

# Hybrid Multi Beam Forming and Multi User Detection Technique for MU MIMO System

A. Mary joy Kinol (✉ [alphonse.kinol@gmail.com](mailto:alphonse.kinol@gmail.com))

Sathyabama Institute of Science and Technology <https://orcid.org/0000-0002-7139-1825>

A Sahaya Anselin nisha

Sathyabama Institute of Science and Technology

Marshiana D

Sathyambama Institute of Science and Technology: Sathyabama Institute of Science and Technology

Krishnamoorthy N.R

Sathyambama Institute of Science and Technology: Sathyabama Institute of Science and Technology

---

## Research Article

**Keywords:** MU-MIMO, Fast Fourier transform, index mapping modulation, multi user detection, bit error rate, ML sub detector, FPGA etc.

**Posted Date:** March 16th, 2021

**DOI:** <https://doi.org/10.21203/rs.3.rs-254314/v1>

**License:** © ⓘ This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

---

# HYBRID MULTI BEAM FORMING AND MULTI USER DETECTION TECHNIQUE FOR MU MIMO SYSTEM

A.Mary Joy Kinol<sup>1</sup>, A.Sahaya Anselin nisha<sup>2</sup> Marshiana.D<sup>3</sup>, Krishnamoorthy.N.R

Alphonse.kinol@gmail.com,

<sup>1,2</sup>. Department of Electronics and Communication Engineering, Sathyabama Institute of science and Technology, Chennai, India

<sup>3,4</sup>. Department of Electronics and Instrumentation Engineering, Sathyabama Institute of science and Technology, Chennai, India

## *Abstract*

Multi user - Multiple-input multiple-output (MU-MIMO) based wireless communication system has several advantage over conventional MIMO systems such as high data rate and channel capacity which drawn great attention recently and prominently preferred for 5G systems. And on the other side interferences due to the multi user mobile environment such as co-channel interference and multiple access interference the overall system performance will be degraded and highly reliable techniques need to be incorporate to improve the Quality of services. Moreover the energy efficiency and compactness requirement of 5G systems presents new challenges to investigate techniques for reliable communications. In this paper we introduce a novel low-complexity radix factorization based fast Fourier transform multi beam former and maximal likelihood –multi user detection (ML-MUD) techniques as signal detector tailored with optimal sub detector systems which results with considerable complexity reduction with intolerable error rate performance. The proposed radix factorized Fast Fourier transform - multi-beam forming (RF-FFT-MBF) architectures have the potential to reduce both hardware complexity and energy consumptions as compared to its state-of-the-art methods while meeting the throughput requirements of emerging 5G devices. Here through simulation results the efficiency of scaled ML sub detector system at the downlink side is compared with the conventional ML detectors. Through experimental results it is well proved that the proposed detector offers significant hardware and energy efficiency with least possible error rate performance overhead.

**Keywords-** MU-MIMO, Fast Fourier transform, index mapping modulation, multi user detection, bit error rate, ML sub detector, FPGA etc.

## *1. INTRODUCTION*

In recent year's multi user - Multiple-input multiple-output (MU-MIMO) system has been emerged as one of the prominent techniques to achieve performance metrics such as quality of services and improved data rate with the limited spectrum availability [1&2]. Since 5G telecommunication system is expected to support prominently many high data rate applications such as IP based mobile broadband applications, IP telephony, live multimedia gaming services etc the demands over maximizing spectral efficiency and channel capacity is keep on increasing. On the other side the energy level and compactness requirements of 5G devices present new challenges such as fully-integrated transceivers with least complexity and energy consumption overhead.

However major challenges are involved in MU-MIMO system over interference caused by multi user channel environment. An efficient MU-MIMO system should provide better performance in reducing the semantic gap between channel capacity enhancement and the attainable quality of services. Several methodologies have been investigated to overcome this co channel interference at the receiver side. In most cases multi beam forming is applied at the transmitter side (uplink MU-MIMO system) and accurate multiuser detection (MUD) is incorporated at the downlink side. Zero-forcing joint beam forming technique to maximize the energy level at the receiver side to satisfy the SINR constraints for each users in MU-MIMO systems [3]. However it requires channel state information to steers the signals into an optimal direction since formulating CSI knowledge for each individual antenna is difficult which results with poor link budget beam forming. Conventional beam forming techniques are not applicable for multi user MIMO interference channel where multiple decentralized users are transmitting too many receivers which is always involved with many challenging and complicated optimization problems compared to many-to-one or one-to-many systems since there is no coordination among the signal transmissions in MU-MIMO systems. Thus, multi-beam forming is essential to increases channel capacity, data rate, as well as the multi user environments which growths explosively in wireless applications which is accomplished using array of multiple antennas. Virtual multi-beam forming for multipoint-to-multipoint transmission system and optimal pre-coding theory is incorporated to nullify the multiple-access interference each among users in MU-MIMO systems [4]. On the side Fast Fourier transform is investigated prominently in recent times which can produces an orthogonal set of RF beams over multi direction. But due to its computationally intense computations and complex natures several complexity reduction and energy-efficiency techniques are incorporated in order to support 5G device requirements. In [5] multiple independent radio frequency (RF) beams are generated using FFT transform and multiplier-less approximation model is used for low-complexity and power efficiency. Developed approximate-FFT algorithms with sparse factorizations to accomplish multi-beam forming for 5G Transceivers [6].

