Analyzing the effect of fixed and moving bottlenecks on traffic flow and car accidents in a two-lane cellular automaton model

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Abstract

In this paper, we studied a two-lane cellular automaton model that considers both types of bottlenecks (i.e., moving and fixed). The slow-moving vehicles are considered as moving bottlenecks in our model. The fundamental diagram and the spacetime configuration were derived. The effect of the bottleneck induces a qualitative and quantitative change in the fundamental diagram of both lanes. Also, the system depends strongly on the length of the fixed bottleneck. Hence, for extremely low densities, vehicles are self-organized into two lanes. The probability of vehicle accidents is computed. It is found that the rear-end and lane-changing collision probabilities were reduced as the length of the fixed bottleneck increased. Furthermore, at low densities the fixed bottleneck has even less impact on the occurrence of lane-changing collisions. Moreover, the revised lane-changing rules reduces the flux at low and moderate density regions and augments it at large densities in the lane where there is a bottleneck. The results also indicate that the revised lane-changing rules maximizes the rear-end collisions and minimizes the unsafe lane changing collisions.

Keywords: fixed bottleneck, slow-moving, fundamental diagram, rear-end collision, lane-changing collision, two-lane, revised lane-changing rules.
1. Introduction

Nowadays, traffic congestion has become a major problem in many countries around the world. Due to its negative consequences on individuals and society, traffic congestion attracted the attention of scientists from different backgrounds\textsuperscript{1-6}. Traffic congestion can be provoked when the demand exceeds the road capacity because of the increase in the density of vehicles. One of the main reasons for the density increase is the bottleneck where it can be a result of a lane drops, accidents area, off-ramp, on-ramp, etc. The name bottleneck means the narrowest point, which is in real traffic can be produced when the density of vehicles in the upstream is higher than that in the downstream. However, traffic bottleneck is considered as one of the major causes of the disturbance in traffic flow. The effect of a bottleneck can induce a local jam that can emerge or persist over time. In the two-lane road, the bottleneck in one lane can also affect the traffic in the other lane by forcing vehicles to change their lane to escape from the dense region. The understanding of the interactions between vehicles and bottlenecks is important in order to improve the traffic situation and road safety. The effect of bottleneck has been studied by researchers to reveals the causality relation between it and the traffic congestion. Implicitly researchers investigated the bottleneck types (i.e., moving and fixed). Chowdhury et al.\textsuperscript{7} studied the effect of the slow-moving vehicles (which can be considered as a moving bottleneck) in a two lanes cellular automaton (CA) model. They found that the higher contribution of the fast vehicles to the throughput is observed when the asymmetric lane-changing rules model is adopted (here the asymmetric lane changing rules refers to the case when the fast vehicles have priority over the slow one in a lane, the slow ones have to shift to the slow lane). Cheng-Jie et al.\textsuperscript{8} discussed the mechanisms for discretionary lane-changing behavior in traffic flow, they found that the classical lane-changing rules cannot explain many cases in the empirical dataset. Yangzexi et al.\textsuperscript{9} analyzed the mixed traffic flow of regular and autonomous vehicles using a CA model.

Knospe et al.\textsuperscript{10} discussed the effect of slow vehicles in a two-lane CA model. They observed that even a small number of slow vehicles can induce the formation of platoons at low densities. Ming-bao et al.\textsuperscript{11} studied the influence of bus stop with left-turn lines between two adjacent signalized intersections based on a symmetric two-lane Nagel–Schreckenberg model, they found that the left-turn lines have a negative effect on the accident rate as well as delay if the stop is located close to the intersections.

Juran et al.\textsuperscript{12} developed a dynamic traffic assignment model which could determine the impact of moving bottleneck. They found that the moving bottleneck can provoke an increase in travel
times and it affects the distribution of traffic among available traveling paths in the network. Using the optimal velocity model\textsuperscript{13,14}, Fang et al.\textsuperscript{15} showed that the effect of moving bottleneck augments with the increase of traffic density, and their effect becomes lower when the maximum speed of slow-moving vehicles increases. Lakouari et al.\textsuperscript{16} studied the interactions between vehicles in bidirectional traffic CA model with two types of vehicles (slow, fast). They found that when the density of one lane increases the slow-moving vehicles control the other lane by imposing their speed over the fast vehicles. Ou and Tang\textsuperscript{17} proposed an extended macro traffic flow with a moving bottleneck model in order to study the effects of a moving bottleneck on the evolution and propagation of traffic flow under uniform flow and a small perturbation. They found that the influences of moving bottleneck depend strongly on the initial traffic density.

