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Research Article

Keywords: Asymmetric seismology, gradient theory, elastic wave equations, numerical modelling

Posted Date: March 6th, 2023

DOI: https://doi.org/10.21203/rs.3.rs-2641701/v1

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Additional Declarations: No competing interests reported.
Numerical Modelling of Elastic Waves based on the Asymmetric Wave Equations

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Abstract--It is a trend in the development of geophysics to seek wave theory closer to the physical reality and derive corresponding wave equations to achieve the high accurate forward modelling, imaging, and inversion of complex structures. The generalized continuum mechanics (GCM) theory enriches the context of the conventional continuum mechanics theory by introducing the additional characteristic length scale parameters to represent the microstructural properties of the medium, and the asymmetric elastic wave equations derived from GCM theory can handle the influence of heterogeneity of medium caused by the microstructure interactions on propagation of seismic waves. Up to the present stage, there are few researches on the numerical and analytical solutions of the elastic wave equations derived from the GCM theory, especially in the frequency band of seismic exploration. In addition, there are few studies in the existing literature that incorporate multiple theories and methods of the GCM theory into an integrated frame. In this paper, we introduce the concept of the multi-scale microstructure interactions and construct quantitative relationship between the characteristic length scale parameter and the characteristic length scale parameter of the micropore reflects the micropore structures, and then integrate the modified couple stress theory and the one-parameter second strain gradient theory into the unified framework for numerical modelling and analysis.

Keywords: Asymmetric seismology, gradient theory, elastic wave equations, numerical modelling.

1. Introduction

The great thing about the establishment of elastic mechanics that is the basis of classical seismology lies in the introduction of the "set system of free particles", nested into the framework of conventional continuum mechanics theory, and then obtains the widely used elastic wave equation without independent free term of rotational motion. Although wave equation derived from conventional continuum mechanics theory has made
great contributions to mankind, anyone engaged in actual work, such as the operators of data processing and
interpreting from petroleum companies and oil service technology companies, know its huge unknow and
unknowable factors. Scholars have been deepening exploration of the field of forward modelling, imaging,
and inversion (Liu, 2019; Liu et al., 2020; Li et al., 2022; Wang et al., 2022), especially the study of wave
theory in rock and soil medium which affects the propagation of seismic waves. It is a trend in the
development of geophysics to seek wave theory closer to the physical reality and derive corresponding wave
equation to achieve the high accurate imaging and inversion of complex structures.

Classical seismology is based on the framework of conventional continuum mechanics theory, which is
considered as a local theory. It means that the stresses in a material point depend only on the strains on the
material point, and the strain energy density function for the medium is assumed to contain only the classical
strain (the first-order derivative of the displacements). It requires an idealized model and assumes that the
material making up the medium are uniformly and continuously distributed, in other words, the material is
represented by a continuous mass rather than as discrete particles (Zhu et al., 2020). In reality, there always
exist complicated internal micro-defects/micro-structures, which is common for both natural rock and soil
medium and man-made materials. In rock and soil medium, micro-defects/micro-structures generally refer
to the structures characterized by the micro-pores, micro-cracks, and micro-voids; while for metal, the micro-
defects/micro-structures refer to the crystal dislocations. It is difficult to describe the complex micro-
structures interactions by the conventional continuum mechanics theory under its continuity assumption, one
feasible solution is to consider every micro-structural separately with conventional continuum theory.
However, it cannot compromise the efficiency and accuracy in such a model. For some complex structures,
il-conditioned problems may occur, especially for the localized deformation, fracture, and the long-range
interactions, the conventional continuum mechanics theory is never capable of adequately handling these
problems (Bonnell and Shao, 2003; Askes and Gutierrez, 2006). It is worth emphasizing that the scale of
micro-defects/micro-structures varies with the observation target, and it is a relative concept. Although the
different scales of observation have straightforward boundary, shorter scales of observation (e.g., microscale,
nanoscale) have an influence on longer scales of observation (e.g., macro-scale) and vice versa, resulting in
heterogeneous responses (Askes and Metrikine, 2005). For rock and soil medium, micro-scale differences in
pore and grains geometry of rocks do bring macro-scale differences of wave responses and characteristics.

