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Fuzzy model multi-attribute decision-making under information systems

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Abstract: This paper combines information systems with fuzzy pattern recognition and proposes an improved fuzzy pattern decision-making method based on it. The decision attributes and conditional attributes are selected according to the decision purpose, and the correlation coefficients of the decision attributes and conditional attributes are derived by combining the original attribute values using the correlation method to determine the weights of each conditional attribute, while the weights of the decision attributes are determined by the decision maker. The standard set and the set to be measured in the fuzzy model are constructed using the affiliation functions of the different attributes. The degree of proximity between the criterion set and the set to be tested is calculated and the solution that best exploits the advantages of the current stage is selected as the optimal solution. The introduction of an information system into the pattern recognition model simplifies the decision-making process by reducing the number of attributes that may be missed in the decision and retaining the characteristics of the original attributes.

Keywords: information system; affiliation function; fuzzy pattern recognition; multi-attribute decision making; closeness

1. Introduction

Information is the cornerstone of decision science, while multi-attribute decision-making (MAMD) approaches are constantly evolving as information becomes increasingly complex. In the 1970s, American scholars proposed the hierarchical analysis method [2], which compares the advantages and disadvantages of each attribute at each level to select a solution. Entropy-TOPSIS method [3-4] is the most widely used multi-attribute decision evaluation method widely used in industry, agriculture, and other fields. However, the actual decision-making process has many reference elements and much information is fuzzy, i.e. uncertain. Therefore fuzzy sets [5] and rough sets [6] have been rapidly applied to decision science after their introduction. In the field of fuzzy decision-making, fuzzy numbers [7-9] combined with the application of the original multi-attribute decision-making methods became more compatible with multi-attribute decision-making. Common types of fuzzy numbers are intuitionistic triangular fuzzy numbers, intuitionistic trapezoidal fuzzy numbers [10-12], Pythagorean fuzzy numbers [13-15], etc. Yue [16] proposed a group decision model based on aggregating dry values into intuitionistic fuzzy numbers. The alternatives are evaluated and normalized by evaluating the attributes of the alternatives, and then by transforming each attribute value into an intuitionistic fuzzy number. Not only does it incorporate dry values for which the decision maker is the subject, but it also simplifies decision-making by
employing intuitionistic fuzzification. Yi et al [17] proposed a new three-way decision model based on Pythagorean fuzzy theory to optimize the score function to propose a new action utility function for risk assessment. Zhang et al [18] proposed a multi-attribute decision-making method based on intuitive trapezoidal fuzzy numbers, calculating the distance between trapezoidal fuzzy numbers, using information entropy and grey scale analysis to determine the attribute weights, and also combining the positive and negative ideal solutions in TOPSIS to calculate the greyscale relational projection values, to select the optimal alternative. Deng et al [19] proposed an adversarial game decision in a decision-making fuzzy environment. Fuzzy adversarial theory based on number theory (DNT) under uncertainty mainly uses the non-exclusivity of fuzzy evaluation to model the decision decision-making of the decision maker. Yang et al [20] introduced foreground theory in the study of the VIKOR intuitionistic trapezoidal fuzzy multi-attribute decision model, which provided a new idea for establishing the reference point of expectation value in the VIKOR model. Meanwhile, Su et al [21] proposed a multi-attribute decision-making method based on entropy and correlation coefficients for a multi-attribute problem with completely unknown attribute weights in a probabilistic pairwise hesitant fuzzy environment. A method based on FAHP-grey correlation TOPSIS [22] was applied in the original evaluation of disassembly solutions, combining the theory of grey correlation with TOPSIS to construct a new closeness index to obtain the optimal decision. The method of determining attribute weights is always a focus in multi-attribute decision problems. Considerable efforts and exploration have been made by previous authors for the problem of attributes with unknown weights. Xu [23] proposed multi-attribute decision-making with linear objective programming, combining subjective and objective assignment methods to balance the decision needs between objective attributes and subjective desires of decision-makers. Chen [24] proposed a multi-attribute combination assignment method based on the sum of squares of deviations. Wang et al. [25] et al. used the experience and preferences of group decision-makers as supervision to build a nonlinear programming model of all decision-makers for all solution sets to balance the determination of weights and the selection of decision solutions. Based on the original nonlinear programming decision model, the influence of decision makers' weights on decision subjects is incorporated to provide a new idea for determining the attribute weights and ranking of solutions in the decision rule. Liu et al [26] proposed a multi-attribute decision-making method for TODIM groups based on trust relationships for existing multi-attribute group decision-making methods that do not consider the influence of social networks and limited rationality of decision-makers. Wang et al [27] proposed a solution to the minimization preference reversal problem in the TOPSIS scheme. Rough sets are commonly used in decision science to solve problems with high uncertainty. Since Yao [28-32] proposed three-way decision-making, i.e. removing the two options of accepting and
rejecting and adding the option of delaying the decision, it is more in line with the decision maker's need to make a decision. The essence of traditional three-way decision-making is that the three-way approach granulates the information as a whole and divides the three-way region through the choice of binary relationships. Multi-attribute decision-making methods based on three-way decision-making are now flourishing. Jiang et al. defined a new fuzzy binary relation in the field of risk decision-making using hesitant fuzzy sets. Pattern recognition, as a scientific theory widely used in artificial intelligence and industrial and agricultural production, still does not have a clear definition. However, in its essence, pattern recognition is the process of comparing existing information with the information to be detected and obtaining the most appropriate choice. Fuzzy pattern recognition is the introduction of the principles of fuzzy sets and closeness based on pattern recognition, i.e. pattern recognition is transformed into fuzzy subsets for comparison of closeness between fuzzy subsets. In economic applications, fuzzy pattern recognition has been a boom in the early 21st century.

