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Mechanical model establishment and energy characteristics analysis for rock material considering the dynamic disturbance effect

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Abstract: An element mechanical assembly model was established with the combined modeling method to study the dynamic damage behaviors of deep rock. In this model, the effect of disturbance and pre-static loading on the rock mechanics and statistical parameters was considered and a modified function was introduced to deduce the damage evolution function, thus a mechanical model of rock was established. This model was verified against the relevant experimental and bibliographic data. The results show that the damage evolution rate curve gradually moves to the right along the strain axis and the peak value of curve decreases first and then increases with increasing parameter $k$. When $k$ is large, the damage evolution rate appears a “plateau” which is more obvious as $k$ gets higher. This “plateau” may correspond to the damage stable development stage. Further, the pre-static loading can improve the resistance to impact of rock. The released energy of rock in the rebound under pre-static loading is higher than that under no pre-static loading with the constant disturbance frequency. It decreases with disturbance numbers rising as the pre-static loading remains unchanged, which indicates the resistance to impact of rock is weakened by adding disturbance frequency. These predictions were consistent with test data in pre-peak stage of “rebound” type curve. For strain-softening type curves, this model did not only fit preferably with test data in the pre-peak stage but also had high fitting accuracy in the post-peak stage. This model can well characterize the mechanical properties and stress-strain behaviors of rock under the combination of static loading and frequent disturbance. These results can provide certain references for the evaluation of the safety and stability of deep rock engineering under high ground stress and frequent dynamic disturbance.

Keywords: Rock; Frequent disturbance; Pre-static load; Damage; Constitutive model; Released energy
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>Y, k</td>
<td>Introduced function and parameter of the function</td>
</tr>
<tr>
<td>Q, T</td>
<td>Calculating parameters in the constitutive equation of rock</td>
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</table>

List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>x, p(x)</td>
<td>Independent variable and probability density function of Weibull distribution</td>
</tr>
<tr>
<td>N, Nf</td>
<td>Total number of micro element and number of destroyed micro element, respectively</td>
</tr>
<tr>
<td>m, F0</td>
<td>Weibull distribution parameters</td>
</tr>
<tr>
<td>σ1, σ2, σ3</td>
<td>First, second, third principal stress, respectively</td>
</tr>
<tr>
<td>σ1*, σ2*, σ3*</td>
<td>First, second, third principal effective stress, respectively</td>
</tr>
<tr>
<td>σ, σ*</td>
<td>Stress and effective stress</td>
</tr>
<tr>
<td>ε, ε1</td>
<td>Strain and first principal strain</td>
</tr>
<tr>
<td>I1</td>
<td>First invariant of the stress tensor</td>
</tr>
<tr>
<td>J2*</td>
<td>Second invariant of effective stress deviation</td>
</tr>
<tr>
<td>φ0, α0</td>
<td>Internal friction angle and the parameter related to internal friction angle when rock yields</td>
</tr>
<tr>
<td>E0, E,</td>
<td>Elastic modulus at initial state, elastic modulus on average, respectively</td>
</tr>
<tr>
<td>E11, Ed</td>
<td>Elastic modulus of the elastic element H1, elastic modulus of the dynamic stress-strain curve, respectively</td>
</tr>
<tr>
<td>ε0</td>
<td>Weibull distribution scale parameters</td>
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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D, D'</td>
<td>Damage variable and damage evolution rate, respectively</td>
</tr>
<tr>
<td>ε11, ε12, ε2εa</td>
<td>Strain of the Maxwell body and viscosity element, elastic element H2, damaged element Dn, respectively</td>
</tr>
<tr>
<td>σ11, σ12, σ2, σa</td>
<td>Stress of the elastic element H1, viscosity element, elastic element H2, damaged element Dn, respectively</td>
</tr>
<tr>
<td>μ</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>ε1, ε12</td>
<td>Strain rate of the Maxwell body and viscosity element, respectively</td>
</tr>
<tr>
<td>σ1*, σ2*, σ3*</td>
<td>Viscosity coefficient and the time corresponding to the strain, respectively</td>
</tr>
<tr>
<td>σf(t), σg(t), σf(t)</td>
<td>The dynamic stress, strain and strain rate of rock samples, respectively</td>
</tr>
<tr>
<td>A, As</td>
<td>The sectional area of elastic bar and rock samples, respectively</td>
</tr>
<tr>
<td>E, v</td>
<td>Elastic modulus of the elastic bar and the longitudinal wave velocity of rock samples, respectively</td>
</tr>
<tr>
<td>εrt, εr0</td>
<td>The start and end strain of the rebound section of stress-strain curve, respectively</td>
</tr>
<tr>
<td>σr, εr</td>
<td>Stress and strain of the rebound section of stress-strain curve, respectively</td>
</tr>
<tr>
<td>Wr</td>
<td>Energy released by the rock after disturbance impact</td>
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1. Introduction

