Simplified formulation to evaluate forces due to shrinkage in composite steel-concrete beams with perfect connection

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Abstract

It has been known for a very long time that time-dependent effects such as creep and shrinkage of concrete greatly influence the behavior of composite steel-concrete beams. It is therefore very important to take these effects into account when calculating the strength and safety of composite steel-concrete beams. To this end, many theoretical and numerical researches have been established to control this phenomenon. Most of this research presents laborious processes and calculations requiring complex techniques. To overcome the excessive amount of calculations and the various difficulties associated with analytical or numerical methods to estimate the additional stresses brought by the shrinkage of concrete in composite steel-concrete beams, we thought of proposing a simplified analytical methodology while ensuring safety desired. In this article, we have tried to simplify an existing analytical model based on the theory of linear viscoelasticity established in 2012. This model consists of combining the static equilibrium equations and the two compatibility relations, in curvature and in deformation, of the composite steel-concrete cross-section with the differential equation resulting from creep rate theory (RCM). Our idea then aims to take this model and simplify it to avoid difficult mathematical transformations. The results from this simplified approach are very satisfactory when compared to those given by the analytical model.

1. Introduction

Due to the complementary performance of steel and concrete, composite steel-concrete beams have become a very suitable structural system for the construction of buildings and bridges. To obtain a monolithic section, shear connectors must be arranged along the beam to connect the concrete slab to the metal beam. This process will lead to a structural system with greatly improved strength, stiffness, ductility and fire protection [1]. Under the application of service loads, the steel beam deforms elastically. On the other hand, the concrete slab will undergo strong inelastic deformations over time, particularly shrinkage, creep and aging of the concrete [2].

Due to the effects of concrete shrinkage and creep on the one hand and the steel-concrete interaction on the other hand, the accurate prediction of the service behavior of composite steel-concrete beams becomes highly complex.

In order to evaluate the structural behavior of composite steel-concrete beams, it is very important to predict the effects of concrete shrinkage and creep [3]. Until now, despite the numerous research carried out in this field, these two phenomena have not yet been mastered [4].

The shrinkage of concrete is, by definition, a physical phenomenon linked to various interdependent factors such as: temperature, type of cement, humidity and transverse dimensions of the element, etc. Total shrinkage includes autonomous shrinkage, drying shrinkage and plastic shrinkage [5, 6]. It affects the time-dependent behavior and reduces the volume of the concrete element. In the long term, it can cause a deformation of concrete structures or a redistribution of internal forces. When the ultimate tensile
limit of concrete is reached, the durability and serviceability of concrete members will be affected [3]. Shrinkage can also cause cracking of the concrete [7–9]. If displacement is prevented or restricted, severe cracking of the concrete structure will occur [10].

With the use of the theory of linear viscoelasticity, many works, analyzing the time-dependent effects on the behavior of composite steel-concrete beams, are the subject of many analytical and numerical researches such as:


2. Presentation Of The Idea

Based on the theory of linear viscoelasticity, Rahal et al [21] proposed an analytical model analyzing the behavior, over time, of composite steel-concrete beams subjected to concrete shrinkage. The process consists in combining the static equilibrium equations and the two compatibility relations, in curvature and in deformation, of the composite steel-concrete cross-section with the differential (constitutive) equation resulting from the creep rate theory (RCM). In this model:

1- to obtain the variation of the normal force $N_c(t)$ and of the bending moment $M_c(t)$ brought by the shrinkage of the concrete and which solicit the concrete slab it is compulsory to solve the system of two differential equations below after:

$$
\left(1 + \frac{I_c}{n I_a}\right) \frac{dM_c(t)}{d\phi} - \frac{a. (1 + n \rho) I_c}{n I_a} \frac{dN_c}{d\phi} + M_c(t) = 0
$$

1
Its solution is as follows:

\[ M_c(t) = C_1 a_1 \dot{\varphi} \cdot e^{\dot{\lambda}_1 \varphi} + C_2 a_2 \dot{\varphi} \cdot e^{\dot{\lambda}_2 \varphi} \quad (3) \]

\[ N_c(t) = C_1 a_2 \dot{\varphi} \cdot e^{\dot{\lambda}_1 \varphi} + C_2 a_2 \dot{\varphi} \cdot e^{\dot{\lambda}_2 \varphi} + \frac{E_c}{A_6} \frac{\varepsilon_{sh\infty}}{\varphi_{\infty}} \quad (4) \]

C_1 and C_2 will be determined from the boundary conditions, and a_1 and a_2 will be determined from the geometric and mechanical characteristics of the composite cross-section.