The generalized continuum mechanics (GCM) theory enriches the context of the conventional continuum
mechanics theory by introducing the additional higher than the first order displacement gradients and the first
order rotation gradients (de Borst and Muhlhaus, 1992; Chang and Ma, 1992; Peerlings et al., 1996; Chang
et al., 1998; Yang et al., 2002; Lam et al., 2003; Kong et al., 2009; Karpavarfarf et al., 2015; De Domenico
et al., 2019), or increasing the degrees of freedom of the material point (Eringen, 1966, 1967, 1990), or taking
the non-local effects into account (Eringen et al., 1977; Ari and Eringen, 1983; Eringen, 1999, 2002); which
are usually accompanied with additional characteristic length scale parameters (or higher order constants) to
represent the microstructural properties of the medium. And the elastic wave equations derived from GCM
theory can handle the influence of heterogeneity of medium caused by the microstructure interactions on
propagation of seismic waves. The term "generalized" in the GCM theory refers to the further relaxation of
the basic assumptions and principles of the conventional continuum mechanics theory. Therefore, these two
theories are not mutually contradictory, rather complementary. Different from the conventional continuum
mechanics theory, the GCM theory empowers the material point with volume properties, allowing the
material point to be driven by forces and couples (a force drives the material point to translate and a couple
drives the material point to rotate). Due to the interactions between the material points, the body couple and
the surface couple are introduced, which leads to the asymmetry of the stress tensors. If the volume of the
material point vanishes, the GCM theory becomes the conventional continuum mechanics theory.

The history of the GCM theory development can be traced back to the end of the 19th century, the existence of the body couple and the surface couple in medium was originally postulated by Voigt (1887), and since then, lots of theories and approaches have been proposed as extensions of the conventional continuum mechanics theory. Among them, the couple stress theory (Toupin, 1962), the strain gradient theory (Toupin, 1964), the second strain gradient theory (Mindlin, 1965), the micro-polar theory and the microstretch theory (Eringen, 1966, 1990), and the non-local theory (Eringen and Edelen, 1972; Eringen, 1983) are considered to be relatively successful and widely used theories. These theories accurately describe the underlying microstructure via introducing factors of certain characteristic length scale parameters that represent the lower scales, while in the meantime, effectively save computing resources compared with the conventional theory. Even though being short of the universally acknowledged physical interpretation and unitive understanding of the characteristic length scale parameters with different definitions and numbers, it yields some difficulties in large-scale application. In 1964, Mindlin (1964) proposed a more generalized theory containing 903 number of the characteristic length scale parameters. In order to further promote the practical application of theories and approaches, pioneer researchers have applied a variety of methods to reduce the number of the characteristic length scale parameters while keeping effectively describing microstructure interactions for decades. Aifantis (1999) proposed one-parameter second strain gradient theory, in which the second order strain gradients are considered as an additional effect of strain energy density. However, the Aifantis’ theory is phenomenological, rather than derived from the microstructure. Yang et al. (2002) introduced an additional equilibrium relation to govern the behavior of the couples, presented a modified couple stress theory with one characteristic length scale parameter. Chakraborty (2008) also presented a non-local extension of Biot’s theory of poroelasticity, which has only one characteristic length scale parameter. The characteristic length scale parameter can be estimated by comparing the theoretical dispersion rate with experimental observation, and Chakraborty’s theory has been successfully applied to the diagnosis of osteoporosis. Wang et al. (2020) derived the asymmetric wave equations based on the modified couple stress theory (Yang et al., 2002) and perform the numerical modelling for seismic waves propagation in the viaduct system. The coefficient reflecting the connection between the characteristic length scale parameter of the medium and the characteristic length scale parameter of the micropore has been obtained through the analysis of field data and synthetic data.

Up to the present stage, relatively few studies have been conducted to derive the elastic wave equations in the frame of the GCM theory with numerical modelling or analytical solutions, especially in the frequency band of seismic exploration. In addition, there are few studies in the existing literature that incorporate multiple theories and methods of the GCM theory into an integrated frame. In this paper, we introduce the concept of the multi-scale microstructure interactions and consider the characteristic length scale parameter as comprehensive parameter affected by multiple factors, including localized deformation phenomenon, stress concentration effect, and the geometric characteristics of the material that make up the medium, and then, incorporate the modified couple stress theory and the one-parameter second strain gradient theory into the unified framework for numerical modelling and analysis. To be specific, first, we derive asymmetric elastic wave equations based on modified couple stress and one-parameter second strain gradient theory under the same definition of characteristic length scale parameter of the medium. Second, we perform numerical modelling for layered model and salt model using three different wave equation and analyze the influence on the propagation of seismic waves caused by the complex microstructure interactions in the medium under different theories. At last, some conclusions are drawn.
2. Asymmetric elastic wave equations

