An assessment of viscoelastic finite-difference models applied to the Brazilian pre-salt

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An assessment of viscoelastic finite-difference models applied to the Brazilian pre-salt

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Abstract

Viscoelastic waves are modeled using a 3D time-domain finite-difference scheme with three classical rheological models – Maxwell (M), Kelvin-Voigt (KV), Standard Linear Solid (SLS) – each of them with a single relaxation mechanism. Attenuation and dispersion effects are introduced in terms of the quality factor $Q$ for a certain angular frequency $\omega$. Initially, seismic attenuation of the 2D poststack seismic data from Brazilian pre-salt is evaluated through a frequency analysis. After that, numerical results are validated with analytical solutions based on a homogeneous model with constant seismic properties. The behavior of each rheological model are discussed based on phase, wave amplitude and spectrum. A simplified multilayered geological model is proposed considering regions of the poststack seismic data from the Brazilian pre-salt, Búzios field. Synthetic data are then generated for each rheological model considering two approaches in terms of $Q$ distribution: a constant $Q$ and a varying $Q$ per layer. Both waveform and spectra are extracted from seismograms and compared with selected regions of Búzios poststack data. The amplitude spectra confirmed that the shallowest post-salt package is more dissipative than deeper regions.
Moreover, based on the analysis of synthetic data, KV and SLS models proved to be effective to mimic seismic wave propagation at Búzios field. Synthetic results also show that a $Q$ distribution per layer can yield a seismic attenuation behavior similar to the one observed within the data.

**Keywords:** viscoelasticity, rheological models, seismic modeling, quality factor, Búzios field, brazilian pre-salt.

### 1 Introduction

The Brazilian pre-salt and its giant oil and gas reservoirs are responsible for about 75% of the total production in Brazil. Generally, the Brazilian pre-salt fields present multiple challenges such as long distance to shore, ultra deepwater, large and variable thickness salt layer, very heterogeneous carbonate reservoirs, lateral variations of lithology and seismic imaging complexity (Johann and Monteiro, 2016). In this instance, an accurate seismic image of subsurface is crucial to minimize the effects imposed by these factors. Seismic modeling considering some geological features, such as stratified salt, is an alternative to resemble the complex geology observed in the real data supporting of the seismic interpretation (Tan et al, 2022).

Attenuation and dispersion are some of the physical phenomena present in the seismic wave propagation in a realistic medium, which cause phase distortion and amplitude decay. Commonly, seismic attenuation is measured in terms of the quality factor $Q$, which is inversely proportional to the attenuation coefficient (Winkler and Nur, 1982). The quality factor is associated with the physical properties of the rocks. It is well known that attenuation and velocity dispersion significantly impact on the seismic interpretation, compromising the resolution of seismic images and decreasing the signal-to-noise ratio (Wang, 2008).

Viscoelastic wave modeling has been an important tool to help understand the physical phenomena behind seismic attenuation. Rheological mechanical models, such as Maxwell, Kelvin-Voigt and Standard Linear Solid, are suitable alternatives to simulate a lossy media behavior. They combine springs and dashpots which represent the elastic and viscous behaviors, respectively. These models are well known and have been reviewed by several authors (e.g. Findley et al (1989); Qausar (1989); Moczo et al (2007); Carcione (2015); Schuster (2017)).

Standard Linear Solid (SLS) is the most applied rheological model in viscoelastic seismic modeling. For example, Emmerich and Korn (1987) tested the SLS model for incorporating attenuation to seismic waves. Carcione et al (1988) used the SLS rheological model to implement a 2D viscoelastic wave modeling in a homogeneous medium using the pseudo-spectral time integration technique. Robertsson et al (1994) used finite-difference methods to simulate 2D and 3D wave propagation through dispersive and attenuative viscoelastic
media represented by SLS. Moczo et al (2007) introduced basic formulations and properties of the finite-difference schemes incorporating realistic attenuation. Zhu et al (2013) evaluated the accuracy of constant-Q wave propagation using a series of SLS mechanisms. Recently, Liu et al (2018) and Ichou et al (2022) used the SLS model to investigate seismic wave propagation in a viscoelastic isotropic media. Kelvin-Voigt (KV) and Maxwell (M) models were also used to represent attenuative media. Carcione et al (2004) developed a numerical approach for 3D wave simulation in anelastic media based on the KV rheological model. Zhu et al (2011) and Brown et al (2018) tested the effects of the Maxwell (M) model on wave propagation. It is not clear whether each of these models (SLS, KV, M) would better suit the characteristics of the Brazilian pre-salt.

