Discrete-Time Noise-Suppression Neural Dynamics for Optical Remote Sensing Image Extraction

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Discrete-Time Noise-Suppression Neural Dynamics for Optical Remote Sensing Image Extraction

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Abstract

Remote sensing is a significant manner of observation at present, and how to extract the target object from optical remote sensing images quickly and accurately is a challenge. Accordingly, a discrete-time noise-suppression neural dynamics (DTNSND) model with an error-accumulation term is proposed to aid the constrained energy minimization scheme for extracting the target object, which restrains the effects of noises in the extraction process. Through theoretical analysis, it is proven that the DTNSND model excels at suppressing noise in diverse noisy environments. Furthermore, numerical simulations are provided to illustrate that the maximal steady-state residual error generated by the DTNSND model during the solution process is markedly lower than that generated by the comparative algorithms. Finally, extraction experiments, using an optical remote sensing image of the Arctic sea ice as an experimental material, confirm the feasibility of the DTNSND model in practice.
Keywords: Target object extraction, discrete-time noise-suppression neural dynamics (DTNSND) model, constrained energy minimization (CEM) scheme, noise-suppression

1 Introduction

Previous researches in the optical remote sensing image recognition target area have been developed [1–4]. Harsanyi et al. [5] present an orthogonal subspace projection (OSP) algorithm, which is a hyperspectral image classification technique based on the linear mixed model. Its operator can detect multiple target feature values of interest and classify these images simultaneously. However, this method lacks practicality owing to the fact that it requires background spectral information of the target. Subsequently, Ren et al. [6] investigate a target-constrained interference-minimized filter, which reduces interfering factors and eliminates other uncorrelated signals while detecting the primary target. A system consisting of airborne sensors and various automated algorithms is investigated in [7], whose main role is to project detection points onto high-resolution images through registration algorithms, employing linear discrimination functions and decision surfaces to classify each detected point. The significance of machine learning has grown exponentially across a wide range of research fields, e.g., in the field of optical remote sensing image processing, mainly due to its advantages, such as efficiency, adaptability, accuracy, automation, and intelligence [8–11]. However, most machine learning methods require the training and testing of data from the same feature space and distribution [12–14], which is often difficult to achieve in practical applications, especially in image processing. The consequence is that when the data distribution changes, re-collection of data and redesign of models are often necessary, leading to a costly and time-consuming process. Moreover, in the case of optical remote sensing images, complex targets and scenes need to be identified, which poses a challenging problem. Meanwhile, selecting appropriate machine learning algorithms and models to handle different situations is complex as well.

Based on the above mentioned issues, the constraint energy minimization (CEM) scheme gives superior performance in optical remote sensing image target recognition and classification. It requires only the target spectral vector but not the spectral information of the background, therefore applied in many fields [15–17]. However, the CEM scheme is deficient under noise conditions, and thus a solver for the CEM scheme is needed, which is capable of restraining noises to obtain optimal filter coefficients for extracting the target. Firstly, the CEM scheme can be transformed into an optimization problem [15]. Several studies in recent years have found that using neural dynamics (ND) to solve optimization problems is valid [18–21]. In [22], a quadratic-programming (QP) based neural dynamic is presented to complete time-varying tasks. To synchronously
handle multiple types of constraints for the time-dependent nonlinear optimization, Chen et al. [23] present a multi-constrained ZNN model, which possesses the effectiveness of exponential convergence property by utilizing the time-derivative information. Xiao et al. construct an adaptive coefficient gradient-based ND and a projected zeroing ND in [24], which decreases the residual error, and relieves the limitation on the available activation function. In addition, Kong et al. [25] establish a generalized varying-parameter recurrent neural network and apply it to control redundant manipulators by solving a QP problem. Although all of the above algorithms have good computational performance, they are not noise-suppression. However, noises are inevitable in practical situations. Therefore, a discrete-time noise-suppression neural dynamics (DTNSND) model with an error-accumulation term is proposed to aid the CEM scheme for extracting optical remote sensing images, and theoretical proofs demonstrate that the DTNSND model maintains a high convergence accuracy and strong stability under noise-free and noise conditions. Moreover, experimental results on the extraction of the Arctic sea ice confirm the feasibility of the DTNSND model in practice. The contributions of this paper are listed as follows:

- The DTNSND model is able to improve the solution accuracy under noise-free and noise conditions.
- The DTNSND model is proposed to aid the CEM scheme to extract optical remote sensing images, presenting a new technique to combine the CEM scheme with discrete-time ND.
- Relevant theoretical analyses and experiments are given to demonstrate that the DTNSND model is globally convergent and robust.