Traditional pattern recognition is divided into standard hierarchies in the application, but the optimal solution needs to be selected in the decision-making process. Also from a decision-making point of view, the choice of decision attributes and the determination of different kinds of decision attributes and conditional attributes are relatively subjective. Therefore, this paper combines information systems with fuzzy pattern recognition and proposes a fuzzy pattern multi-attribute decision-making method based on information systems.

This paper consists of four Sections: Section 1 provides an introduction to the background of the study, Section 2 provides preparatory knowledge, Section 3 provides an algorithmic model, and Section 4 provides an example of a company selecting talent to verify feasibility.

In this section, we will introduce the rules of operations for fuzzy sets, the principles of fuzzy pattern recognition, the steps of fuzzy pattern recognition, and the basic concepts of information systems.

2. Preliminaries

2.1 Fuzzy set operation rules

Definition 1 Let $A, B$ be two fuzzy sets in a theoretical domain $U$ with functions $\mu_A$ and $\mu_B$, then the merge, intersection and complement operations on fuzzy sets are defined as:

$A \cup B : \mu_{A \cup B}(u) = \mu_A(u) \lor \mu_B(u) = \max (\mu_A(u), \mu_B(u))$, taking the larger value,

$A \cap B : \mu_{A \cap B}(u) = \mu_A(u) \land \mu_B(u) = \min (\mu_A(u), \mu_B(u))$, taking the smaller value,

$A^c : \mu_{A^c}(u) = 1 - \mu_A(u)$.

2.2 Principles of fuzzy pattern recognition

Definition 2 Let $F(U)$ be a fuzzy power set of the theoretical domain, if the mapping $\sigma : F(U) \times F(U) \rightarrow [0, 1], (A, B) \rightarrow [0, 1]$ and satisfies
Normalizability: \( \sigma(A, A) = 1, \sigma(U, \ast) = 0; \)
Symmetry: \( \sigma(A, B) = \sigma(B, A), \forall A, B \in U; \)
Inequality: \( A \subseteq B \subseteq C \rightarrow \sigma(A, C) \subseteq \sigma(A, B) \lor \sigma(B, C) \) then \( \sigma(A, B) \) is called the B-approximation to A.

**Definition 3** \(^{[15]}\) Let \( \tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n \) be n fuzzy patterns on a theoretical domain \( U \), and \( \tilde{B} \) be an object to be identified on \( V \) if \( U \cap V \neq \emptyset \), and
\[
\sigma(\tilde{B}, \tilde{A}_i) = \max \{ \sigma(\tilde{B}, \tilde{A}_1), \sigma(\tilde{B}, \tilde{A}_2), \ldots, \sigma(\tilde{B}, \tilde{A}_n) \}
\]
then we say that \( \tilde{B} \in \tilde{A}_i. \)

### 2.3 Fuzzy pattern recognition steps \(^{[18]}\)

Step 1: Select the set of characteristic factors of the pattern \( X = \{x_1, x_2 \ldots x_n\} \); each object \( x_j \) has m sample attributes forming the attribute set \( x_j = (x_{1j}, x_{2j}, \ldots, x_{mj})^T \); this forms the measured attribute matrix \( X = (x_{ij}) \) (\( i = 1, 2 \ldots, m; j = 1, 2 \ldots, n \));

Step 2: Classify the m attributes according to the c-level criterion pattern, then we have the attribute criterion matrix \( Y = (y_{ih})_{m \times c} \) (\( 2 \leq c < m \)).

Step 3: Using the normalization formula to eliminate the influence of the physical dimension of different attribute features and standardize the feature values, we obtain the relative affiliation of the attributes of the sample to be identified \( r_{ij} \) (\( i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \)) to obtain the relative affiliation matrix of the attributes of the sample to be identified \( R = (\tilde{r}_{ij})_{m \times n}, 0 \leq r \leq 1 \) (\( i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \)).

