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The Paradox of Talent: how Chance affects Success in Tennis Tournaments

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ABSTRACT

Individual sports competitions provide a natural setting for examining the relative importance of talent and luck in achieving success. The belief that success is primarily due to individual abilities and hard work rather than external factors is especially strong in this context. Thus, individual talent is regarded as the most important – if not the only – component in ensuring a successful career for players. In this study, we test this belief through detailed data analysis and comparison with an agent-based model, using tennis as a case study due to its popularity and competition structure in direct-elimination tournaments. Our dataset covers the last decade (2010-2019) of main international events in the ATP circuit and consists of tourney results and annual rankings for professional male players. We show that our agent-based model, calibrated on the dataset, can reproduce the main stylized facts observed in real data, including the results of single tournaments and the development of players’ careers in the ATP community. The strength of our approach lies in its simplicity: it requires only one free parameter $a$ to determine the importance of talent in scoring every single point: $a = 1$ indicates the ideal scenario in which only talent matters, whereas $a = 0$ represents the opposite limit case, in which the outcome of each point is entirely due to chance. We find a good agreement with data when talent weights substantially less than luck, i.e. when $a$ is around 0.20. A posteriori, we notice that this surprisingly important role of chance in tennis tournaments is not an exception. On the contrary, it can be explained by a more general paradoxical effect that characterizes highly competitive environments, particularly in individual sports. In other words, when the difference in talent between top players is minimal, chance becomes determinant. Our findings highlight the impact of the – too often underestimated – external factors on athletes' performance in individual tournaments and their careers. Our work points out the unfairness of the “winner-takes-all” reward system that enhances the disparities between the first classified players and the others in major competitions.

Introduction

Understanding the origin of success is a challenging task for researchers in complex systems. Science of Success has recently received a lot of attention because it concerns a huge variety of systems: e.g., paper citations in science1, arts2 and creative careers in general, show business3, and so on.

Investigating what leads to success is inevitably correlated to the role that chance might play in achieving it. We are reluctant to acknowledge the presence of luck in our achievements, as if it could reduce our legitimacy to them. Instead, we adopt two tendencies to neglect the influence of random events: the hindsight bias and the post hoc fallacy. The former describes the human predisposition to think that events are more predictable than they are4. The latter comes from the Latin “post hoc, ergo propter hoc”, which means “after this, therefore because of this” and expresses the propensity to attribute causality to two events just because one happened after the other5. Those two ways of thinking explain why we systematically ignore or underestimate the contribution of chance.

Nevertheless, the influence of unpredictable factors does not necessarily reduce the importance of individual skills. However, the evidence suggests that a combination of talent and effort does not guarantee success4,6,7. Successful people must also be very lucky, other than talented and hard-working4.

Despite the increasing amount of literature on this subject, there has been little attention on analysing success in sports. In principle, sports should be the ideal systems to be examined, given their clear and objective measures of performance and well-defined rules in very controlled environments, other than their ability to attract widespread interest. Yet, only a handful of sports, such as soccer8, baseball9, and tennis10, have been studied, and chance is seldom considered11. Even with our constant exposure to success stories about famous athletes and our intuitive sense that luck plays a role in becoming a top player, we still
assume sports to be fundamentally meritocratic.

Those arguments motivated us to investigate the role of chance in sports, and in particular in individual sports, quantitatively. Specifically, we are interested in individual sports where athletes face an opponent and their competitions are structured as knock-out tournaments.

In this study, we analyse tennis, which has a broad audience and provides easy access to large and detailed datasets about the main international tournaments, inspired by previous work on fencing\cite{12}. In tennis, as in fencing, one of the main goals for a player is to rise in the official ranking. In an ideal talent-oriented view, one could expect a perfect correspondence between talent and ranking order. Yet, athletes very close in talent might end up with totally different rewards for similar performances, particularly when their talent is quite high. The selective mechanisms behind direct-elimination tournaments can systematically amplify little differences between players\cite{13} since those kinds of competitions allow athletes to overcome many opponents at the same time, simply by defeating one of them in a given round.

In individual sports based on knock-out tournaments, small random fluctuations in performances might trigger a rich-get-richer phenomenon, generating a consistent gap in the ranking between individuals with comparable skills. This process is accelerated by the specific scaling of ranking points, gained by players after a competition. Those scales, usually chosen ad hoc, often follow a nonlinear trend which is hard to justify, given that many features included in the word “talent” (intelligence, physical characteristics, skills, and abilities in general) are typically normally distributed in the population\cite{6,14}. This promotes an unfair “winner-takes-all” rationale, thus emphasizing disparities among the athletes and increasing the influence of random events.

In this work, we try to untangle the relative effects of random events and individual abilities by analysing the dynamics of a community of athletes in the ATP circuit. We use an agent-based approach to assess the impact of randomness in tennis tournaments by comparing the results with real data.

Given that «complexity can grow out of simplicity»\cite{15}, we rely on a simple linear model, already adopted in previous works\cite{6,7,11,12}, to simulate a single point in a match. More in detail, we assume a certain weight, expressed by a unique constant parameter $a$, to quantify the relative role of talent and chance for each point in the match.

We show how well our model can reproduce not only athletes’ performance (in terms of match victories/defeats) but also the overall dynamics of their careers over several years. Moreover, within a small range of values for the parameter $a$, our model can provide a strongly nonlinear ranking whose statistical features are in good agreement with those of the real one.

