

# Reliability assessment of cutting tools based on zero-failure data

**Bin Yang**

Shandong University

**Kai Guo** (✉ [kaiguo@email.sdu.edu.cn](mailto:kaiguo@email.sdu.edu.cn))

Shandong University

**Bin Feng**

AVIC Chengdu Aircraft Industrial Co Ltd

**Chang'an Zhou**

Shandong University

**Chao Sun**

AVIC Chengdu Aircraft Industrial Co Ltd

**Jie Sun**

Shandong University

---

## Original Article

**Keywords:** zero-failure data, reliability evaluation, cutting tools, titanium alloys, Weibull distribution

**Posted Date:** May 5th, 2020

**DOI:** <https://doi.org/10.21203/rs.3.rs-25508/v1>

**License:** © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

---

# Reliability assessment of cutting tools based on zero-failure data

Bin Yang<sup>1</sup>, Kai Guo<sup>1\*</sup>, Bin Feng<sup>2</sup>, Chang'an Zhou<sup>1</sup>, Chao Sun<sup>2</sup>, Jie Sun<sup>1</sup>

1. School of Mechanical Engineering, Research Centre for Aeronautical Component Manufacturing Technology & Equipment, Shandong University, Jinan 250061, China

2. AVIC Chengdu Aircraft Industrial (Group) Co.,Ltd, Chengdu 610091, China

\*Corresponding author. Tel.: +86-531-88394593; Fax.: +86-531-88396216.

E-mail address: kaiguo@email.sdu.edu.cn

## Abstract

Titanium alloys are widely utilized in aeronautical monolithic components due to their excellent mechanical properties. However, the low machinability of titanium alloys results in an uncertain lifetime of cutting tools. Reliability assessment on cutting tools has become more and more significant to the effectiveness and stability of machining systems. Considering that cutting tools are often replaced well before the end of their useful lifetime to avoid failures and thus few tools failure data can be obtained during the machining process of titanium alloys, a reliability assessment model based on zero-failure data is developed to evaluate the reliability of cutting tools. A Weibull distribution model is chosen to describe the life distribution of cutting tools for reliability evaluation. Matching distribution curve method and weighted least squares method are used to estimate the distribution parameters. A novel approach using hierarchical Bayesian method for estimating the prior distribution of failure probability with zero-failure data is proposed by combining the characteristics of Weibull distribution and Incomplete Beta distribution. Reliability analysis for cutting tools with zero-failure data is performed using the proposed method. By comparing with the experience of experts and adopting stochastic simulation, the results of point estimation show plausibility of the proposed approach.

**Keywords:** zero-failure data, reliability evaluation, cutting tools, titanium alloys, Weibull distribution.

## 1 Introduction

Nowadays titanium alloys are being extensively used in aeronautical monolithic components thanks to their excellent mechanical properties. However, titanium and its alloys have been recognized as difficult-to-machine materials [1]. The productivity in machining titanium alloys is adversely restricted because of the low machinability [2].

### 1.1 reliability evaluation of cutting tools

Worn tools can bring the deterioration of surface integrity and poor dimensional accuracy of parts, leading to low productivity and reliability of the processing system. Tools are often replaced well before the end of their useful lifetime to avoid failures and related consequences. It has been reported that only 50-80% of the expected tool life is typically used [3]. So it is essential to evaluate the cutting tool life in machining titanium. Generally speaking, tool

wear is considered as a stochastic process and it is a pretty tricky problem to predict tool wear progression accurately. Cutting tool life can be worked out by taking into consideration the observed distribution of the cutting times to failure. Therefore, reliability research on cutting tools is of great importance to manufacturing system effectiveness and stability.

Wager and Barash [4] performed more than one hundred tool life tests using a high speed steel turning tool and found that the tool life values followed a statistical distribution, which approached normal distribution. Liu and Makis [5] derived a recursive formula to calculate the cutting tool reliability in variable conditions if the failure time distribution of the cutting tool can be represented using an accelerated failure time model. Klim et al. [6] proposed a reliability model taking into account both the flank and the face wear on the cutting tool. Wang et al. [7] developed a reliability-dependent failure rate model to predict the reliability of cutting tools. Rodriguez and Souza [8] developed a reliability-based method combined with process planning for estimating the optimum cutting tool change time. Their approach took into account that each cutting tool was used for several different operations and was validated for the case of drilling. Ding and He [9] studied on the cutting tool reliability through a proportional hazards model. Salonitis and Kolios [10] put forward a novel approach for the effective reliability assessment of cutting tool wear based on combinations of stochastic response surface and surrogate modeling methods, coupled with Monte Carlo simulations and first-order reliability methods for the estimation of reliability indices. Wang et al. [11] built a mathematical model of dynamic reliability for machining process under the premise of regarding cutting parameters as random variables. The researchers combined moment estimation using maximum likelihood estimation method with dynamic reliability analysis method.

Besides, the monitored data in the machining process was introduced into reliability analysis to reflect the time-varying characteristics. Chen et al. [12] proposed a novel reliability estimation approach to cutting tools based on logistic regression model by using vibration signals. Li et al. [13] used logistic regression models and acoustic emission signal to evaluate cutting tool reliability. Cutting tools are investigated to determine the best maintenance time based on different conditions estimation. Liu et al [14] applied state space model in cutting tool reliability assessment. The system state is treated as a fuzzy event which is uniquely characterized by membership functions defined over the substitute characteristic variable. The wavelet packet energy extracted from the acoustic emission signal was selected as the feature to estimate the tool state

## **1.2 reliability assessment with zero-failure data**

Because of the variety and complexity of aeronautical structures, there exists a great difference in processing parameters when machining different structural features, which results in the uncertainty of cutting tool life. In addition, the long machining time of aeronautical components and various kinds of cutting tools together lead to the difficulty in understanding tool condition. Considering the large fluctuation of tool life in machining titanium alloys, cutting tools are usually replaced before reaching their failure criterion in actual practice in order to avoid the damage caused by worn tools and ensure the surface quality of components [15]. Consequently, cases of zero-failure data in the actual engineering are usually encountered, which refers to that there is no failure cutter in the specific time. Taking into consideration that few tool failure data can be extracted from the field data in machining process of aerospace components, it is possible to apply the reliability assessment model based on zero-failure data to evaluate reliability of cutting tools.

