Horizontal Mergers and Innovation with Knowledge Spillovers

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Horizontal Mergers and Innovation with Knowledge Spillovers

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Abstract

We introduce knowledge spillovers in a model of innovation competition à la Federico et al. (2017) and Denicolò and Polo (2018), which otherwise features horizontal mergers that harm innovation due to the business stealing effect. With knowledge spillovers, competition discourages investment in R&D due to free-riding, and by internalizing such externalities, a merger can improve the incentives for innovation. Horizontal mergers raise (reduce) innovation if the spillover effect is large (small), in contrast with the standard account of spillover effects on mergers.

Keywords: Horizontal merger, innovation, knowledge spillovers

JEL codes: G34, L13, L40, O30

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1 Introduction

Following some influential antitrust merger cases of late (e.g. Dow/DuPont), there has been a rapidly growing interest in the impact of horizontal mergers on innovation. This literature, spurred by Federico et al. (2017, 2018), has identified various pros and cons of horizontal mergers in terms of productive efficiency (e.g. Denicolò and Polo (2018, 2021), Bourreau et al. (2021), Motta and Tarantino (2021)). The results depend on market structure amongst other factors.

This paper adds to this body of research by introducing knowledge spillovers. While the literature does mention knowledge spillovers, the effects are largely of direct nature and assumed away to crystallize other forces. Diffusion of ideas is hampered by a host of frictions, from imperfect observability of innovative activities to physical and other “distances” between innovators, and by removing these frictions, mergers generate synergies.\footnote{Since the work of Jaffe et al. (1993), numerous evidence of the localization effect of knowledge spillovers have been reported. Moreover, despite the revolution in communication technology, this distance effect seems to be getting stronger (e.g. Kwon et al. (2022)).} Thus, the more imperfect the spillover effect is, the more beneficial is the merger.

We also show that horizontal mergers can improve innovation with knowledge spillovers but the underlying mechanism is distinct from the one discussed above. With spillover effects, competition leads to under-investment in R&D due to the public good nature of R&D. The firms free-ride on each other. A merger may then increase innovation by internalizing such externalities. This means that mergers are more likely to improve innovation when the spillover effect is large, not the other way round, because then the under-investment in R&D is also large.

These results are derived in a two-player version of the model by Federico et al. (2017), due to Denicolò and Polo (2018), in which competition boosts the incentives for investment in R&D via the business stealing effect among market rivals and hence horizontal mergers harm innovation. In other words, we formalize the trade-off between two opposing forces of externality that underpin innovation competition.\footnote{See Bloom et al. (2013) for an empirical investigation of the trade-off. They find that the spillover effect dominates in their sample.}

In the unique symmetric Nash equilibrium of our model with competition, the R&D expenditure decreases with the size of the spillover effect. With a log-convex probability of failure,
the optimal decision of the merged firm is symmetric, and from this, we characterize how the spillover effect interacts with the business stealing effect. A merger raises (reduces) innovation if the spillover effect is large (small). A numerical exercise demonstrates that the corresponding spillover threshold can be low and also the results hold beyond the global log-convexity assumption.

2 A Model of Innovation Competition

The Setup We introduce knowledge spillovers into the two-player model of Denicolò and Polo (2018). There are two ex ante identical firms \( i = 1, 2 \). Each firm \( i \) simultaneously chooses costly R&D effort \( x_i \geq 0 \), which measures its cost. The value of the innovation is \( V \). If only one firm succeeds, it obtains the entire surplus; if both firms succeed, the surplus is split evenly.

For each \( i \), the probability of success is given by the innovation production function

\[
F(x_i + \phi x_j),
\]

where \( j \neq i \) and \( \phi \in [0, 1] \) measures the spillover effect, which we assume to be common.\(^3\) As in Denicolò and Polo (2018), we make the following assumptions on the innovation production function. First, \( \text{supp}(F) = [0, R] \) for some \( R \in \mathbb{R}^+ \cup \{\infty\} \) such that \( F(0) = 0 \) and \( F(R) = 1 \). Second, \( F \) is strictly increasing, concave, and twice continuously differentiable. Third, \( F'(0) = \infty \) and \( F'(R) = 0 \).