Additionally, based on the real and the simulated players’ ranking, we build and compare the directed looser-to-winner networks of the whole sequence of considered matches. Through those networks, we highlight the existence of a paradoxical mechanism in highly competitive environments: as skill improves, performance becomes more consistent, and therefore luck becomes more relevant\cite{5}.

Our results advocate a rethinking of the meritocratic paradigm to align with recent scientific discoveries about the role of chance in individual sports.

**Results**

Some sports, among which tennis, allow players to win a match even after winning less points than their opponent. Such evidences suggest that sport rules might create an inherent advantage during the game, leading to an unbalanced and unfair result (see Material and Methods for more details about tennis rules). This is true especially in sports based on tournaments, where the winner overcomes not only his/her direct opponent, but also all the other losers of that match round. That advantage is reflected on the amount of points gained after a competition, which scales non-linearly as a function of the final ranking.

In order to investigate the aspects that create disparities in ATP (Association of Tennis Professionals) ranking and to estimate the weight of individual abilities on these disparities, as opposed to random external influences, we built and agent-based model able to reproduce the dynamics of several competitive seasons in professional male tennis, starting from a few assumptions at the level of single points.

The calibration and testing of the model is based on data collected from several sources\cite{16–18}; we considered all available matches from 2010 to 2019 of the first thousand players in ATP ranking during the corresponding seasons, related only to main tournaments of the ATP Tour: Grand Slams, Masters 1000, ATP 500 and ATP 250.

For each year, we selected the top 1000 players according to the last update of their ranking. Correspondingly, in our model we simulated a community of 1000 agents, assuming that they do not change over the years (this is a first approximation with respect to the real case). We also fixed the number of seasons $N_S = 15$ for each simulation run, discarding the first five years to ensure ranking reliability. Any agent has a probability of attending a certain number of tournaments ($\leq N_T$) in a given season, conditional to the ranking order of that year. We extracted those probabilities from our dataset and we calibrated the model subsequently (see Material and Methods for more details).
Modelling serve and return points

During a single match of a tournament, organized in sets and games, random external factors act at the local level of single points won or lost, but they can have different impact conditional to the rules of tennis. Moreover, the natural asymmetry of this sport, due to the fact that players always switch between service and return, adds some noise to matches, affecting macroscopically both the outcome of tournaments and the development of players’ career.

In order to take into account these peculiar features at the level of each single point, in our model we considered two distinct performances, $P^S$ and $P^R$ (with $P^S, P^R \in \mathbb{R}$), for players scoring at service and at return, respectively. These performances emerge from a combination of both players’ talent and chance, weighted by a control parameter $a \in [0, 1]$, and are defined as follows:

$$P^S_i = aT_i + (1 - a)L^S_i \quad \quad P^R_j = aT_j + (1 - a)L^R_j$$

(1)

In these equations $i = 1, 2$ and $j = 2, 1$ are the players’ indexes, fixed for a given match; $T_k (k = i, j)$ is the talent of the player, which captures all the individual skills of athletes (height, intelligence, ability, training, etc.); $L^S_i$ and $L^R_j$ ($k = i, j$) are the values of the chance of the agents, which randomly affect the possibility of winning a point for the same athletes when they are at service or at return, respectively.

Imagine that player 1 is at service and, consequently, player 2 is at return: if $P^S_1 > P^R_2$, player 1 gets the point; vice-versa, if $P^S_2 > P^R_1$, the point is assigned to player 2. Once they move to the next game of the set, players 1 and 2 reverse their previous role, so that analogous conditions apply to get a point: if $P^S_2 > P^R_1$, player 2 scores; conversely, if $P^R_2 > P^S_1$, player 1 scores.

Inspired by previous studies, $T \in (0, 1)$ is extracted from a Gaussian distribution with $\mu = 0.6$ and $\sigma = 0.1$ at the beginning of every simulation run, thus it is fixed for every player. $L^S_i \in (0, 1)$ and $L^R_j \in (0, 1)$ are also extracted from Gaussian distributions, with different means, $\mu^S = 0.6$ and $\mu^R = 0.4$, and the same $\sigma^S, R = 0.25$; but, unlike $T$, they change for every point. We put a constraint on $L^S_i$ and $L^R_j$, to consider the possibility of aces or return aces. In particular, if the $i$-th extracted value $L^S_i$ is greater than one, the point is assigned to the server $i$, simulating a winning shot at serve (ace); on the other hand, if the $j$-th extracted value $L^R_j$ is greater than one, the point goes automatically to the receiver $j$, as in the case of a return ace, e.g. a winning shot at the receiver returning a serve.

Values of $\mu^R, \mu^S$ and $\sigma^S, R$ have been chosen after an accurate calibration in order to model the actual advantage of the serving player. In detail, we simulated 60000 matches for 1000 players, at best of 3 sets, with each point scored according to Equation (1), testing several values for $\mu^R, \mu^S$ and $\sigma^S, R$ in order to reproduce the real probability distributions of points won as servers or receivers obtained from our dataset (see Figure 1), deriving from 57877 matches of 2723 players. As expected, these distributions are not centered around 0.5, showing the natural asymmetry –we could say, intrinsic– in the tennis rules, where chances of winning a point at service are much higher than those at receiving.