In the frame of the GCM theory, the characteristic length scale parameter \( l \) of the medium accompanied with the high-order spatial derivative terms of the state variables (strain or stress) is used to describe the influence of the microstructure interactions in the medium on the propagation of seismic waves. In general, the characteristic length scale parameter of the medium is affected by multiple factors, such as localized deformation phenomenon, stress concentration effect in the medium and the geometric characteristics of the materials. In ideal conditions, that is, without considering the influence of other factors, the characteristic length scale parameter \( l_m \) of the micropore can be regarded as the characteristic length scale of the medium, the coefficient \( \gamma \) between the \( l \) and the \( l_m \) can be calculated by comparing the field data with synthetic data (Wang et al., 2020).

In this section, we first define the characteristic length scale parameter \( l_m \) of the micropore as the mean particle diameter, through the quantitative relationship \( l = \gamma l_m, \gamma \geq 1 \), the characteristic length scale parameter \( l \) of the medium can be expressed, and then under the same characteristic length scale parameter \( l \), we derive the asymmetric elastic wave equations based on the modified couple stress theory and one-parameter second strain gradient theory.

2.1. Asymmetric elastic wave equations based on the modified couple stress theory

According to the couple stress theory (Koiter, 1964), the interaction between the material micro-elements is transmitted through the surface stress \( p^{(s)}_i \) and the surface force couple \( m^{(c)}_i \), which can be expressed by stress tensor \( \sigma_{ij} \) and couple stress tensor \( \mu_{ij} \):

\[
p^{(s)}_i = \sigma_{ij} n_j, \quad (1)
\]
\[
m^{(c)}_i = \mu_{ij} n_j, \quad (2)
\]

where, \( n_j \) is the unit normal vector.

At the same time, the stress tensor and the couple stress tensor both satisfy the equilibrium equation of linear momentum and the equilibrium equation of angular momentum:

\[
\frac{d}{dt} \int_{V_a} V_dV = \int_{S_a} \left[ p^{(s)}_i n_j \right] ds + \int_{V_a} F_i dv = 0 \left\{ \rho \frac{\partial^2 u_i}{\partial t^2} \right\}, \quad (3)
\]
\[
\frac{d}{dt} \int_{V_a} \Omega_dV = \int_{S_a} \left[ \epsilon_{ijk} r_j p^{(c)}_k + m^{(c)}_i \right] ds + \int_{V_a} \left[ \epsilon_{ijk} F_i + M_i \right] dv = 0, \quad (4)
\]

where, \( V_a \) is the volume of the material, and \( S_a \) is the boundary. \( V_a, \Omega_a \) are linear momentum and angular momentum, respectively. \( F_i, M_i \) are body force and body couple. \( \epsilon_{ijk} \) is the permutation symbol. \( r_j \) is the position vector. \( u_i \) is the displacement vector, \( \omega_i \) is the rotation vector. \( \rho \) is the density.

Applying the divergence theorem, transforming from surface integral to volume fraction, and ignoring
high-order traces, the differential expression of the equilibrium equations can be obtained:

\[ \sigma_{j,i} + F_i = 0 \left( = \rho \frac{\partial^2 u_i}{\partial t^2} \right), \tag{5} \]

\[ \mu_{j,i} + e_{ii} \sigma_{ii} + M_i = 0, \tag{6} \]

Decompose the stress tensor into the sum of the symmetric stress tensor and the antisymmetric stress tensor (Yang et al., 2002), we obtain:

\[ \sigma_y = \tau_j + r_j, \]

\[ \tau_j = \frac{1}{2} (\sigma_y + \sigma_j), \tag{7} \]

\[ r_j = \frac{1}{2} (\sigma_y - \sigma_j). \]

Thus, the equilibrium equations can be expressed as follow:

\[ \tau_{j,i} + r_{j,i} + F_i = 0 \left( = \rho \frac{\partial^2 u_i}{\partial t^2} \right), \tag{8} \]

\[ \mu_{j,i} + e_{ii} r_{ii} + M_i = 0, \tag{9} \]

According to equation (9), we can derive the relationship between the antisymmetric stress tensor and the couple stress tensor.

\[ r_i = \frac{1}{2} \left( \mu_{j,i} + M_i \right) e_{ii}. \tag{10} \]

Now, the couple stress tensor is decomposed into the sum of the deflection part and the spherical part (Yang et al., 2002):

\[ \mu_i = m_i + \frac{1}{3} \mu_{ii} \delta_i. \tag{11} \]

