Bank's strategic interaction, adverse price dynamics, and systemic liquidity risk

Ulrich Krüger (✉ Ulrich.Krueger@bundesbank.de)
Deutsche Bundesbank  https://orcid.org/0000-0003-4269-324X
Christoph Roling
Deutsche Bundesbank
Leonid Silbermann
Deutsche Bundesbank
Lui-Hsian Wong
Deutsche Bundesbank

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Bank’s strategic interaction, adverse price dynamics, and systemic liquidity risk*

Ulrich Krüger
Deutsche Bundesbank

Christoph Roling
Deutsche Bundesbank

Leonid Silbermann
Deutsche Bundesbank

Lui-Hsian Wong
Deutsche Bundesbank

Abstract
In this paper, we introduce two measures, the Systemic Liquidity Buffer (SLB) and the Systemic Liquidity Shortfall (SLS), to assess liquidity in the banking system. The SLB takes an aggregated perspective on liquidity risks in the banking system. In contrast, the SLS focuses on the problematic banks which suffer a liquidity shortfall. These measures provide an add-on to regulatory liquidity measures such as the LCR because they better incorporate a systemic perspective: (1) They model the impact of a funding shock by valuing assets at depressed market prices, (2) Doing so, they explicitly incorporate banks’ strategic responses to a market undergoing sharp price declines. We test our approach using several applications capturing both a short (5 days) and a medium-term (30 days) stress scenario, a sudden rise in interest rates, the impact of banks’ US dollar business, and the recent COVID-19 crisis.

Keywords: Systemic liquidity risk, market liquidity, funding liquidity, contagion, fire sales.

JEL classification: C63, G01, G17, G21, G28.

*Contact address: Wilhelm-Epstein-Str. 14, 60431 Frankfurt am Main, Germany. E-Mail: ulrich.krueger@bundesbank.de (corresponding author), christoph.roling@bundesbank.de, leonid.silbermann@bundesbank.de, lui-hsian.wong@bundesbank.de. The authors thank Sabrina Baier, Marvin Borsch, Markus Brunnermeier, Sébastien Dereeper, Gerardo Ferrara, Alexandros Gilch, Natalie Kessler, Henry Penikas, Alexander Schmidt, Hermann Schulte-Mattler, participants of the World Finance Banking Symposium 2022 in Miami organised by the Florida International University, participants of the 10th International Conference “Financial Engineering and Banking Society” in 2021, organized by the Université de Lille, participants of the 37th Symposium on Money, Banking and Finance in 2021 organized by the Banque de France, participants of the Joint CNB/ECB/ESRB Workshop “Sources of structural systemic risk in the financial system: identification and measurement” 2019 in Prague, participants of the CCBS Workshop “Managing liquidity and funding risk” 2019 at the Bank of England in London, participants of the CCBS Workshop “Systemic risk assessment: identification and monitoring” 2019 at the Bank of England in London, and seminar participants at the Bundesbank for helpful comments. Discussion Papers represent the authors’ personal opinions and do not necessarily reflect the views of the Deutsche Bundesbank or its staff. On behalf of all authors, the corresponding author states that there is no conflict of interest.
1 Introduction

Financial crises have gone hand in hand with liquidity crises, particularly with a dry-up of liquidity in the banking system.\(^1\) As a consequence of the Great Financial Crisis (GFC) of 2007/2008, policymakers have introduced new liquidity management ratios, such as the Liquidity Coverage Requirement (LCR), which regulate the level and composition of liquid assets and financial liabilities for a single bank.\(^2\) The LCR is based on balance sheet positions weighted by fixed liquidity weights. It is a static concept, which does not properly reflect changes in market prices and liquidity risk in different macroeconomic environments. Policymakers have also developed comprehensive concepts to measure liquidity risk at the single entity level based on stressed cash in- and outflows.\(^3\) Beyond liquidity risk at a single entity, academic research has focused on economic mechanisms that have the potential to destabilise the whole financial system (Morris and Shin, 2004; Brunnermeier and Pedersen, 2009; Allen and Gale, 2010; Krishnamurty, 2010).

These models analyse liquidity risk of the market as a whole and provide insight into a possible source of endogenous liquidity risk: An initial negative shock to asset prices and a run on short-term liabilities may force some financial institutions to reduce their balance sheets. As these institutions shed some of their assets to raise cash and repay debt, they exert downward pressure on market prices. Other institutions see a decline in the market value of their assets and may find it challenging to meet their short-term obligations, urging them to sell assets as well. When institutions’ funding suddenly evaporates, and the system goes through a self-reinforcing cycle of price declines, systemic liquidity risk materialises.\(^4\)

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\(^1\) Examples include the liquidity crisis associated with Long-Term Capital Management (LTCM) in 1998 (Gatev, Schuermann, and Strahan, 2007), the Great Financial Crisis in 2007/2008 (Brunnermeier, 2009) or the European debt crisis in the fall of 2011 (Correa, Sapriza, and Zlate, 2016).

\(^2\) The Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR) were introduced into European law for the first time by the Capital Requirements Regulation (CRR, Regulation (EU) No 575/2013), in keeping with the Basel liquidity framework. For details of these liquidity risk management ratios see Fischer and Schulte-Mattler (2023).

\(^3\) For details see Bank (2019) and Cont, Kotlicki, and Valderrama (2020).

\(^4\) To the best of our knowledge, there is no commonly used definition of systemic liquidity risk. The International Monetary Fund (2011) defines it as "the risk that multiple institutions may face simultaneous difficulties in rolling over their short-term debts or in obtaining new short-term funding".
In this paper, we study this interaction of funding and market liquidity risk both theoretically and empirically. The paper addresses the research question of how to measure short-term systemic liquidity risk when focusing on banks’ strategic interaction via adverse price dynamics in distressed asset sales. We take a widespread bank run as given, in which institutions must repay debt fully and immediately. In our model, this funding shock brings about an endogenous change in the market value of banks’ security holdings as imminent cash outflows force banks to offload securities if their initial cash reserves are insufficient to service their liabilities. Banks have two objectives: First, they stay liquid at any point by selling securities if necessary. Second, they minimise losses on the market value of their securities by considering their impact and the impact of other banks’ sales on the market price.

Two forces then drive endogenous price dynamics: If banks expect the market price of securities to fall because other banks will sell securities to deal with funding shortages, they may dispose of them as early as possible to minimise losses in market value. Banks are cautious, however, not to single-handedly accelerate a price drop with their sales. They can therefore prefer to divide sales into small portions and extend them over a longer period.

To summarise the impact of the funding shock and the induced change in the market liquidity of securities, we introduce the Systemic Liquidity Buffer ($SLB$) and the Systemic Liquidity Shortfall ($SLS$). The $SLB$ is the difference between the liquidity inflows from distress asset sales and the liquidity outflows from on-balance and contingent liabilities. The liquidity inflows are approximated by the stock of available liquid assets in the banking system valued at distress prices. This buffer measures how vulnerable the system is to rollover risk: Low or even negative values indicate that the system does not have enough liquid assets to withstand a system-wide bank run. While some institutions in the system may have sufficient liquidity, other institutions may not be able to meet their financial

through widespread dislocations of money and capital markets”. Our notion above emphasises strains in funding markets and institutions’ reactions to price changes and is similar to Krishnamurty (2010) and Shin (2010).
obligations. The Systemic Liquidity Shortfall (SLS) is the amount of liquidity the system would need to ensure that all institutions withstand the funding shock.

We apply the theoretical model to derive the SLB and SLS of the German banking system. First, we examine the distribution of liquidity risk in the cross-section. In a run on the banking system that takes five days, the total liquidity shortfall SLS is about EUR 40 bn, of which one-third is attributable to systemically important banks. Moreover, the SLB illustrates the consequences of a fire sale spiral: comparing the stock of liquid assets valued at the end of the stress episode at depressed market prices relative to the value of the liquid assets before the run reveals an overall loss in the market value of EUR 51 bn. This loss is significant as it accounts for 8% of banks’ aggregate Tier 1 capital.

Second, we examine the evolution of systemic liquidity risk over time, including the recent COVID-19 crisis. To this end, we aggregate individual excess liquidity according to bank-specific regulatory measures and compare this indicator of liquidity risk with the SLB. Before the most intense period of the GFC in September 2008, this alternative measure of liquidity was positive and rising steadily, showing no sign of possible liquidity risks in the banking system. In contrast, the SLB reached its lowest and negative level in mid-2007, pointing to a vulnerability of the system to liquidity risk. The divergence between the two measures before the crisis stems from the impact of fire sales on security prices: When aggregating individual bank liquidity, we implicitly assume that each bank can sell a particular security at the current market price. However, this assumption may underestimate the downward price pressure banks collectively exert in a liquidity crisis. This latter effect is captured in the SLB.

Third, we study the impact of rollover risk in the US-dollar, the most important foreign currency for internationally active German banks, on liquidity in the banking system. Fourth, we examine the interaction of interest rate risk and liquidity risk. In this application, banks are faced with a repricing of banks’ liquid securities due to an upward shift in the yield curve immediately before they face a run on their debt.