Finally, notice also that the parameter $a$, weighting the relative contribution of talent as opposed to chance in Equation (1), remains the only free global parameter of the model. Of course, $a = 1$ would indicate the ideal scenario where only talent matters while $a = 0$ the opposite, limit case, in which the outcome of each point is completely due to external random forces.

**Figure 1.** Gaussian distributions of points won at service (left) and return (right) over the total extracted from the ATP dataset, see text.
Finding the optimal range of values for the talent strength parameter \( a \)

In what follows, we describe how we tuned the talent strength \( a \) in order to find, through an extended simulations campaign, the optimal range of values making our model able to fit real data over 10 seasons from 2010 to 2019. This allowed us, on one hand, to quantify the role of chance in determining the success of tennis players and, on the other hand, to find the relationship between ranking and talent – hidden in the real world by the unavoidable contribution of random external factors in the athletes’ performance.

At first, we considered the total points cumulated in average during each of the 10 seasons by the players in the ATP circuit as a function of the ranking placement. In the first three panels of Figure 2 we plot the simulated behavior of total points vs ranking placement for several values of the talent weight \( a \), namely \( a = 0.1 \) (a), \( a = 0.2 \) (b) and \( a = 0.8 \) (c), all compared with the real one. At a first sight, the agreement seems quite good only when we consider the top 50 players. This is due to the fact that real ranking positions are also shaped by a lot of minor tournaments not present in our dataset, as better explained in the Material and Methods section. In particular, focusing on the first top 50 players, the best agreement between data and simulations is obtained for \( a = 0.2 \), i.e. when talent matters just at 20% with respect to chance. This value is quite low and, if confirmed, would mean that the impact of random fluctuations on each single point cannot be absolutely neglected.

Figure 2. Trend of the average total points as a function of ranking placements, in both data and simulation, for several values of \( a \): \( a = 0.1 \) (a), \( a = 0.2 \) (b) and \( a = 0.8 \) (c). In panel (d), a zoom of the first part of both the curves shown in panel (b) is reported, together with the correspondent power-law fits.

A further analysis of Figure 2 allows to draw some additional conclusions: first, a non linear scale of ranking points is evident for both data and simulations and, as shown in panel (d) for \( a = 0.2 \), it could be well fitted by power-law curves; second, the slope of the simulation curve for \( a = 0.2 \) slightly underestimates the total amount of points of the first five or eight players in the data. This last effect may be due to the absence of the Nitto ATP finals in our dataset, which involve eight players that are usually placed in the top positions of the ranking. We did not consider those events because they concern very few athletes and their structure differs from all the other tournaments. On the other hand, the simulation curve for \( a = 0.8 \), which seems to
well capture the points of the first ten players in the ranking, largely fails when the ranking decreases.

In order to strengthen the ability of our model to reproduce the ATP tour dynamics, we look at the difference between ranking positions of winners and losers, \( r_{\text{winner}} \) and \( r_{\text{loser}} \) respectively, in the various rounds of the main draw of generic tournament. In detail, we define the difference \( \Delta r \) as follows:

\[
\Delta r = r_{\text{loser}} - r_{\text{winner}}
\]

and remark that higher values of \( r \) mean worse placements in ranking. In that sense, \( \Delta r > 0 \) implies that players with a better position in ranking win; on the other hand, \( \Delta r < 0 \) indicates that players in lower placements win. An increase in \( |\Delta r| \) indicate players become more distant in the ATP ranking. Notice that \( r \) identifies the position in ranking at the time when the match between the two opponents occurred. In Figure 3 we report the distributions of \( \Delta r \), calculated round by round for both ATP data (panel a) and simulation runs in correspondence of three increasing values of \( a \) (panels b, c and d). To enhance visualization, we neglect values of \( |\Delta r| > 200 \).

Figure 3. Distributions of ranking differences in tennis matches, organized (from top to bottom) by rounds of the main draw. Panel (a) shows ATP data results, while the other panels reports simulated results obtained for \( a = 0.1 \) (b), \( a = 0.2 \) (c) and \( a = 0.8 \) (d).

In panel (a) of Figure 3 we observe a general tendency of the peaks of the distributions to shift towards 0 when one goes from the round of 128 to the final \( F \). This trend is likely due to a reduction in the ranking difference between the players as they reach the last rounds of the tournaments and is quite well captured by our model, in particular when \( a = 0.2 \). However, we see a few discrepancies between data and simulations, since in the latter bi-modal distributions emerge for some rounds, while they are less evident in data curves. Those differences might derive from the approximation we made in the number of ATP 250 tournaments per year and, more generally, in the maximum number of participants per tourney type. In any case, simulation outputs correctly reproduce the decrease of the variance going from the first rounds (round of 128, 64, etc...) to the last ones (quarter-finals, semi-finals, finals). The model just fails in replicating the distribution of \( \Delta r \) at round of 128, underlining again that it better captures the evolution of the top 50 players (who are, typically, more likely to reach the final rounds in main tournaments).
In order to explore the range of values of talent strength ensuring the best fit of data for both the total points vs rank and the ranking differences distributions, we can estimate the following root mean squared relative error (RMSRE)\(^2\):

\[
RMSRE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{y_i - \hat{y}_i}{\hat{y}_i} \right)^2}
\]

where \(y_i\) result from simulations, while \(\hat{y}_i\) are real data. In Figure 4 we show the behavior of the RMSRE as function of increasing values of \(a\) for the following quantities: the mean of the total points as a function of the ranking, restricted to the first 50 placements (a); the mean (b), the median (c) and the standard deviation (d) of the distribution of the ranking differences \(\Delta r\), grouped by rounds. One can easily see that, actually, in all the panels the minimum error point falls very close to, and in some cases coincides with, the value \(a = 0.20\) of the talent strength. This is also corroborated by the comparison with the line interpolating all the points, represented in each panel as a green solid line surrounded by its confidence interval, reported as a gray shadow: it clearly appears that the point at \(a = 0.20\) always falls outside, or just at the border, of the confidence interval.