On the other hand receivers should equipped with soft interference canceller (for Multiple accuses and co channel interferences) and an appropriate decoder (to detect each user data over multiple-access system). In general by producing the extrinsic log-likelihood ratio (LLR) for each symbols an efficient implementation of the signal detector was carried out. It is impossible to accommodate multiuser data with output with well approximated Gaussian distribution and this linear MMSE will deteriorate over multiuser environment [7]. In most cases ML has been used as a prominent MUD model which can reduce the performance gap that arises due to lack of coordination among users over MU-MIMO systems. However this nonlinear ML models is always emerged with considerable tradeoff between performance and computational complexity since ML detector is computationally complex and power hungry model. In [8] non coherent maximum likelihood (ML) detector is used which exploits the channel orthogonality approximation to improve the error rate performance with minimized iterative complexity. Though the conventional purpose of using MUD technique is to regulate the co channel interference it is always preferred at the receiver side. Moreover measures to be taken to optimize these signal detection techniques signal detection to meet the complexity reduction at the receiver side with least possible performance degradation overhead [9]. Though optimization technique has been used as powerful technique to accomplish the complexity reduction where it considers only inherent bounds and constraints that arises at the receiver side [10].

In this paper, we combine hardware efficient FFT based multi beam forming with highly optimized maximum likelihood detector which explore following advantages.

- Finite approximation with radix factorization model is used to incorporate critical path reduction due to complex twiddle factor computation and accomplished multi directional beam forming which minimize light to moderate interferences among users.
- Hierarchical sub detector based signal detector is used at the receiver side which will minimize the complexity trade off over higher order modulations such as 16-QAM and 64-QAM.

In addition, to optimize the complexity level multi beam forming and multi user signal detection the proposed framework can offer complete robustness channel variations during multi path propagations.

## **II. MULTI USER MIMO**

Though MIMO systems are having numerous advantages such as improved reliability over fading environment, channel capacity enhancement and spectral efficiency metrics still it has several limitations since it is impractical to equip more number of antennas into mobile devices. The inclusion of additional antennas into the wireless devices may reduce battery life time and increase hardware cost as well. In most cases it suffers with limited diversity which is directly related to number of antennas where it results with poor performance when the channel experience Doppler effect or if it is affected by the channel delay spread. For complete robustness of MU-MIMO system over frequency selective fading channels CCI are widely preferred.

## **III. FFT BASED MULTI BEAM FORMING ARCHITECTURE**

The availability of high end bandwidth ( $>6$  GHz) for improved channel capacity tends to motivate Millimetre-wave (mmW) wireless communication. At mmW communication Radio signal transmission should be more directional and it is essential to accomplish radio propagations using sharp beams. On the other side for multiple access points the transmission need to be done with wide range of sharp beams and also it should results with least power consumption, and maximized system efficiency. For Highly-directional multi beam radio signal propagation in mmW multi-directional beam forming techniques FFT transforms were used. Here FFT is used to generate orthogonal bins driven directional beam forming from a from a uniformly spaced linear antenna array signals. Multiple broadband transmit/receive beams that are orthogonal to each other in space are a critical need for emerging wireless systems as well as defense applications in radar and electronic warfare; as they achieve greater capacity by enabling spatial diversity for MIMO.

### **A. Radix factorization model**

The various issues discussed above in FFT computation tends to invent FFT computation using radix factorizations with improved hardware utilization rate which, in turn, help to improve system performance and energy level as well. The other aspect related to overall latency and energy level assisted transceiver designs are additional performance metrics that can be obtained.

In the proposed radix factorization, the non trivial twiddle factors are converted into trivial twiddle factors which can be easily modeled using simple logical swapping and 2's complemented operations in different stages of FFT computation. Hierarchical index mapping rule is used to decompose the radix-2 DIF FFT into multiple region model and twiddle factors features are extracted using linear index mapping process. This will give reduce number of complex twiddle factor multiplication at different level with order of magnitude of the index map. As stated above, this approach of configuration of the twiddle factors that better adapts to any architectural level changes as like conventional high radix indices.

By integrating decomposition levels in radix-2 DIF FFT through 3-dimensional linear index map number of complex multipliers used for FFT computation is reduces as follows

$$\begin{aligned} n &= \frac{N}{2} n_1 + \frac{N}{4} n_2 + n_3 \{n_1, n_2 = 0, 1, n_3 = 0 \sim \frac{N}{4} - 1\} \\ k &= k_1 + 2k_2 + 4k_3 \{k_1, k_2 = 0, 1, k_3 = 0 \sim \frac{N}{4} - 1\} \end{aligned} \quad (3.1)$$

The DFT has the form of

$$\begin{aligned} X(k_1 + 2k_2 + 4k_3) &= \sum_{n_3=0}^{\frac{N}{4}-1} \sum_{n_2=0}^1 \sum_{n_1=0}^1 x\left(\frac{N}{2} n_1 + \frac{N}{4} n_2 + n_3\right) W_N^{nk} \\ &= \sum_{n_3=0}^{\frac{N}{4}-1} \sum_{n_2=0}^1 \left\{ B_{\frac{N}{2}}^{k_1} \left( \frac{N}{4} n_2 + n_3 \right) \right\} W_N^{(\frac{N}{4} n_2 + n_3)(k_1 + 2k_2 + 4k_3)} \end{aligned} \quad (3.2)$$