With the decrease of shallow high-quality mineral resources, more and more energy exploration is going to deep development. Deep underground engineering, nuclear waste disposal and underground chamber and tunnel construction projects all involve the change of deep rock mechanical behavior. However, deep rock engineering is in a complex environment of high in-situ stress, high karst water pressure and high geothermal temperature. Meanwhile, deep rock is also subjected to dynamic loads such as blasting and mechanical vibration which have a long propagation range and high disturbance energy. Those external disturbances frequently cause the failure of the deep rock mass which performs differently from the shallow engineering, such as the frequent occurrence of rockburst and other geological disasters in the process of deep mining (Li et al., 2019). In addition, underground mining operation is a cyclic process and each cyclic operation will deteriorate the rock mechanical properties, affecting the mechanical properties of rock and the stability of rock mass structure. Therefore, it is necessary to develop a damage constitutive model of rock to consider these disturbance effects on the exploration and development of deep rock.

Rock is a complicated natural geological material and its internal defects (such as micro-cracks and pores) will be gradually evolved under external loads and eventually cause a macroscopic failure. This deterioration of rock mechanical properties is called as damage (Xie et al., 1990). The external loads play an important role in the process of rock damage and failure. The evolution of mechanical properties and damage of rock under dynamic loads have been studied with numerous theoretical and experimental approaches. Ji et al. (2018) and Li et al. (2020) studied the rock damage under external loads and found that the deterioration of
mechanical properties was mainly reflected by the micro-element failure inside rock. In addition, the reduction of the elastic longitudinal wave velocity inside rock implied the rock damage and this damage was spatially unevenly distributed. The combined dynamic and static loading tests on the samples were carried out respectively from the perspectives of magnitude of pre-static load and amplitude of disturbance load to study the rock burst phenomenon, rock characteristic stress, crack development and rock failure mode (Tao et al. (2018), Fei et al. (2022), Feng et al. (2018), Wang et al. (2022)). Liu et al. (2022) conducted a dynamic disturbance triaxial loading test on red sandstone samples via disturbance servo triaxial loading system and found that the disturbance load would cause a sudden rise of rock strain. Such an unexpected increase of strain would decay exponentially with the disturbance numbers. Besides, dynamic disturbance load would also lead to the rapid rock damage under a high pre-static load level but the rock fatigue failure under a low pre-static load level. Zhu et al. (2016) conducted stress relaxation and disturbance tests on rock samples under uniaxial compression to study the influence of dynamic disturbances (such as rock blasting) on the time-varying and rheological deformation of rock. They observed that the stress-drop and strain increase in the tests were related to the residual strain caused by dynamic disturbance. Tang et al. (2014) changed the average stress and the amplitude of disturbance load to explore the mechanical properties of marble under cyclic dynamic disturbance and high stress state. They found that when the upper value of cyclic dynamic disturbance load was within a certain range, the ability of rock resistance to deformation was improved and the rock failed in either shear fatigue or tension fatigue.

The statistical damage constitutive model has been widely adopted to describe the
relationship of stress and deformation of rock. Literature review found that the establishment of constitutive model of rock is relatively plentiful under static load. Jiang et al. (2021), Zhang et al. (2014), Zhu et al. (2019) established statistical damage constitutive models of rock considering the temperature effect based on the Mohr-Coulomb criterion, Mises criterion and Hoek-Brown criterion, respectively. Their models well fitted the test curves and preferably reflected the rock characteristics of compaction stage and residual stage. Some scholars also established their statistical damage constitutive models of rock according to different statistical distribution functions. Wen et al. (2011) proposed a statistical damage softening model based on lognormal distribution in terms of the distribution randomness of rock micro element strength and the testing hypothesis of logarithmic distribution. Chen et al. (2018) compared the damage constitutive model with the power function distribution and Weibull distribution respectively via the triaxial tests data. They pointed out that the model based on the Weibull distribution was more reasonable. However, on account of deep rock affected by the high in-situ stress and the dynamic disturbance load like the blast excavation, the constitutive models of rock under static loads were no longer applicable to the study on the dynamic mechanical properties of deep rocks (Li et al., 2006).

At present, certain achievements have been made in the study of dynamic characteristics, strength and damage characteristics of rock under the coupling of static and dynamic loads (Tian et al., 2021; Tang et al., 2015; Xu et al., 2010), but only a few scholars have conducted preliminary studies on the establishment of rock damage constitutive model under the combined action of pre-static loads and frequent disturbance loads. Zhou et al. (2017) and Wang et al. (2018) proposed constitutive models of rock under dynamic loading-temperature
and cyclic loading-temperature and discussed the effects of model parameters on the
stress-strain curve of rock after reloading. Zhang et al. (2021) carried out dynamic impact
tests on coal samples under the condition of medium strain rate with the split-Hopkinson
pressure bar (SHPB) test system and established a dynamic constitutive model considering the
characteristics of the plastic hardening and softening process of coal. Liu et al. (2012), Shan et
al. (2003) and Jiang et al. (2018) developed the dynamic damage constitutive equation of rock
based on the measured impact failure test results of oblique amphibolite, granite, marble and
sandstone samples, respectively. Their model fitting curves were in good agreement with the
test results, preferably describing the dynamic mechanical properties of rock under impact
loads. These research achievements have laid a foundation for further improving the
theoretical system of rock mechanics, but at present few studies are about the rock dynamic
damage constitutive models considering both the disturbance numbers and the influence of
pre-static load level.

