An alternative technique for fitting the gravimetric geoid for Egypt

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An alternative technique for fitting the gravimetric geoid for Egypt

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Abstract

In this paper, a proposed geoid fitting technique for Egypt's physically determined geoid and GPS/leveling-derived geoid is introduced. First, any errors in the available GPS benchmarks are ruled out. The method relies on the absolute geoid difference, which is the physical geoid minus the geoid obtained through comparing GPS and levelling. The suggested geoid fitting technique uses an optimization algorithm scheme to choose the minimum number of the best-suited GPS benchmarks to be used for fitting the physical geoid. The least-squares prediction method is used to determine each GPS point's impact on the remaining GPS points. The subset of GPS points are used for external validations including the GPS point with the least impact on the other points, till an acceptable limit of the influence of the GPS points on the remaining ones. This step is repeatedly iterated, where the number of available GPS points is reduced by one each time. Also, the results of the first iteration are introduced. The proposed geoid fitting technique is compared with that using polynomial regression of different degrees. The results proved that, the proposed technique gives extremely better results.

Keyword: Egypt – geoid fitting - GPS/leveling – Polynomial Regression – Internal and External Check

Introduction

Gravimetric geoid estimation from dense datasets, such as surface gravity, global geopotential models (GGMs), and topography, is commonly utilized on both regional and local scales (see e.g. Denker et al., 2000; Smith and Roman, 2001). The geoid is precisely recovered by the current models over Egypt at short wavelengths owing to inaccuracies in the truncation techniques and/or geopotential model like EGG97 (Denker et al., 2000), JGEOID2000 (Kuroishi, 2001b), and GEOID93. However, systematic errors in longer wavelengths may occur (Milbert, 1995). The orthometric heights acquired from GPS/leveling measurements, on the other hand, provide exact point-wise geoid undulations that comprise the entire gamut of geoid signals but do not provide the geoid heights strictly speaking. Consequently, they are extremely important to determine gravimetric geoid undulations. Gravimetric data and GPS/leveling geoid undulations should be merged to create a geoid model that is accurate and dependable in terms of spatial resolution (see e.g. Smith and Milbert, 1999). These characteristics make the study of Geoid and GPS/Leveling differences vital for both practical surveying and scientific applications. To this extent, several studies have been carried out in various locations (see e.g.
Forsberg and Madsen (1990), Fotopoulos et al. (1999b), Kearsley et al. (1993), and Mainville et al. (1992)).

Modeling the geoid surface often involves a plane or low order polynomial (Featherstone et al. 1998). For the model utilized in practical geodetic applications, such as large-scale map production, engineering projects, etc., the geometric method has traditionally been preferred. But, the geometrically derived geoid model's correctness is influenced by a number of variables. The distribution of reference stations (GPS/Leveling Stations) must be as uniform as possible over the model area, and these sites must be selected to determine the likelihood that the geoid surface will change. Therefore, it would be advantageous to take the topographic features into account while selecting these reference locations. On the other hand, the number of reference points is stated as at least 1 point/20 km² for modeling the geoid (Anonym 2003). For this reason, the current research suggests an alternative technique for fitting the geoid in Egypt as the number of the available GPS/leveling stations is only thirty.

The precise determination of geoid models has begun to receive more attention in response to the widespread use of satellite-based positioning techniques, GNSS (Global Navigation Satellite System), with the goal of replacing geometric leveling measurements with GPS measurements during geodetic and surveying work. The main aim of this study is to propose a geoid fitting technique for sparsely distributed GPS benchmarks, such as the situation in Egypt. The proposed geoid fitting technique has been compared with the widely used in practice, surface polynomial fitting technique.

2. Methodology and Computation

In this study, the proposed geoid fitting technique uses an automated optimization scheme to select a number of a few best proper GPS benchmarks to be used to fit the physical geoid. The least-squares prediction technique is utilized to calculate each GPS point's impact on the other GPS points.