Tan et al (2022) have recently stated that a viscoelastic model is necessary to match the field data in a Brazilian pre-salt field. Therefore, the introduction of attenuation and velocity dispersion in seismic wave modeling is necessary, mainly in complex geological structures such as post and pre-salt reservoirs. These oil fields have an extensive salt layer that causes a large geological deformation in their structures. Generally, carbonate rocks tend to be more challenging than siliciclastic environments, being the pore system one of those reasons. In this regard, the Búzios field is a good example due to its complex geology and its pre-salt carbonate reservoirs. In sandstones, there is numerous literature focused on seismic attenuation (Toksöz et al, 1979; Winkler and Nur, 1982; Jones, 1986; Tisato and Quintal, 2014; Spencer and Shine, 2016). According to Jones (1986), attenuation is strongly affected by fluid saturation, pore-fluid properties and seismic frequency. Effective pressure also affects attenuation: increasing pressure decreases attenuation (Winkler and Nur, 1982; Jones, 1986; Mayr and Burkhardt, 2006).

In a previous work, seismic data from Búzios oil field was compared in terms of wave spectra to 2D viscoelastic wave simulations using the SLS model (Augusto et al, 2021). This paper aims to assess which classical viscoelastic rheological model (Maxwell, KV and SLS) can better represent a typical Brazilian pre-salt field in the context of a 3D time-domain finite-difference viscoelastic wave propagation modeling. Initially, seismic attenuation of Búzios field is evaluated through a poststack seismic data frequency analysis. This study is described in section 3. In a second moment, 3D seismic wave propagation is simulated considering a viscoelastic media ruled by Maxwell, KV and SLS rheological models. The formulation of wave propagation is developed according to each viscoelastic model and then, the method used to solve numerically the equations is presented in section 4.1. The 3D viscoelastic wave simulation is executed in two tests: a homogeneous and a flat layered geological model. For a homogeneous media, synthetic results for all viscoelastic models are validated with their respective analytical solutions. In addition, amplitude, phase spectra and waveform from each mechanical model are analyzed and compared to the elastic case (section 4.2). Thereafter, a typical flat-layered geological model with geophysical properties - density $\rho$; $P$ and $S$ velocities, $v_p$, $v_s$; $P$
and $S$ quality factors, $Q_p, Q_s$ — is proposed based on well log data from Búzios field (section 4.3). Two distributions of quality factor $Q$ are considered: a constant $Q$ for the typical flat-layered geological model; a constant $Q$ per layer. Synthetic seismograms, waveforms and amplitude spectra obtained from elastic and M, KV, SLS viscoelastic models are discussed and compared to real seismic data from Búzios field.

2 Viscoelasticity

The viscoelasticity is the study of materials which behave simultaneously with elastic and viscous characteristics and, therefore, have hereditary properties that lead to dissipative effects. In the linear viscoelastic hypothesis, the current value of the stress tensor $\sigma$ depends upon the complete history of the components of the strain tensor $\varepsilon$ (Christensen, 1982). The following constitutive relation can express it:

$$\sigma_{ij} = \psi_{ijkl} \ast \dot{\varepsilon}_{kl} = \frac{\psi_{ijkl}}{\varepsilon_{kl}} \dot{\varepsilon}_{kl} \quad (1)$$

where $\psi_{ijkl}$ corresponds to the relaxation function, $\ast$ represents a convolution and the over-dot indicates a short-hand notation of the temporal derivative.

As the relaxation function is time-dependent, it can also be analyzed in the frequency domain through the Fourier transform. The complex modulus $Y(\omega)$ is defined as the Fourier transform of the time derivative of the relaxation function $\dot{\psi}$. Given the viscoelastic complex modulus, the quality factor $Q$ relation can be found using the following definition:

$$Q(\omega) = \frac{\text{Re} \{Y(\omega)\}}{\text{Im} \{Y(\omega)\}}, \quad (2)$$

where $\omega$ is the angular frequency, and $\text{Re} \{Y(\omega)\}$ and $\text{Im} \{Y(\omega)\}$ represent the real and imaginary parts of $Y(\omega)$.

2.1 Formulation of Viscoelastic Wave Propagation

For general viscoelastic formulation, the Boltzmann principle (Eq. 1) can be used. According to Christensen (1982), the stress-strain relation for three dimensional linear viscoelastic isotropic media is given by

$$\sigma_{ij} = (\Pi - 2M) \ast \delta_{ij} \dot{\varepsilon}_{kk} + 2M \ast \dot{\varepsilon}_{ij}, \quad (3)$$

where $M$ is the relaxation modulus corresponding to S-waves and it is analogous to the shear modulus $\mu$, $\Pi$ is the relaxation modulus corresponding to P-waves and it is analogous to $\lambda + 2\mu$, where $\lambda$ is the Lamé’s first parameter.
The derivative of the strain tensor with respect to time (\( \dot{\varepsilon} \)) can be written in terms of the particle velocity vector \( \vec{v} \):

\[
\dot{\varepsilon}_{kl} = \frac{1}{2} \left( \frac{\partial v_l}{\partial x_k} + \frac{\partial v_k}{\partial x_l} \right).
\]

(4)

Taking the time derivative of stress-strain relation (Eq. 3), and adding the equation of motion lead to

\[
\frac{\partial v_i}{\partial t} = \frac{1}{\rho} \left( \frac{\partial \sigma_{ij}}{\partial x_j} + f_i \right),
\]

(5)

which defines the velocity-stress formulation in the seismic wave propagation in time domain. \( \rho \) is the density of the media and \( f_i \) corresponds to the source contribution.