This study focuses on the constitutive relationship of rock under high in-situ stress and
frequent dynamic disturbance to describe the mechanical behaviors in the complex geological
environment of deep rock. First, according to the theory of statistical mesoscopic damage
mechanics, a correction function $Y$ with the same form as Weibull distribution function is
introduced to correct the damage variable and the rock damage evolution equation is further
derived. Subsequently, a mechanical model of rock element assemblage is established via the
combined model research method and then a dynamic statistical damage constitutive model of
rock considering the pre-static load level and disturbance numbers is constructed. Finally, the
present model is comparatively validated through experimental results in literature. The
conclusions derived are helpful and valuable for in-depth understanding the dynamic
properties and damage evolution law of deep rock under the action of frequent disturbance.

2. Statistical damage model of rock

2.1 Load-induced damage variable

Rock as a brittle material may break down under external loads due to the accumulation of
micro-cracks and other internal defects. These micro-cracks and defects are randomly
distributed and can be surveyed on the grounds of statistical theory. The representative volume
element (RVE) is the smallest volume over which the statistical mesoscopic damage analysis
can be made that will yield a value representative of the macroscopic system. Studies show that
the Weibull probability distribution function is usually used to describe the distribution of rock
micro-element strength (Zhang et al., 2014). This study assumes that the mechanical
properties of RVE obey Weibull probability distribution which has the following probability
density function:

\[
p(x) = \frac{m}{F_0} x^{m-1} \exp\left(-\left(\frac{x}{F_0}\right)^m\right)
\]

where \( x \) is some mechanical property and \( F_0 \) is its average value of RVE. \( m \) is the coefficient of
rock material uniformity.

According to the rock strength failure criterion, when the strength of rock micro element
reaches \( F \), the total number of destroyed micro element is regarded as \( N_f \). The damage variable
\( D \) is the ratio of the number of destroyed micro elements to the total number of micro elements
\( N \). Thus, the damage variable \( D \) is

\[
D = \frac{N_f}{N} = \frac{\int_0^F p(x)dx}{N} = 1 - \exp\left(-\left(\frac{F}{F_0}\right)^m\right)
\]
where the strength of rock micro-element $F$ can be calculated by the rock failure criterion.

Previous studies have shown that Drucker-Prager criterion can be better applied to brittle or ductile rocks under compression conditions (Xu et al., 2018). Therefore, the strength of rock micro-element can be described as

$$\begin{align*}
F &= f(\sigma^*) = \alpha_0 I_1 + \sqrt{(J^*_2)} \\
\alpha_0 &= \frac{\sin \phi_0}{\sqrt{9 + 3\sin^2(\phi_0)}}
\end{align*}$$

where $\phi_0$ is the internal friction angle; $\alpha_0$ is the parameter related to internal friction angle when rock yields; $I_1$ is the first invariant of the stress tensor; $J^*_2$ is the second invariant of effective stress deviation.

Based on the continuous damage mechanics and strain equivalence assumption, the relation of stress and strain can be described as

$$\begin{align*}
\sigma &= \sigma^*(1 - D) = E_0\varepsilon(1 - D)
\end{align*}$$

where $\sigma$ and $\sigma^*$ are the nominal axial stress and the effective axial stress, respectively; $\varepsilon$ is the nominal axial strain; $E_0$ is the elastic modulus of rock in undamaged state.

Substituting Eq. (5) into the generalized Hooke’s law yields:

$$\begin{align*}
E_0\varepsilon_i = \sigma_i^* - \mu(\sigma_2^* + \sigma_3^*) = [\sigma_i - \mu(\sigma_2 + \sigma_3)]/(1 - D)
\end{align*}$$

where $\sigma_1$, $\sigma_2$ and $\sigma_3$ are the first, second and third nominal principal stress, respectively; $\sigma_1^*$, $\sigma_2^*$ and $\sigma_3^*$ are the first, second and third effective principal stress, respectively.