Step 4: Similar to Step 3, we obtain the relative affiliation of each standard sample attribute \( S_{ih} \) (\( i = 1, 2, \ldots, m; h = 1, 2, \ldots, c \)), and similarly obtain the relative affiliation matrix \( S = (S_{ih})_{m \times c}, 0 \leq S_{ih} \leq 1, (i = 1, 2, \ldots, m; h = 1, 2, \ldots, c) \).

Step 5: Determine the attribute weights \( \omega_{ij} \), which are generally determined by the entropy weighting method or expert weighting method.

Step 6: Construct a theoretical model for fuzzy pattern recognition, and construct a fuzzy pattern recognition matrix \( U = (u_{ij})_{c \times n}, u_{ij} = f(\omega, S_{ih}, x_{ij}) \).

Step 7: Calculate the comprehensive evaluation index \( \theta_i, (i = 1, 2, \ldots, m) \), which is the closeness of the solution, and rank the best solution according to the principle of monological proximity.

### 2.4 Common closeness formulae

**Hemming closeness** \(^{[18]}\)
\[
\sigma_1(A, B) = 1 - \frac{1}{n} \sum_{k=1}^{n} |A(x_k) - B(x_k)| \tag{1}
\]

**Euclidean closeness** \(^{[18]}\)
\[
\sigma_2(A, B) = 1 - \frac{1}{\sqrt{n}} \left( \sum_{k=1}^{n} (A(x_k) - B(x_k))^2 \right)^{\frac{1}{2}} \tag{2}
\]
2.5 Information System

Call $IS = \{U, AT, V, f\}$ an information system; where $U$ is a non-empty finite set of objects, $U = \{x_1, x_2, \ldots, x_q\}$; denote the set of attributes as $AT$; $f$ is an information function, and for $\forall a \in AT$, $x \in U$ there is $f(x, a) \in V_a$.

On this basis, a fuzzy model multi-attribute decision-making method based on information systems is proposed.

3. Information system-based fuzzy model multi-attribute decision model

3.1 Information system establishment

The attributes are classified according to the characteristics of the attribute values of the different attributes in the information system, and the set of affiliation functions and the set of affiliation values are introduced.

Definition 1 A set of information system $GIS = \{U, AT, V, f, g, G\}$ is established based on the decision object; where $U$ is a non-empty finite set of objects, $U = \{x_1, x_2, \ldots, x_q\}$; the set of attributes is $AT = N \cup AB$, $V = V_n \cup V_{ab}$, $V_n, V_{ab}$ are the value domains of natural attributes and abstract attributes respectively; $f$ is the information function, for $\forall a \in AT$, $x \in U$ have $f(x, a) \in V_a$; $g$ is the set of affiliation functions; $G$ is the set of affiliation values, and for $\forall a \in AT$, $x \in U$ have $g(f(x, a)) \in G_a$.

Where $N$ is a non-empty finite set of natural attributes, i.e. attributes in the attributes that can be quantified, denoted as $N = \{n_1, n_2, \ldots, n_k\}$, and $AB$ is a non-empty set of abstract attributes, i.e. attributes in the attributes that are considered to be defined, denoted as $AB = \{ab_1, ab_2, \ldots, ab_l\}$. $V_n, V_{ab}$ are the value domains of the natural and abstract attributes, respectively. The number of elements in $g$ is as same as the number of elements in $AT$. As shown in Table 1,

<table>
<thead>
<tr>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$\ldots$</th>
<th>$a_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>$f_1(x_1), g_1(f_1(x_1))$</td>
<td>$f_2(x_1), g_2(f_2(x_1))$</td>
<td>$\ldots$</td>
<td>$f_l(x_1), g_l(f_l(x_1))$</td>
</tr>
<tr>
<td>$x_2$</td>
<td>$f_1(x_2), g_1(f_1(x_2))$</td>
<td>$f_2(x_2), g_2(f_2(x_2))$</td>
<td>$\ldots$</td>
<td>$f_l(x_2), g_l(f_l(x_2))$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$x_n$</td>
<td>$f_1(x_n), g_1(f_1(x_n))$</td>
<td>$f_2(x_n), g_2(f_2(x_n))$</td>
<td>$\ldots$</td>
<td>$f_l(x_n), g_l(f_l(x_n))$</td>
</tr>
</tbody>
</table>

The following is an example of how to structure the required information system based on existing information.