Symmetric Nash Equilibrium Under the assumptions, we derive a unique symmetric Nash equilibrium.

**Proposition 1.** Fix any \( \phi \in [0, 1] \). There exists a unique symmetric Nash equilibrium \( x_i = x^* \) for \( i = 1, 2 \), which is characterized by the following first-order condition:

\[
F'((1 + \phi)x^*) \left[ 1 - \frac{1 + \phi}{2} F'((1 + \phi)x^*) \right] = \frac{1}{V}.
\]

\(^3\)One can also imagine a world in which the spillover effect differs across firms, for example, when their innovation capacities diverge. As we shall see, the firms’ equilibrium efforts are continuous in \( \phi \). This will be true even with heterogeneous spillover effects, and hence, our results will remain valid as long as the effects are close.
Proof. Fix $i = 1, 2$. Firm $i$’s profit is given by

$$\Pi_i = F(x_i + \phi x_j) \left[ (1 - F(\phi x_i + x_j))V + F(\phi x_i + x_j)\frac{V}{2} \right] - x_i$$

$$= F(x_i + \phi x_j) \left[ 1 - \frac{1}{2}F(\phi x_i + x_j) \right] V - x_i$$

The first-order and second-order conditions are, respectively,

$$\frac{d\Pi_i}{dx_i} = F'(x_i + \phi x_j) \left[ 1 - \frac{1}{2}F(\phi x_i + x_j) \right] V - \frac{\phi}{2} F(x_i + \phi x_j) F'(\phi x_i + x_j)V - 1 = 0;$$

$$\frac{d^2\Pi_i}{dx_i^2} = F''(x_i + \phi x_j) \left[ 1 - \frac{1}{2}F(\phi x_i + x_j) \right] V - \frac{\phi}{2} F'(x_i + \phi x_j) F'(\phi x_i + x_j)V$$

$$- \frac{\phi^2}{2} F(x_i + \phi x_j) F''(\phi x_i + x_j)V - \phi F(x_i + \phi x_j) F''(\phi x_i + x_j)V < 0.$$

By plugging in $x_1 = x_2 = x^*$, the FOC becomes

$$F'((1 + \phi)x^*) \left[ 1 - \frac{1 + \phi}{2} F((1 + \phi)x^*) \right] = \frac{1}{V},$$

as in the claim. Since $F' > 0$ and $F'' < 0$, the left-hand side of the above equation is monotone decreasing in $x^*$. When $x^* = 0$, the LHS becomes $F'(0) \left[ 1 - \frac{1 + \phi}{2} F(0) \right] = \infty \times 1 = \infty$. When $x^* \to R/(1 + \phi)$, the LHS converges to $F' \left( R \right) \left[ 1 - \frac{1 + \phi}{2} F \left( R \right) \right] = 0 \times \frac{1 - \phi}{2} = 0$. Given the monotonicity of the LHS and the continuity of $F$ and $F'$, the intermediate value theorem proves the existence of unique symmetric Nash equilibrium.

To verify the SOC, setting $x_1 = x_2 = x^*$ turns the condition into

$$F''((1 + \phi)x^*) \left[ 1 - \frac{1 + \phi^2}{2} F((1 + \phi)x^*) \right] - \phi F'((1 + \phi)x^*)^2 < 0$$

This inequality must hold since $F'' < 0$ and $0 \leq F(x^*) \leq 1.$

For $\phi \in [0, 1]$, with some abuse of notation, denote the unique solution to the first-order condition in Proposition 1 as $x^*(\phi)$.

Proposition 2. We obtain the following:

(i) $x^*(\phi)$ is increasing in $V$.

(ii) $x^*(\phi)$ is decreasing in $\phi$. 

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(iii) $x^*(\phi)$ is continuous in $\phi$.

Proof. See Appendix A.1.

The first part of Proposition 2 is obvious and the third part is a technical condition that will be used later to compare pre- and post-merger innovation levels. The second part is one of the main take-aways from our model. It says that the public good nature of R&D generates free-riding. As the spillover effect intensifies, so does the free-riding incentive.