Where the FFT stage one has the form of

$$B_{\frac{N}{2}}^{k_1} \left( \frac{N}{4} n_2 + n_3 \right) = x\left(\frac{N}{4} n_2 + n_3\right) + (-1)^{k_1} x\left(\frac{N}{4} n_2 + n_3 + \frac{N}{2}\right) \quad (3.3)$$

Decomposition of radix-2 DIF FFT is represented as follows.

$$W_N^{(\frac{N}{4} n_2 + n_3)(k_1 + 2k_2 + 4k_3)} = (-j)^{n_2(k_1 + 2k_2)} W_N^{n_3(k_1 + 2k_2)} W_{\frac{N}{4}}^{n_3 k_3} \quad (3.4)$$

Substituting the equation (4) into equation (2) and expanding the summation with regard to index  $n_2$ , we have a set of 4 DFTs of length  $\frac{N}{4}$ .

$$X(k_1 + 2k_2 + 4k_3) = \sum_{n_3=0}^{\frac{N}{4}-1} [H_{\frac{N}{4}}^{k_1 k_2}(n_3) W_N^{n_3(k_1 + 2k_2)}] W_{\frac{N}{4}}^{n_3 k_3} \quad (3.5)$$

Then second stage of FFT  $H_{N/4}^{k_1 k_2}(n_3)$  is described as

$$H_{N/4}^{k_1 k_2}(n_3) = B_{\frac{N}{2}}^{k_1}(n_3) + (-1)^{k_2} (-j)^{k_1} B_{\frac{N}{2}}^{k_1}(n_3 + \frac{N}{4}) \quad (3.6)$$

Decomposition each radix-2 FFT stages are achieved recursively to the remaining length equation 4.10 we will get the radix  $2^2$  FFT algorithms. Here at stage 1 50% of non-trivial twiddle factors are transformed into trivial factors (1,-1, j,-j) where only swapping and sign inversions are required. The algorithm is characterized here has all the merit as that of radix-4 but its structures same as radix-2 butterfly.

#### IV. MULTI USER DETECTION TECHNIQUE

The conventional purpose of using MUD method is to reduce the co channel interference. However, in recent years several methodologies are invented in order to develop a MUD that can perform signal detection with least complexity overhead. Ishihara, Koichi, et al., (2009) developed MUD technique using fast Fourier transform (FFT) by converting received symbols into frequency domain as overlapping the blocks and suppress the interference by deriving weights for each symbols from the minimum mean square error (MMSE) criterion which retain at low complexity irrespective of user rate. In general linear MMSE MUD cannot perform well over multi user environment which is highly sensitive to co-channel interferences since inter user interference has no degrees of freedom.

##### A. Sub detector optimization model

In general linear MMSE MUD cannot perform well over multi user environment which is highly sensitive to co-channel interferences. Since ML detector is computationally complex and power hungry model, while linear MMSE is limited in its inter user interference cancellation capability over multi user environment several nonlinear models are emerged with considerable tradeoff between performance and computational complexity. Here we presents an optimized ML based multi user detector model for discriminating the user's data streams with unique channel parameters in order to reduce the inter user interference.

#### V. PERFORMANCE RESULTS

In this section, both MATLAB simulations and digital architecture implementations are carried out to demonstrate the error performance and design complexity respectively, of the proposed multi beam former and signal detector. The following facts can be observed: (i) All beams formulated with equal gain using FFT transforms are orthogonal to each other and complexity is decrease exponentially as N (FFT point) increases. (ii) Our optimized ML matches with the analytical ML BER very well in moderate and low SNR regimes and end with least discrimination when BER is below  $10^{-3}$ , and explored that our optimal ML has major advantages in terms of complexity and data rate effienecy. In addition, FPGA hardware synthesis results can exploits the finite beneficiary measurements of our radix factorized FFT and optimized ML detectors (Fig 1).