The cases of zero-failure data may occur in the products with high reliability and long-lifetime, which is a special situation in the time-truncated tests. Martz and Waller [16] firstly studied the reliability evaluation models with zero-failure data and presented a Bayesian zero-failure reliability demonstration testing procedure. Han [17] put forward a method to analysis the optimal confidence limits of reliability parameters. The confidence limits were oriented to the products of which life is subordinate to Weibull distribution. Mao [18] proposed the matching distribution curve method to cope with the problem of reliability analysis with zero-failure data, which had been widely used for its excellent characteristics and ease of application. The Bayesian method has been extensively applied in the estimation of failure probability, which makes full use of information obtained from zero-failure data. Fan and Chang [19] conducted a Bayesian zero-failure reliability demonstration test to design the minimum sample size and testing length subject to a certain specified reliability criterion. Jiang et al. [20] proposed a modified maximum likelihood estimation method for estimating the distribution parameters based on the zero-failure data. Xia [21] used the grey bootstrap method in the information poor theory for the reliability analysis of zero-failure data under the condition of a known or unknown probability distribution of lifetime. Han [22] developed an E-Bayesian estimate method based on hierarchical Bayesian estimation, which was proved to be feasible for zero-failure data. Then the E-Bayesian estimate method attracted the attention of many scholars. Yin et al. [23] investigated an E-Bayesian estimation as a reliability analysis method for the seekers to deal with the zero-failure life testing data.

However, reliability assessment for cutting tools based on zero-failure data has not been studied in the past. Within the present paper, Weibull distribution is chosen as the life distribution model of cutting tools for reliability evaluation. Matching distribution curve method and weighted least squares method are applied to the distribution parameters estimate process. A new approach using hierarchical Bayesian method for estimating the prior distribution of failure probability with zero-failure data is proposed by combining the characteristics of Weibull distribution and Incomplete Beta distribution (WIB). The zero-failure data of cutting tool life was extracted from field production data in the machining process of titanium alloy components. Reliability analysis for cutting tools with zero-failure data is performed using WIB method. Point estimation of cutting tool reliability is calculated. By comparing with the experience of experts and using stochastic simulation, the results show that the proposed approach is effective and practicable.

## 2 Reliability assessment of cutting tools

### 2.1 Tool Life Distribution Model

Weibull distribution has been widely used for the analysis of reliability due to its versatility and also shown to be an appropriate life distribution for cutting tools [24]. The probability density function (pdf) of two-parameter Weibull distribution is given by:

$$f(t) = \frac{\beta(t)^{\beta-1}}{\theta^\beta} \exp \left[ -\left( \frac{t}{\theta} \right)^\beta \right] \quad (1)$$

Where  $\beta$  is the shape parameter ( $\beta > 0$ ),  $\theta$  is the scale parameter ( $\theta > 0$ ), and  $t$  is the lifetime.

The parameters  $\beta$  and  $\theta$  have different influence on the density function curves [25]. The larger of the shape parameter  $\beta$ , the steeper of the pdf curve. For the special case  $\beta=1$ , Weibull distribution becomes exponential

distribution. For shape parameter  $\beta=2$ , it is approximate to Rayleigh distribution. While for the value of  $\beta$  in the range  $3 \leq \beta \leq 4$ , the shape of Weibull distribution is close to normal distribution [26]. In the meanwhile, the scale parameter  $\theta$  does not affect the shape of Weibull distribution, and only has an impact on the timescale of pdf curves. Increasing the value of  $\theta$  while holding  $\beta$  constant has the effect of stretching out the pdf.

Weibull reliability function  $R(t)$  and Weibull distribution function is given by:

$$R(t) = \exp \left[ - \left( \frac{t}{\theta} \right)^\beta \right] \quad (2)$$

$$F(t) = 1 - R(t) = 1 - \exp \left[ - \left( \frac{t}{\theta} \right)^\beta \right] \quad (3)$$

Weibull distribution has extensive application in reliability assessment in practical cases, as well as in the cases of reliability analysis based on zero-failure data.

## 2.2 Zero-failure data model

Zero-failure data can be obtained from the time-truncated reliability tests. Assuming that there are  $m$  groups of tools for the test, the truncated time of each cutting tool is  $t_1, t_2, \dots, t_m$  ( $t_1 < t_2 < \dots < t_m$ ). The size of the samples in each group is  $n_i$  ( $i=1, 2, \dots, m$ ). Then the zero-failure data can be described as  $(t_i, n_i)$  ( $i=1, 2, \dots, m$ ). The number of tools with the running time greater than the truncated time  $t_i$  is  $s_i$ , and  $s_i$  can be calculated by  $s_i = n_i + n_{i+1} + \dots + n_m$ , which represents the total number of samples that have not been failed until each end of truncated time  $t_i$ . The cumulative distribution function of cutting tools can be expressed as  $F(t)$ . The failure probability of cutting tool at truncated time  $t_i$  is  $p_i = P(t \leq t_i) = F(t_i)$ . The failure probability  $p_i$  satisfies the following statements:

$$(1) p_0 = P(t \leq 0) = F(0) = 0, \text{ when } t_0 = 0;$$

$$(2) p_1 \leq p_2 \leq \dots \leq p_m.$$

## 2.3 Matching distribution curve method

The process of matching distribution curve method can be seen in Fig. 1. First of all, the estimation of failure probability  $p_i$  at truncated time  $t_i$  is calculated using a certain method. The lifetime probability distribution curve is computed with the aid of least squares method by using the estimation of failure probability  $\hat{p}_i$  at truncated time  $t_i$ . And then, the estimation of reliability model and parameters of probability distribution are obtained based on the lifetime probability distribution curve. It is considered as the most critical step to estimate the failure probability in matching distribution curve method.

