{ip})^n = e^{inp}$ which establishes the fundamental relationship between the trigonometric formula of prime numbers and the complex exponential function.

**Proof**: For an integer $n$, $S(n) = (cos(p) + i \cdot sin(p))^n = cos(np) + i \cdot sin(np)$. For $n > 0$, induction can be used. $S(1)$ and $S(0)$ are true. Our hypothesis is that $S(k)$ is true for some natural $k$. That is

\[(cos(p) + i \cdot sin(p))^k = cos(kp) + i \cdot sin(kp).\]

If $S(k + 1)$ then $(cos(p) + i \cdot sin(p))^{k+1} =

= (cos(p) + i \cdot sin(p))^k (cos(p) + i \cdot sin(p))

= (cos(kp) + i \cdot sin(kp))(cos(p) + i \cdot sin(p))

= cos(kp)cos(p) - sin(kp)sin(p) + i \cdot (cos(kp)sin(p) + sin(kp)cos(p))

Using trigonometric identities, it becomes $cos((k + 1)p) + i \cdot sin((k + 1)p)$. It can be deduced that

$S(k)$ implies $S(k + 1)$, so it is true for all natural numbers. For $n < 0$ consider

\[(cos(p) + i \cdot sin(p))^{-n} = ((cos(p) + i \cdot sin(p))^n)^{-1}

= (cos(np) + i \cdot sin(np))^{-1}

= cos(-np) + i \cdot sin(-np)\]
This last equation can be written as complex $p^{-1} = \bar{p}/|p|^2$. Since $|p|^2$ is unity, $p^{-1} = \bar{p}$. Hence, $S(k)$ holds for all integers $n$.

**Theorem II:** Let $\omega(p) = e^{i\theta}$ a prime oscillator using the Euler notation, which length is unity vector on the complex plane having $p$ angle as a prime number in radian and measured as arc length. Each prime can be represented on the complex plane if we use prime modulo residaum, where $\text{residaum} = p - (t \cdot 2\pi), t \in \mathbb{R}$ and $p$ starts from 7.

The prime numbers below 7, the 2, 3 and 5 can be represented on the complex plane directly as those less than $2\pi$. For example, $13 \mod (13 - 2 \cdot 2\pi) = 0,43$, which lies in the first quadrant, having positive sine and cosine components. Having more prime numbers on the complex plane, those overlap each other (Figure 1 b.).

**Definition:** the rooting of complex $p$ number can be given using the de Moivre theorem $n\sqrt[\omega(p)]{i} = n\sqrt{(\cos(p) + i \cdot \sin(p))} = (e^{i\theta})^{1/n} = e^{i\theta/n}$. Then the $n$th roots are given $(\cos \frac{p+2\pi k}{n} + i \cdot \sin \frac{p+2\pi k}{n})$ where $k \in [0 \ldots n - 1]$, resulting $n$ times $n$ roots.

**Proof:** Since the earlier induction proof holds for all integers $n$ and $n = -1$ is also an $n$th root of unity, we conclude that rooting can results any complex way represented prime number.

Figure 1. Prime numbers representation on the complex plane a) and the first 180 prime numbers b).
Consider the roots of unity which has unit length. The $n$th roots (represented as $+$ sign on Figure 2.) of that unit length complex prime number $\sqrt[n]{\omega(p)}$ lie evenly on the unit circle. The centre of sum of that $n$th roots lie in the origin.

Figure 2. Prime number 2 and its 5th roots on the complex plane.

The sum of the centre of complex way represented prime numbers has helical structure as $z$ value added to the consecutive primes (Figure 3 and 4.).

Figure 3. Sum of centre of prime numbers.
Consider the complex way represented prime numbers and their roots. It can be clearly seen that the prime numbers can be represented as a root of real numbers on the complex plane. It can be concluded that prime numbers can be generated from itself by rooting.
Compute the $\sum_{i=1}^{N} \omega_i (2\pi s)^{-1}$ $s \in \mathbb{Z}$ which localize the prime numbers in the first quadrant as $s$ increasing. The $\sum_{i=1}^{N} \omega_i (2\pi s)^{-1}$ way represented helical structure is more pronounced when $s = 10, 30, 60$ and 100.

Figure 6. The squeezed complex way represented prime numbers.

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