3 Merger Effects

After a merger, the merged firm continues to operate two R&D lines and chooses $(x_1, x_2)$ to maximize profit. The innovation production function remains the same for each R&D line. In other words, we do not model direct synergy effects which may shift $F$ itself. The spillover parameter $\phi$ also remains unaffected.

First, we show that the result of Denicolò and Polo (2018) generalizes to our setting. When the probability of failure is a log-convex function of $x$, the merged firm’s optimal R&D choice is symmetric for any $\phi$.\(^4\)

Lemma 1. Fix any $\phi \in [0, 1]$. If $1 - F(x)$ is log-convex, the merged firm’s optimal R&D choice is symmetric.

Proof. The merged’s firm’s profit function is given by

$$
\Pi = [F(x_1 + \phi x_2) + F(\phi x_1 + x_2) - F(x_1 + \phi x_2)F(\phi x_1 + x_2)]V - (x_1 + x_2)
$$

Fix any R&D plan $(x_1, x_2)$. We want to show that the merged firm can improve the success probability by instead choosing $(\frac{x_1 + x_2}{2}, \frac{x_1 + x_2}{2})$.

We want to show that

$$
F(x_1 + \phi x_2) + F(\phi x_1 + x_2) - F(x_1 + \phi x_2)F(\phi x_1 + x_2) \\
\leq F \left( (1 + \phi) \frac{x_1 + x_2}{2} \right) + F \left( (1 + \phi) \frac{x_1 + x_2}{2} \right) - F \left( (1 + \phi) \frac{x_1 + x_2}{2} \right) F \left( (1 + \phi) \frac{x_1 + x_2}{2} \right),
$$

\(^4\)Denicolò and Polo (2018) derive the result for the case of $\phi = 0$.  

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which is equivalent to
\[
\left(1 - F\left(\frac{(1 + \phi) x_1 + x_2}{2}\right)\right)^2 \leq (1 - F(\phi x_1 + x_2)) (1 - F(x_1 + \phi x_2)) \iff \\
2 \log \left[1 - F\left(\frac{(1 + \phi) x_1 + x_2}{2}\right)\right] \leq \log [1 - F(\phi x_1 + x_2)] + \log [1 - F(x_1 + \phi x_2)].
\]
Since \(\log[1 - F(x)]\) is convex, the final inequality must hold.

Next, we derive the unique symmetric R&D choice of the merged firm. The optimal choice is continuous in the spillover parameter.

**Proposition 3.** Fix any \(\phi \in [0, 1]\). If \(1 - F(x)\) is log-convex, there exists a unique symmetric R&D plan \(x_1 = x_2 = x^m\) for the merged firm, which is characterized by the following first-order condition:
\[
(1 + \phi) F'((1 + \phi)x^m) [1 - F((1 + \phi)x^m)] = \frac{1}{V}.
\]
Furthermore, the solution to the first-order condition, \(x^m(\phi)\), is continuous in \(\phi\).

**Proof.** See Appendix A.2.

We are now ready to present our main result. If the spillover effect is large enough, then a merger increases innovation. Note that since \(2F - F^2\) is strictly increasing in \(F \in [0, 1]\), \(x^m(\phi) > x^*(\phi)\) implies a higher chance of successful innovation under the merger since \(F(x)\) is an increasing function.

**Proposition 4.** If \(1 - F(x)\) is log-convex, there exists some \(\bar{\phi} \in (0, 1)\) such that \(x^m(\phi) > x^*(\phi)\) for all \(\phi > \bar{\phi}\).

**Proof.** By Propositions 2 and 3, we know that \(x^*(\phi)\) and \(x^m(\phi)\) are both continuous functions. Thus, it suffices to show that \(x^m(1) > x^*(1)\).

When \(\phi = 1\), the following two equations hold:
\[
F'(2x^*(1))[1 - F(2x^*(1))] = \frac{1}{V}, \\
F'(2x^m(1))[1 - F(2x^m(1))] = \frac{1}{2V}.
\]
Since our assumptions on \(F\) guarantee that the LHS is decreasing in \(x\), we verify that \(x^m(1) > x^*(1)\).
When there is no spillover effect, i.e. \( \phi = 0 \), our model reduces to that of Denicolò and Polo (2018). In such a case, innovation decreases after a merger because the merger reduces competition and internalizes the negative externality generated by the business stealing effect. While this is true as long as \( \phi \) is sufficiently close to 0, we cannot pin down single-crossing without further assumptions on \( F \). This is because \( x^m(\phi) \) is not necessarily monotone, unlike \( x^*(\phi) \).