Using the least squares prediction technique, the influence at each point is calculated from the neighboring points excluding the value of the computational point. The equation of the least squares prediction is expressed as (Moritz, 1980; Fashir and Kadir 1998; Tscherning 2002):

\[
\begin{align*}
\Delta N_p = & (C(p, p_1) \ C(p, p_2) \ldots \ldots \ C(p, p_n)) \begin{pmatrix}
C(p_1, p_1) & C(p_1, p_2) & \ldots & C(p_1, p_n) \\
C(p_2, p_1) & \ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots & \ldots \\
C(p_n, p_1) & \ldots & \ldots & C(p_n, p_n)
\end{pmatrix}^{-1}
\begin{pmatrix}
\Delta N_{p_1} \\
\Delta N_{p_2} \\
\vdots \\
\Delta N_{p_n}
\end{pmatrix}
\end{align*}
\]

\[ (1) \]

where \( C(P, P_i) \) is the covariance between the point under consideration and the running nearby points and \( C(P_r, P_j) \) represents the covariance between the running nearby points.
In the current investigation, the generalized covariance model of Hirvonen has been identified and tested which is expressed as follows (Moritz 1980, p. 179):

\[ C(P_i, P_j) = C(s) = \frac{C_0}{(1 + A^2 s^2)^p} \]

(2)

where \( s \) refers to the distances between the pair of the considered points and the parameter \( A \) is presented by (Abd-Elmotaal 1992):

\[ A = \frac{1}{\xi} \left( \frac{1}{2^p} - 1 \right)^{1/2} \]

(3)

with the empirically covariance function \( C_0 \), correlation length \( \xi \). The parameter \( P \) depends on the gravity anomalies' type and a value of 0.25 has been used (ibid.).

Equation (2) demonstrates that the covariance function depends mainly on the inverse square distance, and it leads to severely ill conditional covariance matrices. Therefore, it has been replaced by the following local covariance function (Fashir et. al., 1998, eq.3) as:

\[ C(P_i, P_j) = C(s) = C_0 \left( 1 + \frac{s}{R} \right)^{-1} \]

(4)

where \( R \) refers to the mean radius of the Earth. The difference between the estimated values and the data values (residuals) is calculated.

In the first step, the total numbers of points (except the point under consideration) are used for computing the absolute geoid difference of point under consideration. In the second step, the absolute geoid is calculated using all GPS benchmarks apart of the computational point and the running point for which we need to estimate its influence. The standard deviation of the differences at all points between to the above two steps is computed and considered as a key for the decision for choosing the minimum number of the GPS stations used for fitting the geoid. The chosen GPS stations for the fitting have been used to compute the external check at the remaining GPS stations by estimating the absolute geoid differences at these GPS stations and compare them with the original values.

For the purpose of the external check, the GPS points having the minimum influence on the remaining points are added to the subset of the GPS points till an acceptable limit of the influence of the GPS points on the remaining ones is achieved. This step is repeated, where the number of available GPS points is reduced by one each time. For the current application of the proposed geoid fitting technique, this acceptable limit has been set to a couple of decimeters (the number of the available GPS stations is too small compared to the area of Egypt).
The output of the aforementioned scheme is two subsets where the first subset includes the points having the minimum influence, which are used to estimate the external check of the geoid quality. The second subset comprises the GPS points used for the geoid fitting process.

The second proposed technique in this study depends only on the first iteration. The GPS stations having minimum influence on the other stations (having minimum standard deviation of the residuals) are used for the external check of the geoid fitting quality. The remaining GPS points are used for the geoid fitting process.

3. Data used

To validate the proposed techniques, two data sets are used: the first comprises 30 GPS data points with known geoid height in Egypt. These stations are regularly distributed all over the country (Fig. 1), however the total number of GPS stations is too less compared to the Egypt’s surface area.

![Fig. 1 Available GPS Stations](image)

The second data set is the physical geoid undulation for Egypt. The used physical geoid is computed using high-degree tailored geopotential model for Egypt (after, Abd-Elmotaal, 2008) see Fig. 2.
Our computationms are based on the absolute geoid differences between the two surfaces. The absolutat geoid differencvces are shown in Fig. 3.