Example 1 Two companies of different sizes in cities A, A1, and A2, have to choose a partner company for their city B strategy, and there are three companies in city B, B1, B2, and B3, for which they can choose. The relevant information is presented in Tables 2 and 3 below,

Table 2 Companies’ information in City B
Then the information system $\text{GIS}$ is constructed according to Table 2 and Table 3.

The domain is $U = \{B_1, B_2, B_3\}$; the set of attributes $AT = \{N_1, N_2, AB_1, N_3\}$, where the natural attribute $N = \{N_1, N_2, N_3\}$, and the abstract attribute $AB = \{AB_1\}$; the attribute values $V$ are the values corresponding to each attribute in Table 3. Set the set of affiliation functions to $g$.

### 3.2 Establishment of the standard set and the set to be tested

Step 1: Building the information system $\text{GIS}$

If each attribute in $AB = \{ab_1, ab_2, ..., ab_l\}$ is divided into $c_i (i = 1, 2, ..., n)$ levels, and the abstract attribute values are transformed into constants between 0 and 1, then each level is divided as follows. Please refer to Table 4.

<table>
<thead>
<tr>
<th>$ab_1$</th>
<th>$ab_2$</th>
<th>...</th>
<th>$ab_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$i$</td>
<td>$i$</td>
<td></td>
<td>$i$</td>
</tr>
<tr>
<td>$c_2$</td>
<td>$2$</td>
<td>$2$</td>
<td>$2$</td>
</tr>
<tr>
<td>$i$</td>
<td>$i$</td>
<td></td>
<td>$i$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
<td>...</td>
</tr>
<tr>
<td>$c_i$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
</tbody>
</table>

and the value obtained after the division is used as the normalized value of the attribute.

Step 2: Classify the objective function and find the weights

The decision target attributes are selected from the attribute set and classified into attributes. The default target attribute is mixed, i.e., it is the natural attribute $n_\tau, \tau = (1, 2, ..., k)$ and $ab_\epsilon, \epsilon = (1, 2, ..., l)$. For abstract attributes, the normalized values are determined according to the attribute abstraction hierarchy in Step 1. To eliminate the influence of different physical scales on decision-making normalized value is compared with the maximum value of the same attribute.

In a standard information system domain, there are $z$ attributes, $c$ attributes are selected as target attributes, each target attribute is used as a reference sequence, and the remaining $(z - c)$
attributes are used to calculate the correlation \( \gamma_{ij}(i = 1,2,\cdots,c,j = 1,2,\cdots,z-c) \) to the target attribute using correlation analysis, and the sum of the correlations \( R_i \) and the influence weight \( \omega_{ij} \) is obtained. Please refer to Table 5,

\[
R_i = \sum_{j=1}^{z-c} \gamma_{ij} \tag{3}
\]

\[
\omega_{ij} = \frac{\gamma_{ij}}{R_i} \tag{4}
\]

| Table 5 Relative values of natural attributes after removing physical dimensions |
|---------------------|---------------------|---------------------|
|                     | \( n_1 \)           | \( n_2 \)           | \( \cdots \)         | \( n_k \)         |
| \( m_1 \)           | \( \gamma_{11} \)   | \( \gamma_{12} \)   | \( \cdots \)         | \( \gamma_{1k} \) |
| \( m_2 \)           | \( \gamma_{21} \)   | \( \gamma_{22} \)   | \( \cdots \)         | \( \gamma_{2k} \) |
| \( \vdots \)        | \( \vdots \)        | \( \vdots \)        | \( \ddots \)         | \( \vdots \)      |
| \( m_p \)           | \( \gamma_{p1} \)   | \( \gamma_{p2} \)   | \( \cdots \)         | \( \gamma_{pk} \) |

The objective attributes are assigned weights using subjective or expert assignment methods so that the sum of the objective function attribute weights is 1.

Step 3: Determine the criterion set by choosing the affiliation function

To construct a fuzzy set with each target attribute and non-target attribute, all fuzzy sets are merged to obtain the criterion set \((B\bar{Z})\). This can be used to satisfy the decision-makers’ preferences. The criteria matrix \( A \) is constructed by determining the criteria scheme from the target attribute weights.