In the one-dimensional stress state, $\sigma_1=\sigma$, $\sigma_2=\sigma_3=0$, $\varepsilon_1=\varepsilon$, and the $I_1$, $J^*_2$, and $F$ can be described as

$$I_1 = \sigma_1^* + \sigma_2^* + \sigma_3^* = \frac{E\varepsilon_i(\sigma_1 + \sigma_2 + \sigma_3)}{\sigma_i - \mu(\sigma_2 + \sigma_3)} = E\varepsilon$$
According to Eq. (9), the strength of rock micro-element under uniaxial loading is proportional to the axial strain and then Eq. (9) can be transformed into

$$\frac{F}{F_0} = \frac{\varepsilon}{\varepsilon_0}$$  \hspace{1cm} (10)$$

Therefore, Eq. (2) can be described as

$$D = 1 \cdot \exp\left(-\frac{F}{F_0}^{n}\right) = 1 \cdot \exp\left(-\frac{\varepsilon}{\varepsilon_0}^{n}\right)$$  \hspace{1cm} (11)$$

The rock damage evolution rate $D'$ can be obtained by deviation of Eq. (11).

$$D' = \frac{\partial D}{\partial \varepsilon} = \frac{dD}{d\varepsilon} = \frac{m\varepsilon^{m-1}}{\varepsilon_0^n} \exp\left(-\frac{\varepsilon}{\varepsilon_0}^{n}\right)$$  \hspace{1cm} (12)$$

The parameters $m$ and $\varepsilon_0$ in Eq. (11) are the shape and scale parameters of Weibull distribution function respectively, which are generally related to the material properties. The higher the value of $m$, the more uniform the distribution of mechanical properties of RVE.

The mechanical properties of rock under the combined action of static load and frequent disturbance are significantly different from those under the static or dynamic loads solely. It can be considered that the rock under disturbance will have a certain degree of damage (Wang et al., 2017; Wang et al., 2015). Under different pre-static loads and disturbance numbers, the stress-strain curves of rock have great differences in form. Therefore, a modified function $Y$ with the same form as Weibull distribution function was introduced in this study (Liang et al., 2021). Considering the effect of pre-static loads and disturbance frequency, the value of parameter $k$ in the modified function $Y$ is to ensure that the established model can accurately
describe the stress-strain relationship of rock. Thus, the modified rock damage variable and
damage evolution rate can be expressed as

\[ D = 1 - Y \times \exp \left( -\left( \frac{\varepsilon}{\varepsilon_0} \right)^m \right) = 1 - \exp \left( -\left( \frac{\varepsilon}{\varepsilon_0} \right)^k \right) \times \exp \left( -\left( \frac{\varepsilon}{\varepsilon_0} \right)^m \right) \]  \hspace{1cm} (13)

\[ D' = \exp \left( -\left( \frac{\varepsilon}{\varepsilon_0} \right)^k \right) \times \exp \left( -\left( \frac{\varepsilon}{\varepsilon_0} \right)^m \right) \times \left( \frac{m \varepsilon_0^{m-1}}{\varepsilon_0^m} + \frac{k \varepsilon_0^{k-1}}{\varepsilon_0^k} \right) \]  \hspace{1cm} (14)

2.2 Sensitivity analysis of parameter \( k \)

In order to explore the effect of \( k \) in the modified function \( Y \) on the damage variable and
damage evolution rate, \( m=1.5 \) and \( \varepsilon_0=0.01021 \) were set and substituted into Eq.(13) and
Eq.(14) to obtain the damage evolution curve and damage evolution rate curve under different
values of \( k \) (see Fig. 1 and 2). Fig. 1 illustrates that the damage evolution curve presents an
upward convex shape when \( k \) is small and gradually goes through four stages with the
increase of \( k \), namely the initial damage stage, the damage stable development stage, the
damage accelerated development stage and the damage unstable development stage, which
shows a slow first and then a sharp rise and finally the growth rate slows down again
presenting an overall shape of ‘\( S \)’ (Li et al., 2017). Fig. 2 shows the variation curve of damage
evolution rate with \( k \) value. As \( k \) increases, the damage evolution rate curve gradually shifts to
the right along the strain axis, and the peak value of curve decreases first and then increases.
In addition, when \( k \) value is high, the damage evolution rate gradually appears a ‘plateau’,
which means that the damage evolution rate remains approximately unchanged at this stage.
This period of ‘plateau’ corresponds to the damage stable development stage of rock and the
end of ‘plateau’ can be considered as the start of the rock damage accelerated development
stage.
The stress-strain curves of rock corresponding to different $k$ value can be obtained by Eq. (5) and the elastic modulus was set as 22.4 GPa in the Eq. (5) (see Fig. 3). It is obvious that the post peak stage of the stress-strain curves of rock was greatly influenced by $k$. With the increment of $k$, the peak strength of the curve rises while the strain at the peak point of the curve gradually increases and the curve generally shifts to the right. Besides, the stress-drop rate in the post-peak stage of the stress-strain curve can be affected by the value of $k$ and the brittleness of rock is improved as $k$ rising.

3. Dynamic damage constitutive model of rock under one-dimensional pre-static loads

3.1 Basic assumptions

(1) The inertia effect at constant strain rate is ignored.

(2) The micro-element of rock has both statistical damage characteristics and elastic viscosity. That is, the rock micro-element is an aggregation of an elastic element $H_1$ and a viscous element in series to form Maxwell’s body, then connected in parallel with another elastic element $H_2$ to form the Poyting-Thomson’s body and finally connected in parallel with the damaged body $D_a$. This formed mechanical model of the rock micro-element assembly is shown in Fig. 4.