4. Numerical computations

In this research, two techniques are applied. The internal check (residuals at the GPS stations used for the geoid fitting) of the first technique is illustrated in Table 1, where the
statistics for the absolute geoid difference residuals are shown. The internal points used for fitting the gravimetric geoid are checked for different number of benchmarks.

**Table 1. Absolute geoid difference for internal checks for the first technique**

<table>
<thead>
<tr>
<th>Number of GPS stations used for geoid fitting</th>
<th>Min [m]</th>
<th>Max [m]</th>
<th>Avg [m]</th>
<th>St.dev. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>27 stations</td>
<td>-0.05</td>
<td>0.06</td>
<td>0.0012</td>
<td>0.03</td>
</tr>
<tr>
<td>26 stations</td>
<td>-0.05</td>
<td>0.06</td>
<td>0.0014</td>
<td>0.03</td>
</tr>
<tr>
<td>25 stations</td>
<td>-0.05</td>
<td>0.06</td>
<td>-0.0015</td>
<td>0.03</td>
</tr>
<tr>
<td>24 stations</td>
<td>-0.05</td>
<td>0.06</td>
<td>-0.0015</td>
<td>0.03</td>
</tr>
<tr>
<td>23 stations</td>
<td>-0.05</td>
<td>0.06</td>
<td>-0.0015</td>
<td>0.03</td>
</tr>
<tr>
<td>22 stations</td>
<td>-0.05</td>
<td>0.06</td>
<td>-0.0016</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 1 shows that, the internal precision (standard deviation) does not depend on the number of the used GPS stations for geoid fitting.

Table 2 provides statistics for the remaining residuals at the GPS stations that weren't used for the geoid fitting. This process presents the external check which indicates the geoid fitting quality. The rows of Tables 1 and 2 are correspondent (the sum is the total number of GPS stations, i.e, 30).

**Table 2. Absolute geoid difference for external checks for the first technique**

<table>
<thead>
<tr>
<th>Number of check stations</th>
<th>Min [m]</th>
<th>Max [m]</th>
<th>Avg [m]</th>
<th>St.dev. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 stations</td>
<td>-0.17</td>
<td>0.30</td>
<td>0.014</td>
<td>0.28</td>
</tr>
<tr>
<td>4 stations</td>
<td>-0.18</td>
<td>0.31</td>
<td>-0.005</td>
<td>0.22</td>
</tr>
<tr>
<td>5 stations</td>
<td>-0.94</td>
<td>0.32</td>
<td>-0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>6 stations</td>
<td>-0.94</td>
<td>0.3</td>
<td>-0.31</td>
<td>0.42</td>
</tr>
<tr>
<td>7 stations</td>
<td>-0.92</td>
<td>0.22</td>
<td>-0.40</td>
<td>0.42</td>
</tr>
<tr>
<td>8 stations</td>
<td>-1.42</td>
<td>0.19</td>
<td>-0.67</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 2 shows that the best geoid fitting, expressed by the minimum standard deviation of the residual occurs where 26 GPS stations have been used for fitting the gravimetric geoid and only four GPS stations were used for the external check.

The internal check of the second used technique is illustrated in Table 3, where the statistics of the residuals of the absolute geoid differences are shown.
Table 3. Absolute geoid difference for internal checks for the second technique

<table>
<thead>
<tr>
<th>Number of Check Points</th>
<th>Min [m]</th>
<th>Max [m]</th>
<th>Avg [m]</th>
<th>St.dev. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>27 stations</td>
<td>-0.05</td>
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<td>0.0012</td>
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<td>0.06</td>
<td>0.0014</td>
<td>0.03</td>
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<td>0.03</td>
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<td>0.06</td>
<td>-0.0015</td>
<td>0.03</td>
</tr>
<tr>
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<td>-0.05</td>
<td>0.06</td>
<td>-0.0015</td>
<td>0.03</td>
</tr>
<tr>
<td>22 stations</td>
<td>-0.05</td>
<td>0.06</td>
<td>-0.0016</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Here, also the internal precision does not depend on the number of the used GPS stations for the geoid fitting. Comparing Table 1 and 3 shows that the inter checks remains the same for the two techniques.