In addition to the business stealing effect, which generates a negative externality on innovation, our model captures another, positive, externality due to knowledge spillovers. Before a merger, the firms free-ride on each other’s R&D, and this discourages investment in R&D. The free-riding problem is relaxed by the merger, and when the spillover effect is sufficiently large, this spillover channel dominates the business stealing channel.

4 Numerical Example

We want to know whether the spillover effect can be substantial enough to set the threshold \( \phi \) relatively low. Also, global log-convexity (or log-concavity) is a strong assumption on \( 1 - F \). Our results can nonetheless be obtained outside this assumption.

Consider a class of functions for \( F \), or the constant-elasticity innovation production functions, of the form

\[
F(x) = Ax^\theta,
\]

where \( \theta \in (0, 1) \). This function, introduced by Denicolò and Polo (2018), exhibits log-convexity when \( x \) is low and log-concavity when \( x \) is high. We normalize it by setting \( A = 1 \) so that \( \text{supp}(F) = [0, 1] \), and for computational ease, set \( \theta = 1/2 \), i.e. \( F(x) = \sqrt{x} \). Then, \( 1 - F(x) \) is log-convex when \( x \in (0, 1/4) \) and log-concave when \( x \in (1/4, 1) \).

Let us add the following regularity conditions:

\[
V \leq \frac{4}{1 - \phi} \quad \text{and} \quad V < \frac{2}{1 + \phi}.
\]

(1)  

Note that when \( V \) is sufficiently small (\( V < 1 \)), these two inequalities are both satisfied for any \( \phi \).

\(^5\)See the footnote in Appendix A.2.
Pre-merger  Firm 1’s profit is

\[
\Pi_1 = \sqrt{x_1 + \phi x_2} \left[ (1 - \sqrt{\phi x_1 + x_2})V + \sqrt{\phi x_1 + x_2} \frac{V}{2} \right] - x_1
\]

\[
= \sqrt{x_1 + \phi x_2} \left[ 1 - \frac{1}{2} \sqrt{\phi x_1 + x_2} \right] V - x_1
\]

The FOC is

\[
\Pi'_1 = \frac{\left[ 1 - \frac{1}{2} \sqrt{\phi x_1 + x_2} \right] V}{2 \sqrt{x_1 + \phi x_2}} - \frac{\phi \sqrt{x_1 + \phi x_2} V}{4 \sqrt{x_1 + x_2}} - 1 = 0,
\]

which becomes, after plugging in \( x_1 = x_2 = x^* \),

\[
\frac{1}{2 \sqrt{(1 + \phi)x^*}} \left[ 1 - \frac{1}{2} \sqrt{(1 + \phi)x^*} \right] \frac{\phi}{4} = \frac{1}{V}.
\]

This gives

\[
x^*(\phi) = \frac{1}{1 + \phi} \left( \frac{2V}{(1 + \phi)V + 4} \right)^2 . \tag{2}
\]

Given the assumed functional form with the spillovers, we have to make sure that \((1 + \phi)x^*(\phi) \in [0, 1]\). We have

\[
(1 + \phi)x^*(\phi) = \left( \frac{2V}{(1 + \phi)V + 4} \right)^2 \leq 1
\]

\[
\iff 2V \leq (1 + \phi)V + 4
\]

\[
\iff V \leq \frac{4}{1 - \phi},
\]

which gives the first condition of (1).