## A. Simulation results

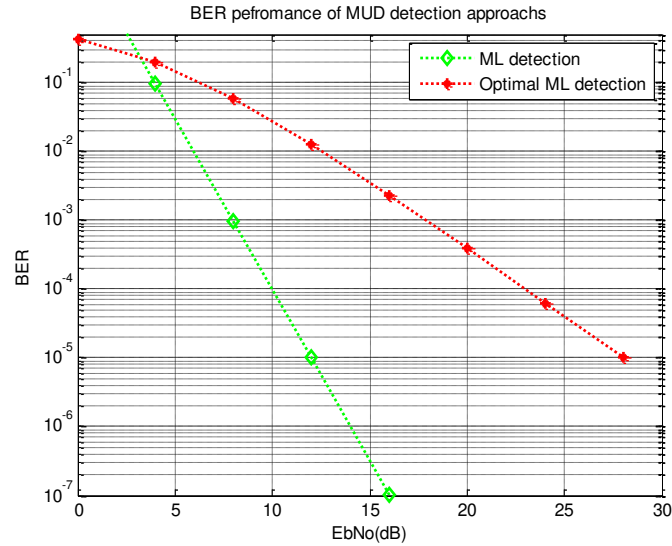
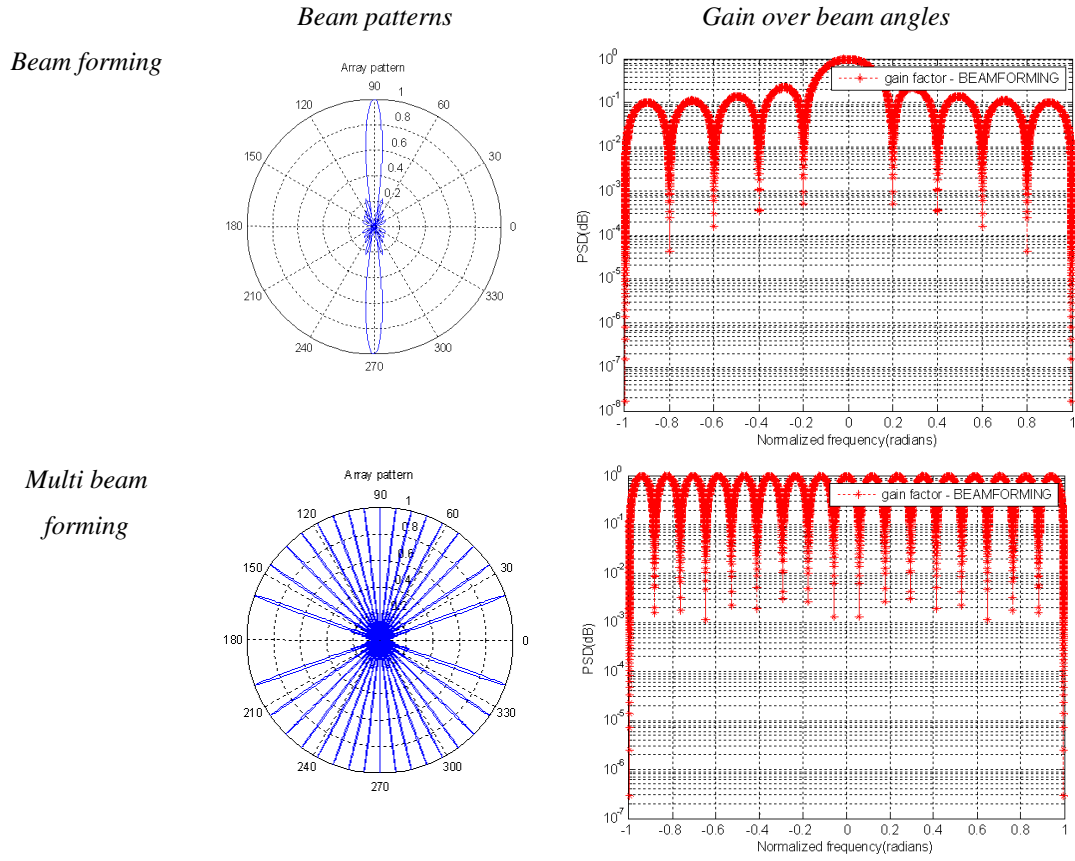


Fig.1 Performance comparison over QoS trade off measure of optimal ML detector over conventional detector

The resultant ML detector is capable of achieving a flexible tradeoff in terms of the achievable BER, least possible design complexity and tolerable error protection. Moreover, by exploiting the benefits of multi beam forming at the transmitter side which can minimized inter-channel interference among users the receiver side can able to detect the signal received using the proposed optimal maximum-likelihood (ML) detector. In this signal detector can able to jointly detect the signals from beneficiary diversity enabled MIMO channels. Though the ML detector complexity is exponentially increased with modulation order and as well as the diversity gain employed at the transmitter side the proposed detector results with least possible design complexity overhead since more simpler mathematical models were used at each sub detectors with least possible iterations and each considers only minimal constellations analytically which is statistically obtained for every modulation order.

As shown in Fig 1 BER vs Eb/No performance of optimal and ML detectors with 16-QAM MU-MIMO system. This comparison results proved the performance level of optimal ML detector has nearly matched to conventional ML detectors with metrics over maximized complexity reduction.

In general the overall performance measure of proposed optimal ML Detector is directly related to the probability of error that can be minimized by using both diverted orthogonal sharp beams and diversified signal transmission using MIMO system.



(a) Array gain over degree of freedom

(b) Multi -beam patterns using FFT

Fig 2. Multi-beam forming

## B. Experimental Results

Here systematic approach of scaling the index of radix with twiddle factor optimized FFT computation is validated over conventional stage optimized FFT radix indices. The area efficiency of radix factorization is analyzed separately. The benefits of critical path delay reduction due to reduced bit level accumulation over precise high radix FFT computation is also proved through delay metrics analyzes.