2 Problem formulation

The CEM scheme can be written as [26]

$$\min \| S(t)w(t) + a(t) \|^2_2 / 2, \quad (1a)$$

subject to

$$N(t)w(t) = u(t), \quad (1b)$$

where \(\| \cdot \|_2\) denotes \(L_2\)-norm operator; \(S(t) \in \mathbb{R}^{r \times m}\) and \(N(t) \in \mathbb{R}^{n \times m}\) are the real-valued full-rank matrices; \(a(t) \in \mathbb{R}^r\) and \(u(t) \in \mathbb{R}^n\) are the real-valued coefficient vectors; \(w(t) \in \mathbb{R}^m\) is the vector to be solved. Subsequently, a Lagrange formula is constructed as follows:

$$F(w(t), v(t), t) = f(w(t), t) + v(t)^T(N(t)w(t) - u(t)), \quad (2)$$

with

$$\begin{cases} f(w(t), t) = \| S(t)w(t) + a(t) \|^2_2 / 2, \\
 v(t) = [v_1(t), \ldots, v_n(t)]^T \in \mathbb{R}^n, \end{cases} \quad (3)$$

where superscript \(^T\) is the transposition operation, and \(v(t)\) is the Lagrange-multiplier operator vector. Then, the model for solving CEM scheme (1) can
be formulated as
\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{\partial F(w(t), v(t), t)}{\partial w(t)} = \frac{\partial f(w(t), t)}{\partial w(t)} + N^T(t)v(t) = 0, \\
\frac{\partial F(w(t), v(t), t)}{\partial v(t)} = N(t)w(t) - u(t) = 0.
\end{array} \right.
\end{align*}
\tag{4}
\]

Let
\[
Q(t) = \begin{bmatrix}
Y(t) & N^T(t) \\
N(t) & 0_{n \times n}
\end{bmatrix} \in \mathbb{R}^{(m+n) \times (m+n)},
\]
\[
x(t) = \begin{bmatrix}
w(t) \\
v(t)
\end{bmatrix} \in \mathbb{R}^{m+n},
\]
\[
z(t) = \begin{bmatrix}
d(t) \\
u(t)
\end{bmatrix} \in \mathbb{R}^{m+n}.
\]

where \(Y(t) = S^T(t)S(t) \in \mathbb{R}^{m \times m}\) and \(d(t) = -S^T(t)a(t) \in \mathbb{R}^m\). Ulteriorly, (4) can be rewritten as
\[
Q(t)x(t) = z(t).
\tag{5}
\]

3 DTNSND model design and simplification

An error function is defined as follows:
\[
e(t) = Q(t)x(t) - z(t).
\tag{6}
\]

Then, a design formula with an integral term is introduced as
\[
\dot{e}(t) = -\varsigma e(t) - \eta \int_0^t e(\gamma)d\gamma,
\tag{7}
\]
where \(\varsigma > 0\) and \(\eta > 0\). Subsequently, associating (6) and (7) generates
\[
\dot{x}(t) = Q^{-1}(t) \left( \dot{z}(t) - \dot{Q}(t)x(t) - \varsigma \left( Q(t)x(t) - z(t) \right) \right)
- \eta \int_0^t \left( Q(\gamma)x(\gamma) - z(\gamma) \right)d\gamma.
\tag{8}
\]

Further, to discretize the continuous ND, the Euler forward difference formula is introduced:
\[
\dot{x}_k = \frac{x_{k+1} - x_k}{t},
\tag{9}
\]
where $\iota$ is the sampling period, and $k$ represents the updating index at time instant $t = k\iota$. Then, the DTNSND model is obtained as

\[
x_{k+1} = x_k + Q_k^{-1}\left(z_k - z_{k-1} - x_k(Q_k - Q_{k-1}) - \varsigma\iota(Q_k x_k - z_k)\right) - \eta^2 \sum_{i=0}^{k} (Q_i x_i - z_i) .
\] (10)

Let $\Theta = \varsigma\iota$ and $\Delta = \eta^2$. Hereto, equation (10) can be finally represented as

\[
x_{k+1} = x_k + Q_k^{-1}\left(z_k - z_{k-1} - x_k(Q_k - Q_{k-1}) - \Theta(Q_k x_k - z_k)\right) - \Delta \sum_{i=0}^{k} (Q_i x_i - z_i) .
\] (11)

In order to facilitate the convergence and robustness analysis of DTNSND model (11), a concise form is derived in the following theorem.