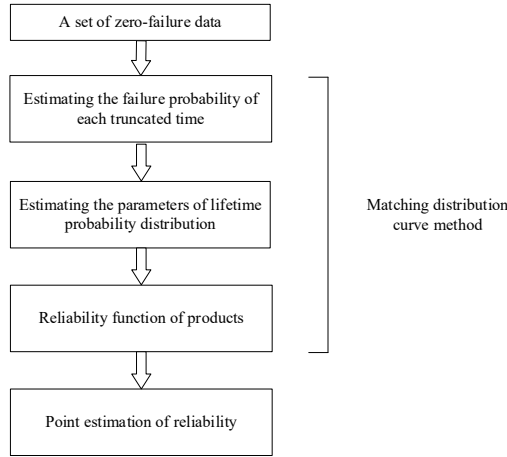


Fig.1 Process of matching distribution curve method

### 2.3.1 Estimation of failure probability

The expression of the traditional method for estimating failure probability is [27]:

$$\hat{p}_{Tm_i} = \frac{0.5}{s_i + 1} \quad (4)$$

Bayesian method is a common approach for estimating the failure probability  $p_i$ . Different application processes of Bayesian method are discussed as follows.

#### 1 Classical Bayesian method

Uniform distribution on  $[0,1]$  is chosen as the prior distribution of failure probability at each truncated time. The expression is  $\pi(p_i)=1/(1-0)=1$ .

Suppose there are  $s_i$  tools for life test, and  $r_i$  tools fail before time  $t_i$ , and the failure probability of each sample is  $p_i$ , and the likelihood estimation expression of failure probability is:

$$L(r_i | p_i) = \binom{s_i}{r_i} p_i^{r_i} (1 - p_i)^{s_i} \quad (i = 1, 2, \dots, m) \quad (5)$$

There is no failure sample at time  $t_i$  under zero-failure situation, which means  $r_i=0$ . Then the likelihood estimate of the failure probability changes into:

$$L(0 | p_i) = (1 - p_i)^{s_i} \quad (i = 1, 2, \dots, m) \quad (6)$$

Based on Bayesian theory, the posterior distribution of the failure probability is given by:

$$\pi(p_i | s_i) = \frac{\pi(p_i) L(0 | p_i)}{\int_0^1 \pi(p_i) L(0 | p_i) dp_i} = (s_i + 1)(1 - p_i)^{s_i} \quad (7)$$

The estimated failure probability  $\hat{p}_{CBi}$  of the classical Bayesian method under the quadratic loss condition is provided by:

$$\hat{p}_{CBi} = \int_0^1 p_i \pi(p_i | s_i) dp_i = \frac{1}{s_i + 2} \quad (i = 1, 2, \dots, m) \quad (8)$$

## 2 Hierarchical Bayesian method

In hierarchical Bayesian method, the unknown parameters in the prior distribution are given another prior distribution. The purpose of adopting hierarchical Bayesian method is to reduce the influence of uncertain parameters.

It is not appropriate to use uniform distribution on  $[0,1]$  as the prior distribution of failure probability because the failure probability is considered to be small under zero-failure situation.

Beta distribution is assumed to be the prior distribution of  $p_i$ . The density function can be shown as:

$$\pi(p_i|a,b) = \frac{1}{B(a,b)} p_i^{a-1} (1-p_i)^{b-1} \quad (9)$$

Beta function  $B(a,b)$  is defined as:

$$B(a,b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt \quad (10)$$

In order to satisfy the precondition of zero-failure data analysis, the failure probability is  $p_1 \leq p_2 \leq \dots \leq p_m$  when the truncated time is  $t_1 < t_2 < \dots < t_m$ . The density function of  $p_i$  must be a monotone decreasing function. Consequently, the range of parameters can be determined as  $0 < a \leq 1$  and  $1 < b < c$  [28], where  $c$  is the upper bound of  $b$ .

According to [29], considering the robustness of Bayesian estimation, the value of hyper parameter  $a$  can be determined as  $a=1$ , and the value of  $c$  should not be large. The range of  $c$  can be determined on 2~9 according to [30]. Uniform distribution is chosen as the prior distribution of  $b$  and the density function is  $\pi(b)=1/(c-1)$ .

When  $a=1$ , the hierarchical prior probability density function of failure probability  $p_i$  can be expressed as:

$$\pi(p_i) = \int_1^c \pi(p_i|b)\pi(b)db = \frac{1}{c-1} \int_1^c b(1-p_i)^{b-1} db \quad (11)$$

Under the quadratic loss condition, the estimated failure probability  $\hat{p}_{HBi}$  using the hierarchical Bayesian method is [28]:

$$\hat{p}_{HBi} = \frac{(1+s_i) \ln\left(\frac{s_i+c+1}{s_i+2}\right) - s_i \ln\left(\frac{s_i+c}{s_i+1}\right)}{c-1-s_i \ln\left(\frac{s_i+c}{s_i+1}\right)} \quad (i=1,2,\dots,m) \quad (12)$$

## 3 WIB method

Suppose  $p_i = P(T \leq t_i) = F(t_i)$ , the failure probability is  $p_i < p_j$  when  $t_i < t_j$ . After logarithm and linear transformation to the Weibull distribution function, the relationship can be deduced as follows:

$$\begin{cases} \ln \ln \frac{1}{1-p_i} = \beta \ln t_i - \beta \ln \theta \\ \ln \ln \frac{1}{1-p_j} = \beta \ln t_j - \beta \ln \theta \end{cases} \quad (13)$$

Suppose that the shape parameter  $\beta$  satisfies  $\beta > 1$ , the following relationship can be get from the two equations above:

$$\ln \frac{\ln R_i}{\ln R_j} = \beta \ln \frac{t_i}{t_j} < \ln \frac{t_i}{t_j} \quad (14)$$