The forces acting on the metal beam will be obtained by the static equilibrium of the cross-section.

\[ A_c : \text{the cross section area of concrete in the slab.} \]
\[ A_s : \text{metallic beam area.} \]
\[ \rho : \text{Reinforcement percentage} \quad (\rho = A_a/A_c). \]
\[ A_a : \text{the area of the longitudinal reinforcement incorporated in the slab.} \]
\[ I_c : \text{moment of inertia of the concrete slab.} \]
\[ I_s : \text{moment of inertia of the metallic beam} \]
\[ \alpha : \text{distance between the neutral axis of the metal beam and that of the reinforced concrete slab.} \]
\[ b_{eff} : \text{effective width of the reinforced concrete slab.} \]
\[ C_c : \text{distance from the slab centre of gravity to the neutral fibre of the mixed section.} \]
\[ C_s : \text{distance from the steel beam centre of gravity to the neutral fibre of the mixed section.} \]
\[ E_c : \text{is the modulus of tensile elasticity of concrete.} \]
\[ E_s : \text{is the modulus of tensile elasticity of steel.} \]
\[ n : \text{equivalence coefficient} \quad (n = E_s/E_c). \]
\( \varepsilon_{sh}(t) \) : is the deformation due to concrete shrinkage. It can be determined using calculation codes for concrete structures such as: EC2, ACI, model code fib,........ Etc

\( M_c(t) \) : bending moment in the concrete slab due to shrinkage.

\( M_s(t) \) : bending moment in the metallic beam.

\( N_c(t) \) : normal force in the concrete slab due to shrinkage.

\( N_s(t) \) : normal force in the metallic beam.

Through this work, we seek to greatly simplify this formulation so that it can be easily used by engineers in design offices.

### 3. Formulation Of The Proposed Approach

Figure 1 shows the different components of the cross-section namely: the concrete slab, the metal beam, the reinforcement embedded in the slab and the shear connectors. For the formulation of the present approach, in Fig. 1 is shown the various forces acting on the composite cross-section.

#### 3.1 Static equilibrium equations

At any instant \( t \), the static equilibrium gives the following system of equations:

\[
\sum F/x = 0 \Rightarrow N_s(t) + N_c(t) = N_0
\]

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Figure 1 : Section transversale mixte acier- béton

\[
\sum M/Gs = 0 \Rightarrow M_s(t) - N_c(t) \times a + M_c(t) = M_0
\]

6

Under the effect of concrete shrinkage, the slab can therefore be subjected on its neutral axis to a normal tensile force. Based on Hook’s law, this force can be obtained by the following relation [59, 60]:

\[
N_c(t) = \frac{1}{n_L} A_c E_c \varepsilon_{sh}(t)
\]

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We introduce the relation (Eq. 7) in the static equilibrium equations (Eqs. 5 and 6) of the model proposed by Rahal et al [21], we will find:
Since the shrinkage does not depend on the external loading, we cancel $N_0$ and $M_0$ and we will have:

\[ N_s(t) = - \frac{1}{n_L} A_c E_c \varepsilon_{sh}(t) + N_0 \]

\[ M_s(t) = \frac{1}{n_L} A_c E_c \varepsilon_{sh}(t) \times a - M_c(t) + M_0 \]

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\[ M_s(t) = \frac{1}{n_L} A_c E_c \varepsilon_{sh}(t) \times a - M_c(t) \]

$nL$ : modular ratio for shrinkage.

### 3.2 Deformation compatibility

It is known that the curvature and the axial deformation of a beam are related to the displacements by kinematic expressions [1].

We have two equations (Eqs. 10 and 11) with three unknowns, $N_s(t)$, $M_c(t)$ and $M_s(t)$. From the first equation, we will have $N_s(t)$. It remains to find the expression of the bending moment $M_c(t)$ acting on the concrete slab and $M_s(t)$ acting on the steel beam.

To solve this problem, we exploit the compatibility of strains at the steel-concrete interface. This condition has been used by several researchers in the formulation of their model, such as: [12–16, 20–23].