**Theorem 1** DTNSND model (11) can be transformed as

\[
(1 + \Theta)e_k - e_{k-1} + \Delta \sum_{i=0}^{k} e_i + O(\iota^2) = 0 ,
\] (12)

where $O(\iota^2)$ denotes the vector of truncation error $O(\iota^2)$.

**Proof** DTNSND model (11) can be rewritten as

\[
Q_k x_{k+1} = z_k - z_{k-1} + Q_{k-1} x_k - \Theta e_k - \Delta \sum_{i=0}^{k} e_i .
\] (13)

Then, employing the Taylor expansion expression to the variable in (13) yields

\[
x_{k+1} = x_k + \iota x_k + O(\iota^2)
\] (14)

and

\[
x_{k-1} = x_k - \iota \dot{x}_k + O(\iota^2) .
\] (15)

Substituting (14) and (15) into (13) as well as combining the definition of $e_k$, one has

\[
e_k + \iota Q_k \dot{x}_k + O(\iota^2) = e_{k-1} + \iota Q_{k-1} \dot{x}_k - \Theta e_k - \Delta \sum_{i=0}^{k} e_i .
\] (16)

Next, applying the Taylor expansion to $Q_k$, it has

\[
Q_k = Q_{k-1} + \iota Q_{k-1} + \psi(\iota^2) ,
\] (17)

where the matrix $\psi(\iota^2)$ consists of $O(\iota^2)$. Substituting (17) into (16) and collating gives

\[
(1 + \Theta)e_k - e_{k-1} + \Delta \sum_{i=0}^{k} e_i + O(\iota^2) = 0 .
\] (18)

The proof is complete. □
4 Convergence analysis

The following theorem illustrates the global convergence of DTNSND model (11).

**Theorem 2** The residual error \( \lim_{k \to \infty} \| e_k \|_2 \) of DTNSND model (11) is \( O(\iota^2) \).

**Proof** In the light of (18), we have

\[
(1 + \Theta)e_{k+1} - e_k + \Delta \sum_{i=0}^{k+1} e_i + O(\iota^2) = 0. \tag{19}
\]

Let the \( \alpha \)th element of \( e_k \) be \( e^a_k \). Then, subtracting the \( \alpha \)th element of (19) from the \( \alpha \)th element of (18) products

\[
(1 + \Theta + \Delta)e^a_{k+1} = (2 + \Theta)e^a_k - e^a_{k-1} + O(\iota^2). \tag{20}
\]

Let \( \chi^\alpha_{k+1} = [e^\alpha_{k+1}, e^\alpha_k]^T \), and (20) can be formulated as

\[
\chi^\alpha_{k+1} = H\chi^\alpha_{k} + O(\iota^2), \tag{21}
\]

with

\[
H = \begin{bmatrix}
\frac{2+\Theta}{1+\Theta+\Delta} & -1 \\
1 & 0
\end{bmatrix}.
\]

From (21), it can be generalized that

\[
\| \chi^\alpha_{k+1} \|_2 \leq \| H\chi^\alpha_{k} \|_2 + \| O(\iota^2) \|_2 \\
= \| H\chi^\alpha_{k} \|_2 + O(\iota^2) \\
\leq \| H \|_2 \| \chi^\alpha_{k} \|_2 + O(\iota^2) \\
\leq \| H \|_2 \| H\chi^\alpha_{k-1} \|_2 + \| H \|_2 O(\iota^2) + O(\iota^2) \\
= \| H \|_2 \| H\chi^\alpha_{k-1} \|_2 + O(\iota^2) \\
\leq \| H \|_2 \| H\chi^\alpha_{k-1} \|_2 + O(\iota^2) \\
\vdots \\
\leq \| H \|_2 \| \chi^\alpha_{1} \|_2 + O(\iota^2).
\]