The use of the memory variables avoids the convolution calculation present in Equation 3 and this was the approach chosen for this work. The velocity-stress formulation and the memory variables for each model are presented in the following sections.

2.2 Mechanical Models

The viscoelastic mechanical models are a usual way to describe anelastic media taking attenuation and waveform distortion effects into account. They combine springs and dashpots as mechanical components to represent elastic and viscous effects, respectively. In the case of isotropic material symmetry, the elastic-viscoelastic correspondence principle is well established to provide the solution of linear viscoelasticity from mechanical models. In this work, we consider the Maxwell, Kelvin-Voigt and Standard Linear Solid models. They play an important role in the literature which is justified by historical development.

2.2.1 Maxwell Model

The Maxwell model (M) is represented by a spring and dashpot in series. The relaxation function that characterizes this model is described as shown in the following equation:

\[
\psi_M(t) = ke^{-t/\tau}H(t),
\]

(6)

where \( k \) is the elasticity constant of the spring, \( H(t) \) is the Heaviside function and \( \tau \) is a relaxation time, expressed by

\[
\tau = \frac{\eta}{k},
\]

(7)

with \( \eta \) as the dashpot viscosity. Then, its complex modulus \( Y_M(\omega) \) is given by:

\[
Y_M(\omega) = \frac{\omega \eta}{\omega \tau - i},
\]

(8)
where $i$ represents the imaginary unit. Applying the relation in Equation 2, the quality factor of Maxwell model is given by:

$$Q_M = \omega \tau. \quad (9)$$

A material ruled by the Maxwell model causes strong attenuation at low frequencies, and has no losses at high frequencies, because the relaxation time is directly proportional to angular frequency.

For viscoelastic wave propagation, the implementation of Maxwell model is straightforward. Since its relaxation function exhibits an exponential stress relaxation, the use of the memory variable is recommended. The relaxation function, described in Eq. 3, assumes

$$\Pi = \pi e^{-t/\tau^p} H(t) \quad (10)$$

and

$$M = \mu e^{-t/\tau^s} H(t), \quad (11)$$

for the relaxation modulus corresponding to P and S-waves, respectively. Then, the velocity-stress formulation for the Maxwell model is

$$\begin{align*}
\dot{\sigma}_{ij} &= \pi \frac{\partial v_i}{\partial x_i} - 2\mu \frac{\partial v_i}{\partial x_j} + r_{ij}, \quad (i = j) \\
\dot{\sigma}_{ij} &= \mu \left( \frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) + r_{ij}, \quad (i \neq j)
\end{align*} \quad (12)$$

The memory variables are given by

$$\begin{align*}
\dot{r}_{ij} &= - \left( r_{ij} + \frac{1}{\tau^p} \right) \pi \frac{\partial v_i}{\partial x_i} - 2\mu \left( \frac{1}{\tau^s} \right) \frac{\partial v_i}{\partial x_j}, \quad (i = j) \\
\dot{r}_{ij} &= - \left( r_{ij} + \frac{1}{\tau^s} \right) \left( \frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right), \quad (i \neq j)
\end{align*} \quad (13)$$

The P and S relaxation times, which follow the Eq. 7, become

$$\tau_{p,s} = \frac{Q_{p,s}}{\omega} \quad (14)$$

### 2.2.2 Kelvin-Voigt Model

The Kelvin-Voigt model (KV) consists of a linear spring and linear dashpot connected in parallel, in which the stress-strain relation is given by

$$\sigma(t) = k \varepsilon(t) + \eta \ddot{\varepsilon}(t), \quad (15)$$
From this relationship, we can obtain the relaxation function associated with the KV model, described as

$$\psi_{KV}(t) = kH(t) + \eta\delta(t), \quad (16)$$

where $\delta(t)$ is Dirac’s delta function. The corresponding complex modulus function is

$$Y_{KV}(\omega) = k + i\omega\eta, \quad (17)$$

The quality factor for the KV model is given by

$$Q_{KV} = \frac{1}{\omega\tau}, \quad (18)$$

where $\tau$ is the relaxation time, the same defined in the Eq. 7. Unlike the Maxwell model, the KV model induces high attenuation at high frequencies and is lossless at low frequencies.