There are \( z \) attributes in the subject of a standard information system domain, and \( c \) attributes are selected as target attributes. The relative affiliation of the target attribute is denoted by \( g_i(x) = mb_i(i = 1,2,\cdots,c) \), and the relative affiliation of the non-target attribute is denoted by,

\[
g_j(x) = fmb_j(j = 1,2,\cdots,z-c) \tag{5}
\]

The target attribute carries a weight \( \rho_i(i = 1,2,\cdots,c) \), and satisfies

\[
\sum_{i=1}^{c} \rho_i = 1 \tag{5}
\]

And,

\[
\bar{p}_i = \frac{mb_i}{MB_i} + \frac{\omega_{ij}fmb_j}{FMB_j}(i = 1,2,\cdots,c)(j = 1,2,\cdots,z-c) \tag{6}
\]

Then the standard set is,

\[
B\bar{Z} = V_{i=1}^{c} \bar{p}_i \rho_i \ (i = 1,2,\cdots,c) \tag{7}
\]

Then the standard matrix is,

\[
A = (mb_i\rho_i, \max(\rho_i(\omega_{ij}fmb_j))(i+j)\times1 \ (i = 1,2,\cdots,c)(j = 1,2,\cdots,z-c) \tag{8}
\]

Example 2 is given below to verify the feasibility of step 2,

**Example 2** Using the direct economic loss of the disaster as the target attribute, the influence weight of other attributes on it was calculated. As shown in Table 6,

| Table 6 2000-2003 Disaster Loss Table |
|-------------------------------|----------------|----------------|----------------|----------------|
| Year                          | 2000           | 2001           | 2002           | 2003           |
| Direct economic losses from disasters (RMB billion) | 2045. 3 | 1942. 2 | 1637. 2 | 1884. 2 |
The maximum value of the same attribute was selected and the physical dimension was eliminated to normalize the attribute value, which was obtained in Table 7.

Table 7 Disaster loss table after eliminating physical dimension

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct economic losses from disasters (RMB billion)</td>
<td>1.00</td>
<td>0.95</td>
<td>0.80</td>
<td>0.92</td>
</tr>
<tr>
<td>Crop Damage Area (thousand hectares)</td>
<td>1.00</td>
<td>0.92</td>
<td>0.79</td>
<td>0.95</td>
</tr>
<tr>
<td>Earthquake disaster losses (RMB billion)</td>
<td>0.31</td>
<td>0.32</td>
<td>0.03</td>
<td>1.00</td>
</tr>
</tbody>
</table>

From this, the correlation between the area of crop damage (thousand hectares) \( x_1 \) and earthquake damage (billion yuan) \( x_2 \) and the direct economic damage (billion yuan) \( x_3 \) was calculated using grey correlation analysis with the correlation degree of 0.59 and 0.48 to calculate the impact weight. Table 8 was obtained.

Table 8 The weights of different attributes affecting the direct economic loss of disasters

<table>
<thead>
<tr>
<th>Attributes</th>
<th>Impact weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of crop damage (thousand hectares)</td>
<td>0.55</td>
</tr>
<tr>
<td>Earthquake disaster damage (billion yuan)</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Set each attribute affiliation function to 
\[
g_i(x) = \frac{f_i}{f_{\max(i=1,2,3,4)}}.
\]

The resulting set of criteria is constructed as 
\[
\hat{p} = \frac{0.5}{x_1} + \frac{0.19}{x_2} + \frac{0.92}{x_3}
\]

The standard matrix is 
\[
A = \begin{pmatrix} 0.5 & 0.19 & 0.92 \end{pmatrix}
\]

Step 4: Constructing the set to be tested

The standard subject and the subject to be tested have the same or similar domain, and the target attributes in the set to be tested are selected according to the target attributes in the standard set and classified.

For the abstract attributes in the target attributes, the values are normalized according to the attribute abstraction hierarchy in Step 2. For the natural attributes of the subject to be measured, the natural attribute with the largest value in the subject to be measured is selected as the basis, and the remaining attributes are compared to the value \( \delta_{mq} \in [0,1], m = (1,2,\cdots,p), q = (1,2,\cdots,k) \).

\[
H_m = \frac{\sum_{m=1}^{p} \delta_{mq}}{p}
\]

as a normalized value.

The influence weight \( \omega_{ij} \) of the non-target attributes on the target attributes in Step 3 is used as the influence weight of the non-target attributes on the target attributes in the subject of the information system domain to be tested. The normalized values are fuzzified by choosing the same affiliation function as in the standard set. The weights of the target attributes in the criterion
set are assigned, and the sum of the attribute weights is 1. A fuzzy set is constructed for each target attribute and non-target attribute, and all fuzzy sets are merged to obtain the criterion set \( (\hat{D}C) \). This is the basis for satisfying the decision maker’s preferences. The target attribute weights are used to determine the solution to be tested, and the matrix \( B \) is constructed.