Here we compare the performance of the proposed radix factorized FFT over conventional high radix regimes and metrics of optimized ML sub detector over using single compound ML as a MUD benchmark schemes which is explored in table 2 with improved hardware efficiency. We performed using verilog HDL implementation and the hardware efficiency was proved using Cyclone III (EP3C16F484C6) ALTERA FPGA implementation. The hardware synthesis was carried without using any degree of tools driven hardware optimization since the core objective of this experiment is to prove the complexity reduction of the aforementioned optimal ML detector designs. Here only architectural level modifications were incorporated as shown in hardware report figure xx which achieved the highest achievable complexity reduction and delay optimization in terms of improved operating frequency.



Table 1 Performance comparison of 8-point radix factorized FFT model.

FFT model used	Logic element used	Fmax report	Power dissipation report(mW)
Radix-2	3405	71.41MHz	178.12mW
Radix-2 <sup>2</sup> methodology	2253	75.18MHz	160.11mW

Table 2 Comparison of proposed optimized ML sub detector model.

Detector model used	Logic cells	Logic registers	DSP elements(9 bit embedded multiplier)	DSP 18x18 elements	LUT's	Fmax report summary(MHz)
ML	2826	1112	112	56	1714	56.59
Optimized ML sub detector	1584	1024	64	32	570	117.9

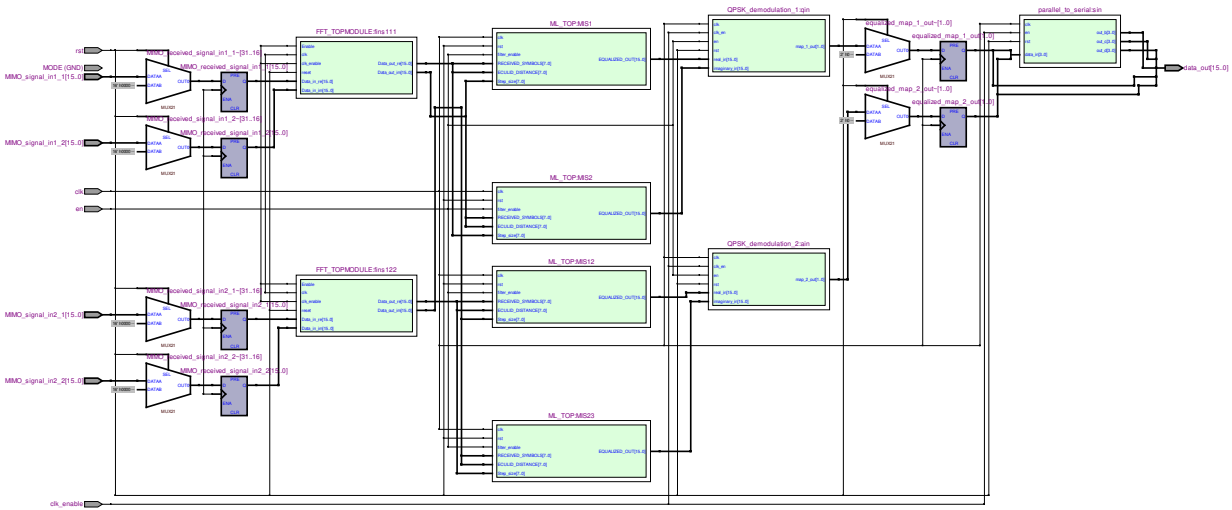


Fig .3 Hardware register transfer level view

## VI. CONCLUSION

The methodologies discussed in this work can provide highly reliable Multi user MIMO radio signal transmission at mm Wave. Here the attenuations caused by multi user dynamic mobile environments are compensated by using directional sharp beams at the transmitter side and appropriate multi user detector at the receiver side. A comprehensive study of the MU-MIMO performance degradation both, implemented based on the IEEE 802.15.3c standard for high data-rate applications, has been presented for the first time. In particular, radix twiddle factors are normalized by trivial conversion and more realistic signal detectors are embedded at the receiver side. The performance assessment of both factorized FFT multi beam former and optimal ML detector was conducted through FPGA hardware synthesis, array gain analyzes over various beam angles and simulated BER analysis over an ITU channel. Moreover, the demands over compactness and high data rate for future 5G devices which employing MU-MIMO radio signal transmission with appropriate methodologies for improved QoS services such as multi beam forming (at the uplink side) and multi user detection (at the downlink side), respectively, has been proposed and analyzed.

## REFERENCES

1. *Abdo Adam Mohamed Ahmed, et al., (2018), "MU-MIMO downlink capacity analysis and optimum code weight vector design for 5G big data massive antenna millimeter wave communication", Wireless Communications and Mobile Computing .*
2. *Ali Ehab et al., (2017), "Beamforming techniques for massive MIMO systems in 5G: overview, classification and trends for future research", Frontiers of Information Technology & Electronic Engineering, Vol. 18.6, pp. 753-772.*
3. *Son Hyukmin and Bruno Clerckx (2014), "Joint beamforming design for multi-user wireless information and power transfer", IEEE Transactions on Wireless Communications, Vol. 13.11, pp. 6397-6409.*
4. *Yu; Wei-Xiao Meng; Ming-Chuan Yang; Li-Ming Zheng; Zhong-Zhao Zhang Virtual multi-beamforming for distributed satellite clusters in space information networks IEEE Wireless Communications ( Volume: 23, Issue: 1, February 2016)*
5. *Dora Suarez Renato J. Cintra, F'abio M. Bayer†Arindam Sengupt, Sunera Kulasekera, Arjuna Madanayake, Multi-Beam RF Aperture Using Multiplierless FFT Approximation Electronics Letters(2014),50(24):1788*
6. *Ariyaratna Viduneth, et al., (2018), "Analog approximate-FFT 8/16-beam algorithms, architectures and CMOS circuits for 5G beamforming MIMO transceivers", IEEE Journal on Emerging and Selected Topics in Circuits and Systems, Vol. 8.3, pp. 466-479.*
7. *Banerjee Smita and Ved Vyas Dwivedi (2018), "Effect of signal to interference ratio on adaptive beamforming techniques", Journal of King Saud University-Engineering Sciences, Vol. 30.4, pp. 313-319.*