Equation (13) is equivalent to  $\frac{\ln R_i}{\ln R_j} < \frac{t_i}{t_j}$  and  $R_j < (R_i)^{\frac{t_j}{t_i}}$ . That means:

$$p_j = 1 - R_j > p_i' = 1 - (R_i)^{\frac{t_j}{t_i}} \quad (15)$$

Where:

$$p_i' = 1 - (R_i)^{\frac{t_j}{t_i}} > 1 - R_i = p_i \quad (16)$$

When  $0 < t_1 < t_2 < \dots < t_m$ , there is:

$$\frac{\ln R_1}{t_1} > \dots > \frac{\ln R_i}{t_i} > \dots > \frac{\ln R_j}{t_j} > \dots > \frac{\ln R_m}{t_m} \quad (17)$$

Where  $R_j > (R_m)^{\frac{t_j}{t_m}}$  equals to:

$$1 - R_j = p_j < 1 - (R_m)^{\frac{t_j}{t_m}} \quad (18)$$

And there is:

$$1 - (R_m)^{\frac{t_j}{t_m}} < 1 - R_m = p_m \quad (19)$$

Generally, the range of  $p_j$  can be expressed as:

$$p_i' < p_j < p_m^* \quad (20)$$

Incomplete Beta distribution  $B(\theta_1, \theta_2, a, b)$  is used to describe the prior distribution of  $p_i$ . The density function is [31]:

$$\pi(x) = \frac{(x - \theta_1)^{a-1} (\theta_2 - x)^{b-1}}{B(a, b) (\theta_2 - \theta_1)^{a+b-1}}, \theta_1 < x < \theta_2 \quad (21)$$

The estimated failure probability  $p_1$  using the hierarchical Bayesian method is given by:

$$\hat{p}_1 = \frac{(1 + s_1) \ln(\frac{s_1 + c + 1}{s_1 + 2}) - s_1 \ln(\frac{s_1 + c}{s_1 + 1})}{c - 1 - s_1 \ln(\frac{s_1 + c}{s_1 + 1})} \quad (22)$$

Suppose that  $p_m^*$  is the upper bound of  $p_m$ . Incomplete Beta distribution  $B(\hat{p}_{i-1}', p_m^*, 1, b)$  is chosen as the prior distribution of  $p_i (i > 1)$ . Then the prior distribution of  $p_i$  can be expressed as:

$$\pi(p_i | b) = \frac{(p_m^* - p_i)^{b-1}}{B(1, b) (p_m^* - \hat{p}_{i-1}')^b}, \hat{p}_{i-1}' < p_i < p_m^* \quad (23)$$

Assuming that  $p_m^* = 1$ , the hierarchical prior probability density function of failure probability  $p_i (i > 1)$  is:

$$\pi(p_i) = \frac{1}{c - 1} \int_1^c \frac{(1 - p_i)^{b-1}}{B(1, b) (1 - \hat{p}_{i-1}')^b} db \quad (24)$$

The estimated failure probability  $\hat{p}_i$  using the WIB method under the quadratic loss condition is given by:

$$\hat{p}_{WIBi} = (1 - \hat{p}_{i-1}') \frac{(1 + s_i) \ln(\frac{s_i + c + 1}{s_i + 2}) - s_i \ln(\frac{s_i + c}{s_i + 1})}{c - 1 - s_i \ln(\frac{s_i + c}{s_i + 1})} + \hat{p}_{i-1}' \quad (25)$$

Where:



$$\hat{p}_{i-1}' = 1 - (1 - \hat{p}_{i-1})^{\frac{t_i}{t_{i-1}}} \quad (26)$$

In WIB method, the estimated failure probability  $\hat{p}_i$  is a revised value which is not only related to  $\hat{p}_{i-1}$ , but also associated with  $t_i$  and  $t_{i-1}$ . Hence, this estimation method fully considers the information of zero-failure data.

In the light of the above analysis, the final expressions for estimating the failure probability of the 4 methods are listed in Table 1.

TABLE 1 Common models to represent failure probability

| Name     | Method                       | Failure Probability Estimation Equation  |
|----------|------------------------------|--|
| Method 1 | Traditional method           | $\hat{p}_{TMI} = \frac{0.5}{s_i + 1}$  |
| Method 2 | Classical Bayesian method    | $\hat{p}_{CBI} = \frac{1}{s_i + 2}$  |
| Method 3 | Hierarchical Bayesian method | $\hat{p}_{HBI} = \frac{(1 + s_i) \ln(\frac{s_i + c + 1}{s_i + 2}) - s_i \ln(\frac{s_i + c}{s_i + 1})}{c - 1 - s_i \ln(\frac{s_i + c}{s_i + 1})}$                                       |
| Method 4 | WIB method                   | $\hat{p}_{WBI} = (1 - \hat{p}_{i-1}') \frac{(1 + s_i) \ln(\frac{s_i + c + 1}{s_i + 2}) - s_i \ln(\frac{s_i + c}{s_i + 1})}{c - 1 - s_i \ln(\frac{s_i + c}{s_i + 1})} + \hat{p}_{i-1}'$ |

### 2.3.2 Estimation of distribution parameters

Estimation of the distribution parameters is carried out by utilizing the least-squares method. In order to take advantage of information get from zero-failure data, the weighted least squares method is applied to the parameters estimation process.

The failure probability of the product is  $p_i$ ; when  $t=t_i$ , the estimation of it is  $\hat{p}_i$ . There will be an equation as follows:

$$\hat{p}_i = 1 - \exp \left[ - \left( \frac{t_i}{\theta} \right)^\beta \right] \quad i = 1, 2, \dots, m \quad (27)$$

Hence, the following result can be obtained by taking logarithm of Equation (26):

$$\ln t_i = \ln \theta + \beta^{-1} \ln \ln(1 - p_i)^{-1} \quad (28)$$

Let  $y_i = \ln t_i$ ,  $x_i = \ln \ln(1 - p_i)^{-1}$ ,  $\mu = \ln \theta$ ,  $\sigma = \beta^{-1}$ , then the equation can be presented as:

$$y_i = \mu + \sigma x_i + \varepsilon_i \quad i = 1, 2, \dots, m \quad (29)$$

Where  $\varepsilon_i$  is the error caused by replacing the true value  $p_i$  with estimated value  $\hat{p}_i$ . The weighted coefficient is:

$$w_i = n_i t_i / \sum_{j=1}^m n_j t_j \quad (30)$$

The parameter  $\hat{\mu}$  and  $\hat{\sigma}$  can be estimated according to the principle of least square when the following equation is minimal:

$$\sum_{i=1}^m w_i (y_i - \hat{\mu} - \hat{\sigma} x_i)^2 \quad (31)$$

Then  $\hat{\mu}$  and  $\hat{\sigma}$  can be computed by:

$$\begin{cases} \hat{\mu} = \frac{BC - AD}{B - A^2} \\ \hat{\sigma} = \frac{D - AC}{B - A^2} \end{cases} \quad (32)$$

Where  $A = \sum_{i=1}^m w_i x_i$ ,  $B = \sum_{i=1}^m w_i x_i^2$ ,  $C = \sum_{i=1}^m w_i y_i$ ,  $D = \sum_{i=1}^m w_i x_i y_i$ .