The eigenvalues of matrix \( H \) are \( g_{1,2} = (\Theta \pm \sqrt{\Theta^2 - 4\Delta + 2})/(2\Theta + 2\Delta + 2) \). On account of the absolute values of the real part of \( g_{1,2} \) are less than 1, it can be obtained that \( \lim_{k \to \infty} \| H \|_2^k = 0 \). Then, the following conclusion is drawn:

\[
\lim_{k \to \infty} \| \chi^\alpha_{k+1} \|_2 \leq \lim_{k \to \infty} \| H \|_2^k \| \chi^\alpha_{k} \|_2 + O(\iota^2) = O(\iota^2).
\]

Therefore, one can further get that the residual error \( \lim_{k \to \infty} \| e_k \|_2 \) of DTNSND model (11) is \( O(\iota^2) \). The proof is complete. \( \square \)
5 Robustness analysis

During the target extraction, the presence of noise is strongly related to the quality of the extraction. The random noise is the most common, and thus the following theorem is presented to demonstrate the effectiveness of DTNSND model (11) in finding the theoretical solution under the random noise condition. Meanwhile, DTNSND model (11) with a noise term $\zeta(t)$ can be expressed as follows:

$$
\begin{align*}
    x_{k+1} &= x_k + Q_k^{-1} \left( z_k - z_{k-1} - x_k (Q_k - Q_{k-1}) - \Theta(Q_k x_k - z_k) \right) \\
    &\quad - \Delta \sum_{i=0}^{k} (Q_i x_i - z_i) + \zeta(t).
\end{align*}
$$

(22)

Theorem 3 Assuming that DTNSND model (11) to aid the CEM scheme (1) for Arctic sea-ice extraction is affected by the constant noise, the steady-state computing error of DTNSND model (11) is $O(\epsilon^2)$, regardless of the magnitude of $\zeta(t) = \epsilon$.

Proof DTNSND model (11) is converted into a linear system (18), and thus the steady-state computing error of the model is the superposition effects of two parts: constant noise $\zeta(t) = \epsilon$ and $O(\epsilon^2)$. Thus, from (18), the $\alpha$th subsystem with a constant noise term can be regarded as

$$
(1 + \Theta + \Delta) e_{k+1}^\alpha = e_k^\alpha - \Delta \sum_{i=0}^{k} e_i^\alpha + \epsilon^\alpha.
$$

(23)

The aforementioned equation can be converted via the Z-transform into the following form:

$$
(1 + \Theta + \Delta) \left( z e^\alpha(z) - z e^\alpha(0) \right) = e^\alpha(z) - \Delta e^\alpha(z) \frac{z - 1}{1 - z^{-1}} + \frac{z - 1}{z e^\alpha(z)},
$$

(24)

where $e^\alpha(0)$ denotes the initial value of $e^\alpha$. The above equation can be further expressed as

$$
e^\alpha(z) = \frac{(1 + \Theta + \Delta) z (z - 1) e^\alpha(0) + z e^\alpha}{(1 + \Theta + \Delta) z (z - 1) - (z - 1) + z \Delta}.
$$

(25)

The roots of the above equation are $z_{1,2} = (\Theta \pm \sqrt{\Theta^2 - 4\Delta + 2})/(2\Theta + 2\Delta + 2)$. Evidently, the absolute values of these roots are not more than 1, that is, (23) is stable. Based on the final value theorem, have

$$
\lim_{k \to \infty} e_k^\alpha = \lim_{z \to 1} (z - 1) e^\alpha(z) = \lim_{z \to 1} \frac{(z - 1) ((1 + \Theta + \Delta) z (z - 1) e^\alpha(0) + z e^\alpha)}{(1 + \Theta + \Delta) z (z - 1) - (z - 1) + z \Delta} = 0.
$$

(26)

Thus, the steady-state computing error for subsystem $\alpha$ can be further obtained as $\lim_{k \to \infty} e_k = 0$. Since the truncation error of (20) is $O(\epsilon^2)$, the steady-state error
The proof is complete. \(\square\)

In addition, considering the effect of the random noise, Theorem 4 is given.