Thus, we can say that the optimal value of the talent strength falls in a small range \(a \in (0.15, 0.25)\) and that we can confidently choose \(a = 0.2\) as reference value for this range. This result, which would confirm the fundamental role of chance in determining the outcomes of tennis competitions (at least for male players), is surely quite unexpected for those considering the success an expression of the individual talent of tennis players. However, a-posteriori it can be easily explained by the characteristics of the data we analysed and we based our model on, since they capture only the behaviour of players at the top positions in ATP circuit. As a matter of fact, in this case a paradoxical mechanism (also conjectured by Mauboussin\(^3\)) clearly takes place: the higher the level of competitiveness, the more similar the individual abilities of players, the less determinant talent becomes to the match outcome. Matches between players in the first 50 placements are often influenced by few, significant
turning points. Our model strongly support such conclusion, thus suggesting its validity for any other individual sport where tournaments are organized in successive rounds, to be validated in future researches.

Finally, we checked that, once calibrated with $a = 0.2$, our model is able to correctly replicate the real average number of tournaments attended in a season, as a function of players’ ranking (see Figure 14 in Material and Methods). Results are shown in Figure 5, where we see a good agreement between data and simulations, within the standard deviations (shaded areas in the figure) of the average number of tournaments per ranking placement in a season.

![Participation as a function of ranking placements](image)

**Figure 5.** Tourney attendance as a function of ATP ranking, comparison between data (blue dots) and simulations (red squares) with talent strength $a = 0.20$.

**Inspecting the features of the agents**

In our model, we assume agents’ talent do not vary during a simulation run. This is a simplification, and one can argue it neglects the role of training, learning and physical changes in the evolution of a professional career in sports\(^\text{11}\). However, we maintain that a dynamic definition of talent might introduce biases, thus reducing the interpretation of model results. Talent should be intended as a combination of multiple intrinsic characteristics of a person, relevant to achieve results: without loss of generality we assume, quite naturally, that the impact of talent variations is negligible in professional performers at the highest level of practice\(^\text{11}\). We interpret athletes’ ability to perform to their highest potential during competitions as talent\(^\text{12}\).

In Figure 6 we observe the average trend of talent as a function of ranking positions, for three distinct scenarios expressed by the talent strength $a$: the reference value $a = 0.20$ (a), the ideal case $a = 1.00$ (b) and the completely random one, $a = 0.00$ (c). Interestingly, panel (a) and (b) are very similar, apart from the smaller standard deviations of the top fifty players when $a = 1.00$. This result, on one hand, confirms that our model can only appreciate talent variations of the first 50 positions in ranking, whereas it shows no changes for the other placements. On the other hand, it reinforces the paradoxical mechanism of the influence of talent, which is distributed almost in the same way, either when it weights less than chance ($a = 0.20$) or when it is the main contribution, thus revealing that other factors affect the dynamics of ATP ranking. Panel (c), finally, displays the expected random distribution of talent, given that it does not contribute at all ($a = 0.00$).

It is worth noting that Figure 6 have no equivalent in data, being talent an hidden variable of players. Moreover, the set of agents is constant during a given simulation run, as opposite to the community of players in our dataset. Perhaps, that aspect might favour the correlation between the first ranking positions and athletes’ talent. However, such a correlation overall is not as strong as one could expect after the selective processes of tournaments. This suggests that talent may be neither well preserved nor fairly rewarded in sports like tennis, and assumptions of this kind should be avoided in general.

**Focusing on ranking interactions**

In individual sports based on direct elimination tournaments, pairwise interactions among the players are crucial, since winning a competition depends on the outcome of single matches, from the point of view of the individuals. For this reason, we study the network of matches from 2010 to 2019 of male tennis players in the ATP tour and compare it to the network built by simulated matches of our agent-based model.

In the previous sections we investigated the dynamics of ranking, considering how tournaments affect it but neglecting the details of the head-to-head results, as a function of ranking placements. Rather, we focused on the influence of tournament
Figure 6. Talent as a function of ranking for different values of $a$. Black dots mark the mean values, while the blue bars represent their standard deviations.

rounds, precisely the main draw ones. We consider a weighted directed network where nodes are positions in ATP ranking, and a link $(a, b)$ is created if $a$ loses against $b$ in any round of the main draw of the top-tier ATP tour without distinction, thus disregarding qualification stages. This choice for the structure of the network follows the rationale of\(^{22}\); it sounds reasonable since it highlights the importance of nodes in terms of their ingoing connections, which is common practice in network science literature.