Beyond banks’ liquidity situation, the COVID-19 crisis also affected banks’ capital buffers. For details, see Neisen and Schulte-Mattler (2021).
Systemic liquidity risk has been studied extensively recently, and our paper complements this literature. Bakoush, Gerding, Mishra, and Wolfe (2022) study the interaction of liquidity and solvency risk in the US banking system, emphasising the interbank network structure. Similarly, Zhang, Fu, Luc, Wang, and Zhang (2021) studied systemic liquidity risk in a network of publicly listed Chinese banks.

The plan of the paper is as follows. Section 2 presents the theoretical model, and Section 3 explains how we calibrate this model for data analysis. Section 4 discusses four empirical applications of this approach to the German banking system. Section 5 concludes.

2 Conceptual model

First, we explain the concept of the SLB. Then, we describe the model to compute distress prices for liquid securities. Finally, we formally define the SLB.

The SLB is calculated at the individual bank level and consists of the following building blocks:

$$\text{SLB} = \text{current stock of cash (equivalents)} + \text{current stock of liquid securities measured at distress prices} - \text{expected net outflows from liabilities and contingent liabilities.} \quad (1)$$

The SLB follows a cash flow concept. The stock of liquid assets valued at distress prices is a proxy for cash inflows from distress asset sales. The SLB indicates that the bank is prone to liquidity risk if it is minor or even negative. By aggregating the SLB across all banks, we obtain an indicator of the banking system’s resilience to liquidity risk. The SLB is constructed in a similar manner to established excess liquidity requirements, which demand banks to hold enough liquid assets to cover expected liquidity outflows in a specified stress scenario over a certain period.\(^6\) Notably, all variables of the SLB are

\(^6\) This allows us to quickly compare the SLB with established microprudential indicators, see Section 4.
exogeneous\textsuperscript{7}, except for the distress prices of liquid securities, which a model endogenously determines.

To model the distress prices, we assume an exogenous shock to funding liquidity where all banks simultaneously face liquidity outflows from liabilities and contingent liabilities over a certain period. Banks’ counterparties withdraw cash and do not roll over credit. We model an endogenous shock to market liquidity based on the exogenous funding shock. We assume two objectives characterise banks’ decision-making. First, they aim to stay liquid at any point by drawing on existing cash reserves and generating cash through sales of securities. Second, they aim to minimise the market value declines of securities by considering the price impact of their and other banks’ strategies. Assuming that banks take into account the projected liquidity outflows of other banks and their decision-making, the constellation described above leads to a coordination problem among banks in terms of the timing and the volume of sale of liquid securities. The model determines banks’ optimal selling strategies in Nash equilibrium to derive the distress prices.

### 2.1 Modelling distress prices

After having explained the concept of the \textit{SLB}, we now formulate the underlying model.

**Price impact ratio**

The total volume of sales by the entire banking system invokes a price reaction on asset markets. Let $p_{k,t}$ denote the market price of asset $k$ at time $t$. The gross return of an asset class $k$ is denoted as $R_{t,t+1}^k := p_{k,t+1}/p_{k,t}$. A widely used empirical measure of market liquidity suggested by Amihud (2002) considers the (absolute value of the) price change per nominal amount traded in the market. If even small trading volumes are associated with large price changes, the Amihud measure is large and indicates illiquid markets. We consider a modified version of the Amihud measure in our model. We restrict attention to

\textsuperscript{7} Stock of cash, quantities of liquid securities and net outflows are readily available from reports for regulatory liquidity measures. See Section 3 for more details.
falling prices in the stress episode, i.e. $R_{t,t+1}^k < 1$, and assume the constant relationship

$$\lambda_k = \frac{R_{t,t+1}^k - 1}{V_{t,t+1}^k}$$

(2.1)

between returns and the trading volume.\(^8\) $V_{t,t+1}^k$ denotes the trading volume for asset class $k$ measured at market prices at time $t + 1$. Rearranging Equation 2.1, for a given price impact ratio $\lambda_k$ we obtain the relative price decline between the two periods $t$ and $t + 1$:

$$R_{t,t+1}^k (V_{t,t+1}^k) = R_{t,t+1}^k = 1 + \lambda_k V_{t,t+1}^k.$$  

(2.2)

Note that while $\lambda_k$ is constant the simulated price decline $R_{t,t+1}^k$ varies depending on the total sales volume $V_{t,t+1}^k$ (measured by market prices at time $t + 1$). Total sales in asset class $k$ in period $t$ are then given by $V_{t,t+1}^k = \sum_{i=1}^N v_{i,k,t}$, in which $v_{i,k,t}$ denotes the sales volume of bank $i$ in asset $k$ at time $t$. Bank’s sales volume is determined endogenously by our optimisation model.

**Optimisation model**

We formulate the bank’s liquidity management for the above-described shock scenario in a stylised version and consider two banks ($i = 1, 2$), two periods ($t = 1, 2$) and one asset class ($k = 1$) for this purpose. While this simplified version is sufficient to explain the model’s main features, extending it to $N$ banks and $T$ periods is straightforward. We assume that in the optimisation problem below, bank 1 takes sales of bank 2 as given.

\(^8\) Since $\lambda_k$ is assumed to be a constant, it is reasonable to determine the parameter in the most conservative way possible for the empirical model. For more details refer to Section 3.2.
Regarding the sale volume $v_{1,t}$, the optimisation problem for bank 1 is then

$$
\min_{\{v_{1,1},v_{1,2}\}} \sum_{t=1}^{2} a_{1,t}(1 - R_{t,t+1}(V_{t,t+1})) = \min_{\{v_{1,1},v_{1,2}\}} \sum_{t=1}^{2} (-a_{1,t} \cdot \lambda \cdot (v_{1,t} + v_{2,t}))
$$

(2.3)

w.r.t. the transition equations

$$
c_{1,t+1} = c_{1,t} + v_{1,t} - l_{1,t},
$$

$$
a_{1,t+1} = a_{1,t} \cdot (1 + \lambda \cdot (v_{1,t} + v_{2,t})) - v_{1,t},
$$

and additional constraints

$$
-c_{1,t} \leq 0,
$$

$$
-v_{1,t} \leq 0,
$$

$$
-v_{1,t} - c_{1,t} + l_{1,t} \leq 0,
$$

$$
v_{1,t} \leq \left( \frac{a_{1,t}}{1 - \lambda a_{1,t}} \right) \cdot (1 + \lambda v_{2,t}), \text{ for } t = 1, 2.
$$

The objective function minimises bank 1’s losses in market value on its holdings of liquid securities $a_{1,t}$. The transition equations show the evolution of cash holdings $c_{1,t}$ and the market value of the asset $a_{1,t}$ that bank 1 has. $l_{i,t}$ denotes outflows from deposits and other forms of debt. The first two inequalities are non-negativity constraints on the cash holdings and sales volumes. The third constraint ensures that the bank serves its liquidity providers. The fourth constraint is an upper bound to the sales volume which results from the availability of assets and the reduction in market prices.

In the remainder of this section, we rely on two assumptions.

**Assumption 2.1**

- a) $\lambda = (R_{t,t+1} - 1)/V_t$ is strictly negative, i.e. $\lambda < 0$,

- b) $l_{i,1} - c_{i,1} > l_{i,2}$ for $i = 1, 2$. 
The first assumption says that we focus on falling prices in the stress episode. The latter assumption considers that banks’ liabilities net of liquid funds follow a monotonic decreasing pattern over time. The assumed liability profile should be consistent with the actual conditions since banks’ short-term liabilities usually exceed their long-term liabilities (see also Figure 1(a) and 1(b)). The second assumption is helpful for technical reasons. In such a setting, it can be proved that at least one equilibrium always exists. Before discussing possible equilibria, we state the following result.

**Theorem 2.1**

Let

\[
\begin{align*}
d_{1,1}(v_{2,1}, v_{2,2}) &= \frac{1}{2} (l_{1,1} + l_{1,2} - c_{1,1}) + \frac{1}{2} \left( v_{2,2} + \left( \frac{\lambda a_{1,1}}{1 - \lambda a_{1,1}} \right) v_{2,1} \right) \quad \text{and} \\
d_{1,2}(v_{2,1}, v_{2,2}) &= \frac{1}{2} (l_{1,1} + l_{1,2} - c_{1,1}) - \frac{1}{2} \left( v_{2,2} + \left( \frac{\lambda a_{1,1}}{1 - \lambda a_{1,1}} \right) v_{2,1} \right).
\end{align*}
\]

Then, under assumption 2.1, the optimal solution to problem (2.3), denoted by \((v_{1,1}^*, v_{1,2}^*)\), is given by

1. **Just-in-time**: \(v_{1,1}^* = l_{1,1} - c_{1,1}, \ v_{1,2}^* = l_{1,2}\) if \(d_{1,1}(v_{2,1}, v_{2,2}) < l_{1,1} - c_{1,1},\)

2. **Smoothing**: \(v_{1,1}^* = d_{1,1}(v_{2,1}, v_{2,2}), \ v_{1,2}^* = d_{1,2}(v_{2,1}, v_{2,2})\) if \(l_{1,1} - c_{1,1} \leq d_{1,1}(v_{2,1}, v_{2,2}) < l_{1,1} + l_{1,2} - c_{1,1},\)