Taking the divergence of equation (10), and substituting equation (11), we have:

\[ r_{i,j} = \frac{1}{2} (m_{j,i} + M_{i,j}) e_{ii}. \tag{12} \]

Substitute equation (7) and (12) into equation (8), to eliminate the antisymmetric stress tensor, the equilibrium equation containing the symmetrical stress tensor and the deflection part of the couple stress tensor has following relationship:

\[ \tau_{j,i} + \frac{1}{2} e_{ii} m_{ii,i} + \frac{1}{2} e_{ii} M_{i,i} + F_i = 0 \left( = \rho \frac{\partial^2 u_i}{\partial t^2} \right), \tag{13} \]

Further, we consider the boundary conditions in the frame of couple stress theory.

Then, applying the principle of virtual work, the constitutive relations of the modified couple stress theory (Yang et al, 2002) are derived. The works done by the external loads through the virtual displacement \( \delta u_i \) and the virtual rotation \( \delta \omega \) are as below (Hadjesfandiari and Dargush, 2011):

\[ \int_{V_i} \left( \sigma_{j,i} + F_i \right) \delta u_i dv = 0, \tag{14} \]
According to the principle of virtual work, the change in the deformation energy is equal with the works done by the external loads through the virtual displacement and the virtual rotation.

\[
\int_{V} \left[ \sigma_{ij,ij} + e_{ii} \sigma_{ii} + M_{i} \right] \delta \omega_{ij} dv = 0, \tag{15}
\]

where, \( w \) is the deformation energy density.

Under the assumption of small deformation, the relative displacement between two points in the Cartesian coordinate system is expressed as follow (Aki and Richards, 2002):

\[
du_{i} = u_{i,j} dx_{j}, \tag{17}
\]

where, \( u_{i,j} \) is the displacement gradient tensor and can be decomposed into the sum of symmetric strain tensor \( e_{ij} \) and antisymmetric rotation tensor \( \omega_{ij} \).

\[
e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \tag{18}
\]

The rotation vector \( \omega_{i} \) corresponding to the antisymmetric rotation tensor \( \omega_{ij} \) is defined as:

\[
\omega_{i} = \frac{1}{2} e_{ik} u_{k,j} = -\frac{1}{2} e_{ik} \omega_{k,j}. \tag{19}
\]

In the frame of couple stress theory, the rotation gradient tensor \( \omega_{ij} \) is introduced to describe the torsional and bending deformation of the material. We define the symmetric curvature tensor \( \chi_{ij} \) and the antisymmetric curvature tensor \( \kappa_{ij} \), respectively, as (Yang et al., 2002):

\[
\chi_{ij} = \frac{1}{2} (\omega_{i,j} + \omega_{j,i}) = \frac{1}{2} \left( e_{ii} u_{i,j} + e_{ij} u_{i,j} \right),
\]

\[
\kappa_{ij} = \frac{1}{2} (\omega_{i,j} - \omega_{j,i}) = \frac{1}{2} \left( e_{ii} u_{i,j} - e_{ij} u_{i,j} \right). \tag{20}
\]

Substituting the equilibrium equation (equation (13)) and equation (18) and (20) into equation (16), we obtain,

\[
\int_{V} \delta w dv = \int_{V} \left[ \delta u_{i} \left( \sigma_{i,j,i} + F_{i} \right) + \delta \omega_{i} \left( \mu_{i,j} + M_{i} \right) + \sigma_{ii} \delta u_{i} + m_{i} \delta \omega_{i,j} \right] dv
\]

\[
= \int_{V} \left( \delta e_{ij} e_{ij} + \delta \chi_{ij} m_{i} \right) dv, \tag{21}
\]
Then, we derive the conjugate relationship between the deformation energy density and the displacement and the torsional and bending deformation. And the deformation energy density only depend on the symmetric strain tensor and the symmetric curvature tensor.

\[
\tau_{ij} = \frac{\partial w}{\partial e_{ij}}, \\
m_{ij} = \frac{\partial w}{\partial \chi_{ij}},
\]

As for isotropic medium, the deformation energy density can be written as:

\[
w = \frac{1}{2} \lambda (\varepsilon_{ij})^2 + \mu \varepsilon_{ij} + \mu l^2 \chi_{ij} \chi_{ij} = \frac{1}{2} \lambda (\varepsilon_{ij})^2 + \mu \varepsilon_{ij} + \eta \chi_{ij} \chi_{ij},
\]

where, \(\lambda, \mu\) are the Lamé constants. \(\eta\) is a parameter reflecting the characteristics of the medium's rotational movement, \(\eta = \mu l^2\), and \(l\) is the characteristic length scale parameter of the medium.