**Theorem 4** Considering that DTNSND model (11) is disturbed by the bounded random noise \(\zeta(t) = \tau \in \mathbb{R}^{m+n}\), its residual error is \(\lim_{k \to \infty} \|e_k\|_2 < 2(m + n) \sup_{1 \leq i \leq k, 1 \leq \alpha \leq (m+n)} |\tau_i^\alpha|/(1 - \|H\|_2) + O(\tau^2)\).

*Proof* DTNSND model (11) is a linear system, and thus the residual error of it can be regarded as a superposition of the impacts of \(\zeta(t) = \tau\) and \(O(\tau^2)\). Considering the random noise \(\zeta(t) = \tau\) only, it has

\[(1 + \Theta + \Delta)e_{k+1} = e_k - \Delta \sum_{i=0}^{k} e_i + \tau.\]  

(27)

Similar to getting (20), the following equation is available:

\[(1 + \Theta + \Delta)e_{k+1}^\alpha = (2 + \Theta)e_k^\alpha - e_{k-1}^\alpha + \tau_k^\alpha - \tau_{k-1}^\alpha.\]  

(28)

Let \(\beta_k^\alpha = [\tau_k^\alpha - \tau_{k-1}^\alpha, 0]^T\), and equation (28) can be rewritten as

\[x_{k+1}^\alpha = Hx_k^\alpha + \beta_k^\alpha.\]  

(29)

Therefore, one has

\[
\|x_{k+1}^\alpha\|_2 \leq \|Hx_k^\alpha\|_2 + \|\beta_k^\alpha\|_2 \\
\leq \|H\|_2 \|x_k^\alpha\|_2 + \|\beta_k^\alpha\|_2 \\
\leq \|H\|_2 \|x_k^\alpha\|_2 + \|H\|_2 \|\beta_{k-1}^\alpha\|_2 + \|\beta_k^\alpha\|_2 \\
\leq \|H\|_2 \|x_{k-1}^\alpha\|_2 + \|H\|_2 \|\beta_{k-1}^\alpha\|_2 + \|\beta_k^\alpha\|_2 \\
\vdots \\
\leq \|H\|_2 \|x_1^\alpha\|_2 + \|H\|_2 \|\beta_{k-1}^\alpha\|_2 + \cdots + \|\beta_k^\alpha\|_2 \\
< \|H\|_2 \|x_1^\alpha\|_2 + \max_{1 \leq i \leq k} \|\beta_i^\alpha\|_2/(1 - \|H\|_2) \\
< \|H\|_2 \|x_1^\alpha\|_2 + 2 \max_{1 \leq i \leq k} |\tau_i^\alpha|/(1 - \|H\|_2).
\]

Note that \(\lim_{k \to \infty} \|H\|_2^k = 0\), and the aforementioned equation can be rearranged as

\[
\|x_{k+1}^\alpha\|_2 < 2 \max_{1 \leq i \leq k} |\tau_i^\alpha|/(1 - \|H\|_2).
\]

Then, the following conclusion can ultimately be drawn

\[
\lim_{k \to \infty} \|e_k\|_2 < 2(m + n) \sup_{1 \leq i \leq k, 1 \leq \alpha \leq (m+n)} |\tau_i^\alpha|/(1 - \|H\|_2) + O(\tau^2).
\]

The proof is complete. \(\square\)
6 Numerical experiments

In this section, numerical experiments are carried out to demonstrate the excellent robustness and stability of DTNSND model (11). In detail, in the following experiments, the time region \([t_0, t_f]\) of the variable \(x(t)\) is set to be \([0, 20)\) and the matrices \(Q(t)\) and \(z(t)\) are defined as

\[
Q(t) = \begin{bmatrix}
Q_{11}(t) & Q_{12}(t) & \cdots & Q_{1(m+n)}(t) \\
Q_{21}(t) & Q_{22}(t) & \cdots & Q_{2(m+n)}(t) \\
\vdots & \vdots & \ddots & \vdots \\
Q_{(m+n)1}(t) & Q_{(m+n)2}(t) & \cdots & Q_{(m+n)(m+n)}(t)
\end{bmatrix} \in \mathbb{R}^{(m+n)\times(m+n)},
\]