3. **Front-Servicing**: \(v_{1,1}^* = l_{1,1} + l_{1,2} - c_{1,1}, \ v_{1,2}^* = 0, \) if \(l_{1,1} + l_{1,2} - c_{1,1} \leq d_{1,1}(v_{2,1}, v_{2,2})\) and \(\frac{a_{1,1}}{1 - \lambda a_{1,1}} \geq v_{2,2},\)

4. **Distress-Sale**: \(v_{1,1}^* = \left( \frac{a_{1,1}}{1 - \lambda a_{1,1}} \right) (1 + \lambda v_{2,1}), \ v_{1,2}^* = 0, \) if \(l_{1,1} + l_{1,2} - c_{1,1} \leq d_{1,1}(v_{2,1}, v_{2,2})\) and \(\frac{a_{1,1}}{1 - \lambda a_{1,1}} < v_{2,2}.\)

A key variable in the theorem is \(d_{1,1}\). It determines the decision of bank 1, which of the four possible strategies is optimal. Note that the optimal strategy for bank 1 in period 1 in the Smoothing strategy coincides with the variable \(d_{1,1}\), which determines which of the four strategies is optimal. The four strategies ensure that bank 1 can raise sufficient
cash and simultaneously minimises the price drops caused by its selling behaviour. If it anticipates significant sales by bank 2 in period 2, it may prefer to sell some of its holdings immediately to trade at a relatively favourable price. Aware of its influence on the market price, the bank may restrain sales in period 1 to avoid driving down the price. After all, slumping prices affect traded securities and reduce the market value of the remaining portion of the assets. We refer to Appendix A.1 for more detailed explanations regarding the intuition of the expressions for each of the four cases and to Krüger, Roling, Silbermann, and Wong (2022) for the proof of Theorem 2.1. A symmetric version of the theorem holds for bank 2, given that bank 1 has decided on its strategy.

The strategic interaction created through the bank’s influence on the price impact of the asset means that the problem takes the form of a 2-period game in pure strategies under complete information, in which banks choose selling strategies to minimise their losses in the market value of its portfolio and to meet liquidity outflows. Next, we show that Nash equilibria in this setup always exist so that the individually optimal selling strategies are mutually compatible and banks avoid becoming illiquid.

**Theorem 2.2**

Under assumption 2.1, a Nash equilibrium exists. In other words, for each initial parameter setting \( w = (a_{1,1}, a_{2,1}, c_{1,1}, c_{2,1}, l_{1,1}, l_{1,2}, l_{2,1}, l_{2,2}, \lambda) \), a combination of strategies \( (v_{1,1}, v_{1,2}) \) for bank 1 and \( (v_{2,1}, v_{2,2}) \) for bank 2 exists such that simultaneously the following two statements hold:

1. The strategy vector \( (v_{1,1}, v_{1,2}) \) of bank 1 is an optimal solution to the optimisation problem w.r.t. the strategy \( (v_{2,1}, v_{2,2}) \) of bank 2.

2. The strategy vector \( (v_{2,1}, v_{2,2}) \) of bank 2 is an optimal solution to the optimisation problem w.r.t. the strategy \( (v_{1,1}, v_{1,2}) \) of bank 1.

We refer to Krüger et al. (2022) for the proof.

Two opposing incentives determine banks’ optimal strategy in this model. On the one hand, banks will individually strive to sell their assets as quickly as possible to be ahead
of competing banks and secure favourable prices. On the other hand, they will try to divide the sale into small portions so as not to accelerate the price drop singlehandedly. Hence, large banks tend to act more cautiously than those with little influence over the market price. A banking system in which the portfolio of assets and the liabilities of one bank is much more extensive relative to those of the other bank tends to reach a Nash equilibrium where the large bank chooses Just-in-time or Smoothing and the small bank chooses Front-Servicing or Distress-Sale.

2.2 The Systemic Liquidity Buffer

At the end of the stress period, we take stock of a bank’s remaining liquid funds and the market value of the remaining security portfolio. We define the $SLB$ for bank $i$ as

$$SLB_{i,T+1} = c_{i,T+1} + a_{i,T+1} = c_{i,1} + \sum_{t=1}^{T} (v_{i,t} - l_{i,t}) + a_{i,T+1}, \quad (2.4)$$

where $a_{i,T+1}$ denotes the security portfolio’s market value, evaluated at prevailing prices after the stress period has ended. While $c_{i,T+1}$ includes actual cash flows regarding outflows and asset sales, this second term $a_{i,T+1}$ is a hypothetical cash flow. Here, we ask how much cash the bank could generate if it sold the remaining portfolio.

By aggregating the $SLB$ across all banks, we obtain the liquidity buffer of the entire banking system,

$$SLB_{T+1} = \sum_{i=1}^{N} SLB_{i,T+1} \quad (2.5)$$

where $N$ denotes the total number of banks in the system.

We also introduce the systemic liquidity shortfall ($SLS$),

$$SLS_{T+1} = \sum_{i=1}^{N} \min \{ SLB_{i,T+1}, 0 \}, \quad (2.6)$$

including only those banks with insufficient liquidity in a crisis. In this way, banks with
sufficiently large liquidity positions do not offset illiquid banks. While the SLB measures the number of liquid assets available in the system after the funding shock, the shortfall SLS is informative about the level of liquidity needed in the banking system to ensure that all banks withstand a liquidity shock. Notice that the SLS is directly derived from the SLB, so for brevity, we will sometimes refer to the SLB in the following sections.

We refer to the buffer as a systemic liquidity buffer for two reasons. First, we examine an extreme scenario in which all banks, or a substantial part of the banking system, face liquidity outflows simultaneously. Therefore, the buffer results from a system-wide liquidity shock. Second, a bank’s asset sales can push market prices down affecting the market value of securities held by other banks and is, therefore, likely to affect the liquidity management of other banks in the system.

### 3 Implementation

First, we introduce a generalised model, which can be used for numerical analyses of the German banking system. Then, we describe the regulatory and market data of net outflows, liquid assets and cash. Finally, we discuss the model assumptions.

#### 3.1 Generalised problem

To perform numerical analyses of the German banking system using regulatory and market data, we need to expand the model considering $N$ banks, $K$ asset classes and $T$ days. In this setup, a bank $i$ has to decide on the sales volume of its assets $a_{i,k,t}$ for each asset class $k = 1, \ldots, K$ and in each point in time $t = 1, \ldots, T$. In order to simplify the problem, we will consider a pro-rata approach to asset sales and assume that the sales volume of bank $i$ in asset class $k$ at day $t$ is given by

$$ v_{i,k,t+1} = \omega_{i,t} \cdot a_{i,k,t} \cdot P_{t,t+1}^k(V_{k,t+1}), $$

(3.1)
The variable $\omega_{i,t}$ describes the fraction which bank $i$ spreads equally across asset classes at day $t$. The decisions by bank $i$ to be made are captured by the strategy vector $\omega_t = (\omega_{i,1}, \ldots, \omega_{i,t}, \ldots, \omega_{i,T})$. The gross return per asset class $k$ is denoted as
\[
R^k_{t,t+1}(V_{k,t+1}) = 1 + \lambda \cdot V_{k,t+1}. 
\] (3.2)

Combining 3.1 and 3.2, we obtain
\[
R^k_{t,t+1}(\omega_t) = \frac{1}{1 - \lambda \cdot \sum_{i=1}^{N} \omega_{i,t} \cdot a_{i,t,k}}. 
\] (3.3)

Taking into account 3.3, the optimisation problem for the implementation model for bank $i$ is

\[
\min_{(\omega_{i,t})_{t=1}^{T}} \sum_{t=1}^{T} \sum_{k=1}^{K} a_{i,t,k}(1 - R^k_{t,t+1}(\omega_t)), 
\] (3.4)

such that, for all $t = 1, 2, \ldots, T$, and $k = 1, 2, \ldots, K$,

\begin{itemize}
  \item[(L)] $c_{i,t+1} \geq 0$,
  \item[(C)] $c_{i,t+1} = c_{i,t} + \left( \sum_{k=1}^{K} \omega_{i,t} \cdot a_{i,t,k} \cdot R^k_{t,t+1}(\omega_t) \right) - l_{i,t}$,
  \item[(B)] $0 \leq \omega_{i,k,t} \leq 1$,
  \item[(V)] $a_{i,k,t+1} = a_{i,k,t} \cdot R^k_{t,t+1}(\omega_t) - \omega_{i,t} \cdot a_{i,t,k} \cdot R^k_{t,t+1}(\omega_t)$.
\end{itemize}

Due to its complexity finding a closed-form solution to the optimisation problem goes beyond the scope of this paper. To reach a satisfactory solution, we develop a heuristic approach that iteratively determines each bank’s optimal strategy conditional on the strategies of the other banks. Details are laid out in Appendix A.2.