Then, we derive the constitutive relations for isotropic medium.

\[
\tau_{ij} = \lambda \delta_{ij} \varepsilon_{kk} + 2\mu e_{ij}, \\
m_{ij} = 2\mu l^2 \chi_{ij} = 2\eta \chi_{ij},
\]

Substituting the constitutive relations (equation (24)) into the equilibrium equation (equation (13)), the elastic wave equations based on the modified couple stress theory (Yang et al., 2002) are given as follow (without consideration of body force and body force couple).

\[
(\lambda + \mu) u_{j,j} + \mu u_{k,k} + \frac{1}{2} e_{ij} \eta \left( \frac{1}{2} e_{lm} u_{n,ml} + \frac{1}{2} e_{lm} u_{n,ml} \right) = \rho \frac{\partial^2 u_i}{\partial t^2},
\]

2.2. Asymmetric elastic wave equations based on the one-parameter second strain gradient theory

According to the non-local theory, the state variables of a point are weighted by the state variables of all points in its neighborhood (Eringen, 1972).

\[
\tilde{A}(x) = \frac{1}{V} \int h(n) A(x + n) dV,
\]

where, \(V\) is the volume of neighborhood, and for a spherical neighborhood, we consider the characteristic length scale parameter \(l\) of the medium as the radius of the spherical neighborhood, \(V = \frac{4}{3} \pi l^3\). \(\tilde{A}(x)\) an equivalent state variable of a point, \(A(x + n)\) is the state variable of each point in the neighborhood. \(h(n)\) is an empirical weighting function subject to the normalization condition \(\int_V h(n) dV = V\). And when \(A(x + n)\) is constant, \(\tilde{A}(x) = A(x + n)\). Assuming \(h(n) = 1\), equation (26) can be rewritten as:

\[
\tilde{A}(x) = \frac{1}{V} \int A(x + n) dV,
\]
Applying the Taylor series expansion algorithm, keeping the derivative term below the second order, and substituting the volume of neighborhood, we derive:

\[
A(x) = A(x) + \frac{t^2}{10} V^2 A(x) = \left[1 + \frac{t^2}{10} V^2\right] A(x).
\]  

(28)

According to the generalized Hooke's law, the expression of constitutive equation is as follow:

\[
\sigma_{ij} = C_{ijkl} \varepsilon_{kl},
\]

(29)

where, \( \sigma_{ij} \) is the stress tensor, \( \varepsilon_{ij} \) is the strain tensor, \( C_{ijkl} \) is the elastic coefficient tensor.

Thus, we define the non-local strain tensor as (Voyiadjis and Dorgan, 2004):

\[
\tilde{\varepsilon} = \left[1 + \frac{t^2}{10} V^2\right] \varepsilon = \left[1 + cV^2\right] \varepsilon.
\]

(30)

where, \( c \) is the coefficient of second order gradient tensor, \( c = t^2/10 \).

Substituting the non-local strain tensor and the non-local stress tensor for the strain tensor and the stress tensor, respectively, we obtain:

\[
\tilde{\sigma}_{ij} = C_{ijkl} \tilde{\varepsilon}_{kl} = C_{ijkl} (\varepsilon_{ij} + cV^2 \varepsilon_{ij}).
\]

(31)

As for isotropic medium, the constitutive equation can be rewritten as:

\[
\tilde{\sigma}_{ij} = \lambda (\varepsilon_{ij} + cV^2 \varepsilon_{ij}) \delta_{ij} + 2\mu (\varepsilon_{ij} + cV^2 \varepsilon_{ij}) = \left[1 + cV^2\right] \left(\lambda \delta_{ij} \varepsilon_{ij} + 2\mu \varepsilon_{ij}\right).
\]

(32)

The equilibrium equation is given as:

\[
\sigma_{\mu,ij} + F_i = \rho \frac{\partial^2 u_i}{\partial t^2},
\]

(33)

In the case of equivalent stress, the equilibrium equation can be rewritten as:

\[
\tilde{\sigma}_{\mu,ij} + F_i = \rho \frac{\partial^2 u_i}{\partial t^2},
\]

(34)