Post-merger  The profit of the merged firm is

\[
\Pi = \left[ \sqrt{x_1 + \phi x_2} + \sqrt{\phi x_1 + x_2} - \sqrt{x_1 + \phi x_2 \phi x_1 + x_2} \right] V - (x_1 + x_2).
\]

By differentiating and plugging in \( x_1 = x_2 = x^m \), we obtain

\[
\left[ \frac{1 + \phi}{2 \sqrt{(1 + \phi)x^m}} - \frac{1 + \phi}{2} \right] V - 1 = 0,
\]

\footnote{It is straightforward also to check the validity of the SOC.}
which gives
\[ x^m(\phi) = \frac{1}{1 + \phi} \left( \frac{(1 + \phi)V}{(1 + \phi)V + 2} \right)^2. \]  
(3)

It is obvious that \( x^m(\phi) \in [0, 1] \).

We want to ensure that \((1 + \phi)x^m(\phi) < \frac{1}{4}\) so that it lies in domain where \(1 - F(x)\) is log-convex. Note from (3) that
\[ (1 + \phi)x^m(\phi) < \frac{1}{4} \iff V < \frac{2}{1 + \phi}, \]
which gives the second condition of (1). In Appendix A.3, we show that \( x^m(\phi) \) is indeed optimal given (1).

**Spillover Threshold**  When \( \phi = 0 \), we have
\[ x^*(0) = \left( \frac{2V}{V + 4} \right)^2 > x^m(0) = \left( \frac{V}{V + 2} \right)^2, \]
which is consistent with Denicolò and Polo (2018) given small \( V \).\(^7\) When \( \phi = 1 \), we have
\[ x^*(1) = \frac{1}{2} \left( \frac{2V}{2V + 4} \right)^2 < x^m(1) = \frac{1}{2} \left( \frac{2V}{2V + 2} \right)^2. \]

To calculate \( \phi \), from (2) and (3), we obtain
\[ x^*(\phi) \leq x^m(\phi) \iff \frac{1}{1 + \phi} \left( \frac{2V}{(1 + \phi)V + 4} \right)^2 \leq \frac{1}{(1 + \phi)} \left( \frac{(1 + \phi)V}{(1 + \phi)V + 2} \right)^2 \]
\[ \iff 2(1 + \phi)V^2 + 4V \leq (1 + \phi)^2V^2 + 4(1 + \phi)V. \]

Since \( V > 0 \), this is equivalent to
\[ V\phi^2 + 4\phi - V \geq 0, \]
and, since \( \phi \geq 0 \), we derive
\[ \phi \geq \frac{\sqrt{4 + V^2} - 2}{V}. \]

\(^7\)Denicolò and Polo (2018) show that when \( \phi = 0 \) and \( V < 2 \), the merged firm’s optimal R&D allocation is symmetric and the probability of success becomes lower after the merger. However, the post-merger allocation is asymmetric and improves the chance of innovation when \( V \geq 2 \).
We plot below $(V, \phi)$ that satisfy all the required conditions (the shaded area). When $V = 1$, $\phi = \sqrt{5} - 2 \approx 0.236$.

5 Conclusion

This paper formally identified the spillover channel of mergers in a stylized two-player model of innovation competition. While the business stealing effect means that a merger harms the incentives for innovation by reducing competition, there are opposing forces when the spillover effect generates free-riding externality in a competitive setting. Horizontal mergers increase innovation when the spillover effect dominates the business stealing effect, and vice versa. This finding contrasts with the standard account of knowledge spillovers, which suggests that mergers facilitate knowledge spillovers directly within the merged entity and hence are more beneficial when such effects are small.

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8 The first inequality of (1) is satisfied in the shaded area. Dashed boundary means that the points on the boundary are not included.
A Proofs

A.1 Proof of Proposition 2

(i) Recall that $x^*(\phi)$ solves

$$F'((1 + \phi)x^*) \left[ 1 - \frac{1 + \phi}{2} F((1 + \phi)x^*) \right] = \frac{1}{V}. \tag{4}$$

As $V$ increases, the LHS must decrease to satisfy the equality. Since the LHS is a decreasing function of $x^*$, to compensate the change in $V$, $x^*(\phi)$ increases.

(ii) When $\phi$ increases, the LHS of (4) decreases since $F' > 0$ and $F'' < 0$, while the RHS is fixed. Thus, $x^*$ must decrease.