We remind that the adjacency matrix for a directed network has elements\(^{23}\):

$$A_{ij} = \begin{cases} 1 & \text{if there is an edge from } j \text{ to } i \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (4)

When we consider the weights of the links, $A_{ij} = 1$ can be replaced by $A_{ij} = w_{ij}$, where $w_{ij}$ is the number of times $i$ defeats $j$. Specifically, we normalize $w_{ij}$ taking into account the total number of times $i$ faces $j$:

$$\tilde{w}_{ij} = \frac{w_{ij}}{w_{ij} + w_{ji}}$$  \hspace{1cm} (5)

We study two networks: one built from our dataset, with $N = 667$ nodes and $L = 25413$ edges, with the condition that the ATP ranking of nodes must be less than or equal to 1000; the other graph results from simulation runs with the reference value of the talent strength $a = 0.20$, sampling links until we obtain the same value of $L$, resulting in a network with $N_{\text{sim}} = 986$.

In Figure 7 we first visualize\(^ {24}\) the two networks for a restricted set of nodes, the first 200 in ranking, for both real data (a) and simulated ones with $a = 0.20$ (b). By indicating the nodes’ rank with a colour scale, in both cases one can spot the
Figure 7. Network visualization of the top 200 ranked nodes. Panel (a) shows the data layout, while panel (b) refers to simulations with $a = 0.20$. In both cases a core-periphery structure can be easily identified.
presence of a denser core of top-ranked nodes surrounded by a periphery of less connected nodes indicating lower positions in
the ranking. This common feature is not surprising, since top-ranked players more likely end to play one against each other in
the final rounds of the main competitions. However, it confirms the ability of our model to reproduce, also from a topological
perspective, the behaviour of the ATP dataset when a talent strength \( a = 0.20 \) is assumed.

To further reinforce this last statement, we compare the in-degree and out-degree distributions of both the networks. The
in-degree is the number of ingoing links connected to a node and the out-degree is the number of outgoing links. Bearing in
mind the adjacency matrix Equation (4), we can write:

\[
\begin{align*}
    k_{in}^i &= \sum_{j=1}^{N} A_{ij}, \\
    k_{out}^j &= \sum_{i=1}^{N} A_{ij}
\end{align*}
\]

(6)

In Figure 8 we observe a very good agreement between data and simulation for the in-degree distribution (a), which scales
as a power law if we exclude the tail (see Figure 9); instead, for the out-degree distributions (b), the two trends differ for low
values of \( k \).

\[\text{Figure 8. In- and out-degree distributions for the networks of both real (a) and simulated (b) matches.}\]

\[\text{Figure 9. Power law fit of the in-degree distribution for both real ATP data (a) and simulations (b).}\]

To show the relationship between the importance of nodes in the network and their ranking positions in the ATP, we test
two centrality measures. The former is the in-degree centrality:

\[
c_{in}^i = \frac{1}{N-1} \sum_{j=1}^{N} A_{ij}
\]

(7)

which in this case captures how central certain nodes, representing the ranking placements of players with many wins, become
in the topology of the network.
Panel (a) of Figure 10 shows the in-degree centrality as a function of players’ ranking placements in both data and simulations. We observe an overall concordance in trend, although simulation curve slightly underestimates in-degree centrality values. In more detail, the top fifty positions have approximately the same centrality, meaning that they have a comparable number of wins, on average.

A natural extension of the in-degree centrality is the eigenvector centrality, which awards nodes not only because they have many connections, but also because they are connected to other nodes which are themselves important. In the network of tennis matches, it underlines the fact that winning many times can be relevant but not sufficient: a victory turns out to be crucial also when the winner player defeats an opponent who has already a lots of wins at his credit.

The eigenvector centrality for a node in a directed network is proportional to the centralities of the nodes that point to :

\[ x_i = \kappa_1^{-1} \sum_j A_{ij} x_j \]  

(8)

where \( \kappa_1 \) is the largest eigenvalue of the adjacency matrix. We specify that Equation (8) holds for weighted networks. Using such a definition, given that we consider ranking placements as nodes, we expect a monotonic decreasing relationship between ranking and eigenvector centrality. That is exactly what we observe in panel (b) of Figure 10; also, eigenvector centrality curve is correctly predicted by the model simulations. It is worth noting that the decreasing trend of eigenvector centrality is evident only for the top 100 players, whereas it is less significant for the other placements.

Interestingly, the comparison between the two analyzed centrality measures captures some features of our specific context, which is men’s professional tennis. On one hand, in-degree centrality, which can be interpreted as the importance of how much players win, put all the top ranked positions on the same level; in other words, one could not distinguish between them, based on in-degree only. On the other hand, eigenvector centrality adds the not negligible weight of defeating high centrality players: by taking this into account, a scale emerges, thus making the top placements distinguishable from each other. Interestingly, both those behaviors are correctly reproduced by our model with \( a = 0.20 \): despite the limited weight of talent, a selective mechanism still arises.

To better understand the interplay between the ranking and the structure of the network of matches, we inspect their adjacency matrices representing the weight of each element with a color scale. Figure 11 shows the two weighted matrices for real data (a) and simulations with \( a = 0.20 \) (b): in line with the visualization of Figure 7, they appear both sparse and asymmetrical.