\[
z(t) = \begin{bmatrix} z_1(t) & z_2(t) & \cdots & z_{m+n}(t) \end{bmatrix}^T \in \mathbb{R}^{m+n},
\]

where \(Q_{ij}\) denotes the \(ij\)th element of \(Q(t)\), and \(z_i\) denotes the \(i\)th element of \(z(t)\). To make the experiment work effectively, matrix \(Q(t)\) is described as follows:

\[
\begin{align*}
Q_{ij}(t) &= i + \sin(t), \quad i = j, \\
Q_{ij}(t) &= \sin\left(\frac{t}{i-j}\right), \quad i > j, \\
Q_{ij}(t) &= \cos\left(\frac{t}{i-j}\right), \quad i < j.
\end{align*}
\]

Then, the vector \(z(t)\) is described as

\[
\begin{align*}
\{ z(t) = \sin(t), \quad &\text{when } i \text{ is even} \\
\{ z(t) = \cos(t), \quad &\text{when } i \text{ is odd}
\end{align*}
\]

Besides, let both \(m\) and \(n\) be 10. For comparison, Newton-Raphson iterative (NRI) algorithm [27] and Z-type model [28] are introduced below.

- **NRI algorithm:**
  \[
x_{k+1} = x_k - Q_k^{-1}(Q_kx_k - z_k).
  \]

- **Z-type model:**
  \[
x_{k+1} = 1.5x_k - x_{k-1} + 0.5x_{k-2} + Q_k^{-1}\left(i\dot{z}_k - i\dot{Q}_kx_k - \mu(Q_kx_k - z_k)\right),
  \]

where \(\mu\) denotes the step size. With the steady-state error \(\|e_k\|_2\) being the measure for the convergence performance of these algorithms, comparative results are shown in Figs. 1 through 3, and Table 1.

6.1 Noise-free condition

The accuracy of the model is an important indicator for evaluating the performance of a solving algorithm. Figure 1 evaluates the steady-state error of three models with different sampling periods under the noise-free condition. The results show that all three models converge. However, the proposed
Table 1  MSSRE generated by Z-type model (32), NRI algorithm (31), and DTNSND model (11) for solving Eq. (30) with different sampling periods under noise conditions

<table>
<thead>
<tr>
<th>$\tau$ (s)</th>
<th>Algorithm</th>
<th>MSSRE under different noise conditions</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Noise-free $\zeta(t) = 0$</td>
<td>Random noise $\zeta(t) \in 5 \times [-1, 1]$</td>
<td>Constant noise $\zeta(t) = 100$</td>
</tr>
<tr>
<td>0.1</td>
<td>Z-type (32)</td>
<td>$4.313642 \times 10^{-1}$</td>
<td>$6.325087 \times 10^{3}$</td>
<td>$8.240402 \times 10^{2}$</td>
</tr>
<tr>
<td></td>
<td>NRI (31)</td>
<td>$2.654581 \times 10^{-1}$</td>
<td>$3.281694 \times 10^{3}$</td>
<td>$3.279462 \times 10^{2}$</td>
</tr>
<tr>
<td></td>
<td>DTNSND (11)</td>
<td>$8.988422 \times 10^{-3}$</td>
<td>$8.240402 \times 10^{-1}$</td>
<td>$8.988422 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.01</td>
<td>Z-type (32)</td>
<td>$4.426232 \times 10^{-2}$</td>
<td>$6.324835 \times 10^{3}$</td>
<td>$6.324997 \times 10^{2}$</td>
</tr>
<tr>
<td></td>
<td>NRI (31)</td>
<td>$2.630418 \times 10^{-2}$</td>
<td>$3.174315 \times 10^{3}$</td>
<td>$3.173803 \times 10^{2}$</td>
</tr>
<tr>
<td></td>
<td>DTNSND (11)</td>
<td>$8.756398 \times 10^{-6}$</td>
<td>$9.074429 \times 10^{-1}$</td>
<td>$8.756398 \times 10^{-6}$</td>
</tr>
<tr>
<td>0.001</td>
<td>Z-type (32)</td>
<td>$4.427403 \times 10^{-3}$</td>
<td>$6.324310 \times 10^{3}$</td>
<td>$6.324599 \times 10^{2}$</td>
</tr>
<tr>
<td></td>
<td>NRI (31)</td>
<td>$2.627958 \times 10^{-3}$</td>
<td>$3.163786 \times 10^{3}$</td>
<td>$3.163428 \times 10^{2}$</td>
</tr>
<tr>
<td></td>
<td>DTNSND (11)</td>
<td>$8.762132 \times 10^{-9}$</td>
<td>$9.666709 \times 10^{-1}$</td>
<td>$8.762145 \times 10^{-9}$</td>
</tr>
</tbody>
</table>
DTNSND model (11) possesses the smallest steady-state error, which is much smaller than those of the other two comparative models. Moreover, as shown in Table 1, when the sampling period \( \iota \) decreases from 0.1 s to 0.01 s and then to 0.001 s, the maximal steady-state residual errors (MSSREs) generated by DTNSND model (11) exhibit a variation proportional to \( O(\iota^2) \), decreasing from \( 8.988422 \times 10^{-3} \) to \( 8.756398 \times 10^{-6} \) and then to \( 8.762132 \times 10^{-9} \), which are smaller than those of comparative models, validating the correctness of Theorem 2. It is worth noting that truncation and rounding errors can introduce calculation noise even in the absence of external disturbances. In this regard, the proposed DTNSND model (11) demonstrates the superiority due to its noise suppression characteristics, and thus it outperforms the comparative models.