Unlike Maxwell’s implementation, the Kelvin-Voigt model implementation does not use the relaxation function. For KV model, the stress-strain relation (Eq. 15) is combined to Eqs. 4 and 5 to yield the first and second order velocity-stress equations given by:

$$\dot{\sigma}_{ij} = \pi \frac{\partial v_i}{\partial x_i} \left( 1 + \frac{\tau_p}{\rho} \frac{\partial v_i}{\partial x_i} \right) - 2\mu \frac{\partial v_i}{\partial x_j} \left( 1 + \frac{\tau_s}{\rho} \frac{\partial v_i}{\partial x_j} \right), \quad (i = j)$$

$$\dot{\sigma}_{ij} = \mu \left( \frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) + \frac{\mu \tau_s}{\rho} \left( \frac{\partial^2 v_j}{\partial x_i^2} + \frac{\partial^2 v_i}{\partial x_j^2} \right), \quad (i \neq j) \quad (19)$$

where relaxation times are given by

$$\tau_{p,s} = \frac{1}{\omega Q_{p,s}}. \quad (20)$$

### 2.2.3 Standard Linear Solid Model

The Standard Linear Solid (SLS), also known as Zener model (Zener, 1948), was introduced by Poynting and Thomson (1902). It is a series combination of a spring and a Kelvin-Voigt model. The relaxation function of a material ruled according to the SLS model is given by:

$$\psi_{SLS}(t) = M_R \left[ 1 - \left( 1 - \frac{\tau_{\varepsilon}}{\tau_{\sigma}} e^{-t/\tau_{\sigma}} \right) \right] H(t), \quad (21)$$

where $M_R$ is the relaxation modulus of the medium and $\tau_{\sigma}$ and $\tau_{\varepsilon}$ correspond to the relaxation times. In the frequency domain, its complex modulus is given
by
\[ Y_{SLS}(\omega) = M_R \left( \frac{1 + i\omega\tau_\varepsilon}{1 + i\omega\tau_\sigma} \right). \] (22)

The effects of absorption and dispersion can be quantified using the quality factor \( Q \), which for the Zener model can be written in terms of the relaxation times \( \tau_\sigma \) and \( \tau_\varepsilon \):
\[ Q_{SLS}(\omega) = \frac{1 + \omega^2 \tau_\varepsilon \tau_\sigma}{\omega(\tau_\varepsilon - \tau_\sigma)}. \] (23)

The viscoelastic wave propagation using SLS model follows the same strategy used in Maxwell’s implementation and can also be found in the work of Robertsson et al (1994). The relaxation function assumes the following definition:
\[ \Pi = \pi \left[ 1 - \left( 1 - \frac{\tau_\varepsilon^p}{\tau_\sigma} e^{-t/\tau_\sigma} \right) \right] H(t), \] (24)
and
\[ M = \mu \left[ 1 - \left( 1 - \frac{\tau_\varepsilon^s}{\tau_\sigma} e^{-t/\tau_\sigma} \right) \right] H(t), \] (25)
where \( \tau_\varepsilon^p \) and \( \tau_\varepsilon^s \) are the strain relaxation times for P and S-waves, respectively, and \( \tau_\sigma \), the stress relaxation time for both, P and S-waves.

It gives then:
\[
\begin{cases}
\dot{\sigma}_{ij} = \frac{\tau_\varepsilon^p}{\tau_\sigma} \left( \frac{\partial v_i}{\partial x_i} \right) - 2\mu \frac{\tau_\varepsilon^s}{\tau_\sigma} \left( \frac{\partial v_i}{\partial x_j} \right) + r_{ij}, (i = j) \\
\dot{\sigma}_{ij} = \mu \frac{\tau_\varepsilon^s}{\tau_\sigma} \left( \frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) + r_{ij}, (i \neq j)
\end{cases}
\] (26)

The memory variables are obtained from the solution of the following differential equations:
\[
\begin{cases}
\dot{r}_{ij} = -\frac{1}{\tau_\sigma} \left( r_{ij} + \left( \pi \left( \frac{\tau_\varepsilon^p}{\tau_\sigma} \right) - 2\mu \left( \frac{\tau_\varepsilon^s}{\tau_\sigma} \right) \frac{\partial v_i}{\partial x_i} + 2\mu \left( \frac{\tau_\varepsilon^s}{\tau_\sigma} \right) \frac{\partial v_i}{\partial x_j} \right) \right), (i = j) \\
\dot{r}_{ij} = -\frac{1}{\tau_\sigma} \left( r_{ij} + \mu \left( \frac{\tau_\varepsilon^s}{\tau_\sigma} \right) \left( \frac{\partial v_j}{\partial x_i} + \frac{\partial v_i}{\partial x_j} \right) \right), (i \neq j)
\end{cases}
\] (27)