(3) The damaged body $D_a$ had isotropic damage characteristics and was linearly elastic with the average elastic modulus $E$ before being damaged. The strength in damaged body $D_a$ obeyed the Weibull distribution with parameters $(m, \varepsilon_0)$. According to the stress state of rock, the damage variate $D$ could be derived by referring to the results in Section 2.2. The constitutive relation of the damaged body can be described as

$$\sigma_a = E\varepsilon_a(1 - D) \quad (15)$$
where $\varepsilon_a$ and $\sigma_a$ are the strain and stress in the damaged body $D_a$, respectively.

(4) The viscous element has no damage characteristics (Wang et al., 2017). Thus, the constitutive relation of rock can be expressed as

$$\sigma_{12} = \eta \frac{d\varepsilon_{12}}{dt}$$

where $\varepsilon_{12}$ and $\sigma_{12}$ are the strain and stress of the viscosity element, respectively; $\eta$ is the viscosity coefficient; $t$ is the time corresponding to the strain.

(5) The stress-strain relationship of the rock micro-element before damage conforms to the linear differential equation and the strain superposition principle is still valid (Wang et al., 2015).

(6) The dynamic damage constitutive relation of rock under one-dimensional pre-static loads can be deduced based on the strain equivalence principle and viscoelastic constitutive relation.

### 3.2 Damage constitutive model of rock subjected to dynamic disturbance under one-dimensional pre-static loads

When rock is subjected to disturbance under one-dimensional pre-static loads, the stress-strain relationship between damaged body, elastic body and viscosity body in the combination mechanical model of rock has the following:

$$\begin{align*}
\sigma &= \sigma_a + \sigma_{11} + \sigma_2 \\
\varepsilon &= \varepsilon_a = \varepsilon_2 = \varepsilon_1 \\
\varepsilon_1 &= \varepsilon_{11} + \varepsilon_{12} \\
\sigma_{11} &= \sigma_{12}
\end{align*}$$

where $\varepsilon$ and $\sigma$ are the strain and stress in the combination mechanical model of rock respectively; $\varepsilon_1$ is the strain of the Maxwell body; $\varepsilon_{11}$ and $\sigma_{11}$ are the strain and stress of the...
elastic element \( H_1 \) respectively; \( \epsilon_{12} \) and \( \sigma_{12} \) are the strain and stress of the viscosity element, respectively; \( \epsilon_2 \) and \( \sigma_2 \) are the strain and stress of the elastic element \( H_2 \), respectively; \( \epsilon_a \) and \( \sigma_a \) are the strain and stress of the damaged body \( D_a \), respectively.

The Maxwell’s body gives the relation in series:

\[
\epsilon_{i1} = \sigma_{i1} / E_{i1}, \quad \dot{\epsilon}_{i1} = \sigma_{i1} / \eta
\]  

\[
\dot{\epsilon}_i = \frac{\sigma_{i1}}{E_{i1}} + \frac{\sigma_{12}}{\eta}
\]  

where \( \dot{\epsilon}_i \) is the strain rate in Maxwell’s body.

Taking the Laplace transform of Eq. (19) gets:

\[
L[\dot{\epsilon}_i(t)] = L\left[\frac{\dot{\sigma}_{i1}(t)}{E_{i1}}\right] + L\left[\frac{\sigma_{12}(t)}{\eta}\right]
\]

\[
\frac{sF_{\sigma_{i1}}(s)}{E_{i1}} + \frac{F_{\sigma_{12}}(s)}{\eta} = \frac{\dot{\epsilon}_i(t)}{s}
\]  

Under the initial conditions, \( t=0, \epsilon_i=0, \sigma_i=0 \), Eq. (20) can be transformed as

\[
F_{\sigma_{i1}}(s) = \frac{E_{i1}\eta\dot{\epsilon}_i(t)}{(s\eta + E_{i1})s}
\]

Taking the inverse Laplace transform of Eq. (22) gives

\[
\sigma_i(t) = \eta\dot{\epsilon}_i(t) - \eta\dot{\epsilon}_i(t)\exp\left(-\frac{E_{i1}\epsilon_i(t)}{\eta\dot{\epsilon}_i(t)}\right)
\]  

where \( E_{i1} \) is the elastic modulus of \( H_i \).

With regard to the damaged body, the constitutive relation can be described as

\[
\sigma_a = E_a\epsilon_a(1-D)
\]  

where \( E_a \) is the elastic modulus of \( D_a \).