Thus, the estimation of the scale parameter and shape parameter is:

$$\hat{\theta} = \exp \hat{\mu} \quad (33)$$

$$\hat{\beta} = \hat{\sigma}^{-1} \quad (34)$$

### 2.3.3 Estimation of reliability model

The estimate of reliability  $\hat{R}(t)$  expression for the product is:

$$\hat{R}(t) = \exp \left[ - \left( \frac{t}{\hat{\theta}} \right)^{\hat{\beta}} \right] \quad (35)$$

It can be seen from Equation (35) that point estimate of reliability model is obtained through the existing zero-failure data.

## 3 Case study

The zero-failure data of cutting tools was extracted from the field production data in titanium alloy milling process. All the milling work was carried out on a CNC Vertical Machining Center (RAMBAUDI RAMMATIC-1201G) under fluid cooling condition. Down milling method was adopted. The axial depth of cut  $a_p$  was 20mm and the radial depth of cut  $a_e$  was 1mm. The parameters of milling tool are shown in Table 2. The cutting speed and feed rate are shown in Table 3. The material used for the milling job was Ti-6Al-4V alloy.

TABLE 2 Details of the milling tool


| Details            | Value |  |
|--------------------|-------|---|
| Teeth              | 4     |   |
| Diameter(mm)       | 20    |   |
| Cut length(mm)     | 40    |   |
| Overall length(mm) | 125   |   |
| Helix angle        | 48°   |   |
| Corner radius (mm) | 0     |   |

TABLE 3 Cutting parameters

| Cutting speed<br>$v_c(\text{m/min})$ | Spindle speed<br>$n(\text{r/min})$ | Feed rate<br>$f(\text{mm/min})$ | depth of cut<br>$a_p(\text{mm})$ | width of cut<br>$a_e(\text{mm})$ |
|--------------------------------------|------------------------------------|---------------------------------|----------------------------------|----------------------------------|
| 94.2                                 | 1500                               | 600                             | 20                               | 1                                |

Each milling job was stopped at a specific time, and 30 sets of data were collected. None of the milling tools was failure at last.

All of the milling tools are divided into 6 groups, the cutting times are shown in Table 4.

According to the definition of zero-failure data model above, the minimum time in each group is regarded as the truncated time. Then the zero-failure data can be obtained, which is shown in Table 5. According to the experience of the experts, the reliability of the milling tool is no more than 0.3 after 180min and no more than 0.2 after 240min. It can be seen that only two cutters have been worked for more than 180 minutes, which indicates that the failure probability is high when the cutting time reaches 180 minutes. The experience of the experts is used for evaluating the accuracy of the 4 models proposed above.

TABLE 4 30 sets of milling tool data

| Group | Cutting time(min)                 |
|-------|-----------------------------------|
| 1     | 8.1; 8.1; 8.1; 41.6; 41.6         |
| 2     | 57.7; 57.7; 80.7; 80.7; 80.7      |
| 3     | 80.7; 80.7; 80.7; 80.7; 80.7      |
| 4     | 83.1; 83.1; 83.1; 83.1; 88.7      |
| 5     | 96.8; 96.8; 122.2; 138.4; 138.4   |
| 6     | 148.9; 161.4; 161.4; 188.0; 210.9 |

TABLE 5 Zero-failure data and sample number of milling tools

| $i$ | $t_i$ | $n_i$ | $s_i$ |
|-----|-------|-------|-------|
| 1   | 8     | 5     | 30    |
| 2   | 57    | 5     | 25    |
| 3   | 80    | 5     | 20    |
| 4   | 83    | 5     | 15    |
| 5   | 96    | 5     | 10    |
| 6   | 148   | 5     | 5     |

Assuming that the shape parameter  $\beta > 1$  [32]. The failure probability of milling tools at each truncated time is calculated by using the 4 methods mentioned above, and the results are shown in Table 6. The parameter value of  $c$  is set as 5 when estimating  $\hat{p}_{HBi}$  and  $\hat{p}_{WIBi}$ .

TABLE 6 Reliability estimation of milling tools at each truncated time

| Truncated time      | $\hat{P}_{TMi}$ | $\hat{P}_{CBI}$ | $\hat{P}_{HBI}$ | $\hat{P}_{WIBi}$ |
|---------------------|-----------------|-----------------|-----------------|------------------|
| $t_1=8\text{min}$   | 0.983871        | 0.968750        | 0.970910        | 0.970910         |
| $t_2=57\text{min}$  | 0.980769        | 0.962963        | 0.965945        | 0.782712         |
| $t_3=80\text{min}$  | 0.976190        | 0.954545        | 0.958926        | 0.679916         |
| $t_4=83\text{min}$  | 0.968750        | 0.941176        | 0.948239        | 0.635462         |
| $t_5=96\text{min}$  | 0.954545        | 0.916667        | 0.929931        | 0.550426         |
| $t_6=148\text{min}$ | 0.916667        | 0.857143        | 0.890874        | 0.354863         |

Comparing with the experiences of specialists, the failure probability estimation results of the milling tools are more acceptable in engineering applications using the WIB method proposed in this paper. The estimated reliability is about 0.35 at the truncated time  $t_6=148\text{min}$ . It can be found that in the other three cases, the results of reliability estimation are more than 0.8 at  $t_6=148\text{min}$ , which is out of accord with the actual situation. In conclusion, the accuracy of these models have a certain degree of inadequacy when used for reliability estimation of milling tools. Therefore, the WIB method is chosen as the failure probability estimation method in the following.