### 6.2 Random noise condition

In practical applications, random noise is typically caused by quantization error, measurement error, transmission error, and other factors. Although these errors can be minimized in the design and implementation of algorithms, they can still affect the performance of algorithms to some extent. In this subsection, a random noise \( \zeta(t) = 5 \times [-1, 1] \) is introduced for the comparison between three different algorithms. Compared with the other two models, as shown in Fig. 2 and Table 1, DTNSND model (11) still exhibits excellent performance in accuracy. In contrast, both Z-type model (32) and NRI algorithm (31) display
large steady-state errors at the order of $10^3$, and thus they are less favorable than DTNSND model (11).

### 6.3 Constant noise condition

As illustrated in Fig. 3 and Table 1, even under the constant noise condition, the proposed DTNSND model (11) still exhibits with small steady-state error, almost the same as that under the noise-free condition. Besides, DTNSND model (11) has the MSSREs at the orders of $10^{-3}, 10^{-6},$ and $10^{-9}$ for sampling periods of $\tau = 0.1$ s, $\tau = 0.01$ s, and $\tau = 0.001$ s, respectively. This finding is consistent with the analytical results of Theorem 3. By contrast, the comparative models have significant steady-state errors due to the effect of constant noise, which means that both models have low accuracy and poor robustness under the constant noise condition. Overall, the proposed DTNSND model (11) outperforms the other two models in terms of solution accuracy as well as robustness under the constant noise condition.

![Figure 3](image)

**Figure 3** Steady-state error $\|e_k\|_2$ of Z-type model (32), NRI algorithm (31), and DTNSND model (11) under the constant noise $\zeta(t) = 100$ condition. (a) $\tau = 0.1$ s. (b) $\tau = 0.01$ s. (c) $\tau = 0.001$ s.

### 7 Extraction experiments

To further verify the noise suppression of DTNSND model (11), we acquire a remote sensing image of the Arctic sea ice for the following extraction experiments under noise-free and random noise $\zeta(t) \in 5 \times [-1, 1]$ conditions. The remote sensing image is from HY-1C satellite, which is equipped with the Chinese ocean colour and temperature scanner. Extraction results are shown in Fig. 4 and Table 2.

### 7.1 Dataset and experimental parameters

Applying a preconditioned original remote sensing image of the Arctic sea ice as an experimental material, experiments are performed using MATLAB 2018b on a computer with Windows 11, AMD Ryzen 5 4600G, Radeon Graphics 3.70 GHz, and 16 GB RAM. In addition, to illustrate the excellent performance of DTNSND model (11) in detail, some of the indicators are discussed such
as overall classification accuracy (OA), average classification accuracy (AA), product precision (PP), MSSRE, and Kappa coefficient. Then, their calculation formulas are listed below.

\[
\begin{aligned}
    OA &= \frac{TP+TN}{TP+FP+TN+FN}, \\
    AA &= \frac{1}{2} \left( \frac{TP}{TP+FN} + \frac{TN}{TN+FP} \right), \\
    PP &= \frac{TP}{TP+FP}, \\
    \text{Kappa} &= \frac{p_0 - p_e}{1 - p_e}, \\
    p_0 &= \frac{TP+TN}{TP+TN+FP+FN}, \\
    p_e &= \frac{(TP+FN)(TP+FP)+(FP+TN)(FN+TN)}{(TP+TN+FP+FN)^2}.
\end{aligned}
\]

According to the definition of the confusion matrix, TP, FP, TN, and FN mean true positive, false positive, true negative, and false negative, respectively.