8. *Han Gangtao, et al., (2017), "Orthogonal binary modulation division for two-user uplink massive MIMO systems with noncoherent ML detection", IEEE Communications Letters, Vol. 21.2, pp. 294-297.*
9. *Al-Hussaibi Walid .A and Falah H Ali (2018), "Extending the user capacity of MU-MIMO systems with low detection complexity and receive diversity", Wireless Networks, Vol. 24.6, pp. 2237-2249.*
10. *Dahrouj Hayssam and Wei Yu (2010), "Coordinated beamforming for the multicell multi-antenna wireless system", IEEE Transactions on Wireless Communications, Vol. 9.5, pp. 1748-1759.*
11. *Björnson Emil, Erik G Larsson and Thomas L Marzetta (2016), "Massive MIMO: Ten myths and one critical question", IEEE Communications Magazine, Vol. 54.2, pp. 114-123.*
12. *Qian Zhuo and Martin Margala (2016), "Low-power split-radix FFT processors using radix-2 butterfly units", IEEE Transactions on Very Large Scale Integration (VLSI) Systems, Vol. 24.9, pp. 3008-3012.*
13. *Sanguinetti Luca, et al., (2014), "Large system analysis of the energy consumption distribution in multi-user MIMO systems with mobility", IEEE Transactions on Wireless Communications, Vol. 14.3, pp. 1730-1745.*
14. *Vorobyov Sergiy .A (2013), "Principles of minimum variance robust adaptive beamforming design", Signal Processing, Vol. 93.12, pp. 3264-3277.*
15. *Yedavalli Pavan .S et al., (2017), "Far-field RF wireless power transfer with blind adaptive beamforming for Internet of Things devices", IEEE Access, Vol. 5, pp. 1743-1752.*

Author Contributions: "All authors have read and agree to the published version of the manuscript".

Funding: "This research received no external funding".

Conflicts of Interest: "The authors declare no conflict of interest".



Dr. .A.Mary Joy Kinol. She received her B.E degree in Electronics and communication Engineering from Sathyabama university in 2006 and M.Tech degree in VLSI Design from Sathyabama university in 2008 She is a life member of the ISTE. She is currently working as Assistant Professor in the Department of Electronics and communication engineering in JEPPIAAR SRR Engineering College, Chennai. Her area of interest includes RF Circuit Design. MIMO, UWB Communication.



Dr. A.Sahaya Anselin Nisha. She received her B.E. degree in electronics and communication engineering in April 2002 , M.E. degree in applied Electronics with 1<sup>st</sup>rank from sathyabama University in 2006, India and PhD from Sathyabama University in 2013. She has 15 years of teaching experience in various Engineering Colleges. She is currently working as head of department of Electronics and control engineering in Sathyabama University, Chennai, India. She has published more than 20 papers in various journals and Conferences .Her research interests include wireless communication, UWB communication systems, and multiple antenna systems ,RF Circuit design and Microwave Communication . She is a life fellow of the Institution of Electronics and Telecommunication Engineers (IETE), India, Antenna Test and Measurement Society (ATMS), and Institute of Electrical and Electronics Engineers (IEEE).



Dr. Marshiana.D received her B.E degree in Instrumentation and Control in 2002 and Completed her M.E from Annamalai University in 2004. She completed her Ph.D from Sathyabama Institute of Science and Technology in 2018. Her areas of interest include Process control, controller design and soft computing techniques with 30 Publications. She is currently working as Assistant Professor and having teaching experience of 15 years. Her areas of interest include control system and process control, Soft computing techniques and control algorithms. She attended various workshops conducted by the industrials. She guided several student projects and she made research publications in diverse Journals and Conferences.



Dr.N.R.Krishnomoorthy graduated from Jerusalem College of Engineering, Chennai, affiliated to Madras University in 2004 with Bachelor's Degree in Electronics and Communication Engineering. Later he completed his M.E degree in Electronics and Control from Sathyabama Institute of Science and Technology, Chennai in 2007. He has joined as Lecturer in Sathyabama Institute of Science and Technology in 2008. His passion towards teaching made as Associate professor in the department of Electronics and Instrumentation Engineering in Sathyabama Institute of Science and Technology, Chennai. He Completed his Ph.D in 2019 from Sathyabama Institute of Science and Technology. He played vital role in the sathyabamasatnano-satellite project as the payload subsystem incharge. He acquired a teaching experience of more than 13 years. His areas of interest include signal processing, wireless communication, coding techniques and integrated circuits. He attended various workshops conducted by the industrials. He guided several student projects and he made research publications in diverse Journals and Conferences. He is a life member of Indian Society of Technical Education. (ISTE).