The parameters of the lifetime probability distribution of the milling tool can be gained using weighted least square method and the results are  $\hat{\beta}=1.37$  and  $\hat{\theta}=147.31$ , which satisfy the assumption that  $\beta>1$ .

The reliability  $R(t)$  of the milling tool can be expressed as:

$$\hat{R}(t) = \exp \left[ - \left( \frac{t}{147.31} \right)^{1.37} \right] \quad (36)$$

According to the experiences of specialists, the reliability of the milling tool is expected no more than 0.3 when the cutting time is more than 180min and no more than 0.2 when the cutting time is more than 240min. The reliability at 180min and 240min can be obtained from the reliability function:

$$\begin{cases} \hat{R}(180) = 0.27 \\ \hat{R}(240) = 0.14 \end{cases} \quad (37)$$

From Equation (37), the WIB method proposed in this paper is more suitable for engineering application and consistent with the experts' knowledge.

The expectation of cutting tool life can be obtained through Weibull distribution function, which can be expressed as:

$$\hat{E}(T) = \hat{\theta} \Gamma \left( 1 + \frac{1}{\hat{\beta}} \right) = 147.31 \Gamma \left( 1 + \frac{1}{1.37} \right) = 134.67 \text{ min} \quad (38)$$

The expectation of tool life is considered as the reference value of the milling tool life under specific working condition.

Considering the value of  $c$  is determined based on experience, the influence of  $c$  on the estimation result should be evaluated. By using different values of  $c$ , the estimated reliability is shown in Table 7. The estimation of the parameters of lifetime probability distribution as well as the reliability of the milling tool at 180min and 240min are

shown in Table 8.

It can be found that with the increase of  $c$ , the reliability increases gradually. The change of  $c$  has influence on the estimation of reliability. In the meanwhile, the estimation results utilizing different values of  $c$  are consistent with expert knowledge yet. So it is concluded that the WIB method has good robustness, which is still applicable even if the value of  $c$  is different due to empirical information.

TABLE 7 Estimation on reliability under different values of  $c$

| $c$ | $t_1=8$  | $t_2=57$ | $t_3=80$ | $t_4=83$ | $t_5=96$ | $t_6=148$ |
|-----|----------|----------|----------|----------|----------|-----------|
| 2   | 0.969279 | 0.771594 | 0.664123 | 0.616734 | 0.526182 | 0.322273  |
| 3   | 0.969832 | 0.775356 | 0.669464 | 0.623094 | 0.534486 | 0.333592  |
| 4   | 0.970379 | 0.779083 | 0.674758 | 0.629371 | 0.542605 | 0.344484  |
| 5   | 0.970910 | 0.782712 | 0.679916 | 0.635462 | 0.550426 | 0.354863  |
| 6   | 0.971422 | 0.786224 | 0.684909 | 0.641340 | 0.557924 | 0.364739  |
| 7   | 0.971915 | 0.789614 | 0.689730 | 0.646998 | 0.565102 | 0.374147  |
| 8   | 0.972390 | 0.792881 | 0.694380 | 0.652441 | 0.571976 | 0.383125  |
| 9   | 0.972845 | 0.796030 | 0.698865 | 0.657677 | 0.578562 | 0.391710  |

TABLE 8 Estimation on Weibull distribution parameters and reliability

| $c$            | 2      | 3      | 4      | 5      | 6      | 7      | 8      | 9      |
|----------------|--------|--------|--------|--------|--------|--------|--------|--------|
| $\hat{\beta}$  | 1.40   | 1.39   | 1.38   | 1.37   | 1.37   | 1.36   | 1.36   | 1.35   |
| $\hat{\theta}$ | 138.77 | 141.62 | 144.48 | 147.31 | 150.09 | 152.82 | 155.51 | 158.15 |
| $\hat{R}(180)$ | 0.24   | 0.25   | 0.26   | 0.27   | 0.28   | 0.29   | 0.30   | 0.30   |
| $\hat{R}(240)$ | 0.12   | 0.12   | 0.13   | 0.14   | 0.15   | 0.16   | 0.17   | 0.17   |
| $\hat{E}(T)$   | 126.47 | 129.22 | 131.96 | 134.67 | 137.32 | 139.93 | 142.49 | 145.01 |

In order to evaluate the performance of the presented method, stochastic simulation is adopted to calculate the estimation results of distribution parameters. By using MATLAB, 30 pseudorandom numbers obeying Weibull distribution with  $\beta = 1.37$  and  $\eta = 147.31$  are created. The pseudorandom numbers are grouped according to TABLE 9, which is the same as the grouping condition of zero-failure data of cutting tools. The minimum time in each group is regarded as the truncated time. Subsequently, estimates of distribution parameters were performed based on the 30 pseudorandom numbers. The processes of pseudorandom numbers generation and parameters estimation are repeated ten times and average values are taken as the results. TABLE 10 shows the simulation results using different methods mentioned above. It is worthy noted that WIB method shows the minimum error comparing with other method, which can be chosen for reliability assessment of cutting tools coping with zero-failure data.

TABLE 9 Grouping condition of 30 random numbers

| $i$ | $n_i$ | $s_i$ |
|-----|-------|-------|
| 1   | 5     | 30    |
| 2   | 5     | 25    |
| 3   | 5     | 20    |
| 4   | 5     | 15    |
| 5   | 5     | 10    |
| 6   | 5     | 5     |

TABLE 10 Results of estimation on distribution parameters

| Method                       | $\hat{\beta}$ | $\hat{\theta}$ |
|------------------------------|---------------|----------------|
| Traditional method           | 1.16          | 2466.47        |
| Classical Bayesian method    | 1.08          | 1635.06        |
| Hierarchical Bayesian method | 0.91          | 3332.43        |
| WIB method                   | 1.30          | 201.03         |

## 4 Conclusion

(1) In the case of zero-failure data, this paper proposed a novel methodology for estimating the prior distribution of failure probability of cutting tools. The new method is based on the hierarchical Bayesian method by combining the characteristics of Weibull distribution and Incomplete Beta distribution.