The relaxation times of SLS model can be written in terms of quality-factor and angular frequency, when the Equation (23) reaches its maximum value:
\[ \tau_\sigma = \sqrt{\frac{1 + \frac{1}{Q_p^2} - \frac{1}{Q_p}}{\omega_0}} \] (28)
and
\[ \tau_\varepsilon^{p,s} = \sqrt{\frac{1 + \frac{1}{Q_{p,s}^2} + \frac{1}{Q_{p,s}}}{\omega_0}}. \] (29)
where the angular frequency is
\[
\omega_0 = \frac{1}{\sqrt{\tau_\varepsilon \tau_\sigma}}.
\] (30)

### 3 Brazilian Pre-Salt Field Data

In order to extract information on seismic attenuation of a real seismic data, a high-resolution seismic reflection data from Búzios is used. The Búzios oil field is located in the central region of Santos Basin, about 180 km from Rio de Janeiro’s coast, with water depth of 1940 m and area of 852.2 km². Today, Búzios field is considered the largest oil field in the ultra-deep waters in the world. This field presents a complex geology structure due to its great heterogeneity, which can cause problems in seismic wave propagation.

A frequency analysis of the seismic section was executed in order to investigate seismic attenuation behavior in the Búzios field. In this regard, we used the near offset seismic data, which has an offset range from 147 to 158 m. The data already counts with preconditioning workflow such as low-cut filter, swell noise attenuation, de-bubble filter and resampling 1 ms to 4 ms. For this study, three regions were selected in the poststack seismic section close

![Fig. 1 Near offset seismic data from Búzios field. In highlight, the regions selected according to the proximity to an exploratory well.](image.png)
to an exploratory well, as shown in Figure 1. The Fourier transform was performed for each region and the resulting amplitude and dB power spectra are displayed in Figure 2.

![Amplitude and dB Power Spectra](image)

**Fig. 2** (a) Amplitude and (b) dB Power spectra extracted from Búzios field seismic data in the selected regions, where each color corresponds to the same region color shown in the Figure 1.

According to the amplitude spectra observed in Figure 2, we can notice that there is a loss in the frequency content between shallow and intermediate regions. Thus, the shallow sedimentary package probably has higher attenuation level than the following regions. Commonly, shallow sedimentary packages in offshore fields have a poor compaction level which can affect factors that strongly contribute to attenuation such as fluid saturation, pore fluid viscosity, confining pressure, and pore geometry (Jones, 1986). On the other hand, the spectra analysis of the subsequent packages (intermediate and deep regions) displays a slight loss in frequency content, suggesting lower seismic attenuation between these regions. Therefore, the evaluation of the amplitude spectra from the poststack seismic data suggests that shallower sediments can substantially attenuate more than deeper ones.

4 Numerical Experiments

In this section, we present the results obtained using our viscoelastic code according to each rheological model in the seismic wave propagation. At the first, general information about the discretization method used to solve numerically the seismic wave propagation in both homogeneous and layered media. Thereafter, the results for homogeneous media are presented, which are the validation and the seismic attributes analysis comparing the elastic and viscoelastic cases. The results for layered model are shown and discussed in the sequence, where they are compared to the Búzios seismic data.
4.1 Model Discretization

The first-order stress-velocity viscoelastic wave equations (Eqs. 12, 19 and 26 for Maxwell, KV and SLS models, respectively) are used to generate synthetic seismograms. The staggered-grid Finite Difference (FD) solver has 2nd-order accuracy in time and 8th-order accuracy in space. The discretization applied to the equations follows the works from Virieux (1986), Levander (1988), Robertsson et al (1994) and Moczo et al (2018). To avoid nonphysical reflections at the limits of the computational domain, the absorbing boundary condition by Cerjan et al (1985) is used.

For the homogeneous media, the model used has $201 \times 201 \times 201$ grid-points in x, y and z directions. The grid spacing is 5 m and the time step is 0.25 ms. The signal source is a zero-phase Ricker wavelet with a peak frequency of 20 Hz. The geological model used for flat layered model has $401 \times 401 \times 401$ grid-points in x, y and z directions. In this simulation, the zero-phase Ricker wavelet used has a peak frequency of 45 Hz. The grid spacing and sampling rate must be lower than those used in the homogeneous test. Based on numerical dispersion criteria, as used Robertsson et al (1994), the grid spacing chosen is 2 m in all spatial directions and the time step is 0.15 ms.