There are \( z \) attributes in the subject of an information system domain to be tested, and \( c \) attributes are selected as target attributes, the relative affiliation of the target attribute is denoted by \( g_i(x) = mb_i (i = 1,2, \ldots, c) \) and the relative affiliation of the non-target attribute is denoted by \( g_j(x) = fmb_j (j = 1,2, \ldots, z-c) \), and the weight of the target attribute is \( \alpha_i (i = 1,2, \ldots, c) \), which is satisfied,

\[
\sum_{i=1}^{c} \alpha_i = 1
\]  
and,

\[
\bar{y}_i = \frac{mb_i}{MB_i} + \frac{\omega_{ij}fmb_j}{FMB_j} (i = 1,2, \ldots, c)(j = 1,2, \ldots, z-c)
\]

Then the set to be measured is,

\[
\hat{D}C = \bigvee_{i=1}^{c} \bar{y}_i \alpha_i (i = 1,2, \ldots, c)(j = 1,2, \ldots, z-c)
\]

Then the matrix to be measured is,

\[
B = (mb_i \alpha_i, max(\alpha_i, \omega_{ij} fmb_j))(i+j)_{x=1} (i = 1,2, \ldots, c)(j = 1,2, \ldots, z-c)
\]

### 3.3 Selecting the fuzzy pattern recognition criterion and choosing the optimal solution
The combined attribute value of two fuzzy vectors calculated by the closeness formula \( \sigma(A,B) \) is called the closeness of these two fuzzy vectors.

### 2.4 Information system-based fuzzy model multi-attribute decision-making process
A table of steps and a flowchart of the information system-based fuzzy model multi-attribute decision making is given below. Shown in Table 9 and Figure 1.

**Table 9 Multi-attribute decision-making process based on the fuzzy model of information system**

| Step 1: Collect the set of attributes \( AT \) in the subject \( U \), and construct the information system GIS; |
| Step 2: Divide the attribute set \( AT \), divide the natural attributes to divide the attribute set \( AT \), divide the natural attributes \( N = \{n_1, n_2, \ldots, n_k\} \) and abstract attributes \( AB = \{ab_1, ab_2, \ldots, ab_l\} \); |
| Step 3: Determine the division of target attributes and non-target attributes; |
| Step 4: Classify the abstract attributes into levels, using normalized representation, and use the level classification as the basis to determine the affiliation of different attributes; |
| Step 5: Degradation of natural attributes in the criterion set; |
| Step 6: Calculate the influence weight \( \omega_i \) for each attribute in the criterion set and determine the attribute affiliation according to the affiliation function \( g_i(f(x,a)) (where i = 1,2, \ldots, l+k) \); |
| Step 7: Construct the standard set \( (BZ) \); |
| Step 8: De-quantize the natural attributes in the set to be measured; |
| Step 9: Construct the set to be measured \( (\hat{D}C) \) from the influence weights of the non-target and |
Step 10: Find the appropriate closeness calculation method $\sigma(A, B)$, and calculate the closeness of the set to be tested to the standard set for the standard matrix and the vector to be tested.

Step 11: The calculated closenesses are ranked and the best solution is selected.

The proposed information system-based fuzzy pattern decision model has the following advantages:

1) Compared to the traditional fuzzy pattern recognition approach where the attributes required for pattern recognition are given directly by experts or processes, this method combines information systems with pattern recognition to reduce the possible omission of attributes in the decision-ranking process.

2) The model's decision-making capability is enhanced by combining the information system to classify attribute requirements and attribute value characteristics when the information system is of mixed semantics.

3) The use of substitutable correlation analysis to calculate attribute weights, which allows for flexibility in the determination of non-target attribute weights by varying the measure and method of determining weights according to the decision needs under different conditions. For the determination of the objective function weights, the expert assignment method is used. The subjective opinions of decision-makers and objective facts in the decision-making process are fully considered.

4) Using raw data normalization combined with correlation analysis to determine non-objective attribute weights, avoiding the effects of fuzziness of physical units and attribute values.

5) Combining the advantages of AHP and TOPSIS, and making the decision ranking data available by calculating the closeness compared to the three-branch decision and ranking.

4. Example of an algorithm

The correlation method used in this example is the grey correlation analysis. The first step is to introduce grey correlation analysis, which is the degree of influence of different factors on a particular factor in a grey system. The essence of the idea is to determine whether a series of curves are closely related to each other based on their similarity in geometry. Shown in Table 10,

![Diagram of the decision-making process based on information system fuzzy mode](image-url)
Table 10 Grey relational analysis process

Step 1: The original data is processed and the data is dimensionless for different physical scales;

Step 2: Calculate the list of corresponding differences and the maximum-minimum difference $\Delta$ according to the values required by the formula;

Step 3: Calculate the correlation coefficient $\delta_i(k) = \frac{\Delta_{\text{min}} + \gamma \Delta_{\text{max}}}{\Delta_{\text{min}} + \gamma \Delta_{\text{max}}} \text{ factor, } 0 < \theta < 1$;

Step 4: Find the correlation $r_i = \frac{1}{n} \sum_{k=1}^{n} \delta_i(k)$.