To further inspect the two matrices, let us consider a triangular matrix whose elements are \( \hat{A}_{ij} = A_{ij} - A_{ji} \). In this way, we evaluate whether players in ranking positions or won more times when they faced each other, since the elements are the fraction of wins of the node and, vice-versa, is the ratio of victories of the node (see Equation (5)). Results are shown in Figure 12. Provided that \( \hat{A}_{ij} = 0 \ \forall i < j \), this representation gives us insights about the ranking difference between and , as underlined in the color-bars of the heat-maps: following the definition given in Equation (2), red elements of the matrix indicate better ranked players win more, while values in blue mean worse ranked players win more. The case of , which would represent a balanced number of victories between and , cannot be distinguished from the absence of matches with this formalism; however, this occurs only in 0.5% of the dataset.
Figure 11. Adjacency matrix of 500 nodes for real ATP data (a) and simulations for $a = 0.2$ (b). Colorbars highlight the different weights of the elements.

Comparing panels (a) and (c) in Figure 12, one can immediately see that both data and simulations show a higher density for the first 100 nodes. However, while for the real matches top players have a low probability to face athletes beyond the 200th placement, for the simulated matches top agents seem more likely to face opponents in the bottom part of the ranking. Focusing on panels (b) and (d), we see that our model correctly captures the unpredictability of a match outcome as we approach the diagonal of the matrix, i.e. when ranking positions are closer. In fact, in addition to the general color mixing with no patterns characterizing both the networks, the common presence of many blue areas indicates higher possibilities for better ranked players of being defeated. On the other hand, the simulation network does not exactly reproduce the pattern off-diagonal: the dominance of the top 10 placements is evident in data (dark red area in panel (b)), whilst it is less pronounced in panel (d).

Discussion and conclusions

In this work we studied the impact of luck in individual sports. Specifically, we investigated the role of talent and chance in tennis, by focusing on the male ATP circuit and quantifying the influence of unpredictable factors, from the level of single points in a match to the macroscopic evolution of the whole ATP ranking.

We built an agent based model, calibrated on data from international rankings and tournaments, which allowed us to estimate the relative weight of chance with respect to talent in ATP tournaments. The starting hypothesis was that small random fluctuations at the level of each single point, along with the structural “bias” characterising tennis and resulting in an asymmetrical probability of winning points when at service or receiving, could become relevant especially when the level of competitiveness is very high and the selective mechanism of tournaments brings athletes with comparable, very high, talent to confront each other.

Due to the availability, the amount and the organization of data, we limited our consideration to the main tourneys of the ATP tour (Grand Slam, Masters 1000, ATP 500 and 250) only. We collected matches and rankings of the first thousand male athletes in ATP circuit from 2010 to 2019 and, through a comparative analysis where the weight $a$ of talent was the only free parameter, we showed that the capability of our model of reproducing several relevant features of that community of athletes – e.g. the distributions of points as function of ranking and the distributions of ranking differences in matches – is maximum around $a = 0.20$. This result would imply that the role of external random events in determining tennis matches is relatively consistent, in average around 80% for men’s professional players.

Under this assumption, we also showed that our agent based model captures the main elements of the directed network built from the match results between ranking positions. In particular, the network obtained from our numerical simulations shows centrality trends comparable with the network of real data, either for in-degree centrality and the eigenvector centrality. The former reflects the importance of how much players win. The latter underlines the influence of defeating high centrality players. Finally, simulated networks allowed us to explore the dominance of certain ranking positions on the others.

Although our model gives valuable insights about the evolution of careers in tennis, it has also some evident limitations. Firstly, we simulate a community of 1000 players/agents that not change in time along the 10 considered seasons, while the real community of athletes is of course an open system, where someone continually withdraws and someone else enters the circuit. Furthermore, ATP rankings are not limited to the main tournaments of the ATP tour in reality; for this reason, the model cannot reproduce the overall ranking dynamics. This explains why some results apply to a limited set of players. Secondly, the talent of each simulated athlete is also fixed in time, thus representing the maximum level of ability he can reach along his entire career, while taking into account both individual skills and efforts. Of course, this is a strong approximation, since along...
10 years any player can surely increase the level of his performance with the help of hard-training and cumulated experience. Moreover, in our model the influence of chance is limited to the outcome of each single point and to the asymmetry of service and return, which means that an external source of noise, independent from tennis rules, is absent. Such an external source could be represented, for example, by the random occurrence of injuries or accidents that can condition, interrupt or even defeat the promising career of many brilliant players.

Despite such limitations of the model, we believe that our study could provide several valuable insights. In fact, by fixing the relative weight of talent around 20%, it gives further support to the existence of a paradoxical mechanism (already claimed in\(^3\)): i.e. the higher the level of competitiveness, the closer the talent of athletes involved, the more determinant random fluctuations become in order to win a match and to develop a successful career. Those evidences of a “rich-get-richer” effect should question the current awarding methods, rooted in the dominating meritocratic paradigm, and discourage the application of an unfair “winner-takes-all” logic. In particular, we consider that the configuration of rewards and their distribution is worth a specific investigation. Divergences in monetary rewards – and consequent non-monetary rents – appear to depict a profoundly different picture than the one suggested by the talent comparison. This might have a role in reducing excessively the remuneration of the looser in finals or semifinals rounds, when compared to the difference in talent with the opponent. Consequently, athletes with similar skills end up having totally different outcomes and wealth profiles, simply because of small fluctuations, at the level of single points of a match.