4.2 Homogeneous Model

For the homogeneous test, seismic wave simulation is performed to ensure our viscoelastic implementation, comparing the numerical results with an analytical solution. After that, some seismic attributes are computed in order to evaluate how each rheological model can affect the seismic trace. The physical properties of this model are $V_p = 3500$ m/s, $V_s = 2000$ m/s, $\rho = 2000$ kg/m$^3$, $Q_p$ and $Q_s = 30$, for viscoelastic case; and $Q_p$ and $Q_s = \infty$, for elastic case.

For the validation, a vertical particle velocity ($V_z$) source is placed in the middle of the model. Both analytical and numerical synthetic seismograms of vertical particle velocity component ($V_z$) are generated and each central trace is selected and normalized. The analytical solution is available in Appendix A. Figure 3 shows the comparison between the analytical and numerical solutions obtained using our algorithm. In addition, Figure 3b shows the difference between numerical and analytical solutions. The numerical solutions match their respective analytical solutions, taking into account that the absolute differences were less than 0.025%.

After performing a code validation, a seismic wave simulation is performed using the same homogeneous model. In this case, instead of a vertical particle velocity source, a pressure source is injected in this numerical experiment. As used in the validation test, the center frequency of the zero-phase Ricker wavelet is 20 Hz. Three seismic attributes were investigated: waveform, amplitude and phase spectrum. Here, we can observe how much the waveform is distorted, the amplitude is absorbed and how long the phase is shifted. All these attributes were extracted from the central trace of synthetic seismograms.
for the three viscoelastic cases: Maxwell, KV and SLS. The elastic case was also performed and is used for comparison purposes.

The first seismic attribute investigated is the waveform, whose results are presented in Figure 4a. Comparing the resulting waveforms with the elastic case, we notice that the Maxwell and KV cases presented minimal waveform distortion, while the SLS case yielded considerable distortion. Furthermore, only SLS viscoelastic model caused a shift in the maximum peak of the waveform; hence the wave travels slower than in an elastic media.

In Figure 4b, the amplitude spectra are extracted. Note that all viscoelastic models generated a similar amplitude attenuation compared to elastic spectrum. On the other hand, we can also observe a frequency peak shift for the KV (17.58 Hz) and SLS (18 Hz) viscoelastic cases compared to the elastic case (20 Hz). It confirms that dispersion effects were more effective in KV and SLS viscoelastic cases. For the Maxwell viscoelastic case, there was no peak frequency shift.

The Figure 4c shows the phase spectra. In Maxwell’s case, we can notice the phase shift up to 10 Hz, i.e., at low frequencies. Otherwise, KV viscoelastic
Fig. 4 Comparison of rheological models in homogeneous seismic simulation: (a) waveform, (b) amplitude and (c) phase analysis. Elastic simulation is in black, viscoelastic Maxwell is in orange, viscoelastic Kelvin-Voigt is in magenta and viscoelastic SLS is in cyan.

The case presented a phase shift at high frequencies. The behavior of both Maxwell and KV cases is expected due to their attenuation characteristics (Eqs. 9 and 18), where the Maxwell model has strong attenuation at low frequencies and KV has it at high frequencies. For the SLS case, the maximum phase shift occurred close to the frequency range where is its frequency peak.

4.3 Layered Model

A flat layered geological model is used to generate a seismic attenuation behavior similar to the observed in real seismic data. In this test, seismic wave modeling is also performed in order to analyze waveforms and amplitude spectra of each seismic interface reflection, considering two scenarios in terms of quality factor Q distribution. To fill over all the physical properties of the geological model used in the seismic wave simulation, density and interval velocity are extracted from the same well log data used to select the regions on the
viscoelastic models applied to the brazilian pre-salt seismic section (see figure 5a). therefore, the geological model was divided into four layers. the values of p- and s-wave velocity and density for each layer are shown in table 1. two approaches were adopted for quality-factor values in the multilayered model; here called scenarios 1 and 2. in the first, we considered different q values for each layer. based on qualitative seismic attenuation analysis from a seismic section of the búzios field and the literature, the chosen values are shown in table 2. these values indicate that the most substantial intrinsic attenuation is usually present in saturated, unconsolidated marine sediments (jones, 1986). in scenario 2, it was assumed a constant q-factor value to all layers, based on average values of scenario 1, being \( q_p = 70 \) and \( q_s = 50 \). this approach is widely used in the industry as a q compensation option. the attenuation properties for water layer in both scenarios are \( q_p = 10000 \) and null for \( q_s \). all viscoelastic models were performed for each scenario. the elastic case was also simulated as a benchmark.

unlike the homogeneous model simulation, the source position is located at 30 m of depth and at the center of the model in northing and easting. the geometry acquisition of the horizontally layered model simulation used a pressure source and one receiver every 2 m, totaling 401 receivers along the y-axis at 22 m depth. this source-receiver configuration generated 2d synthetic seismograms for each test simulation. p-wave synthetic seismograms were computed using the pressure component’s wavefield separation procedure. this
### Table 1  Physical properties of the flat layered model