![Figure 4](image)

**Figure 4** Results of CEM scheme (1) for the Arctic sea ice extraction with three algorithms. (a) Original image. (b)-(d) Extraction results with DTNSND model (11), NRI algorithm (31), and Z-type model (32) under the noise-free $\zeta(t) = 0$ condition. (e) Ground-truth image. (f)-(h) Extraction results with DTNSND model (11), NRI algorithm (31), and Z-type model (32) under the random noise $\zeta(t) \in 5 \times [-1, 1]$ condition.
Table 2  Classification accuracy indicators and results of three algorithm-assisted CEM scheme (1) for the Arctic sea ice: MSSRE, OA, AA, PP, and Kappa

<table>
<thead>
<tr>
<th>Accuracy Indicator</th>
<th>Noise-free $\zeta(t) = 0$</th>
<th>Random noise $\zeta(t) \in 5 \times [-1, 1]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DTNSND (11)</td>
<td>NRI (31)</td>
</tr>
<tr>
<td>MSSRE</td>
<td>$8.788956 \times 10^{-14}$</td>
<td>$2.929643 \times 10^{-14}$</td>
</tr>
<tr>
<td>OA</td>
<td>0.970293</td>
<td>0.970293</td>
</tr>
<tr>
<td>AA</td>
<td>0.980305</td>
<td>0.980305</td>
</tr>
<tr>
<td>PP</td>
<td>0.892185</td>
<td>0.892185</td>
</tr>
<tr>
<td>Kappa</td>
<td>0.923019</td>
<td>0.923019</td>
</tr>
</tbody>
</table>
7.2 Comparison of extraction results

Figure 4(a) shows the original remote sensing image of the Arctic sea ice with a size of 174 × 255 pixels and a resolution of 50 m × 50 m. To visually compare the extraction results of three algorithms, Fig. 4(a) is first processed into a ground-truth image shown in Fig. 4(e). It can be observed from Fig. 4(b)-(d) that three algorithms to aid CEM scheme (1) show good performance in extraction experiments under the noise-free condition, and the Arctic sea ice can be accurately identified. Moreover, from Table 2, classification accuracy indicators for three algorithms are OA=0.970293, AA=0.980305, PP=0.892185, and Kappa=0.923019, identically. Besides, the MSSREs are of order 10^{−14}, 10^{−14}, and 10^{−13}, which shows that three algorithms have a high and similar convergence accuracy. In contrast, Fig. 4(f)-(h) illustrate the effects of three algorithms to extract the Arctic sea ice under the random noise ζ(t) ∈ 5 × [−1, 1] condition. Distinctly, DTNSND model (11) still extracts the Arctic sea ice with essentially the same quality as the one under the noise-free condition, and the metrics in Table 2 can further support this result. On the contrary, there exist misidentifications on the left side in Fig. 4(g), while Fig. 4(h) has a large missing area. It means that NRI algorithm (31) and Z-type model (32) can not successfully overcome the perturbation of the noise in the solving process.

8 Conclusion

In this paper, we have constructed a DTNSND model for solving the QP problem converted from the CEM scheme, so as to extract the target from the optical remote sensing image. The proposed DTNSND model with an integration term suppresses the noise and finds optimal filter coefficients for the target extraction efficiently and stably. After conducting theoretical analysis and a series of numerical simulations, the noise suppression capabilities of the constructed model is confirmed, as well as its exceptional computational performance. Additionally, an experimental comparison of optical remote sensing image target extraction under different noise conditions was performed using an optical remote sensing image of the Arctic sea ice. The results demonstrated the high accuracy and strong stability of the DTNSND model.

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