Having observed similar results in other sports based on tournaments\(^1\), the general methods of awarding should be seriously questioned, both in terms of ranking points and prizes, which typically follow an exponentially decreasing trend, from the first classified to the last one. Such a configuration is probably more desirable in order to attract a more sensationalistic environment for each challenging match, but it remains harmful for the sport discipline itself. The assumption of a one-to-one perfect correspondence between talent of athletes and their performance in competitions is unlikely to exist, because randomness can significantly influence any athlete’s performance and the match itself. Our future studies will focus on the need for a revision in the current awarding rule, aimed at enabling a more authentic rewarding of talent\(^2\).
Material and Methods

Tennis is a racket sport that is usually played individually against a single opponent, in a rectangular court with a net across the center. The objective is to hit the ball over the net so that it lands inside the court’s margins and cannot be returned by their opponent. When that happens, players score a point. One need at least four points to win a game and six games to win a set, in both cases with a margin of two in case of equal score. A match is usually played as best of three or five sets (Slam tournament).

If a game score of 6-6 is reached, players must play a tie-break game in order to decide who wins the set. In a tie-break game, a player must reach 7 points with a two point advantage to win.

Since every point weights differently in the economy of a match, and one cannot simply score more points than their opponent to win, the influence of external factors can either be marginal or crucial. In this framework, the interchange between serve and return become relevant and must be taken into account when modelling a single point in tennis.

In this respect, the key idea is that the representation of a single rally (series of hits after service) between two opponents lets the whole dynamics emerge. In other words, a player’s performance is not modelled explicitly, but it is the result of the sequence of scored points.

Calibrating athletes’ participation to ATP tournaments

With our agent-based model we aim to reproduce, within the NetLogo environment, the dynamics of the main tournaments of ATP Tour, which is the most relevant circuit for men’s professional tennis players. In order to do this, we simulate a community of 1000 agents (athletes) playing for a certain number of events in a given season, conditional to his ranking order in that year. As shown in the three panels of Figure 13, we calibrated these probabilities exploiting the information present in our dataset and related to the first 1000 athletes in the ATP ranking who attended the main tournaments in seasons from 2010 to 2019. From data in panels (a) and (b) one may assume that the higher their ranking in a given season, the more events athletes played in that season. In tennis, this is not generically correct, because of the very different weight characterizing distinct types of tournaments. In fact, looking at the average number of the attended competitions, reported in Figure 14 as function of the ranking placement, and comparing this number calculated for all the tournaments (panel d) with the same one disaggregated per each tournament category (panels a, b and c), different patterns emerge: the top 10 ranked players tend to compete less than athletes in slightly lower positions (from 10 to 50), and have a comparable number of tourneys with respect to players in the range [50, 100].

This might be explained by the large amount of ATP 500 and 250 (sometimes we refer to them as “minor” tourneys to be concise) in contrast with Grand Slams and Masters 1000, but also by ATP rules: only the best eighteen results are considered in ATP ranking, hence top 10 players, who have easier access to major events, tend to compete in the minimum amount of tournaments required; on the other hand, athletes reasonably close to the top 10 (up to the 100th position), try to improve their placement by increasing the number of minor tourneys they can attend. External factors could also affect athletes’ participation, like prize money, funding availability, country of origin and of the tournament, injuries, and so on. All those elements result in the non-monotonic trend displayed in Figure 14.

Since we restrict our dataset to the main tournaments of the ATP tour, one might ask whether they are enough to explain the evolution of ATP ranking in our dataset. Figure 15 shows the difference between the total number of tournaments considered in the ranking (blue dots) and the number of ATP tournaments present in our dataset (red triangles), as a function of ranking placements. There is a gap from the 50th position, which widens for lower placements, meaning that ranking is shaped by other tourneys, not present in our dataset. Given that, we expect our model can better capture the trend of the first fifty athletes, in terms of ranking points, while it might highlight other features of the system.

Simulating ATP tournaments

As anticipated, in our model we only consider the most important tourneys in terms of ranking points, prizes and prestige: Grand Slams, Masters 1000, ATP 500 and ATP 250. Tournament draws in these competitions are arranged in rounds that halve players at each step of the tournament. Each competition has two knock-out phases, thus two distinct draws: a qualifying one followed by the main one. The qualifying draw allows those players who cannot directly access the main phase of a certain tournament to qualify for a place in the starting line-up of that tournament, after facing a preliminary set of rounds. Once the qualifiers have been determined, the main draw can start. Both draws are arranged in an analogous way, placing seeded players first, then all the others (see Figure 16 for a visual example).

The number N of participants per tournament is fixed and related to its type. To compete in a given tournament, players are primarily selected according to their ranking placement, but there are also other criteria, which might leave space to randomness. This is the case of wild cards, which are a small number of reserved spots that can be assigned to randomly selected players.
Figure 13. Probabilities of attending different kind of tournaments in our dataset (panels (a), (b), (c) refer to Grand Slams, Masters 1000, ATP 500 and 250 respectively), as a function of ranking order. Ranking positions are grouped in intervals to enhance visualization.
Figure 14. Average number of played tournaments as a function of ranking placements in ATP tour. Panels (a), (b), (c) refer to Grand Slams, Masters 1000, ATP 500 and 250 respectively. Panels (d) refers to all the tournaments.