# Figures

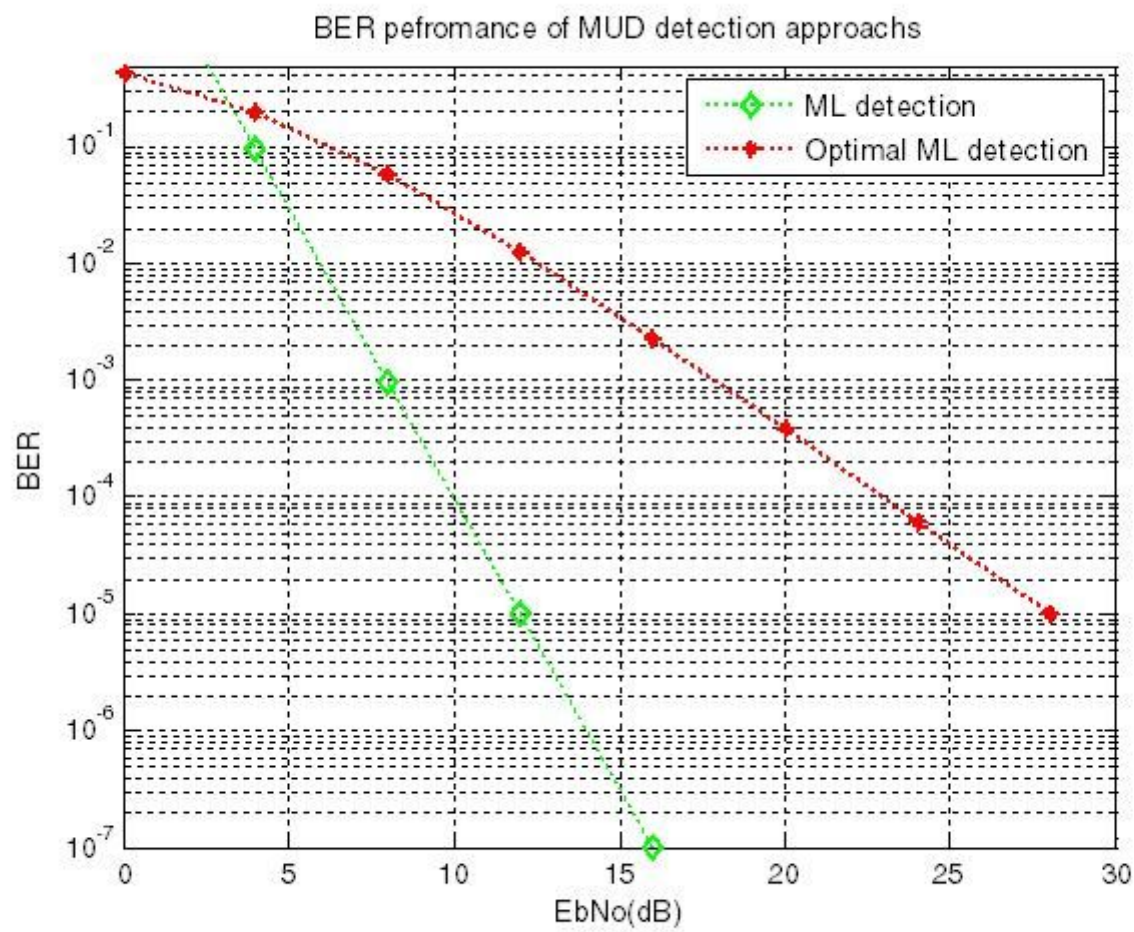


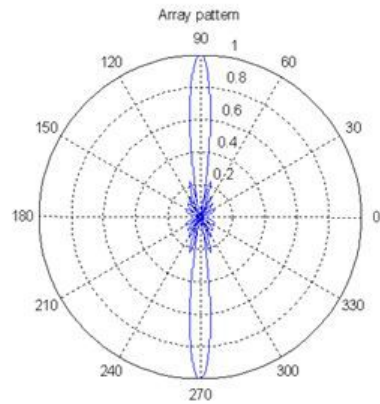
Figure 1

Performance comparison over QoS trade off measure of optimal ML detector over conventional detector