<table>
<thead>
<tr>
<th>Layers</th>
<th>$v_p$(m/s)</th>
<th>$v_s$(m/s)</th>
<th>$\rho$(kg/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>1520</td>
<td>0</td>
<td>1020</td>
</tr>
<tr>
<td>Post-salt #1</td>
<td>1870</td>
<td>1120</td>
<td>2100</td>
</tr>
<tr>
<td>Post-salt #2</td>
<td>3000</td>
<td>1730</td>
<td>2400</td>
</tr>
<tr>
<td>Salt</td>
<td>4500</td>
<td>2580</td>
<td>2050</td>
</tr>
</tbody>
</table>

### Table 2  Attenuation properties of the flat layered model for scenario 1 and 2

<table>
<thead>
<tr>
<th></th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layers</td>
<td>$Q_p$</td>
<td>$Q_s$</td>
</tr>
<tr>
<td>Water</td>
<td>10000</td>
<td>0</td>
</tr>
<tr>
<td>Post-salt #1</td>
<td>40</td>
<td>35</td>
</tr>
<tr>
<td>Post-salt #2</td>
<td>70</td>
<td>50</td>
</tr>
<tr>
<td>Salt</td>
<td>120</td>
<td>100</td>
</tr>
</tbody>
</table>

procedure was done by applying the divergence operator. The main idea is to obtain a single trace from the P-wave synthetic seismogram. Figure 6 shows the P-wave synthetic seismograms for each modeling case in scenarios 1 and 2.

![Fig. 6 Viscoelastic P-wave synthetic seismogram for both scenarios 1 and 2.](image)
From these seismograms, the central traces were selected. The pulses, which correspond to each interface reflection, were isolated and analyzed. Waveform and frequency analysis were performed for each scenario. Figure 7 displays the waveform of the traces selected, where each column corresponds to each scenario. In addition, the viscoelastic traces were compared to the elastic trace for purposes of seismic attenuation assessment. As expected, there is no difference between the elastic and viscoelastic cases at the first reflection due to the acoustic layer. Comparing the second and third pulses, we notice that seismic attenuation level in each layer caused amplitude losses and slight distortion. Nevertheless, as well as in the homogeneous test, the SLS viscoelastic model is the one that presents a more effective deformation in its waveform than the Maxwell and KV. Figure 8 shows the reflections referring to the second and third interfaces in more detail.

The amplitude spectra for the elastic case are shown in Figures 9b and 9c. As expected, there is no difference among the spectra for the different layers, which shows the absence of attenuation and dispersion effects in the medium. According to the amplitude spectra, we only have amplitude loss caused by geometric spreading. When the spectra were normalized (see Figure 9c), we clearly observed no frequency loss.

Figure 10 shows the amplitude spectra of all viscoelastic models for scenarios 1 and 2. For simplicity purposes, we only compared normalized spectra. Considering the Maxwell viscoelastic case, the frequency analysis is displayed.
Fig. 8 Elastic and viscoelastic waveform comparison: (a) second and (b) third interface reflections.

Fig. 9 Elastic wave simulation using 3D layered model: (a) P-wave synthetic seismogram, (b) amplitude spectrum and (c) normalized amplitude spectrum. Color code follows as Figure 5.

in Figures 10a and 10d. The amplitude spectra showed minimal shifted frequency band differences compared to the elastic case, observed at low frequencies in scenarios 1 and 2. The same behavior was also observed in the homogeneous test.

Figures 10b and 10e display the amplitude spectra for KV viscoelastic simulation. In both scenarios, we notice a loss in the frequency content characterized by peak frequency shift per layer. In scenario 1, the Post-Salt#2 and salt reached 34 Hz and 30 Hz, respectively. While in scenario 2, the peak frequencies get around 38 Hz for the Post-Salt#2 and 33 Hz for the salt interface.
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Fig. 10 Amplitude spectrum for 3D layered model for (a) and (d) viscoelastic Maxwell case, (b) and (e) viscoelastic KV case and, (c) and (f) viscoelastic SLS case. In the first row, using different Q-factor values per layer (scenario 1), and in the second row, using a same Q-factor value to all layers (scenario 2). Color code follow as Figure 5.

Therefore, in the scenario where each layer has its own seismic attenuation level (scenario 1), there was greater loss in frequency content between the post-salt 2 (blue curve) and salt (red) interfaces, when compared to scenario 2.