For example, wild cards may be exploited by local players who do not gain direct acceptance, or by players who are just outside the ranking range required to gain direct acceptance, or even for players whose ranking has dropped due to a long-term injury\textsuperscript{27}.

There are other reserved places for the so-called seeded players we mentioned above. Seeds are players whose position in a tournament is arranged based on their ranking in order to prevent them from encountering other seeds in the early rounds of the competition. For a given tournament, there is a specified number of seeds, depending on the size of the draw. For ATP tournaments, typically one out of four players are seeds. The seeds are usually chosen and ranked by the tournament organizers\textsuperscript{28}.

For the sake of simplicity, we refer to the official ATP rulebook\textsuperscript{16}, but we make some approximations to model the dynamics of several tennis tournaments. In Table 1 we summarise the values we use in our agent-based model to arrange tournament draws, based on ATP rules.

<table>
<thead>
<tr>
<th></th>
<th>Grand Slam</th>
<th>Masters 1000</th>
<th>ATP 500</th>
<th>ATP 250</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-participants</td>
<td>128</td>
<td>128/64</td>
<td>32</td>
<td>32</td>
</tr>
<tr>
<td>N-seeds</td>
<td>32</td>
<td>32/16</td>
<td>8</td>
<td>8</td>
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<tr>
<td>N-qualif-participants</td>
<td>128</td>
<td>64/32</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>N-qualif-seeds</td>
<td>32</td>
<td>16/8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>N-qualifiers</td>
<td>16</td>
<td>16/8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>N-wild-cards</td>
<td>8</td>
<td>5/4</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 1. Summary of the composition of tournaments we used in our model.

In our model we limit the number $N_T$ of competitions during each season to one tournament per week (except for Grand
Slams which can last two weeks), so that there are $N_T = 52$ events in total. In particular, we simulate four Grand Slams, nine Masters 1000, thirteen ATP 500 and twenty-six ATP 250. This choice avoids general tourney overlaps during a simulated year, and is consistent with a real tennis season. The only exception concerns ATP 250 tournaments, which should be around 36. However, such an underestimation of ATP 250 contribution improves model interpretability, without decreasing its predictive power; moreover, it requires less computational resources.

We assume there are no byes in the first rounds, meaning that all the players must face an opponent. Consequently, we increase the number of participants in Masters 1000 to 128 and 64, while they should be 96 and 56, in order to complete the draws. It is worth noting that Masters 1000 are the only tournaments with two alternative lengths of their draws, whereas in every other case we consider only one possible draw size.

In Table 2 we show the specific scales of points for different types of competitions. We can see that they all decrease in a non-linear way. Accumulating points let athletes rise in the official ATP ranking, but they are only valid for one season: any new result cancels out the corresponding one of the previous year, if present.

<table>
<thead>
<tr>
<th></th>
<th>Grand Slam</th>
<th>Masters 1000</th>
<th>ATP 500</th>
<th>ATP 250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winner</td>
<td>2000</td>
<td>1000</td>
<td>500</td>
<td>250</td>
</tr>
<tr>
<td>Final</td>
<td>1200</td>
<td>600</td>
<td>300</td>
<td>150</td>
</tr>
<tr>
<td>Semi-Final</td>
<td>720</td>
<td>360</td>
<td>180</td>
<td>90</td>
</tr>
<tr>
<td>Quarter-Final</td>
<td>360</td>
<td>180</td>
<td>90</td>
<td>45</td>
</tr>
<tr>
<td>Round-of-16</td>
<td>180</td>
<td>90</td>
<td>45</td>
<td>20</td>
</tr>
<tr>
<td>Round-of-32</td>
<td>90</td>
<td>45</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Round-of-64</td>
<td>45</td>
<td>20</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Round-of-128</td>
<td>10</td>
<td>10*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Qualif-1st</td>
<td>25</td>
<td>20</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>Qualif-2nd</td>
<td>16</td>
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<td>5</td>
</tr>
<tr>
<td>Qualif-3rd</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2. Allocation of points per tournament and round.

Every run of our simulations starts with a random allocation of the 1000 players in an initial ranking, which is updated as soon as agents attend events and earn points, following the updating rule we explained above. Once the last tournament of the last season ends, the simulation can stop (see Figure 17). Then, it is possible to extract several outcomes as function of the chosen talent strength $a$, which is the only free control parameter of the model (see Equation (1)).
Figure 16. Example of a main draw (upper part), from Wimbledon 2015 (source: https://www.dotspor.it/). Seeds are highlighted in bold, placed in such a way they do not face each other until the 3rd round. Wild cards and qualifiers are underlined by the symbols (W) and (Q) respectively.
Figure 17. Example of the output in the NetLogo interface at the end of a single tournament simulated with our agent-based model. At the bottom, the agents/athletes who did not pass the qualification phase are visible; at the higher levels the agents who passed the various rounds are also visualized until, at the top level, the winner is highlighted with a greater size. The color intensity of the agents is proportional to their talent, while the numbers in the labels represents the ranking of the agents at the beginning of the tournament. In this case, the first agent in the ranking was also the winner.

References


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**Author contributions statement**

A.P. conceived the model, C.Z. analysed the data and made the numerical simulations. A.P., A.R. and A.E.B. supervised the study. All authors reviewed the manuscript.

**Data availability statement**

Data analysed in this work are available online 16–18.

**Additional information**

**Competing interests** The authors declare no competing interests.