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9 For the particular case of one asset class, i.e. $K = 1$, and two banks, i.e. $N = 2$, the optimisation problem (2.3) and the more general problem (3.4) below are equivalent. We refer to Krüger et al. (2022) for the proof.
3.2 Data

Net outflows, liquid assets and cash

Data on net outflows, the initial stock of liquid assets and cash are obtained from the Common Reporting Framework (COREP), which includes two different data sets: (1) the Additional Monitoring Metrics for Liquidity (AMM) and (2) the Liquidity Coverage Requirement (LCR). The AMM data set is the basis for our analyses assessing the banking system’s resilience to liquidity risks in the short run (see Section 4.1), and the LCR data set is used to compare the SLB with microprudential liquidity requirements over time (see Section 4.2). To conduct a long-term study before 2018 reports complement the LCR data set by the German Liquidity Directive, which covers the period from the end of 2000 to the end of 2017.

German banks report both data sets on a monthly basis. Generally, we use reporting information at the banking group level.\textsuperscript{10} For banks not part of a banking group we use reporting information at the solo level. Furthermore, we consider the unique properties of the banking associations’ liquidity management into account. As the savings and cooperative banks are integrated into a central cash management controlled by their respective central institutions, it is not reasonable to model them as independently acting players in a systemic liquidity crisis. Thus, we assume that savings banks coordinate with their regional head bank (Landesbank), and cooperative banks coordinate with their respective central institute so that each association would act like a single banking group when selling securities.

We aggregate banks’ reported liquid assets into five classes: government bonds, uncovered bonds, covered bonds, shares and asset-backed securities.\textsuperscript{11} We assume that these

\textsuperscript{10} The implicit assumption is that liquidity can quickly be transferred within the same banking group in a stressful period. The assumption could be substantial for large international banks, which conduct worldwide business operations across several jurisdictions. See, for example, the \textit{Financial Stability Board (2018)} on the ongoing regulatory discussion regarding complexities associated with liquidity in resolution for global systemically important banks.

\textsuperscript{11} The reported instrument breakdown for the German Liquidity Directive differs. We construct the following asset classes using the national liquidity regulation: bonds, covered bonds, money market papers, equities and collateral eligible for refinancing at central banks.
classes properly reflect differences in liquidity in times of stress, and we assign different price impact ratios to each class. Further details regarding their computation can be found below. For each asset class, market values are reported, which we use as initial values when modelling distress sale losses.

We determine the daily net outflows for each bank based on outflows from funding obligations net of inflows from non-fungible assets (e.g. inflows from interest income). We obtain outflows for the AMM data set according to their contractual obligations. For deposits, we also consider so-called behavioural outflows (see Section 4.1 for further details). The AMM data set provides a granular breakdown of banks’ maturities of obligations. In particular, inflows and outflows are reported daily for the first seven days. To ensure that the stress scenario is sufficiently severe, we restrict daily net outflows to be floored by zero, i.e. inflows from non-fungible assets cannot overcompensate outflows from payment obligations. \(^{12}\) The LCR data set refers to a period of 30 calendar days for the net outflows, and the German Liquidity Directive refers to a period of one month, respectively. As no more detailed breakdown by maturity is provided, the daily net outflows for each bank are calculated based on the simplifying assumption of uniformly distributed outflows over the referred period.\(^{13}\)

**Price impact ratio**

A vital parameter of the empirical model is the price impact ratio \(\lambda\). Since our price impact ratio is a constant value in the empirical model, it is reasonable to determine \(\lambda\) for each asset in the most conservative way possible. We calculate the daily associations between the aggregated trading volume and the trading volume weighted average price decline across all securities that belong to a particular asset class \(k\) over a specific observation period. We select the smallest value (which reflects the most significant daily price impact)

\(^{12}\) In contrast, according to the LCR the amount of inflows that can offset outflows is capped at 75% of total expected cash outflows. This requires that a bank maintains a minimum amount of stock of HQLA equal to 25% of the total cash outflows.

\(^{13}\) We confirmed the robustness of our results by a distribution of net outflows over 30 days skewed to the right, where most outflows occur during the first few days of financial stress.
among the calculated daily associations and assign it to the price impact ratio $\lambda_k$.

In principle, the approach follows the concept of the Amihud-Ratio, which is defined as the average daily association between a unit of trading volume (measured in USD) and the relative price change for an individual security over a certain period, e.g. one year (see Amihud (2002)).\footnote{Due to high computational effort and insufficient data granularity that prevents us from applying price impact ratios on an individual security basis we have to aggregate assets and assign common price impact ratios to each asset class. The choice to aggregate assets implies that all assets within one class correlate equally to one. This assumption tends to overestimate the simulated distress sale losses. However, the imprecision should not be very large. First, banks’ largest security class is ‘government bonds’, which primarily consists of only one asset, namely ‘Bundesanleihen’ (Bunds). Second, in times of severe market liquidity stress, downward price pressure is often exerted simultaneously on different securities, i.e. correlations are usually high in a liquidity crisis.}

For government bonds (which make up by far the largest part of bank’s portfolio of liquid assets) we use MTS data\footnote{MTS is one of Europe’s leading electronic fixed-income trading markets and a significant fraction of Italian government bonds is said to be traded via this market. Italy was one of the countries most affected by the European Sovereign debt crisis.} on daily bond prices and turnovers and calculate the average price impact ratio for Italian government bonds during the period from the beginning of May to the end of June 2012, which was just prior to the ECB president Mario Draghi’s famous speech on July 26, 2012 at UKTI’s Global Investment Conference over the irreversibility of the Euro and ECB’s preparedness to do ‘whatever it takes’. During this turbulent European Sovereign debt crisis, spreads of 10-year Italian bonds over the corresponding German government bonds interest rate have been the largest in recent history. The trading volume and price data for corporate bonds and covered bonds not traded on a centralised exchange are captured based on data from the TRACE reporting system with Bloomberg’s TACT analysis and valuation function. The sample covers the period from June 2016 to August 2016.

### 3.3 Discussion of the model assumptions

Our model makes three specific assumptions. First, we disregard the role of the central bank as a lender of last resort, as we want to test the banking system’s resilience to a widespread funding shock without assistance from the central bank. Consequently, we as-
sume banks do not have access to central bank funding. In particular, repo transactions vis-à-vis the central bank are excluded.\footnote{Having said that, we do not wholly disregard central banks. Banks’ reserves with the central bank have become an important part of banks’ liquidity buffers. As central banks worldwide have adopted extraordinary monetary policy measures in recent years, their decisions and actions have implications for the data we use in our paper applications (see Section 4).} This assumption is guided by the macroprudential, i.e. preventive focus of our liquidity metrics. They are supposed to pick up liquidity risks that do not anticipate lender of last resort activities in the spirit of Bagehot (1873), as addressing those liquidity risks in a timely manner would ideally make central bank intervention less likely and necessary.\footnote{Additionally, distinguishing between liquidity and solvency problems might be challenging in a crisis (see, for example, Thakor (2015)).} Second, we assume that banks cannot rely on interbank credit as a potential funding source during episodes of stress. In particular, banks cannot offset the cash outflows through the interbank repo market. Banks depend on outright asset sales as a short-term funding source to service their liabilities in such a scenario.\footnote{While in Germany, the majority of all repo transactions are traded on the interbank repo market (see ECB (2017) and ICMA (2021)), some large non-bank financial intermediaries, such as investment funds and insurances, also have access to the repo market and could provide short-term liquidity to the banking system through the repo channel. We leave it for future research to incorporate private repo markets in the analyses.} Third, we assume banks restrict asset sales to securities designated as liquid assets according to the Capital Requirements Regulation (CRR), i.e. high-quality liquid assets (HQLA), which are eligible for the LCR. We are thus following the underlying concept of the LCR where banks should build up a buffer of HQLA that can be used in times of stress. Adopting such a regulatory perspective allows us to readily compare the results produced by our model with the LCR.

4 Applications

We use the SLB to address four policy issues. First, we examine the impact of a severe funding shock on systemic liquidity over five days. We supplement the baseline funding shock with two alternative shock scenarios covering the German banks’ US dollar business, and suddenly rising interest rates in the banking system. Finally, we contrast the SLB
with microprudential liquidity requirements, including the LCR.

4.1 How resilient is the banking system in the short run?

As Gorton and Metrick (2012) point out, the GFC was a system-wide run on the banking system. Funding risks materialised at various stages of the crisis and affected several segments of wholesale funding markets, including a dry-up of the (asset-backed) commercial paper market in August 2007 and in September 2008 (Kacperczyk and Schnabl, 2010; Brunnermeier, 2009) and a sharp increase in haircuts in the repo markets (Gorton and Metrick, 2012). Moreover, at the height of the crisis in September 2008, non-financial firms heavily relied on existing credit lines or loan commitments (Ivashina and Scharfstein, 2010; Cornett, McNutt, Strahan, and Tehranian, 2011). Some institutions even faced a decline in their retail deposits (Shin, 2009). Empirical evidence of distress sales in the market for residential mortgage-backed securities during the GFC is provided by Merrill, Nadauld, Stulz, and Sherlund (2014).