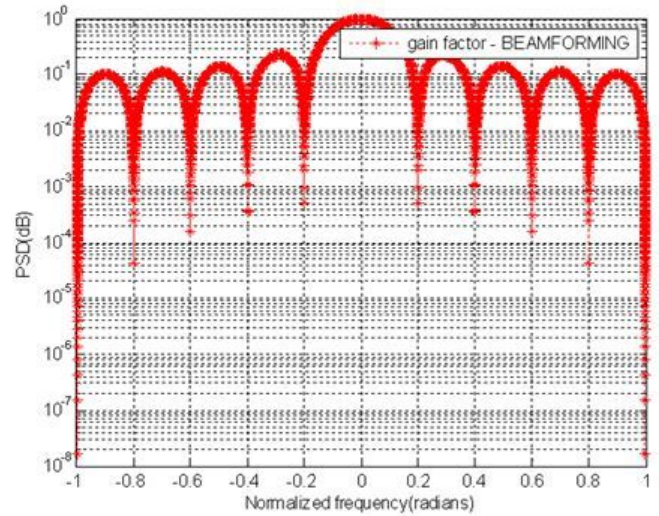


## Beam patterns

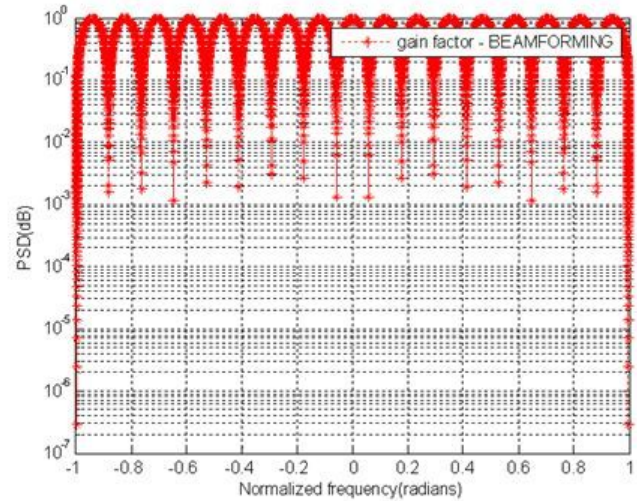
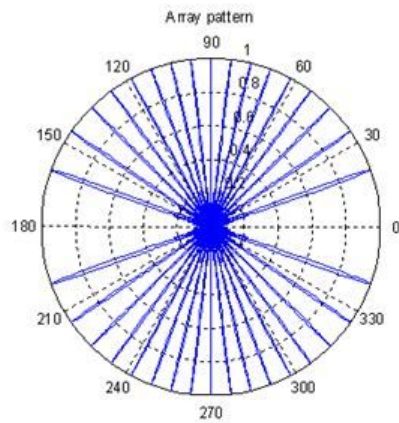
### Beam forming



## Gain over beam angles



### Multi beam forming

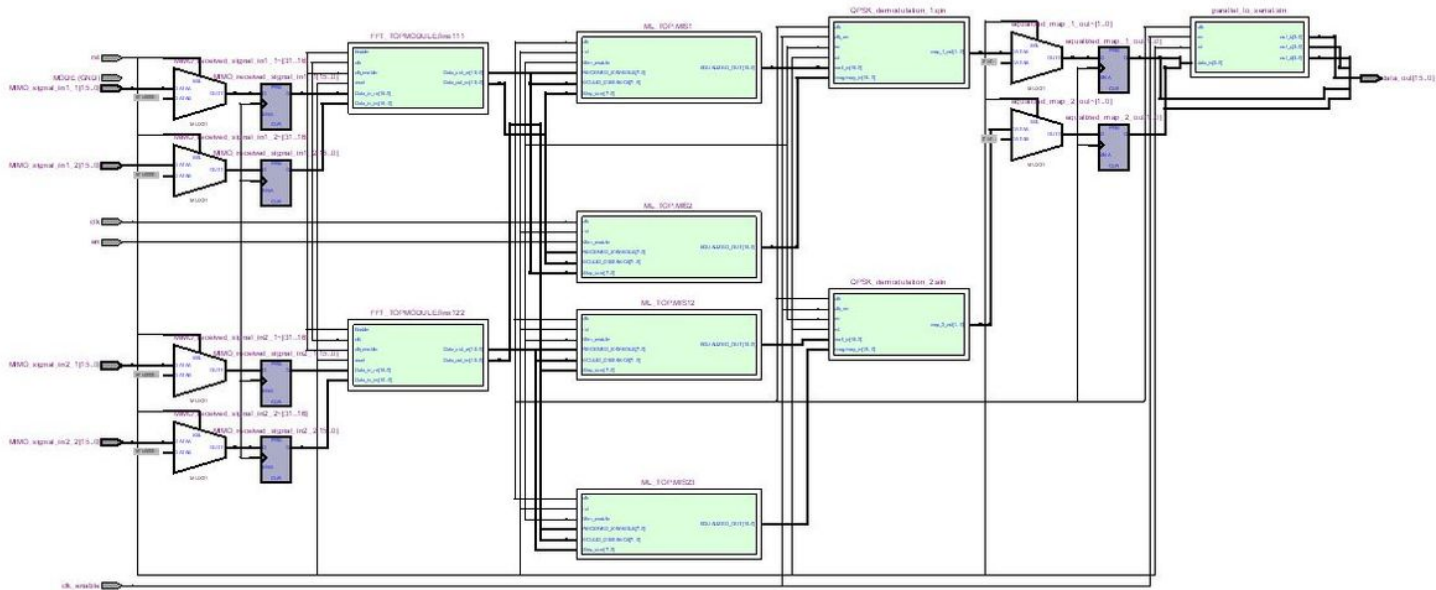


(a) Array gain over degree of freedom

(b) Multi -beam patterns using FFT

Figure 2

Multi-beam forming



**Figure 3**

Hardware register transfer level view