Substituting the constitutive equation based on the non-local theory into the equilibrium equation, we derive the asymmetric elastic wave equations based on the one-parameter second strain gradient theory (without consideration of body force):

\[
\left(1 + cV^2\right) \left[\left(\lambda + \mu\right) u_{ij,j} + \mu u_{ij,ij}\right] = \rho \frac{\partial^2 u_i}{\partial t^2},
\]

(35)

According to the equation (25) and (35), it's worth noting that the independent free terms in the asymmetric wave equations derived from the two different theories are not exactly the same. For the equation (25), the independent free term is \( \frac{1}{2} e_{ijkl} \eta \left(\frac{1}{2} \varepsilon_{ij,mn} u_{m,n,ij} + \frac{1}{2} \varepsilon_{ij,mn} u_{n,m,ij}\right) \), for the equation (35), the independent free term is \( cV^2 \left(\lambda + \mu\right) u_{ij,j} + \mu u_{ij,ij} \). The independent free term is used to describe the internal microstructure interactions, compared with the modified couple stress theory, the one-parameter second strain gradient theory in whose strain energy density function contains higher order spatial derivative of strain can describe the heterogeneity of medium caused by smaller scale microstructure interactions. Assuming that the
scale of the internal microstructure of the medium is the characteristic length scale parameter $l$ of the medium, the asymmetric elastic wave equation based on the modified couple stress theory can represent the microstructure interactions with the scale $l$, and the asymmetric elastic wave equation based on the one-parameter second strain gradient can represent the smaller scale microstructure interactions inside the spherical neighborhood with the scale $l$ as the radius. Therefore, the concept of multi-scale microstructure interactions can be introduced.

In next section, by setting the same characteristic length scale parameter of the micropore, we perform numerical modelling for layered model and salt model using the conventional elastic wave equations, the asymmetric elastic wave equations based on the modified couple stress theory and one-parameter second strain gradient theory, analyze the influence of the microstructure interactions described by the different theories on the propagation of seismic waves.

3. Numerical modelling

3.1. Layered Model

First, we perform numerical modelling of two layered model, as shown in Fig. 1. Both the horizontal and vertical sizes of the two layered model are 3.2 km, and the grid interval is $dx = dz = 8$ m. For the first layer of the model, the parameters (P-wave velocity, S-wave velocity, density) are (2.4 km/s, 1.3856 km/s, 1400 kg/m$^3$); for the second layer, the parameters (P-wave velocity, S-wave velocity, density) are (4.2 km/s, 2.4249 km/s, 2600 kg/m$^3$). The characteristic length scale of the micropore of the first layer is set to 700 μm, and the second layer is set to 300 μm. A Ricker wavelet with a dominant frequency of 25 Hz is located at (1.6 km, 0 km), as shown in the red pentacle in Fig. 1. The time interval is 0.001 s, and the record length is 3 s.

We perform numerical modelling for the two layered model using the conventional elastic wave equations and the asymmetric elastic wave equations based on the modified couple stress theory. Fig. 2 shows the snapshots generated by different elastic wave equations at a moment of 1.25 s. Fig. 2a and 2d are the snapshots ($x$ and $z$ component) generated by the conventional elastic wave equations, Fig. 2b and 2e are the snapshots ($x$ and $z$ component) generated by the asymmetric elastic wave equations based on the modified couple stress theory, and Fig. 2c and 2f are the difference between Fig. 2a and 2b, Fig. 2d and 2e, respectively.

As shown in Fig. 2, due to the addition of independent free term containing the characteristic length scale
parameters of the medium to the asymmetric elastic wave equation based on the modified couple stress theory, some new components of wavefields appear in both the $x$ and $z$ components of the synthetic snapshots, which can be regarded as the displacement disturbance caused by the heterogeneity of the medium. In Fig. 2c and 2f, the changes of wavefields can be observed more clearly. However, the changes are only reflected in the propagation of S-wave and have no effect on the propagation of P-wave.

The numerical modelling results of layered model show that the elastic wave equation derived from the modified couple stress theory, containing the independent free term embedded the characteristic length scale parameter of the medium, can describe the propagation of displacement disturbances generated by considering microstructure interactions in the medium. The propagation of displacement disturbances can be clearly observed in the synthetic snapshots. At the same time, the heterogeneity of the medium caused by the microstructure interaction described by the modified couple stress theory has no effect on the propagation of the P-wave, however, it will cause the wavefields of the S-wave to appear new components, making the S-wave propagate in a dispersive manner.