Table 4 shows the external check for the second GPS fitting technique. It shows that the best geoid fitting expressed by the minimum standard deviation of the residual occurs where 27 GPS stations have been used for the fitting and only 3 GPS were used for the external check. It is also remarkable that the standard deviation remains constant where 6 to 8 GPS stations used for external check.

Table 4. Absolute geoid difference for external checks for the second technique

<table>
<thead>
<tr>
<th>Number of Check stations</th>
<th>Min [m]</th>
<th>Max [m]</th>
<th>Avg [m]</th>
<th>St.dev. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 stations</td>
<td>-0.40</td>
<td>0.10</td>
<td>-0.26</td>
<td>0.15</td>
</tr>
<tr>
<td>4 stations</td>
<td>-0.67</td>
<td>-0.31</td>
<td>-0.43</td>
<td>0.16</td>
</tr>
<tr>
<td>5 stations</td>
<td>-0.67</td>
<td>-0.15</td>
<td>-0.38</td>
<td>0.19</td>
</tr>
<tr>
<td>6 stations</td>
<td>-0.67</td>
<td>0.29</td>
<td>-0.25</td>
<td>0.33</td>
</tr>
<tr>
<td>7 stations</td>
<td>-0.67</td>
<td>0.31</td>
<td>-0.29</td>
<td>0.33</td>
</tr>
<tr>
<td>8 stations</td>
<td>-0.67</td>
<td>0.31</td>
<td>-0.25</td>
<td>0.33</td>
</tr>
</tbody>
</table>

For the purpose of comparison, the fitting of the gravimetric geoids within the current study was also done by subtracting a polynomial regression surface of first, second and third orders. Table 5 lists the residuals for the 30 GPS points after the polynomial surface subtracting. Given that all GPS stations were employed in the fitting strategies, Table 5 indicates the fitted geoids' internal precision.

Table 5. Geoid fitting using polynomial regression

<table>
<thead>
<tr>
<th>Polynomial order</th>
<th>Min [m]</th>
<th>Max [m]</th>
<th>Avg [m]</th>
<th>St.dev. [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st order</td>
<td>-4.294752701</td>
<td>4.507983807</td>
<td>-5.80277E-15</td>
<td>2.243857501</td>
</tr>
<tr>
<td>2nd order</td>
<td>-2.894399368</td>
<td>4.192150822</td>
<td>2.57059E-05</td>
<td>1.99744508</td>
</tr>
<tr>
<td>3rd order</td>
<td>-2.237132697</td>
<td>2.416441762</td>
<td>2.97443E-05</td>
<td>1.235629134</td>
</tr>
</tbody>
</table>
Comparing Tables 2, 4 and 5 illustrates that the suggested two geoid fitting techniques in this paper are more accurate than the polynomial regression fitting technique till the third order.

Figures 4 and 5 shows the fitted geoid (A) and the absolute geoid difference (B) using the two proposed geoid fitting techniques with the best number of GPS stations used for the geoid fitting (4 and 3 respectively). Figures 4 and 5 confirm again that the second proposed geoid fitting technique gives better geoid quality.

Fig. 4 Final geoid after using 26 GPS stations for case of the first technique
Fig. 5 Final geoid after using 27 GPS stations for case of the second technique
5. Conclusion

Two powerful alternative geoid fitting techniques have been proposed in the current investigation. They have been successfully applied to fit the gravimetric geoid for Egypt. The quality of the fitted geoid using the proposed geoid fitting techniques expressed by the residual at the external check points is one and half decimeters. This quality is relatively too good compared to the very limited number of the available GPS stations in the country.

For the sake of comparison, the gravimetric geoid for Egypt has been fitted to the GPS/levelling derived geoid using surface polynomials regression fitting techniques. In this case about 75% of the available GPS stations have been used to fit the gravimetric geoid and the remaining 25% GPS stations have been used to estimate the quality of the fitted geoid. The results proved that the proposed geoid fitting techniques within the current investigation are superior.

The implanted tests of the proposed geoid fitting techniques illustrate their capability to fit the gravimetric geoid in case of sparse GPS stations. The proposed techniques need to be tested in case of dense GPS stations coverage to determine their suitability.

References