The amplitude spectra for the SLS viscoelastic case are presented in Figures 10c and 10f. In general, the results of KV and SLS viscoelastic models were quite similar. In scenario 1, the Post-Salt #2 and salt peak frequencies and salt reached 35 Hz and 32 Hz, respectively. While in scenario 2, the peak frequencies corresponding to the Post-Salt #2 and salt interfaces reached 40 Hz and 35 Hz, respectively. Comparing both SLS and KV spectra, we can also note that the attenuation was more severe using KV model, where the bandwidth is slightly thinner than the SLS model.

Comparing amplitude spectra extracted from real and synthetic data (Figures 2 and 10, respectively), we can observe that Maxwell viscoelastic model did not meet the seismic attenuation characteristics. Otherwise, KV and SLS viscoelastic models showed considerable correspondence with the spectra.
observed in the Búzios field. Furthermore, scenario 1 obtained better similarity to the real data in spectra analysis. Thus, it confirms that the shallower sedimentary package should be more dissipative than the deeper ones.

5 Conclusion

In this paper, we implemented a 3D viscoelastic modeling based on three rheological models: Maxwell (M), Kelvin-Voigt (KV) and Standard Linear Solid (SLS). The algorithm was developed using staggered grid FD technique in time domain. For each model two tests were performed: a homogeneous and a flat layered media. The comparison of numerical FD results against analytical solutions delivered satisfactory accuracy in accordance with the attenuation characteristics of each viscoelastic rheological model.

The seismic attributes investigated in the homogeneous test were waveform, phase and amplitude spectra. The results showed how each rheological model acts on the seismic wave propagation. The amplitude spectra revealed that all viscoelastic models yield a similar intrinsic amplitude attenuation. For the waveform attribute, the SLS model was the only case that presented considerable distortion when compared to the elastic case. The analysis of phase spectrum helped to understand where the phase shift occurred in each viscoelastic case. While the Maxwell and KV models suffered phase shifts at low and high frequencies, respectively, only the SLS model showed a phase shift in the main frequency range.

The Búzios field case study presents evident attenuation and dispersion characteristics noticed in the analysis of the amplitude spectra extracted from 2D near offset seismic data. The evaluation of amplitude spectra suggested that the shallowest post-salt package is more dissipative than the deepest ones. Wave propagation simulation in a layered model was driven to reproduce a similar attenuation behavior. As expected, Maxwell viscoelastic model presented only attenuation of the amplitude, compared with the elastic case. It is worth mentioning that both the KV and SLS models were able to reproduce characteristics observed in the seismic data of the Búzios field, in terms of amplitude spectra. Furthermore, the amplitude spectra similarity between the real and synthetic seismic data can also allow us to obtain a qualitative relation of the $Q$ factor distribution in the region under study. Moreover, using an effective quality factor value for all lithologies was not a reasonable alternative to incorporate in the $Q$ seismic compensation. On the other hand, using quality factor values per layer or per sedimentary packages yielded results close to the real data.

We conclude that using KV and SLS viscoelastic models to simulate wave propagation for the Búzios field is fully plausible. In particular, the SLS viscoelastic seismic modeling is an efficient approach to model constant $Q$. Nevertheless, the KV mechanical model is an attractive alternative to simulate an anelastic wave propagation because it has less computational cost and yields quite similar results compared to SLS.
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Appendix A Analytical solution for homogeneous viscoelastic media

The viscoelastic analytical solution is obtained using the correspondence principle, which takes into account the elastic solution (Carcione, 2015). The solution presented here is for a force acting in the positive z-direction and an homogeneous medium. Pilant (1979) derives the equation, and its simplification can be found in Gosselin-Cliche and Giroux (2014):

$$W = \frac{F(\omega)}{4\pi \rho R^3 \omega^2} \left\{ \begin{array}{l} x z \left[ \frac{(R^2 k_p^2 - 3 - 3i R k_p) e^{-ik_p R} +}{(3 + 3i R k_s - R^2 k_s^2) e^{-ik_s R}} \right] \hat{i} + \\ y z \left[ \frac{(R^2 k_p^2 - 3 - 3i R k_p) e^{-ik_p R} +}{(3 + 3i R k_s - R^2 k_s^2) e^{-ik_s R}} \right] \hat{j} + \\ (x^2 + y^2 - 2z^2)(e^{-ik_p R} - e^{-ik_s R}) + \\ (z^2 R^2 k_p^2 + i((x^2 + y^2 - 2z^2) R k_p) e^{-ik_p R} + \\ ((x^2 + y^2) R^2 k_s^2 - i(x^2 + y^2 - 2z^2) R k_s) e^{-ik_s R}) \hat{k} \end{array} \right\} \tag{A1}$$

The correspondence principle acts by substituting viscoelastic (complex) wavenumbers $k_p$ and $k_s$, contained in the complex modulus. Thus, the complex modulus of the Maxwell (Equation 8), Kelvin-Voigt (Equation 17) and SLS (Equation 22) model are used in order to simulate each analytical solution.

References


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