(2) Weibull distribution is chosen as cutting tool life distribution model for reliability evaluation. Matching distribution curve method and weighted least squares method are applied to estimate the distribution parameters.

(3) The failure probability of each truncated time can be get from WIB method, which fully considers the information of zero-failure data collected from the workshop. Subsequently, point estimation of cutting tool reliability is obtained. By comparing with the experience information given by specialists, estimation results show good agreement with expert knowledge. Finally, the robustness of this method is discussed. The proposed method is still applicable even if the value of  $c$  is different due to empirical information.

(4) Random simulation is adopted to evaluate the performance of different methods. WIB method shows the minimum error comparing with other method, which can be chosen for reliability assessment of cutting tools based on zero-failure data.

## Declarations

## Availability of data and materials

There is no data and materials available because all the data mentioned in this paper is listed in the manuscript.

## Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Funding

This work was supported by the National Natural Science Foundation of China under Grant 51975335. This work was also supported in part by the Major Scientific and Technological Project of Shandong Province under Grant 2019GGX104008 and Grant 2019GGX104006.

## Author contribution

Bin Yang performed theoretical analysis and data analysis. The manuscript was written by Bin Yang. The project was conceived, planned and supervised by Kai Guo and Jie Sun. Bin Feng and Chang'an Zhou conducted the case study. The manuscript was corrected by Kai Guo.

## Acknowledgments

The authors would like to acknowledge the financial support from the National Natural Science Foundation of China under Grant 51975335 as well as the Major Scientific and Technological Project of Shandong Province under Grant 2019GGX104008 and Grant 2019GGX104006.

## Reference

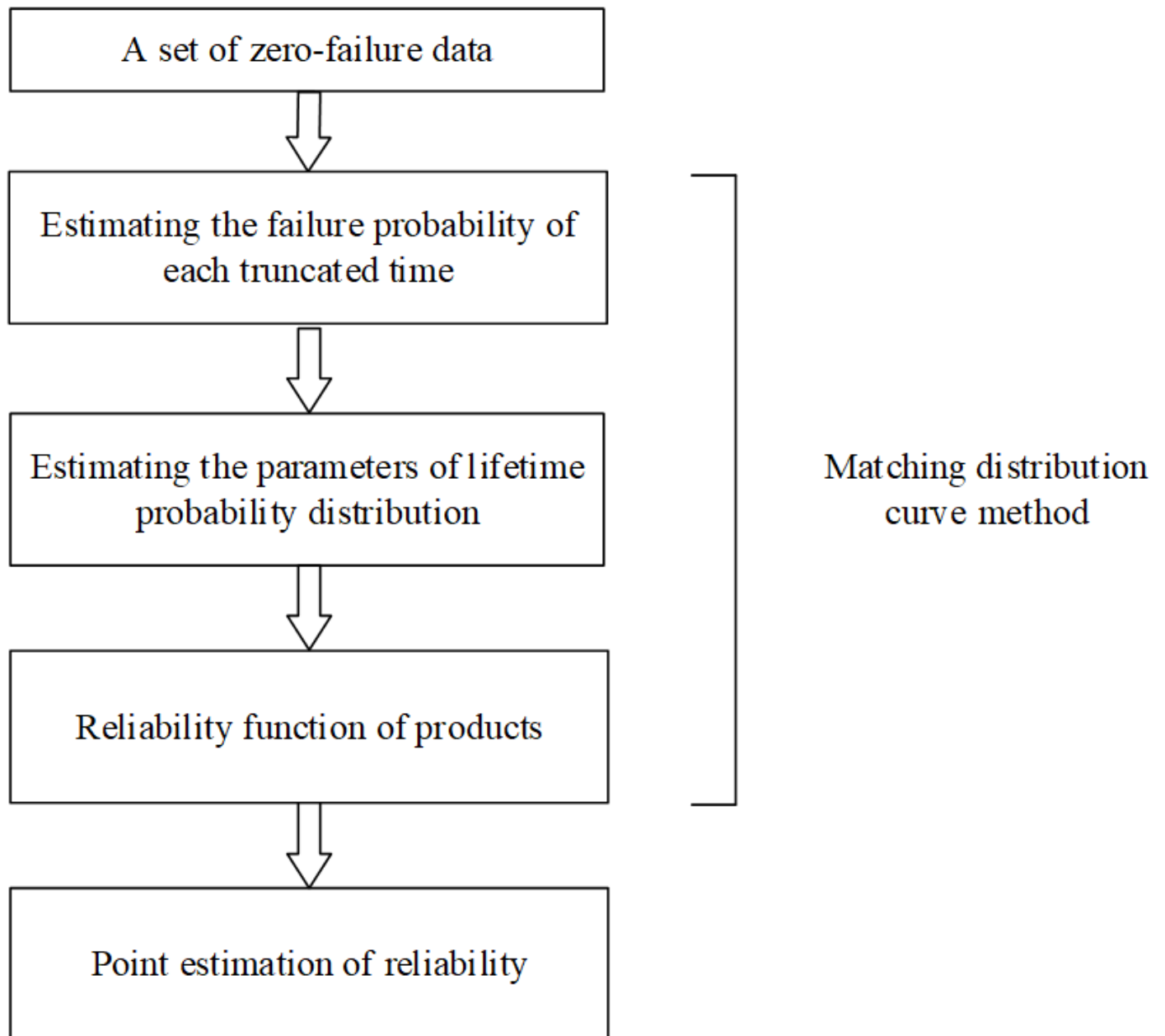
- [1] Zhu Z, Guo K, Sun J, et al. Evolution of 3D chip morphology and phase transformation in dry drilling Ti6Al4V alloys[J]. *Journal of Manufacturing Processes*, 2018, 34:531-539.
- [2] Li A, Zhao J, Luo H, et al. Progressive tool failure in high-speed dry milling of Ti-6Al-4V alloy with coated carbide tools[J]. *International Journal of Advanced Manufacturing Technology*, 2012, 58(5-8):465-478.
- [3] Wiklund H . Bayesian and regression approaches to on-line prediction of residual tool life[J]. *Quality and Reliability Engineering International*, 1998, 14(5):303-309.
- [4] Wager J G, Barash M M. Study of the Distribution of the Life of HSS Tools[J]. *Journal of Engineering for Industry*, 1971, 93(4): 1044-1050.
- [5] Liu H, Makis V. Cutting-tool reliability assessment in variable machining conditions[J]. *IEEE Transactions on Reliability*, 1996, 45(4): 573-581.
- [6] Klim Z, Ennajimi E, Balazinski M, et al. Cutting tool reliability analysis for variable feed milling of 17-4PH stainless steel[J]. *Wear*, 1996, 195(1-2): 206-213.
- [7] Wang K S, Lin W S, Hsu F S. A new approach for determining the reliability of a cutting tool[J]. *The International Journal of Advanced Manufacturing Technology*, 2001, 17(10): 705-709.
- [8] Rodriguez C E P, de Souza G F M. Reliability concepts applied to cutting tool change time[J]. *Reliability Engineering & System Safety*, 2010, 95(8): 866-873.
- [9] Ding F, He Z. Cutting tool wear monitoring for reliability analysis using proportional hazards model[J]. *The International Journal of Advanced Manufacturing Technology*, 2011, 57(5-8): 565-574.
- [10] Salonitis K, Kolios A. Reliability assessment of cutting tool life based on surrogate approximation methods[J]. *The International Journal of Advanced Manufacturing Technology*, 2014, 71(5-8): 1197-1208.