Baseline funding shock

In our base case, banks experience a combination of some of the funding difficulties illustrated above. We examine an extreme scenario, in which there is a widespread run on financial institutions for a short time, say, five days. We assume liabilities become due according to their contractual maturity during this period. We deviate from this approach concerning deposits. Deposits include sight deposits held by retail investors, some of which are subject to deposit insurance and are, therefore, less run-prone. Banks provide contractual and so-called behavioural outflows for deposits in their data reports. These behavioural outflows take banks’ estimates of actual business dynamics of deposit outflows into account and reflect the experience that they are a lot stickier than their contractual maturities suggest. Therefore, we consider this scenario with behavioural deposit outflows to be the baseline case, but we also examine results when the contractual deposit outflows are used instead.
Panel (a) of Figure 1 shows that outflows account for more than 20% of liquid assets in this horizon of five days, and that outflows beyond five days up to 30 days would add little stress in this scenario. Note the significant difference between net outflows according to contractual maturities (blue) and net outflows adjusted to incorporate behavioural outflows from deposits (red). Taking these behavioural outflows into account, we obtain a scenario that may resemble a breakdown of wholesale funding markets.

In Panel A of Table 1, we explore the cross-sectional distribution of liquidity risk for December 2021, by evaluating the $SLB$ and the $SLS$ for several institutions according to their business model. We learn that there is substantial heterogeneity in the cross-section: while there is no shortfall among cooperative and savings banks, commercial banks have a shortfall of about EUR 40 bn. If we sort the banking system according to the systemic importance of institutions, we observe a shortfall of about EUR 15 bn for systemically important institutions in this scenario. Overall, the $SLS$ is helpful in assessing the liquidity needs of the banking system and also helps to find potentially vulnerable institutions within the system.

In addition, we consider the loss in market value that banks experience during the funding shock. Here, we compare the stock of liquid assets valued at the end of the stress episode at possibly depressed market prices relative to the value of the liquid assets before the run on the banking system started. There is an overall loss in the market value of EUR 51 bn in this period. While this loss is 3% of liquid assets, it is significant for most banks in terms of Tier 1 capital. The loss is also relevant for systemically important institutions, which suffer a loss of 9% of their Tier 1 capital.

Finally, in Figure 2, we depict the evolution of the gross returns by asset class to illustrate the dynamics of the simulated downward price spiral. Note that we adopt the pro-rata assumption in our model, so that asset sales are distributed across the whole portfolio of the banks. Therefore, we observe a price impact for all types of assets. Figure 2

\footnote{In addition, we examine the $SLB$ when contractual deposit outflows are used. In this case, the scenario becomes even more severe and the $SLB$ aggregated across all banks becomes negative (EUR -1,200 bn). This result shows that the treatment of deposit outflows can greatly impact any liquidity analysis.}
shows a sharp price fall over the first two days, then a flattening. This is because most banks, in particular smaller institutions, engage in distress sales, in which they sell their stock of liquid assets as early as possible. The cumulative price declines vary between 2% and 12%. Notably, government bonds suffer the most significant decline, despite having the most minor (absolute) price impact ratio. Significantly, the price impact across asset classes depends on the commonality and level of banks’ security holdings. As government bonds account for the bulk of banks’ liquid securities (roughly two-thirds), their large total selling volume drives the relatively large price drop experienced by sovereign bonds.20

**US-Dollar funding shock**

Thus, the US dollar business plays an important role for internationally active banks, and funding of European institutions in this currency has tended to be vulnerable in times of market-wide distress.21 Therefore, we specifically analyse systemic liquidity risk in the US dollar. To this end, we reapply the baseline funding shock outlined above, which is limited to liquid assets and net outflows in the US dollar.

Panel (b) of Figure 1 shows the net outflows in US dollars across maturity bands, as a percentage of liquid assets in US dollars. Note that in contrast to the case in which we consider all maturities, adjusted net outflows now exceed the net outflows with contractual outflows. There are two reasons for this result. First, banks do not maintain sizeable retail business in US dollars. Consequently, there is not much impact from replacing contractual outflows from deposits with behavioural outflows. Second, the adjusted net outflow also incorporates contingent outflows from credit lines, pushing the adjusted outflow over the net outflow without contingent outflows. This figure also highlights that banks may be vulnerable to liquidity shocks in certain currencies, as in this case, the system faces net outflows over five days which exceed liquid assets in that currency.

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20 It is necessary to add, however, that in a crisis, government bonds may be subject to flight-to-liquidity effects as a result of increased demand by institutional investors, which could dampen the price declines shown here if these effects are not fully captured in the price impact parameter (Santis, 2014; Beber, Brandt, and Kavajecz, 2008).

The results of this exercise are presented in Panel B of Table 1. We see that the overall SLB is positive, indicating that the banking system is resilient to a funding shock with distress sales for exposures in US dollars. However, the overall SLS is large relative to the SLS in the baseline case, which includes all currencies. The shortfall of USD 29 bn is primarily due to a shortfall in systemically important institutions. These institutions dominate the US dollar business conducted by German banks. As this shortfall is mostly concentrated on systemically important institutions, these findings may suggest that the German banking system is vulnerable to a funding shock in US dollars.\(^{22}\)

**Combined interest rate and funding shock**

We examine the interaction between interest rate and liquidity risk. We combine the baseline funding shock studied above with an exogenous interest rate shock, which shifts the yield curve upwards. In this way, we add another risk channel that impacts the banking system’s resilience. In addition to the exogenous increase in outflows on banks’ liability side and the endogenous distress sales by banks to restore liquidity, there is a new effect: Rising interest rates lower the present value of banks’ assets immediately. Most importantly, from a liquidity management perspective, the market value of fixed-income securities decreases. In this sense, the asset valuation channel opens up in two ways, an instantaneous repricing effect and a distress-sale effect that materialises throughout the stress horizon.\(^{23}\)

We start by describing the interest rate shock that affects the market value of the bond portfolio. Using data from the Bundesbank’s Securities Holdings Statistics (SHS), we obtain a sample of the government bonds that German banks held in March 2018.

\(^{22}\) Note that a simplification had to be considered. The analysis assumes that banks only use liquid assets in US dollars to deal with the funding shock. In practice, banks can issue additional debt in euros, and then transform these funds into US dollars by using FX swaps. Similarly, they can use existing Euro cash to buy US dollars on the spot market. Incorporating these features into the model requires additional assumptions on the nature of the EUR/USD swap market or the evolution of the EUR/USD spot rate. We leave these extensions for future work.

\(^{23}\) Notice that this combined shock is adopted in a pure ad-hoc fashion. We do not claim that rising interest rates may cause a run on the banking system or vice versa. Furthermore, we neither model the effect of a rise in interest rates on the value or composition of banks’ liabilities, nor consider income-related effects on capital.
We accompany these securities with market data from Thomson Reuters Datastream, including the modified duration. According to the modified duration, an overnight increase in interest rates of 100 basis points is associated with an average loss in the market value of 8.5% and a median loss of 6%. We view this measure of the sensitivity of banks’ bond holdings to changes in interest rates as a guideline in this exercise: We examine a scenario in which a rise in interest rates results in an instantaneous loss in the market value of banks’ bonds of 10%. Given the information from the sample described above, this loss is larger and more widespread, as we adopt this drop in the value of bonds to the entire portfolio, including government and other types of bonds.

In Panel C of Table 1, we present the combined interest rate and funding shock results. The initial shock of 10% corresponds to a market value loss of about EUR 51 bn, while the distressed sales result in a loss in the market value of about EUR 44 bn. The total shortfall in the system increases only slightly, but the losses in market value on banks’ securities increase substantially relative to the baseline funding shock, from EUR 51 bn to EUR 95 bn. Accordingly, the loss as a share in aggregate Tier 1 capital increases from 8% to 15%. Hence, in this exercise, the additional interest rate shock increases the level of market value losses, but does not change the dynamics of the funding shock significantly, as the shortfall in the combined scenario (Panel C) does not change much relative to the baseline scenario (Panel A).

4.2 How does the systemic liquidity buffer differ from the microprudential view on liquidity risk?

In principle, the microprudential regulation requires banks to hold a certain amount of liquid assets to cover their expected net outflows in case of a stress scenario over a certain period, say 30 days. Typically, liquid assets are weighted depending on how easily an

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24 We focus on bond issues by the following countries: Austria, Germany, Spain, France, Greece, Great Britain, Italy, Portugal and the US.
25 In June 2020, systemically relevant institutions held government bonds with a market value of EUR 256 bn. Smaller institutions (less significant institutions) had a bond portfolio with a market value of EUR 197 bn.
asset can be expected to raise cash at short notice. For example, cash has a weight of 100%, while corporate bonds are assigned lower weights, say of 50% or 85%, depending on the rating of the bond. The net outflows are the difference between cash outflows and cash inflows a bank faces. Outflows are derived from the bank’s liabilities and weighted depending on their rollover risk and run risk. For instance, it is typically assumed that retail deposits are more stable than deposits of other commercial banks in a stress period and hence attract a smaller weight. An analogous approach applies to the inflows a bank expects, such as repayments on interest and principal made by non-financial customers.

**Time series analysis from 2000 to 2017**

Until 2017 banks' liquidity was regulated nationally in the German Liquidity Regulation. According to the national regulation, an institution’s liquidity was deemed sufficient if available liquid assets cover the expected net outflows for the next month. We investigate systemic liquidity risk over a longer period of time from 2000 to 2017, using data from the German liquidity regulation. This period is particularly interesting as it included the period before the GFC in 2008. This exercise aims to present a long-run view of systemic liquidity risk.