According to Fig. 4, substituting Eqs. (13), (23) and (24) into Eq. (17) deduces the dynamic damage constitutive model of rock as
\[
\sigma(t) = E_1 \varepsilon(t) + E_2 \varepsilon(t) \exp\left( \frac{\varepsilon(t)}{\varepsilon_0} \right) \exp\left( \frac{\varepsilon(t)}{\varepsilon_0} \right)^m + \eta \dot{\varepsilon}_i(t) (1 - \exp\left( - \frac{E_{11} \varepsilon(t)}{\eta \varepsilon_i(t)} \right))
\] (25)

This established model has seven parameters of \(E_{11}, E_2, \sigma_m, \varepsilon_0, \eta, k\). They are to be determined for numerical calculation of the damage constitutive equation of rock under one-dimensional pre-static load and disturbance load. The value of \(E_{11}\) in the constitutive model is approximately similar to the deformation modulus of the stress-strain curve at the initial stage under dynamic and static loading, which can be used as the preset value for fitting. The value of \(E_a\) is probably equivalent to the dynamic deformation modulus \(E_d\) (secant modulus is defined as the dynamic deformation modulus to reflect the compression deformation characteristics of rock in dynamic loading section) (Wang et al., 2015). Therefore, the value of \(E_d\) in fitting is preset as the value of \(E_a\). In order to calculate conveniently, set \(E_a = Q \cdot E_{11}\), and the \(Q\) is the calculating parameter. In addition, \(E_2\) is the elastic modulus of elastic elements and it was assumed that there exists a proportional relationship between \(E_2\) and the deformation modulus at the initial stage of the stress-strain curve of rock under dynamic and static loading. Thus, a calculating parameter \(T\) was introduced and can be described as: \(T = E_2 / E_{11}\). The value range of \(\eta\) is generally from 0.1 to 0.5 MPa·s (Unteregger et al., 2015) and the \(\eta = 0.3\) MPa·s was adopted in this study.

4. Validation of this damage constitutive model for rock

In order to verify this proposed damage constitutive model for rock, the results of disturbance impact test on granite under one-dimensional pre-static loads were adopted (Tian et al., 2021). The rock sample was grayish-white in color from the Huashan area of Shaanxi Province and its main components were micro-plagioclase, plagioclase, quartz and biotite.
The sample was a cylinder of 50 mm by 25 mm in diameter with a difference of less than 0.02 mm between the two ends. An improved split-Hopkinson pressure bar (SHPB) device as shown in Fig. 5 was adopted in the disturbance tests. This device was mainly composed of an air chamber, a projectile, an incident bar, a transmission bar, a buffer bar, a pre-static loading device and a data acquisition and display device. The mode of homalographic loading between the rock sample and bars were adopted and the loading wave is the stress pulse of half-sine wave under constant strain rate.

4.1 Analysis of the dynamic stress-strain curves of rock

Previous studies show that the stress-strain curve of rock under dynamic load can be generally divided into four stages as shown in Fig. 6: the stable development of microcracks, unstable development of microcracks, fatigue damage and fatigue failure (Wang et al., 2017). In the stage of the stable development of micro-cracks (see the OA section in Fig. 6), the curve is approximately linear and reflects that the rock is in the elastic deformation stage and the internal micro-cracks of rock develop steadily. In the stage of unstable development of micro-cracks (see the AB section in Fig. 6), the curve presents an upward convex shape which shows the rock is in the plastic deformation stage and the internal micro-cracks of rock fully develop under the external load and result in some internal damage. In the fatigue damage stage (see the BC section in Fig. 6), the curve shows a downward trend and the impact pressure is in the unloading state which implies there exist both elastic deformation and plastic deformation and the damage accumulates continuously. Although there is no macro failure of rock, the internal structure of rock changes, leading to the weakening of its impact resistance. In the fatigue failure stage, there are two main forms of rock stress-strain curve,
rebound section (see the CD section in Fig. 6) and strain-softening section (see the CD’ section in Fig. 6). The rebound phenomenon (the CD section) is that the rock is not completely broken under the combined action of pre-static load and disturbance impact and a certain amount of elastic strain energy is still stored in the rock. When the impact stress is unloaded, the kinetic energy from disturbance impact is less than the elastic strain energy stored in the rock sample, causing the decrease of rock strain. In turn, the reason why there is no rebound phenomenon (the CD’ section) is the kinetic energy from disturbance impact is always greater than the elastic energy stored in the rock sample during unloading, thus resulting in the increased rock strain.

Fig. 7 shows the stress-strain curves of the disturbance impact test on granite under one-dimensional pre-static load. It is obvious that these dynamic stress-strain curves appear rebound phenomenon, indicating the elastic strain energy stored in the samples is greater than the kinetic energy from disturbance impact. When the pre-static load is constant, the dynamic peak stress decreases and the strain corresponding to the peak point of the dynamic stress-strain curve increases with the increase of disturbance numbers. According to Fig. 7a-7e, the dynamic disturbance number has a significant influence on the rebound part of stress-strain curve of rock. With the increase of disturbance number, the stress-drop rate of the rebound curve decreases and the semi-closed area formed by the curve envelope increases. This phenomenon becomes more obvious when the pre-static load is higher.