(iii) Let

$$G(\phi, x^*) := F'((1 + \phi)x^*) \left[ 1 - \frac{1 + \phi}{2} F((1 + \phi)x^*) \right] - \frac{1}{V}. \tag{9}$$

The implicit function theorem implies that if $\frac{\partial}{\partial x^*} G(\phi, x^*(\phi)) \neq 0$, there exists a unique differentiable function locally solving $G$ defined on $U_\phi$, an open neighborhood of $\phi$.

Since $F' > 0$, $F'' < 0$, and $0 \leq F(x) \leq 1$, for any $(\phi, x^*(\phi))$, we obtain

$$\frac{\partial G}{\partial x^*} = (1 + \phi)F''((1 + \phi)x^*) \left[ 1 - \frac{1 + \phi}{2} F((1 + \phi)x^*) \right] - \frac{(1 + \phi)^2}{2} (F'((1 + \phi)x^*))^2 < 0. \tag{10}$$

Therefore, we can apply the implicit function theorem to conclude that for any $\phi \in [0, 1]$, there exists an open neighborhood $U_\phi$ such that there exists a unique differentiable function $x^*(\phi)$ such that $G(\phi, x^*(\phi)) = 0$.

Note that $\cup_{\phi \in [0,1]} U_\phi$ is an open cover of the compact set $[0,1]$. Thus, a finite subcover, \{ $U_{\phi_1}, \ldots, U_{\phi_K}$ \}, exists. Note also that $x^*(\phi_k)$ and $x^*(\phi_l)$ coincide on $U_{\phi_k} \cap U_{\phi_l}$ for any $k, l \leq K$ since we have shown that the FOC admits a unique solution for each $\phi$. By applying the pasting lemma,\(^{10}\) gluing all $x^*(\phi_k)$ gives the desired function $x^*(\phi)$, which must be a continuous function since every $x^*(\phi_k)$ is.

Note that $(1 + \phi)x^*(\phi)$ is in the interior of $\text{supp}(F)$. As $F'$ is a strictly decreasing function and $F'(\overline{R}) = 0$, $F'((1 + \phi)x^*(\phi)) \neq 0$.

\(^{10}\)The pasting lemma gives a condition under which two continuous functions can be “glued together” to obtain a continuous function on the union of domains. For a formal statement, see Munkres (2014). By selecting appropriate closed subsets of $U_\phi$'s, the lemma can be applied to our setup.
A.2 Proof of Proposition 3

The FOC of the merged firm is

\[
\frac{\partial \Pi}{\partial x_1} = [F'(x_1 + \phi x_2) + \phi F'(\phi x_1 + x_2) - F'(x_1 + \phi x_2)F(\phi x_1 + x_2) - \phi F(x_1 + \phi x_2)F'(\phi x_1 + x_2)] V^{-1} = 0.
\]

Given Lemma 1, plugging in \(x_1 = x_2 = x_m\) yields

\[
(1 + \phi)F'((1 + \phi)x_m)[1 - F((1 + \phi)x_m)] = \frac{1}{V}.
\] (5)

Given the assumption that \(F' > 0\) and \(F'' < 0\), when \(\phi\) is fixed, the LHS of (5) is a decreasing function of \(x_m\). When \(x_m = 0\), the LHS is \((1 + \phi) \times \infty \times 1 = \infty\). When \(x_m \to R/(1 + \phi)\), the LHS converges to \((1 + \phi) \times 0 \times 0 = 0\). Given the monotonicity of the LHS and the continuity of \(F\) and \(F'\), the intermediate value theorem shows the existence of unique \(x_m\).

Now, we check the SOC of the optimization problem. Let \(H\) be the hessian of \(\Pi\). We want to show that when \(\phi \in [0, 1)\), \(H\) is negative definite, and when \(\phi = 1\), \(H\) is negative semi-definite. This amounts to showing the following:

\[
H_{11}(= H_{22}) = (1 + \phi^2)F''((1 + \phi)x_m)[1 - F((1 + \phi)x_m)] - 2\phi(F'((1 + \phi)x_m)^2 < 0
\]

\[
H_{12}(= H_{21}) = 2\phi F''((1 + \phi)x_m)[1 - F((1 + \phi)x_m)] - (1 + \phi^2)(F'((1 + \phi)x_m))^2 < 0
\]

\[
|H| = H_{11}H_{22} - H_{12}H_{21} = H_{11}^2 - H_{12}^2 \geq 0
\]

The first two inequalities are evident from the assumption that \(F' > 0\), \(F'' < 0\), and \(0 \leq F(x) \leq 1\). For the last inequality, observe first that

\[
0 \leq |H| \iff 0 \leq (H_{11} + H_{12})(H_{11} - H_{12}) \iff 0 \geq H_{11} - H_{12},
\]

where the last equivalence holds since \(H_{11} < 0\) and \(H_{12} < 0 \Rightarrow H_{11} + H_{12} < 0\).