Figure 3(a) presents the evolution of the aggregated $SLB$ across all banks in Germany and the aggregated excess liquidity according to the national, microprudential regulation from 2000 to 2017. Notably, both indicators are constructed in a similar manner: They equal the sum of stock of cash and liquid securities less of net outflows. The only difference is the measurement of liquid securities. While simulated distress prices for the $SLB$ measure liquid securities, the microprudential approach assigns fixed weights which do not consider a reduction for most types of securities. We make three observations.

(i) The $SLB$ is lower than the aggregate excess liquidity throughout. Hence, considering the impact of potential distress sales, systemic liquidity is lower than the aggregate of the individual liquidity measures. This is plausible given that the microprudential approach does not consider reducing the actual market price for most types of liquid
assets.

(ii) The $SLB$ falls below zero in the second quarter of 2006 and reached its low point in the second quarter of 2007, while the microprudential excess liquidity continues to rise. Therefore, ahead of the most intense period of the GFC in September 2008, the $SLB$ indicates that the banking system is vulnerable to liquidity risk. The reason for the diverging patterns of the $SLB$ and the microprudential excess liquidity is that the banking system increased its short-term funding on a large scale in the run-up to the crisis from June 2003 to June 2007, resulting in an increase of net outflows of nearly 65% to EUR 763 bn. Notably, an increase in net outflows has a twofold effect on the $SLB$: Net outflows are directly deducted from the $SLB$ (as for the microprudential excess liquidity), and, in addition, an increase in net outflows results in lower prices of liquid assets once banks start selling assets to raise cash, which depresses the $SLB$ further. The latter effect is not reflected in the microprudential view. In this sense, the $SLB$ has the potential to serve as an early warning indicator of systemic liquidity risk induced by excessive short-term refinancing in the banking system.

(iii) In 2008, both measures of liquidity risk decreased since the GFC. German banks drastically reduced their interbank borrowing and hoarded liquidity. These effects result in substantially lower net outflows and a more prominent cash position, which increases both the $SLB$ and aggregate excess liquidity.

**Time series analysis from 2018 to 2022**

After the GFC, liquidity requirements for banks were substantially revised and harmonised, resulting in the Basel III regulatory standard. In Germany, the LCR replaced the German Liquidity Regulation in 2018 and related a liquidity buffer to a net liquidity outflow over a 30-calendar-day period.\(^{26}\)

We adopt an analogous approach to compare the aggregated $SLB$ with the aggregated

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\(^{26}\) The LCR was introduced into European law for the first time by the CRR. For details on further regulatory reforms, such as the CRR II and the CRR III, and their impact on banks see Neisen and Schulte-Mattler (2022), Neisen and Schulte-Mattler (2020a) and Neisen and Schulte-Mattler (2020b).
excess liquidity according to the LCR, which builds the foundation for microprudential liquidity requirements. Again, the two measures differ only in the valuation of liquid securities. We depict the evolution of the SLB and the aggregate excess liquidity since 2018 in Figure 3(b). We make two observations.

(i) As the microprudential excess liquidity and the SLB are larger than zero, both measures indicate sufficient liquidity in the system to withstand the underlying funding shock. For most of the period under review, the SLB is, however, lower than the excess liquidity. Again, this is plausible given that for the microprudential excess liquidity, the reduction of the actual market price is very small for most securities that banks hold for liquidity management. Government bonds account for more than two-thirds of all securities designated as HQLA. Most government bonds receive a weight of 100% and are thus measured at their current market price.\(^\text{27}\) Thus, each bank individually assumes that government bonds, say, can be sold at the current market price, but this assumption may neglect the downward price pressure exerted by banks collectively. In contrast, the SLB takes the effect of distress sales on market prices, resulting in a reduction of up to 35% during the observation period.

(ii) Over time the difference between the microprudential excess liquidity and the SLB has decreased, and has even temporarily disappeared after the outbreak of the COVID-19 pandemic in 2020 and 2021. The reason why the gap has narrowed is that most banks have built up large central bank funds (cash reserves) following the introduction of extraordinary monetary policy measures in the euro area, such as the Targeted Longer Term Refinancing Operations (TLTRO-III) and the Pandemic Emergency Longer-Term Refinancing Operations (PELTROs). Accordingly, in June 2020, cash reserves accounted for 58% of aggregate HQLA. Cash reserves are taken into account equally in both the SLB and the LCR framework. As they form such a large part of banks’ liquid assets, the need to liquidate other assets in times of stress is relatively low. Thus, a large share of cash reserves causes the level of both indicators to converge.

\(^{27}\) Equivalently, they are subject to a haircut of 0% if their risk weight for credit risk is also 0% under the Basel Capital Adequacy Rules.
The Bridge between LCR and $SLB$

Table 2 provides a comparative calculation between the LCR excess liquidity and the $SLB$ to illustrate the differences between the two measures. The calculation is done for December 2019 and December 2021 because the difference between the two measures has decreased significantly during the outbreak of the COVID-19 pandemic. As shown in Table 2, the difference between the $SLB$ and the LCR excess liquidity is mainly driven by the different valuations of government bonds. While for government bonds, the LCR haircut is zero or very small, their simulated distress sale loss is immense. As government bonds account for the bulk of banks’ liquid securities (roughly two-thirds), their large total selling volume drives the relatively large price drop experienced by sovereign bonds. Notably, for uncovered bonds, shares, and ABS, the losses due to distress sales are lower than the LCR haircut. While the (absolute) price impact ratios for uncovered bonds, shares and ABS are relatively high, their stock of securities held by banks is relatively low. Consequently, the simulated selling volume is relatively low, and so is the downward pressure on market prices.\footnote{As the stock of covered bonds has almost halved between December 2019 to December 2021 the simulated selling volume and the downward pressure on market prices has reduced respectively. This explains why the losses due to distress sales are higher than the LCR haircut in December 2019, but lower in December 2021.}

These observations regarding the haircuts reveal an interesting point regarding our model: The haircuts computed for the $SLB$ are not necessarily more conservative than the LCR haircuts, but instead they are dynamic and reflect the characteristics of a particular stress situation. The percentage drop in prices increases over proportionally with the amount of securities to be sold.\footnote{We are aware that our model does not capture the entire financial system. Rather it represents the sales of the banking system during a liquidity crisis. The price drops for covered and sovereign bonds should be quite accurate given the large holdings of the banking system in these segments. The share of government bonds held by German banks in the amount of all German government bonds outstanding has been around 15% for the past decade and recently declined to 12%. As this market share is not negligible, considerable price reactions are to be expected if many banks were to sell government bonds at the same time. The price movements determined for the other asset classes with smaller portfolios held by banks might be slightly upwardly biased. Nevertheless, since the volumes of securities that need to be sold properly reflect the banks’ funding needs in a crisis scenario, our statements regarding the strategic interaction of banks and the numeric results should represent good proxies.}
5 Conclusions

This paper measures systemic liquidity risk by analysing banks’ strategic interaction via adverse price dynamics. The model tests the banking system’s resilience to an exogenous funding shock. It gauges the collective impact of a funding liquidity shock and distress sales on financial institutions, illustrating how strategic bank behaviour can further amplify price declines. In addition, we propose two indicators termed $SLB$ and $SLS$. The first metric measures the resilience of the banking system to such a funding shock scenario with distress sales, and the latter metric measures the aggregate liquidity need in such an extreme event. Both measures are expressed in nominal terms and are therefore easy to interpret.

We demonstrate the practicality of our framework with four examples: First, we compute the impact of a severe funding shock in the short run. We supplement the baseline funding shock with two alternative shock scenarios covering the German banks’s US dollar business and suddenly rising interest rates in the banking system. Finally, we compare the $SLB$ with microprudential measures.

This framework is helpful for policy makers in the context of macroprudential surveillance. Like the $SLB$, the microprudential LCR assumes a stress event where funding suddenly evaporates, and banks face projected outflows over a specified time. However, the LCR assigns fixed liquidity weights (haircuts) to securities designated as HQLA, whereas the $SLB$ assigns distress prices that vary over time depending on system-level factors, particularly the aggregated short-term funding in the banking system. The higher the aggregated short-term funding, the lower the simulated distress prices for securities according to the model underlying the $SLB$. In this respect, the $SLB$ is more sensitive than the LCR to changes in the aggregated short-term funding. It has the potential to provide early warning of mounting vulnerabilities in the banking system caused by excessive short-term borrowing. The $SLB$ signalled an increase in systemic liquidity risks ahead of the GFC 2007-08 by a decline in the corresponding liquidity buffers.
In addition, established microprudential indicators might be too optimistic regarding systemic liquidity because they do not account for distress sales. For example, the LCR applies a haircut of zero to most government bonds eligible for refinancing at the central bank. While from a microprudential point of view, a liquidity risk-weight of zero for these safe and liquid securities is meaningful, from a macroprudential view, such an approach may underestimate systemic liquidity risk at times. In a financial crisis, the market liquidity of government bonds can deteriorate suddenly. Likewise, refinancing eligibility at the central bank may be restrained (as was the case for Greek government bonds during the European Sovereign debt crisis in 2012). In this respect, our contribution is to provide an indicator that signals systemic liquidity stress in time. Finding suitable macroprudential instruments to deal with the identified systemic liquidity risks is beyond the scope of this paper and is left for future research.30

When working with complex strategic interactions between multiple decision-makers, we must make simplifying assumptions. For example, we take a narrow view of the banks’ strategies. We assume that banks sell securities to maintain liquidity but do not become buyers in these markets. Therefore, the above scenario assumes that funds leave the German banking system entirely and are shifted to other banking systems. Such a setting does not consider a reallocation of funds within the German banking system, which may have a stabilising effect. In times of stress, investors may shift funds that they have provided to some institutions to other institutions that are perceived as high-quality banks, see Pérignon, Thesmar, and Vuillemey (2018) for such effects in the market for certificates of deposits.