In order to study the energy dissipation characteristics in the post-peak rebound section of rock under different disturbance numbers and pre-static loads, the rebound section of stress-strain curve was integrated to obtain the energy released of rock after the disturbance
impact. The integral region was that the starting point was the reverse bending point which first appeared at the post-peak stage of the stress-strain curve and the last strain point recorded in the test was the end point. The energy released is calculated as

\[ W_r = \int_{\varepsilon_{rt}}^{\varepsilon_{r0}} \sigma_r \, d\varepsilon_r \]  \hspace{1cm} (26)

where \( \varepsilon_{rt} \) and \( \varepsilon_{r0} \) are the start and end strains of the rebound section of stress-strain curve respectively; \( W_r \) is the energy released by the rock after disturbance impact; \( \sigma_r \) and \( \varepsilon_r \) are the stress and strain of the rebound section of stress-strain curve, respectively.

The energy variations in the rebound section released by rock under different pre-static loads and disturbance numbers were shown in Fig. 8. As the strain of the rebound section curve decreases, the energy released to resist the kinetic energy from disturbance impact increases. The strain corresponding to the end of the curve represents the strain at the reverse bending point of the total stress-strain curve of the sample. According to Fig. 8b, the energy released by the sample after the disturbance impact is related to the value of pre-static load and the number of disturbance impact. When the number of impact is constant, the energy released in the rebound section under 30 MPa, 90 MPa and 120 MPa is significantly higher than 0 MPa (when the number of disturbances is 1). This indicates that the existence of pre-static load improves the ability of impact resistance of rock. When the value of pre-static load remains unchanged (except for 90 MPa), the energy released in the rebound section of the stress-strain curve decreases with the increment of the disturbance numbers, manifesting that the higher the disturbance numbers are, the more seriously the mechanical properties of the samples deteriorate, and thus the ability to resist impact deformation of rock weakens.

4.2 Comparative validation and analysis of dynamic damage constitutive model of rock
According to this proposed one-dimensional dynamic damage constitutive equation of rock under high pre-static loading (Eq. (25)), the stress-strain curves of granite with biotite under the combination action of pre-static load and disturbance load were fitted. The disturbance impact test data of rock samples under one-dimensional pre-static load were analyzed before fitting. These test data were computed to determine the fitting parameters in the constitutive equation and the calculation was listed in Table 1.

Fig. 9 shows the variation curves of the constitutive fitting parameters $k$, $m$, $\varepsilon_0$ and $\sigma_d$ with the disturbance number. Fig. 9a illustrates that the constitutive parameter $k$ decreases with the increment of disturbance number, indicating that the brittleness of the rock gradually decreases and the stress-drop rate in the post-peak stage decreases with the increase of disturbance frequency. However, when the axial pressure is 60 MPa, the value of $k$ at the 9 times disturbance is higher than that at 2 times disturbance. The reason is that the internal structure of the rock is complex and the internal damage distribution is more discrete. In Weibull distribution function, parameter $m$ is the non-uniformity coefficient of rock material, which adds with the increment of disturbance numbers on the whole, implying that the homogeneity of rock material under disturbance load decreases. As can be seen from Fig. 9b, $\varepsilon_0$ generally increases while the dynamic peak intensity gradually decreases with the increase of disturbance number.

The above parameters were substituted into the damage constitutive equation (Eq. (25)) and the corresponding dynamic stress-strain curves were fitted and compared with the test curves. Fig. 10 shows that the fitted and test curves of granite with biotite under one-dimensional pre-static load and frequent dynamic disturbance have a high degree of
agreement in the pre-peak stage. This can fully reflect the deformation characteristics of rock under pre-static load and disturbance loads. However, the constitutive relationship established in this study is based on the Weibull distribution function and the stress-strain relationship is a one-to-one mapping function. Therefore, the rebound section of the stress-strain curve cannot be presented, which makes the poor consistency of the test curve in the fitting of the rebound section. Although the fitted curves and the experimental curves cannot reach an ideal state of complete consistency, the fitted curves can reflect the overall change trend of test curves for the pre-peak part in the dynamic stress strain curve. This indicates that the proposed model can be extensible to predict the rock stress-strain relationship under three-dimensional engineering conditions.

4.3 Applicability verification and analysis of model

In order to verify the applicability of the proposed damage constitutive model for rock, the test results of skarn samples (Wang et al., 2015) and sandstone samples (Gong et al., 2010) under different pre-static loads were compared and shown in Fig. 11 and Fig. 12. The trend of the calculated curve is basically consistent with the measured curve. For the rebound section of the curve, the prediction of the dynamic peak strength is in general agreement with the test result. As to the stain softening section of the curve, the established model by introducing the modified function $Y$ can preferably predict the total stress-strain curve of rock. In addition, the effects of pre-static loads and disturbance number were considered into the constitutive model, implying that this model has obvious advantages. However, the failure process of rock under the action of coupling dynamic and static loads is complicated. The stress-strain relationship of rock is affected not only by the magnitude of pre-static load and disturbance number, but
also by confining pressure, disturbance time, amplitude of disturbance load and disturbance waveform, and so on. Thus, relevant theories should be further explored.