Figure 2
Snapshots generated by different elastic wave equations: a The conventional elastic wave equations ($x$ component); b the elastic wave equations based on the modified couple stress theory ($x$ component); c the difference between a and b; d the conventional elastic wave equations ($z$ component); e the elastic wave equations based on the modified couple stress theory ($z$ component); f the difference between d and e

Then, a comparison of the synthetic snapshots generated by the conventional elastic wave equation and the asymmetric elastic wave equation based on the one-parameter second strain gradient theory is shown in Fig. 3. Similarly, all the snapshots are computed at 1.25 s. Fig. 3a and 3d are the snapshots ($x$ and $z$ component) generated by the conventional elastic wave equations, Fig. 3b and 3e are the snapshots ($x$ and $z$ component) generated by the asymmetric elastic wave equations based on the one-parameter second strain gradient theory, and Fig. 3c and 3f are the difference between Fig. 3a and 3b, Fig. 3d and 3e, respectively.
Whether it is in the $x$ component or the $z$ component of the synthetic snapshots, the influence of displacement disturbance caused by the heterogeneity of the medium which is described by the second order strain gradient on propagation of seismic waves can be clearly observed from the Fig. 3. That is, new components of wavefields appear in the synthetic snapshots. Comparing the differences of the snapshots as shown in Fig. 3c and 3f, we can observe the changes of the wavefields more clearly. The changes are not only reflected in the propagation of S-wave, but also in the propagation of P-wave, the P-wave and S-wave both propagate in a dispersive manner. At the same time, under the framework of one-parameter second strain gradient theory, the influence of microstructure interactions in the medium on S-wave is significantly greater than that on P-wave.

Further, we extract single-trace synthetic seismograms at the position ($x, z$) = (0.8 km, 0.4 km) for analysis, as shown in Fig. 4. Fig. 4a-c are the $x$ component of single-trace synthetic seismograms, and Fig. 4d-f are the $z$ component of single-trace synthetic seismograms. The blue, red, green lines represent the seismograms generated by the conventional elastic wave equations, the elastic wave equations based on the modified couple stress theory and the one-parameter second strain gradient theory, respectively.
In order to facilitate comparison, the single-trace synthetic seismograms generated by different elastic wave equations are put together in the same figure, as shown in Fig. 4d and 4h. Fig. 4d indicates that, the spatial derivative term of rotation introduced by the modified couple stress theory only affects the S-wave and converted wave, new information appears in the S-wave and converted wave, and both the amplitudes
and the travel time have changed. While the second-order gradient of strain introduced by the one-parameter second strain gradient theory affects the S-wave, P-wave and converted wave, although the changes in seismograms are relatively weaker, the changes of the amplitudes and the travel time of the S-wave, P-wave and converted wave can still be observed in Fig. 4h.

In addition, we notice that, even though the characteristic length scale parameters of the micropore are setting to the same, the heterogeneity of the medium described by the modified couple stress theory has a relatively larger influence on the seismograms, which also shows that the modified couple stress theory and the one-parameter second strain gradient theory derived from the non-local theory describe the heterogeneity with different scales. Compared with the modified couple stress theory, the one-parameter second strain gradient theory can reflect the heterogeneity with smaller scale, however, the influence on seismic waves is relatively weaker.

We capture 7 regions (black dotted bordered rectangle) of Fig. 4d and 4h to zoom in, respectively, as shown in Fig. 5b-f and Fig. 6b-f. The influence of the heterogeneity of the medium caused by the microstructure interactions on propagation of seismic waves can be observed more clearly, and the conclusions drawn are consistent with the above analysis.

Figure 5
Magnification of seismograms generated by different elastic wave equations ($x$ component)
3.2. Salt Model

Then, we perform additional numerical modelling of the SEG/EAGE salt model to further examine the different elastic wave equations. The grid dimensions of the salt model are $n_x = 676$, $n_z = 201$, and the grid spacing in the $x$- and $z$-axis are both 8 m. The P-wave velocity model is shown in Fig. 7. The seismic source is a Ricker wavelet with a dominant frequency of 25 Hz and is located at the red pentacle in Fig. 7. The record-length is 5 s with a time step of 0.0005 s. The characteristic length scale parameters of the micropore of the salt model is set to 700 μm.