We have

\[
H_{11} - H_{12} = (\phi - 1)^2F''((1 + \phi)x_m)[1 - F((1 + \phi)x_m)] + (\phi - 1)^2(F'((1 + \phi)x_m))^2 \leq 0.
\]

This inequality holds since by the log-convexity of \(1 - F(x)\), we have

\[
F''((1 + \phi)x_m)[1 - F((1 + \phi)x_m)] + (F'((1 + \phi)x_m))^2 < 0
\]
for any \((1 + \phi)x^m\). This verifies the SOC. Let \(x^m(\phi)\) be the solution to (5).

Let \(G(\phi, x^m) := (1 + \phi)F'((1 + \phi)x^m)\left[1 - F((1 + \phi)x^m)\right] - 1/V\). By similar arguments to those behind part (iii) of Proposition 2, to prove continuity of \(x^m(\phi)\), it suffices show that 
\[
\frac{\partial}{\partial x^m} G(\phi, x^m(\phi)) \neq 0 \text{ for } (\phi, x^m(\phi)),
\]
which holds since
\[
\frac{\partial}{\partial x^m} G = (1 + \phi)^2 F''((1 + \phi)x^m) \left[1 - F((1 + \phi)x^m)\right] - (1 + \phi)^2 F'((1 + \phi)x^m)^2 < 0.
\]

The continuity of \(x^m(\phi)\) follows from the implicit function theorem and the pasting lemma.\(^{11}\)

### A.3 Optimality of \(x^m\) in Section 4

We proceed with the following steps. First, we show that if the profit-maximizing pair \((x_1, x_2)\) belongs to the interior of \([0, 1]^2\), then the optimal solution is symmetric under the second condition of (1). Second, we show that there cannot be a boundary solution under the same regularity condition.

The FOCs are
\[
\begin{align*}
\frac{\partial \Pi}{\partial x_1} = 0 \Rightarrow \frac{1}{2\sqrt{x_1 + \phi x_2}} + \frac{\phi}{2\sqrt{\phi x_1 + x_2}} - \frac{\sqrt{\phi x_1 + x_2}}{2\sqrt{x_1 + \phi x_2}} - \frac{\phi \sqrt{x_1 + \phi x_2}}{2\sqrt{x_1 + \phi x_2}} &= \frac{1}{V}; \\
\frac{\partial \Pi}{\partial x_2} = 0 \Rightarrow \frac{\phi}{2\sqrt{x_1 + \phi x_2}} + \frac{1}{2\sqrt{\phi x_1 + x_2}} - \frac{\sqrt{\phi x_1 + x_2}}{2\sqrt{x_1 + \phi x_2}} - \frac{\phi \sqrt{x_1 + \phi x_2}}{2\sqrt{x_1 + \phi x_2}} &= \frac{1}{V}.
\end{align*}
\]

By substituting \(y_1 := \sqrt{x_1 + \phi x_2}\) and \(y_2 := \sqrt{\phi x_1 + x_2}\), we obtain
\[
\frac{1}{2y_1} + \frac{\phi}{2y_2} - \frac{y_1}{2y_1} - \frac{\phi y_1}{2y_2} = \frac{1}{V};
\]
\[
\frac{\phi}{2y_1} + \frac{1}{2y_2} - \frac{\phi y_2}{2y_1} - \frac{y_1}{2y_2} = \frac{1}{V};
\]
and equating these two equations gives
\[
(1 - \phi)y_1 - (1 - \phi)y_2 - (1 - \phi)y_1^2 + (1 - \phi)y_2^2 = 0.
\]

