**Phase Analysis on the Error Scaling of Entangled Qubits in a 53-Qubit System**

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Figures SA, SB, SC1, SC2, SC3, SC4,SD.

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**Supplementary Information**

**Supplement A**



Figure SA: Four states |11>, |01>, |10> and |00> of a 2-qubit pair. is a quantum state in the subspace spanned by and , which is usually mixed. It can be written as

 ,

Here *r*, *a*, *θ* are parameters that determines the density matrix of *ρ'*. Possible transitions between states are shown for an energy dissipative system, and the ground state |00> is assumed to be always alive. γ1 represents the transition rate from |11> to , while γ0 is from to |00>.

**Supplement B**



Figure SB: Operating on q[0]⊗q[1] with the Hadamard gate and the CNOT gate.

After operating on with the Hadamard gate and the CNOT gate, possible transitions in a noisy environment are shown in Fig. SB. The resultant state will usually be not , the state first assigned to the quantum computer. The noise-induced transitions generate . However, operator acting on this state will impart a phase of to state |1> in q[0]. This usually results in the final state of .

**Supplement C**

In this section, we present a method with uncorrelated errors. We know if the circuit shown in Fig. 1 is noiseless, the corresponding circle in Fig. 2 would be the largest circle with radius . The circuit is accompanied by uncorrelated noise, as shown in Fig. SC1, where we have considered models for three different noise sources: the depolarizing channel, the dephasing channel and the amplitude damping channel 29,30.

1.  (b) 

(c)

Figure SC1: Noisy Circuit with noise channels and . (a) Noise before the CNOT gate. (b) Noise after the CNOT gate. (c) Noise after the phase rotation gate.

Denote the noise rate as *p*, the depolarizing noise channel is modeled by

The dephasing noise is modeled by

where , ,. The amplitude damping noise is modeled by

where ,. Suppose is the density matrix of the state generated from the noisy circuit, and are the noise rates of channels and , respectively.

When the noise channel is located prior to the CNOT gate,

the density matrix for the depolarizing noise is given by

Notice that here has parameters ,

which means .

A special case is if = 0, then with .

The density matrix for the dephasing noise is

Last, the density matrix for the amplitude damping noise is

When the noise channel is located after the CNOT gate or the phase rotation gate, the density matrix for the depolarizing noise is given by

Notice that here for the excited state , .

The density matrix for the dephasing noise is given by

Last, the density matrix for the amplitude damping noise is

Notice that here the parameters for are

We conclude that if there is depolarizing noise before CNOT gate (which is usually true in experiments), .

Therefore, we can easily calculate the relation between radius and noise rates , . For example, when the amplitude damping channel is located after the CNOT gate, . In addition to the depolarizing noise, in the case of noise before the CNOT gate, R does not depend on the phase ϕ, therefore the phase trajectory is always a circle with radius R for different noise channels and noise rates. If we assume , the radii for the different noise channels and noise rates will be as shown in Fig. SC2.

1.  (b) 

Fig SC2: Radii for different noise channels and noise rates. (a) Result for system of noise channel located before the CNOT gate. (b) Result for system of noise channel located after the CNOT gate or after the phase rotation gate.

If the amplitude damping channel is located after the CNOT or the phase rotation gate, we will have . It is also known that noise rate is for the amplitude damping noise. From the two equations above, we have , which is consistent with the simulation results in Fig.2(b). The trajectories of the noise rates in Fig. SC3 correspond to the trajectories in Fig. 2(b) with different .



Fig SC3: Phase trajectories for the uncorrelated amplitude damping channels located after the CNOT/phase gate.

When the depolarizing channel is located before the CNOT gate, the trajectory will be ellipses instead of circles, as shown in Fig SC4.



Fig SC4: Phase trajectories for the uncorrelated depolarizing channel is located before CNOT gate.

We will now also take into consideration. If the noise channel is a combination of the amplitude damping noise and the dephasing noise, it will transform a single qubit density matrix through

.

For a 2-qubit state , where , we denote and as the relaxation time and dephasing time of the first qubit, and as, respectively, the “times” for the second qubit. The combined noise channel transforms to

The phase trajectories will still be circles.

**Supplement D**

**Classical simulation results of phase angle and orthogonal measurements for state**

 Fig. SD: The classical simulation results of superposition states with . (a) The relationship of , with phase angle *ϕ* . (b) The relationship between a.