At the same time, this setting does not consider adverse effects stemming from predatory trading, which may amplify price declines in a liquidity crisis, as modelled by Brunnermeier and Pedersen (2005). We also assume that banks sell their securities in propor-

30 From the policy perspective, the question arises of whether regulators should consider a (possibly time-varying) requirement to the current microprudential measures such as the LCR and/or the NSFR or whether other complementary instruments are needed to address systemic liquidity risk adequately (see, e.g. European Systemic Risk Board (2014)). This issue is currently being discussed in different regulatory forums (see, e.g. European Central Bank Task Force on Systemic Liquidity (2018)).
tion to their actual holdings (pro-rata) but do not follow a pecking order. While there is empirical evidence\textsuperscript{31} that banks tend to sell securities in such a way in a crisis event, this certainly leaves room for future research.

Another area for improving the current framework is to integrate the interactions between banks and other sectors of the financial system or the real economy. Caccioli, Ferrara, and Ramadiah (2021) find that ignoring the common asset holdings between banks and the non-banks financial sector can significantly underestimate losses in distressed sales. In this vein, Deutsche Bundesbank (2020) shows that other financial intermediaries, such as insurance companies or mutual funds, often played a key role during past liquidity crises. The same applies to the role of the central bank. In a liquidity crisis, central banks may provide emergency liquidity assistance as lenders of last resort. Integrating such crisis responses by the central bank in the model would allow an ex ante policy evaluation.

\textsuperscript{31} Van den End and Tabbae (2012).
References


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Figure 1: Net liquidity outflows across maturities (as a percentage of liquid assets, June 2020)

This figure shows the aggregate net outflow (liquidity outflow - liquidity inflow) of the German banking system in June 2020, as a percentage of liquid assets (blue). The net outflow is sorted into four maturity buckets, spanning 30 calendar days in total. The net outflow is based on contractual maturities, and similarly to the liquidity coverage ratio (LCR), net outflows are restricted to be non-negative, so that a net inflow is not allowed. In addition, the figure displays the net outflow when contractual outflows from deposits are replaced by behavioural outflows from deposits (red). These behavioural outflows are reported by banks. Furthermore, this adjusted net outflow adds contingent outflows from committed credit and liquidity facilities, denoted as credit lines. Panel (a) shows overall positions (all currencies), while Panel (b) reports positions only in US dollars.
Table 1:
Systemic liquidity risk in the cross section
This table presents the Systemic Liquidity Buffer (SLB) and the Shortfall (SLS) for several banking groups in column (1) and column (2), respectively. In Panel A, we present the results of the baseline funding shock for a 5-day period. Here, net outflows (outflows - inflows) materialise according to their contractual maturity, except for deposits, for which behavioural maturities are used. This baseline shock incorporates liquid assets and net outflows in all currencies of 1,362 banks. In Panel B, we consider a funding shock similar to the shock underlying the results shown in Panel A, but we restrict attention to liquid assets and net outflows denominated in US dollars of 69 banks. In all cases, the net outflows and liquid assets are based on supervisory data (Additional Monitoring Metrics for Liquidity) as of December 2021. In Panel C, we combine the funding shock in Panel A with an instantaneous repricing of banks’ bond portfolio, in which all bonds lose 10% of their market value at the beginning of the stress horizon. For details on the computation of the SLB and the SLS, see Section 3. The loss in market value in column (3) is the decline in the value of the portfolio of assets when they are evaluated at the market prices at the end of the scenario horizon relative to the market value of the portfolio before the stress event. This loss is expressed as a percentage of aggregate liquid assets in column (4) and as a percentage of aggregate Equity Tier 1 capital assuming fair-value accounting in column (5).

<table>
<thead>
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<th>Panel A: Baseline funding shock</th>
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<td>Liquidity Risk</td>
<td>SLB (EUR bn)</td>
<td>SLS (EUR bn)</td>
<td>Loss in market value (EUR bn)</td>
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<td>0</td>
<td>7</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td>1,215</td>
<td>-40</td>
<td>51</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>of which: Systemically important banks</td>
<td>779</td>
<td>-15</td>
<td>38</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: US-Dollar funding shock</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>56</td>
<td>-29</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>of which: Systemically important institutions</td>
<td>37</td>
<td>-23</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C: Combined interest rate and funding shock</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>1,172</td>
<td>-43</td>
<td>95</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>of which: Systemically important institutions</td>
<td>746</td>
<td>-17</td>
<td>73</td>
<td>6</td>
<td>18</td>
</tr>
</tbody>
</table>
Figure 2:
The gross returns for liquid assets according to the Systemic Liquidity Buffer (SLB)
This figure shows the gross returns $R_{t,t+1}$ attached to several types of assets classes over the course of the scenario horizon for the funding shock in December 2021. See also Section 2 for a definition of the gross return.
The Systemic Liquidity Buffer (SLB) over time (in EUR bn)

This figure shows the SLB over time as discussed in section 4.2 from 2000 to 2021. In addition, we depict the aggregate excess liquidity (liquid assets - net outflows) derived from microprudential requirements. The excess liquidity is derived from a German liquidity measure from 2000 to 2017 in Figure 3(a), and from the Liquidity Coverage Ratio (LCR) in Figure 3(b). In either case, the net outflows underlying the SLB and the excess liquidity coincide, but the two approaches differ in the valuation of liquid assets, as the SLB takes distress sales into account.
Table 2: **Bridge between microprudential excess liquidity (LCR) and SLB**

This table bridges the microprudential excess liquidity and the SLB before and after the outbreak of the COVID-19 pandemic (i.e. December 2019 and December 2021) to illustrate the evolution of the differences between the two liquidity indicators. Data on liquid assets and net outflows are extracted from regulatory reports on the LCR.

<table>
<thead>
<tr>
<th>Amounts in EUR bn</th>
<th>Dec 2021</th>
<th>Dec 2019</th>
</tr>
</thead>
<tbody>
<tr>
<td>I HQLA at current market prices</td>
<td>1,744</td>
<td>1,322</td>
</tr>
<tr>
<td></td>
<td>thereof</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cash</td>
<td>1,282</td>
</tr>
<tr>
<td></td>
<td>Government bonds</td>
<td>348</td>
</tr>
<tr>
<td></td>
<td>Covered bonds</td>
<td>83</td>
</tr>
<tr>
<td></td>
<td>Uncovered bonds</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>Shares</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>ABS</td>
<td>5</td>
</tr>
<tr>
<td>II LCR haircut deduction from HQLA</td>
<td>33</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>thereof</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cash</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Government bonds</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Covered bonds</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Uncovered bonds</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>Shares</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>ABS</td>
<td>1</td>
</tr>
<tr>
<td>III Net Outflows</td>
<td>1,025</td>
<td>774</td>
</tr>
<tr>
<td>I-II-III = IV LCR excess liquidity</td>
<td>718</td>
<td>505</td>
</tr>
<tr>
<td>V Delta between distress sale losses and LCR haircut</td>
<td>16</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>thereof</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cash</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Government bonds</td>
<td>40</td>
</tr>
<tr>
<td></td>
<td>Covered bonds</td>
<td>-5</td>
</tr>
<tr>
<td></td>
<td>Uncovered bonds</td>
<td>-10</td>
</tr>
<tr>
<td></td>
<td>Shares</td>
<td>-8</td>
</tr>
<tr>
<td></td>
<td>ABS</td>
<td>-1</td>
</tr>
<tr>
<td>IV-V=VI SLB</td>
<td>702</td>
<td>355</td>
</tr>
</tbody>
</table>
A Appendix

A.1 Further explanations on the four selling strategies in Theorem 1

The goal of this subsection is to develop some intuition regarding the four possible strategies $v_{1,2}, v_{1,2}$ and their drivers. Where the Just-in-time solution applies, bank 1 sells the precise amount of the asset that just satisfies the outflows in each period. In the Smoothing solution, bank 1 chooses sales volumes such that the total liquidity need $l_{1,1} + l_{1,2} - c_{1,1}$ is distributed in a certain way across both periods depending on the strategy chosen by bank 2. To be more precise, bank 1 makes two adjustments to this simple rule to balance two opposing motives.