The deformation compatibility condition between steel and concrete was used by Rahal [21] in his model, which we seek to simplify. It translates into the following expression:

\[ \varepsilon_c(t) = \frac{N_c(t)}{E_c A_c} + \frac{M_c(t)}{E_c I_c} C_c = \frac{N_s(t)}{E_s A_s} - \frac{M_s(t)}{E_s I_s} C_s \]

In the equation (Eq. 12), we replace $N_s(t)$ and $M_s(t)$ by their respective expressions (Eqs. 10 and 11), we will easily find the expression of $M_c(t)$ given by the equation (Eq. 13):
Once $M_c(t)$ is known, it is very simple to calculate the expression of $M_s(t)$ by averaging the expression (Eq:11)

\[ M_c(t) = \frac{\left( A_c E_c \varepsilon_{sh}(t) \right)}{n_L} \left[ \frac{c_x C_s}{E_s I_s} + \frac{1}{E_s A_s} + \frac{1}{E_c A_c} \right] \]

4. Validation Of The Proposed Approach

In order to validate the present approach, we will use the same composite beam used by rahal et al [21] and Baghdad et al [23] to validate their proposed models. This beam was also analyzed (prediction of time-dependent effects) and dimensioned in accordance with Eurocode 4 [60]. In this example, the shrinkage parameters were calculated according to Eurocode 2 [61]. The geometric and physical characteristics of the treated example are as follows:

- $b_{eff} = 3100$ mm, $t_c = 250$ mm, $b_f = 400$ mm, $t_f = 20$ mm, $b_{fb} = 400$ mm, $t_{fb} = 30$ mm, $h_w = 1175$ mm, $t_w = 12.5$ mm,
- $A_c = 0.785$ m$^2$, $A_s = 0.0346875$ m$^2$, $A_a = 58.47$ cm$^2$, $\rho = 0.0074$, $I_c = 0.004223633272$ m$^4$, $I_s = 0.0346875$ m$^2$, $C_c = 0.375$ m, $C_s = 0.451$ m, $a = 0.826$m, $E_c = 33 \times 10^4$ MPa, $E_s = 2.1 \times 10^5$ MPa, HR = 70%, Classe de béton = C30/37, HR est l’humidité relative en %

5. Results

The results obtained by this approach are compared to the existing model formulated by Rahal et al [21] and represented on the diagrams of Figs. 3 to 7.

6. Conclusion

The work presented in this article consists in simplifying an existing analytical model based on the aging theory of concrete. The use of the analytical model requires laborious efforts, and several constants must be calculated to determine the additional stresses brought by the shrinkage of the concrete in the composite beams. Following these difficulties, it seemed very logical to us to simplify it and make it applicable with a minimum of effort while ensuring the desired security.

Our current approach is very simple in practical utility by the direct application of the developed expressions. By referring to the existing analytical model, the present approach does not require, as the procedure shows, neither complicated mathematical calculations to be made nor constants to be determined.
The main advantage of this simplified formulation is its compatibility with any code or regulation for the calculation of composite steel-concrete structures used in the world. This possibility is clear in the equation (Eq. 7), in which it will be enough for us to calculate the value of the specific deformation of the shrinkage $\varepsilon_{sh}(t)$ by the calculation regulation to be used such as: EC2, ACI, model code fib, ……etc.

It is clear that at any time $t$, the results obtained by applying our simplified approach that we have formulated (Fig. 2 to 6) are completely comparable to those resulting from the existing analytical model.

We then seek to expand this idea in the case of composite beams in partial connection under the effect of concrete shrinkage. In addition, in the case of composite beams in full and partial connection under concrete creep.

Declarations

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Conflict of interest: No conflict of interest

References


**Figures**

![Figure 1](image)

*Figure 1*

Section transversale mixte acier- béton
Figure 2

Diagrams comparing the final stress (MPa) due to shrinkage of concrete.
Figure 3

Variation, in time, of the concrete slab normal effort $N_c(t)$
Figure 4

Variation, in time, of the concrete slab bending moment $M_c(t)$
Figure 5

Variation, in time, of the normal effort $N_s(t)$ recovered by the steel beam
Figure 6

Variation, in time, of the moment $M_s(t)$ recovered by the steel beam.