\(^{11}\)Unlike Proposition 2, we cannot conclude that \(x^m\) is a monotone function of \(\phi\). Even if \(\phi\) increases, the \((1 + \phi)\) term compensates the decrease in the other two terms of the LHS of (5). Thus, without further assumptions, we cannot pin down the response of the LHS of (5) to a change in \(\phi\). Note that our proof of Proposition 2 relied on monotonic behavior of the LHS of (4).
Provided that $\phi < 1$,\textsuperscript{12} we then find

$$y_1 = y_2 \quad \text{or} \quad y_1 + y_2 = 1.$$ 

Since the first case corresponds to our solution, we show that the second case leads to a contradiction under (1). Note that if $y_1 + y_2 = 1$,

$$\frac{1}{V} = \frac{1}{2y_1} + \frac{\phi}{2y_2} - \frac{y_2}{2y_1} - \frac{\phi y_1}{2y_2} = \frac{1 - y_2}{2y_1} + \frac{\phi(1 - y_1)}{2y_2} = \frac{1 + \phi}{2},$$

which contradicts the assumption that $V < \frac{2}{1+\phi}$, the second condition of (1).

Next, we rule out boundary solutions. It is obvious that the merged firm would not choose $(x_1, x_2)$ from $\{1\} \times (0, 1]$ or $(0, 1] \times \{1\}$ since then shutting down one R&D line is more efficient. Thus, it suffices to consider the firm operating just one of the two R&D lines, i.e. maximizing

$$\Pi = \left[ \sqrt{x} + \sqrt{\phi x} - \sqrt{x} \sqrt{\phi x} \right] V - x.$$ 

Straightforward calculation shows that optimal expenditure is

$$x^\partial = \left( \frac{(\sqrt{\phi} + 1) V}{2(\sqrt{\phi} V + 1)} \right)^2.$$ 

By allocating $(x^\partial, 0)$, the merged firm’s optimized profit is

$$= \left[ \left( \sqrt{\phi} + 1 \right) \sqrt{x^\partial (\phi)} - \sqrt{\phi x^\partial (\phi)} \right] V - x^\partial (\phi)$$

$$= \left[ \left( \frac{(\sqrt{\phi} + 1)^2 V}{2(\sqrt{\phi} V + 1)} - \sqrt{\phi} \left( \frac{(\sqrt{\phi} + 1) V}{2(\sqrt{\phi} V + 1)} \right)^2 \right] V - \left( \frac{(\sqrt{\phi} + 1) V}{2(\sqrt{\phi} V + 1)} \right)^2$$

$$= \frac{(\sqrt{\phi} + 1)^2}{4(\sqrt{\phi} V + 1)} V^2. \quad (6)$$

By allocating symmetric R&D investments across two R&D lines, the merged firm’s optimized

\textsuperscript{12}When $\phi = 1$, the merged firm chooses an aggregate level of R&D expenditure and the optimal R&D plan is not unique.
profit is
\[
\begin{align*}
&= \left[ 2\sqrt{(1 + \phi)x^m(\phi)} - (1 + \phi)x^m(\phi) \right] V - 2x^m(\phi) \\
&= \left[ 2 \frac{(1 + \phi)V}{(1 + \phi)V + 2} - \left( \frac{(1 + \phi)V}{(1 + \phi)V + 2} \right)^2 \right] V - \frac{2}{1 + \phi} \left( \frac{(1 + \phi)V}{(1 + \phi)V + 2} \right)^2 \\
&= \frac{1 + \phi}{(1 + \phi)V + 2} V^2.
\end{align*}
\]

We want to show that (7) exceeds (6), or
\[
\frac{(\sqrt{\phi} + 1)^2}{4(\sqrt{\phi}V + 1)} < \frac{1 + \phi}{(1 + \phi)V + 2} \iff V\phi^2 - 2V\phi\sqrt{\phi} + (2V - 2)\phi - (2V - 4)\sqrt{\phi} + (V - 2) < 0 \\
\iff \left( \sqrt{\phi} - 1 \right)^2 ((1 + \phi)V - 2) < 0
\]

The last inequality holds by the second condition of (1).

References