First, the optimal sale of bank 1 in the first period increases in $v_{2,2}$: if there is a large drop in the price in period 2 due to a sale by bank 2, bank 1 sells a larger amount in period 1 to trade at the relatively high price at that time. Second, the sale of bank 1 in the first period decreases in $v_{2,1}$. The expression $\lambda a_{1,1} / (1 - \lambda a_{1,1})$, which appears in the last constraint in 2.3, measures the potential impact on the market price that bank 1 has in the first period. As bank 2 increases its sale in the first period, bank 1 tends to curb its sale so that it does not accelerate the price decline. The larger the price impact of bank 1 (in absolute value), the larger this tendency to restrain the asset’s sale.

As the first adjustment is positive and the second adjustment is negative, it is not clear which effect prevails. In any case, the negative value of any adjustment to $v_{1,1}^*$ applies to the optimal amount $v_{1,2}^*$, which bank 1 sells in the second period such that the total liquidity need $l_{1,1} + l_{1,2} - c_{1,1}$ is served.

Before discussing the last two cases, let us take a closer look at $d_{1,1}(v_{2,1}, v_{2,2})$, which drives the decision made by bank 1. It has two components: the liquidity need of bank 1 and the impact of the actions of bank 2 on bank 1. Regarding the second component, $\lambda v_{2,1}$ describes the price drop stemming from a sale by bank 2 in the first period. Thus, $a_{1,1} \lambda v_{2,1}$ is the resulting loss for bank 1 due to the asset’s market value decline. As pointed
out above, the level of $v_{2,2}$ determines the magnitude of the price reduction in the second period. Both components are combined in the expression

$$
\frac{1}{2} \left( v_{2,2} + \left( \frac{\lambda a_{1,1}}{1 - \lambda a_{1,1}} \right) v_{2,1} \right),
$$

which describes the impact of the actions of bank 2 that are relevant to bank 1 in absolute terms. Roughly speaking, if (A.1) is small relative to the liquidity need of bank 1, the actions of bank 2 have only a limited impact on bank 1's decision-making. Then, if $l_{1,2}$ is also small, bank 1 focuses on meeting the dominant short-run liquidity need $l_{1,1} - c_{1,1}$ in the first period. As $l_{1,2}$ rises, bank 1 splits the liquidity need more equally among both periods. Therefore, bank 1 decides to increase its sales in the first period above the short-run need and generates more cash at the relatively high price.

In contrast, as (A.1) rises, $d_1$ eventually surpasses the bank’s total liquidity need, and so sales by bank 2 have a significant bearing on the decision made by bank 1. In the Front-Servicing solution, banks 1 already sells an amount equal to the total liquidity needed in the first period and does not make a sale after that. Finally, in the Distress-Sale solution, bank 1 sells the maximum amount available in the first period. So, in contrast to the Just-In-Time or Smoothing solution, bank 1 restricts its sales entirely to the first period in both two cases.

As a consequence, any change in the market price in the second period is driven entirely by bank 2 as $v_{1,2}^* = 0$ in these two cases. Note further that the additional condition $a_{1,1}/(1 - \lambda a_{1,1}) \geq v_{2,2}$ is equivalent to $\lambda a_{1,1}/(1 - \lambda a_{1,1}) \leq \lambda v_{2,2}$. As explained above, $\lambda a_{1,1}/(1 - \lambda a_{1,1})$ is the drop in the market price in the first period induced solely by bank 1 in the extreme scenario that bank 1 sells all assets in period 1. Moreover, as $v_{1,2}^* = 0$, $\lambda v_{2,2} = r_{2,3}$ is the price drop in the second period.

Thus, bank 1 compares a price drop in the first period with the price drop in the second period: if $\lambda v_{2,2}$ was large (in absolute value) relative to $\lambda a_{1,1}/(1 - \lambda a_{1,1})$, then the price would decline very sharply in the second period. Consequently, bank 1 would
suffer a large loss in the asset’s market value at that time. To avoid such a loss, bank 1 prefers to liquidate as much as possible of the asset in the first period, leading to the Distress-Sale solution. Conversely, in the Front-Servicing solution, bank 1 just covers its total liquidity need in the first period, while the potential loss in the market value of the assets held by bank 1 occurring in period 2 is comparatively low.

A.2 Distress sale algorithm

We apply an iterative procedure: Before the iteration process starts, banks are numbered based on a random ranking to determine which bank optimises first, given the strategies of the other banks. The starting values of the iteration are the initial values of banks’ strategies which are set to zero. In each iteration step, the algorithm iteratively calculates each bank’s optimal selling strategy given the selling strategies of the other banks. The implementation of this optimisation step relies on numerical procedures from the Matlab software.

The algorithm stops after a finite number of $M$ iterations, once for all banks, the change in their strategies from iteration step $m$ to iteration step $m+1$ is smaller than a small, positive value $\epsilon$, which we set to 0.001. If the abort criterion is not fulfilled after $m = 50$ iterations, a second (less strict) abort criterion checks if the simulated $SLB$ aggregated across all banks does not change by more than 1%. The second criterion ensures that at least the overall result remains stable and reliable conclusions regarding the overall liquidity situation of the banking system can be made.\footnote{In our applications introduced below, one abort criterion is always satisfied during the iteration process. Our simulations have demonstrated that the more complex the application becomes in terms of a longer shock period or a larger number of banks, the more likely it is that the first criterion is not fulfilled, but the second criterion is.}

Another critical aspect of the empirical model is the treatment of illiquid banks. If a bank has few liquid funds or security holdings, it may become technically illiquid at a specific iteration step. Technically speaking, this means that the non-negative constraints ($L$) and ($B$), as introduced in Section 3.1 cannot be met by the bank, and no feasible
solution exists given the other banks’ strategies as determined during the iteration. For such a case, we need to make specific assumptions about the selling strategy of such illiquid banks. First, we assume that once a bank becomes illiquid during the iteration, those banks are immediately liquidated by a hypothetical resolution authority, and the banks’ entire security holdings are sold on day one for the following iterations. The chosen behavioural assumption reflects a conservative approach and ensures that illiquid banks will tend to further decrease the SLB compared with the impact liquid banks have on the SLB.\textsuperscript{33} Second, we assume that once a bank becomes illiquid during the iteration, it stays in that state until the final iteration. That means when the algorithm calculates a new iteration and banks optimise their strategies based on the updated strategies of the other banks, those banks found to be illiquid in the previous iteration will stay in that state. This approach ensures that banks do not keep switching back and forth between the liquid and illiquid state from iteration to iteration, thereby supporting the algorithm’s convergence.

The box below includes the implementation of our heuristic approach to tackling the optimisation problem 3.4.

\textsuperscript{33} One might argue that the chosen assumption reflects a non-realistic extreme scenario. Another possible alternative could be to allow illiquid banks more time to liquidate their assets, e.g. for illiquid banks their optimisation problem should be applied without the non-negative constraints. It would ensure that these banks can still minimise losses during the distress sale spiral (and therefore would act according to the interests of the banks’ investors). However, this alternative may eventually lead to stark, perverse effects on the SLB. Specifically, illiquid banks would ‘contribute’ to a higher SLB than liquid banks. In other words, the space of feasible selling strategies for liquid banks is bound by constraints which induce those banks to sell securities earlier in the distress sale spiral than they otherwise would and exacerbate the market price decline. Instead, the behavioural assumption we choose ensures that illiquid banks will tend to decrease the SLB relative to liquid banks further.
Distress-sale algorithm

In this box we use $m$ to count the number of iterations. We use the symbol $|| \cdot ||$ to assign the maximum norm to a vector of $N$ components, i.e. $||\omega|| = \max_{i=1}^{N} \{\omega_i\}$. We rely on two different termination criteria which have to be tested before a new iteration step, denoted by $m$, is carried out.

**Criterion 1** $||\omega_m - \omega_{m-1}|| < 0.001$ for all banks $i = 1, \ldots, N$.

**Criterion 2**

$$\left| \frac{\sum_{i=1}^{N} SLB_{m+1} - \sum_{i=1}^{N} SLB_m}{\sum_{i=1}^{N} SLB_m} \right| < 1\%.$$

Note that criterion 1 is equivalent to the requirement that all components of the vector $\omega_m$ should be smaller than 0.001.

**Initialise** the strategy vectors for all banks, i.e. $\omega_{i,t} = 0$ for $i = 1, \ldots, N$ and $t = 1, \ldots, T$. Set $SLB_0 = 0$. Set $m = 1$.

**While** both of the two termination criteria are not satisfied (to be tested for $m > 1$)

**Begin iterate**

For $i = 1$ to $N$

1. If the bank cannot service outflows, it is forced to sell all liquid assets in $t = 1$, i.e. $\omega_{i,1} = 1$ and $\omega_{i,t} = 0$ for $t = 2, \ldots, T$. This assignment is kept fixed throughout all remaining iterations.

2. Determine a strategy vector $\omega_i$ such that $\omega_i$ is optimal w.r.t. optimisation problem (3.4) in the main text under the additional assumption that all other strategy vectors $\omega_j$ for $j \neq i$ are kept fixed. For the $\omega_j$’s the strategy vectors from the previous iteration step are taken into consideration for all banks with index $j > i$ and from the current iteration step for banks with index $i < j$.

**End**

$SLB_m = \sum_{i=1}^{N} c_{i,T+1} + a_{i,T+1}$ in line with formula (2.4) in the main text.

$m = m + 1$

**End iterate**