5. Conclusions

This study modified the damage variable by introducing the correction function $Y$ with the same form as Weibull distribution based on the statistical damage mechanics. Meanwhile, according to the idea of unit assembly mechanics model, the evolution equation of rock dynamic damage was derived by comprehensively considering the influence of disturbance number and the pre-static loads. The statistical damage constitutive model of rock under one-dimensional pre-static load and frequent disturbance was established. The main conclusions are as follows:

(1) The stress-drop rate in the post-peak stage of stress-strain curve of rock is affected by the value of $k$ and the brittleness of rock increases when $k$ rises. The damage evolution rate curve gradually moves to the right along the strain axis and the peak value of the curve decreases first and then increases with the increment of $k$ value. When the value of $k$ is large, the damage evolution rate curve gradually appears a ‘plateau’ and the larger the value of $k$, the more obvious the ‘plateau’ phenomenon of the curve. This ‘plateau’ may correspond to the damage stable development stage of rock.

(2) The energy released by rock in the rebound part is related to the level of pre-static load and the number of disturbance in the stress-strain curves. When the disturbance number is constant, the energy released by rock under pre-static loads is obviously higher than that under no pre-static load. The existence of pre-static load improves the ability of impact resistance of the rock. When the pre-static load remains unchanged, the energy released
decreases with the increase of disturbance number. The ability of rock resistance to impact deformation gets weaker with more frequent disturbance.

(3) This damage constitutive model considers both the influence of disturbance number and pre-static load level. Its predictions in the elastic stage are in a high agreement with the test curves. As to the strain-softening curve, the fitted results are consistent with the test results at both pre-peak and post-peak stages. This model can well characterize the mechanical properties and stress-strain behaviors of rock under high in-situ stress and frequent disturbance under coupling action of dynamic and static loads.

Data Availability Statement

All data used during the study appear in the submitted article.
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Fig. 6 Stress-strain curve of rock under dynamic loading
Fig. 7 Total damage evolution curves of rock under different confining pressures

(a) $\sigma_A = 0$ MPa

(b) $\sigma_A = 30$ MPa

(a) $\sigma_A = 60$ MPa

(b) $\sigma_A = 90$ MPa

(e) $\sigma_A = 120$ MPa
Fig. 8 Relationship of $W_r$ with $\varepsilon_1$ and $n$ under different pre-static loads

Fig. 9 Variation of constitutive parameters with disturbance numbers under different pre-static loads
(a) $\sigma_A=0$ MPa, $n=1$

(b) $\sigma_A=0$ MPa, $n=2$

(c) $\sigma_A=30$ MPa, $n=1$

(d) $\sigma_A=30$ MPa, $n=3$

(e) $\sigma_A=30$ MPa, $n=5$

(f) $\sigma_A=60$ MPa, $n=2$
Test data  
Present model

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(g) \( \sigma_A = 60 \text{ MPa}, n=9 \)

(h) \( \sigma_A = 60 \text{ MPa}, n=12 \)

(i) \( \sigma_A = 90 \text{ MPa}, n=1 \)

(j) \( \sigma_A = 90 \text{ MPa}, n=5 \)

(k) \( \sigma_A = 90 \text{ MPa}, n=6 \)

(l) \( \sigma_A = 120 \text{ MPa}, n=1 \)
Fig. 10 Comparison of the fitted and measured stress-strain curves of granite under different pre-static loads

- (m) $\sigma_A = 120$ MPa, $n = 2$

Fig. 11 Comparison of fitted and measured stress-strain curves of skarn under different pre-static loads

- (a) $\sigma_A = 75$ MPa, $n = 16$
- (b) $\sigma_A = 75$ MPa, $n = 21$
- (c) $\sigma_A = 85$ MPa, $n = 6$
Test data (Gong 2010) Present model
Stress(MPa)
Strain(%)

(a) $\sigma_A=60$ MPa, $\dot{\varepsilon}=82$ s$^{-1}$

(b) $\sigma_A=60$ MPa, $\dot{\varepsilon}=82$ s$^{-1}$

(c) $\sigma_A=80$ MPa, $\dot{\varepsilon}=53$ s$^{-1}$

(d) $\sigma_A=80$ MPa, $\dot{\varepsilon}=78$ s$^{-1}$

(e) $\sigma_A=80$ MPa, $\dot{\varepsilon}=87$ s$^{-1}$

(f) $\sigma_A=80$ MPa, $\dot{\varepsilon}=116$ s$^{-1}$
Fig. 12 Comparison of fitted and measured stress-strain curves of sandstone samples under different pre-static loads.

(g) $\sigma_0 = 90$ MPa, $\dot{\varepsilon} = 78$ s$^{-1}$

(h) $\sigma_0 = 90$ MPa, $\dot{\varepsilon} = 93$ s$^{-1}$

(i) $\sigma_0 = 90$ MPa, $\dot{\varepsilon} = 114$ s$^{-1}$

(j) $\sigma_0 = 90$ MPa, $\dot{\varepsilon} = 132$ s$^{-1}$

(k) $\sigma_0 = 90$ MPa, $\dot{\varepsilon} = 139$ s$^{-1}$
Table 1 Fitting parameters of constitutive equation

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