- [11] Wang X, Wang B, Lv C, et al. Research on tool change time and the dynamic reliability of the machining process based on sensitivity analysis[J]. The International Journal of Advanced Manufacturing Technology, 2017, 89(5-8): 1535-1544.
- [12] Chen B, Chen X, Li B, et al. Reliability estimation for cutting tools based on logistic regression model using vibration signals[J]. Mechanical Systems and Signal Processing, 2011, 25(7): 2526-2537.
- [13] Li H, Wang Y, Zhao P, et al. Cutting tool operational reliability prediction based on acoustic emission and logistic regression model[J]. Journal of Intelligent Manufacturing, 2015, 26(5): 923-931.
- [14] Liu S, Zhang H, Li C, et al. Fuzzy reliability estimation for cutting tools[J]. Procedia CIRP, 2014, 15: 62-67.
- [15] Guo K, Yang B, Sun J, et al. Investigation on the Tool Wear Model and Equivalent Tool Life in End Milling Titanium Alloy Ti6Al4V [C]. ASME 2018 13th International Manufacturing Science and Engineering Conference, MSEC 2018. V004T03A033.
- [16] Martz Jr H F, Waller R A. A Bayesian zero-failure (BAZE) reliability demonstration testing procedure[J]. Journal of Quality Technology, 1979, 11(3): 128-138.
- [17] Han M. Confidence limits of reliability parameters for Weibull distribution[J]. Journal of Mechanical Strength, 2009, 31(1): 59-62.
- [18] Mao S S, Xia J, Guan W. The reliability analysis of zero-failure data for bearing life test[J]. Chinese Applied Probability and Statistics, 1993, 9(3): 326-331.
- [19] Fan T H, Chang C C. A Bayesian zero-failure reliability demonstration test of high quality electro-explosive devices[J]. Quality and Reliability Engineering International, 2009, 25(8): 913-920.
- [20] Jiang P, Lim J H, Zuo M J, et al. Reliability estimation in a Weibull lifetime distribution with zero-failure field data[J]. Quality and Reliability Engineering International, 2010, 26(7): 691-701.
- [21] Xia X. Reliability analysis of zero-failure data with poor information[J]. Quality and Reliability Engineering International, 2012, 28(8): 981-990.
- [22] Han M, Ding Y. Synthesized expected Bayesian method of parametric estimate[J]. Journal of systems science and systems engineering, 2004, 13(1): 98-111.
- [23] Yin Y C, Huang H Z, Peng W, et al. An E-Bayesian method for reliability analysis of exponentially distributed products with zero-failure data[J]. Eksploatacja i Niezawodność-Maintenance and Reliability, 2016, 18(3): 445-449.
- [24] Vagnorius Z , Rausand M , et al. Determining optimal replacement time for metal cutting tools[J]. European Journal of Operational Research, 2010, 206(2):407-416.
- [25] Li H, Xie L, Li M, et al. Reliability assessment of high-quality and long-life products based on zero-failure data[J]. Quality and Reliability Engineering International, 2019, 35(1): 470-482.
- [26] Prakash G. On shrinkage estimation for the scale parameter of Weibull distribution[J]. Data Science Journal, 2008, 7: 125-136.
- [27] Jia X, Wang X L, Guo B. Reliability assessment for very few failure data and zero-failure data[J]. Journal of Mechanical Engineering, 2016, 52(2): 182-188.
- [28] Ming H. E-Bayes reliability analysis for liquid rocket engine with zero-failure data [J]. Acta Aeronautica et Astronautica Sinica, 2011, 32(12): 2213-2219.



- [29] Berger J O. Statistical decision theory and Bayesian analysis[M]. Springer Science & Business Media, 2013.
- [30] Ng H K T. Parameter estimation for a modified Weibull distribution, for progressively type-II censored samples[J]. IEEE Transactions on Reliability, 2005, 54(3): 374-380.
- [31] Chen W, Cui J, Fan X, et al. Reliability statistical analysis of data only one failure[J]. Chinese Journal of Mechanical Engineering, 2003, 39(9): 44-48.
- [32] Cai G, Chen X, Li B, et al. Operation reliability assessment for cutting tools by applying a proportional covariate model to condition monitoring information[J]. Sensors, 2012, 12(10): 12964-12987.

## Figures



**Figure 1**

Process of matching distribution curve method