Two departments of a school, A and B, are recruiting in the job market. There are five applicants, and department A plans to recruit one member and department B also plans to recruit one member. Listed in Tables 11 & 12,

Table 11 Recruitment of departments A and B in the past three years

<table>
<thead>
<tr>
<th>Recruiters</th>
<th>Interview results</th>
<th>Written test results</th>
<th>Work experience</th>
<th>Communication skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Department A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>90</td>
<td>85</td>
<td>2 years</td>
<td>Excellent</td>
</tr>
<tr>
<td>B</td>
<td>92</td>
<td>83</td>
<td>3 years</td>
<td>Excellent</td>
</tr>
<tr>
<td>C</td>
<td>95</td>
<td>83</td>
<td>5 years</td>
<td>Excellent</td>
</tr>
<tr>
<td>Department B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>80</td>
<td>100</td>
<td>1 year</td>
<td>Qualified</td>
</tr>
<tr>
<td>B1</td>
<td>83</td>
<td>96</td>
<td>4 years</td>
<td>Medium</td>
</tr>
<tr>
<td>C1</td>
<td>90</td>
<td>92</td>
<td>2 years</td>
<td>Good</td>
</tr>
</tbody>
</table>

Table 12 Information of five candidates to be hired

<table>
<thead>
<tr>
<th>Interview results</th>
<th>Written test results</th>
<th>Work experience</th>
<th>Communication skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number 1</td>
<td>90</td>
<td>90</td>
<td>1 year</td>
</tr>
<tr>
<td>Number 2</td>
<td>84</td>
<td>99</td>
<td>1 year</td>
</tr>
<tr>
<td>Number 3</td>
<td>85</td>
<td>96</td>
<td>2 years</td>
</tr>
<tr>
<td>Number 4</td>
<td>98</td>
<td>83</td>
<td>4 years</td>
</tr>
<tr>
<td>Number 5</td>
<td>78</td>
<td>98</td>
<td>3 years</td>
</tr>
</tbody>
</table>

To construct an information system based on Table 10 and Table 11, the domain $U$={accepted person} $AT$={interview score($x_1$), written test score($x_2$), work experience($x_3$), communication skills($x_4$)}. Now distinguish between the set of natural attributes $N$={interview scores ($x_1$), written scores ($x_2$)} and the set of abstract attributes $AB$={work experience ($x_3$), communication skills ($x_4$)} based on the characteristics of the attributes in the argument domain $U$.

The target attributes for department $A$ are interview performance ($x_1$) and communication skills ($x_4$); for department $B$, the target attributes are written test performance ($x_2$), work experience ($x_3$). Given an affiliation function of $g_i = \frac{f_i}{f_{\text{max}}}$ for each attribute, the abstract attributes were ranked as shown in Table 13 below,

Table 13 Abstract Attribute Hierarchy

<table>
<thead>
<tr>
<th>Work experience</th>
<th>Grades</th>
<th>Communication skills</th>
<th>Grades</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>0.2</td>
<td>Excellent</td>
<td>1.0</td>
</tr>
<tr>
<td>2 years</td>
<td>0.4</td>
<td>Good</td>
<td>0.75</td>
</tr>
<tr>
<td>3 years</td>
<td>0.6</td>
<td>Medium</td>
<td>0.50</td>
</tr>
<tr>
<td>4 years</td>
<td>0.8</td>
<td>Qualified</td>
<td>0.25</td>
</tr>
<tr>
<td>5 years</td>
<td>1.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

De-quantizing all attributes gives Table 14,
Table 14 Hiring information obtained after de-dimensioning

<table>
<thead>
<tr>
<th>Recruiters</th>
<th>Interview results</th>
<th>Written test results</th>
<th>Work experience</th>
<th>Communication skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Department A</td>
<td>A</td>
<td>0.95</td>
<td>1.00</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>0.97</td>
<td>0.98</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>1.00</td>
<td>0.98</td>
<td>1.00</td>
</tr>
<tr>
<td>Department B</td>
<td>A1</td>
<td>0.89</td>
<td>1.00</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>0.92</td>
<td>0.96</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>1.00</td>
<td>0.92</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Using grey correlation analysis to obtain the weight of non-target attributes on target attributes in sector $A$ as Table 15,

Table 15 Remaining attributes in department $A$ affect the weight of target attributes

<table>
<thead>
<tr>
<th>Written test results</th>
<th>Work experience</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interview results</td>
<td>0.61</td>
</tr>
<tr>
<td>Communication skills</td>
<td>0.62</td>
</tr>
</tbody>
</table>

The interview results and communication skills are given equal weight in the target attributes, both at 0.5; combined with the fuzzy concentration algorithm to construct a set of criteria for hiring personnel in department $A$ as follows,

$$\bar{p}_1 = \frac{0.97}{x_1} + \frac{0.61}{x_2} + \frac{0.26}{x_3} + \frac{1}{x_4}$$