![The SEG/EAGE salt P-wave velocity model](image)

The SEG/EAGE salt P-wave velocity model

![Synthetic shot records of the SEG/EAGE salt model, generated by the different elastic wave equations](image)

Figure 8

Synthetic shot records of the SEG/EAGE salt model, generated by the different elastic wave equations: a The conventional elastic wave equations ($x$ component); b the elastic wave equations based on the modified couple stress theory ($x$...
component); e the difference between a and b; d the conventional elastic wave equations (z component); e the elastic wave equations based on the modified couple stress theory (z component); f the difference between d and e

Figure 9
Synthetic shot records of the SEG/EAGE salt model, generated by the different elastic wave equations: a The conventional elastic wave equations (x component); b the elastic wave equations based on the one-parameter second strain gradient theory (x component); c the difference between a and b; d the conventional elastic wave equations (z component); e the elastic wave equations based on the one-parameter second strain gradient theory (z component); f the difference between d and e.

Figure 8 shows the x and z components of the synthetic shot records for the Salt model generated by the conventional elastic wave equation and the asymmetric elastic wave equation based on the modified couple stress theory. Fig. 8c is the record of Fig. 8a subtracted from Fig. 8b, and the Fig. 8f is the record of Fig. 8d subtracted from Fig. 8e.

We noticed that even when the model is more complex, the displacement disturbance caused by the heterogeneity of the medium described by the spatial derivative of rotation still has an obvious influence on the propagation of seismic waves, as shown in Fig. 8. Compared with the shot records generated by the conventional elastic wave equation, the shot records generated by the asymmetric elastic wave equation have obvious differences in some regions marked with the red arrows. It is easier to observe from the results of the subtraction, that is, as shown in Fig. 8c and 8f, new components appear in the records of S-wave and surface wave. These new components are generated due to considering the heterogeneity of the medium caused by the microstructure interactions.

Figure 9 shows the x and z components of the synthetic shot records for the Salt model generated by the conventional elastic wave equation and the asymmetric elastic wave equation based on the one-parameter second strain gradient theory. Fig. 9c is the record of Fig. 9a subtracted from Fig. 9b, and the Fig. 9f is the record of Fig. 9d subtracted from Fig. 9e.

Similarly, for complex models, the displacement disturbance described by the second-order gradient of strain also has obvious influence on the propagation of seismic waves, see the regions marked by the red
arrows in Fig. 9. If the shot records generated by the two elastic wave equations are subtracted, as shown in Fig. 9c and Fig. 9f, not only the records of S-wave and the surface wave, but also the records of P-wave also appear new components, which shows that the smaller scale heterogeneity of the medium has an influence on the P-wave.

In summary, whether for layered model or more complex Salt model, compared with the numerical modelling results generated by the conventional elastic wave equations, the results generated by the asymmetric elastic wave equations contain the new components, which are very closely related to the characteristic length scale parameter of medium. It’s worth noting that, in the frequency band of seismic exploration, the wavefield responses caused by the multiple scales heterogeneity of the medium can still be clearly observed.

4. Conclusions

There always exist complicated internal micro-defects/micro-structures, whether for natural earth medium and man-made materials. However, it is difficult to describe the complex microstructure interactions by the conventional continuum mechanics theory under its continuity assumption. In addition, it is worth emphasizing that the scale of micro-defects/micro-structures varies with the observation target, and it is a relative concept. In this paper, we integrate the modified couple stress theory and the one-parameter second strain gradient theory into the unified framework for analyzing the complex microstructure interactions in the medium. Then we derive the asymmetric elastic wave equations based on the modified couple stress theory and the one-parameter second strain gradient theory under the same characteristic length scale parameter and perform numerical modelling. And some conclusions can be drawn.

1. The complex microstructure interactions in the medium have an influence on the propagation of seismic waves. In the frequency band of seismic exploration, the influence of the microstructure interactions on the propagation of seismic waves can be obviously observed in seismic waves responses.

2. For the modified couple stress theory, the spatial derivative of rotation is introduced to describe the complex microstructure interactions, which causes the S-wave propagates in a dispersive manner and has no effect on the P-wave propagation.

3. For the one-parameter second strain gradient theory, the second-order gradient of strain is introduced, which can represent smaller-scale microstructure interactions in the medium. And new components appear in P-wave and S-wave responses, meanwhile, the amplitude and travel time have also changed.

4. The influence of the microstructure interactions described by the one-parameter second strain gradient theory on the propagation of seismic waves is more weak, smaller micro-scale differences in pore and grains geometry of rocks result in smaller macro-scale differences of wave responses and characteristics.

Acknowledgements

This work was funded by the Joint Funds of the National Natural Science Foundation of China under grant U20B2014.

References