The non-target attributes in sector $B$ are weighted against the target attributes as Table 16,

Table 16 Influence weight of remaining attributes in department $B$ on target attributes

<table>
<thead>
<tr>
<th>Written test results</th>
<th>Communication skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interview results</td>
<td>0.59</td>
</tr>
<tr>
<td>Work experience</td>
<td>0.43</td>
</tr>
</tbody>
</table>

The written test score and communication ability are given the same weight in the target attribute, both being 0.5; combined with the fuzzy set algorithm to construct the criteria set of hiring personnel in department $B$ as,

$$\bar{p}_2 = \frac{0.55}{x_1} + \frac{0.96}{x_2} + \frac{0.47}{x_3} + \frac{0.27}{x_4}$$

From this, write the standard matrix,

$$P = \begin{pmatrix} 0.97 & 0.61 & 0.26 & 1 \\ 0.55 & 0.96 & 0.47 & 0.27 \end{pmatrix}$$

In the following, we construct the set to be tested and de-quantize the information about the people to be hired, showed in Table 17,

Table 17 Normalisation of the information of the people to be hired

<table>
<thead>
<tr>
<th>Interview results</th>
<th>Written test results</th>
<th>Work experience</th>
<th>Communication skills</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number 1</td>
<td>0.92</td>
<td>0.91</td>
<td>0.2</td>
</tr>
<tr>
<td>Number 2</td>
<td>0.86</td>
<td>1.00</td>
<td>0.2</td>
</tr>
<tr>
<td>Number 3</td>
<td>0.87</td>
<td>0.97</td>
<td>0.4</td>
</tr>
<tr>
<td>Number 4</td>
<td>1.00</td>
<td>0.84</td>
<td>0.8</td>
</tr>
<tr>
<td>Number 5</td>
<td>0.80</td>
<td>0.99</td>
<td>0.6</td>
</tr>
</tbody>
</table>

This was used to construct a matrix for each candidate to be hired,

$$A_1 = \begin{pmatrix} 0.92 & 0.55 & 0.07 & 0.75 \\ 0.49 & 0.91 & 0.2 & 0.28 \end{pmatrix}$$
\[ A_2 = \begin{pmatrix} 0.86 & 0.52 & 0.07 & 0.5 \\ 0.5 & 1 & 0.2 & 0.21 \end{pmatrix} \]
\[ A_3 = \begin{pmatrix} 0.87 & 0.51 & 0.14 & 0.5 \\ 0.5 & 0.97 & 0.4 & 0.2 \end{pmatrix} \]
\[ A_4 = \begin{pmatrix} 1 & 0.5 & 0.31 & 1 \\ 0.54 & 0.84 & 0.8 & 0.46 \end{pmatrix} \]
\[ A_5 = \begin{pmatrix} 0.8 & 0.48 & 0.19 & 0.25 \\ 0.47 & 0.99 & 0.6 & 0.1 \end{pmatrix} \]

The Euclidean approximation is chosen for the standard set and the set to be tested, and the following ranking is obtained, showed in Fig 2.

**Fig. 2 Histogram of Approximate Degree of Candidates**

<table>
<thead>
<tr>
<th>Number 1</th>
<th>Number 2</th>
<th>Number 3</th>
<th>Number 4</th>
<th>Number 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Department A</td>
<td>0.73</td>
<td>0.68</td>
<td>0.59</td>
<td>0.91</td>
</tr>
<tr>
<td>Department B</td>
<td>0.81</td>
<td>0.79</td>
<td>0.75</td>
<td>0.68</td>
</tr>
</tbody>
</table>

From Figure 2 and Table 18, we can see that for department A: No. 4 > No. 1 > No. 2 > No. 3 > No. 5; for department B: No. 1 > No. 5 > No. 2 > No. 3 > No. 4; Therefore, it is recommended that department A used number 4 and department B use number 1.

**5. Conclusion**

In this paper, an information system-based fuzzy pattern decision method is proposed for the first time to combine information system and fuzzy pattern recognition applications for multi-attribute decision problems with unknown attribute weights. An affiliation function is added to the original information system to fuzzify the original data. After normalizing the data, the attribute weights are determined using correlation analysis. The criteria set and the set to be tested are determined by combining the fuzzy numbers and weights, and the closeness is calculated and ranked. Compared to traditional multi-attribute decision-making methods with unknown weights, this method is computationally simple, easy to understand, and flexible enough to change the criterion set and the set to be tested according to the type of correlation analysis and the affiliation function, making the decision more universal.

**6. References**


7. Data Availability.
Notice: All data generated or analysed during this study are included in this published article.