On the other hand, the effect of the fixed bottleneck was also investigated. Zhu et al.\textsuperscript{18} analyzed the effect of speed bottleneck on the spatial-temporal evolution characteristics of traffic and the influence of the position and length of bottleneck on the traffic flow state. They suggested that the use of the feedback signal can improve the traffic situation. Hanaura et al.\textsuperscript{19} derived the fundamental diagrams of a two-lane highway with a few slowdown sections. They found the dependence of jam lengths and the density numerically and analytically. Zeng et al.\textsuperscript{20} proposed a CA model to simulate the traffic flow characteristics under the combined bottleneck of accidents and on-ramp. They observed that the congestion caused by the accident will propagate to the upstream under the saturated flow. Chau et al.\textsuperscript{21} investigated the effect of tollbooths on the traffic flow in a single lane CA model based on Fukui and Ishibashi\textsuperscript{22} as well as the green wave model proposed by Torok and Kertesz\textsuperscript{23}. Fei et al.\textsuperscript{24} proposed a two-lane CA model to analyze the traffic congestion induced by the work zone. The simulation result in the form of speed–flow diagram is in good agreement with that obtained from the empirical data. Furthermore, on-ramps and off-ramps have been studied using CA models in Refs\textsuperscript{25-29}. As the bottleneck can provokes jams in the real traffic it can also induce accidents. Marzoug et al.\textsuperscript{30} investigated the conflict caused by two vehicles simultaneously entering a bottleneck, this conflict can induce an accident if the vehicles do not pay attention to each other. Pang et al.\textsuperscript{31} investigated the influence of rainy weather on the sideswipe collisions and side slip collisions in two lane CA model.

In this paper we propose two categories of collisions probability (i.e., rear-end and lane changing) in unidirectional two lanes CA traffic follow model, where the two types of bottlenecks were considered (i.e., moving and fixed). The initial configuration of the both types of vehicles is homogenous (i.e., the slow and fast vehicles occupy the both lanes with the same
probability which means that we do not consider a lane as a fast or slow). The fixed bottleneck is sited in one lane, vehicles that impeded by the fixed bottleneck can change their lanes in order to increase their speed. the influence of the length of bottleneck on the occurrence of rear-end and lane changing collision probabilities in a two- lane CA model are described. This study can also be considered as an attempt to find out the relationship between the length of the bottleneck and the traffic proprieties in a two-lane CA model. In addition, a revised lane-changing rules is used to simulate a realistic behavior that induced when a vehicle approached to a fixed bottleneck.

The rest of the paper is organized as follow, in section 2 we will present the methodology of this investigation, hence for the section 3 we will discuss the results, the conclusion is given in section 4.

2. Methodology

The two-lane CA model mimics a two adjacent unidirectional road, here each lane in the model is composed of L cells. Each cell can be either empty or occupied by only one vehicle. The vehicles are characterized by their speed \( v_i \) and position \( x_i \). The vehicles differ according to their maximum allowed speed, where the maximum speed of the slow (fast) vehicles is \( v_{max}^s \) \( (v_{max}^f) \).

2.1 NaSch model in single lane

The vehicles are uniformly distributed all over the two lanes according to the fraction of fast \( f \) and slow vehicles \( f_s \). The longitudinal motion of vehicles is described by the Nagel and Schreckenberg model (NaSch) according to the following rules\(^{32}\):

\( R_1 \): Speed adaptation: \( v_i \rightarrow \text{Min}(v_i + 1, v_{max}^f \text{ or } s) \).

\( R_2 \): Safety braking: \( v_i \rightarrow \text{Min}(v_i, d_i) \).

\( R_3 \): Disturbance: \( v_i \rightarrow \text{Max}(v_i - 1,0) \). With probability \( p \).

\( R_4 \): Motion: \( x_i \rightarrow x_i + v_i \).

\( d_i \) denote the number of free cells in front of the vehicle \( i \), where \( d_i = x_{i+1} - x_i - 1 \).

2.2 Symmetric lane-changing rules

The interaction between the two lanes is introduced by the lane-changing rules. The movement of vehicles is divided in two sub-steps. In the first sub-step: vehicles change lanes in parallel
according to lane-changing criteria. In the second sub-step: the both lanes are treated as a separate single-lane where the NaSch model is applied. We considered the model proposed by Rickert et al. where the rules of lane-changing taking into account two criteria.

- Incentive criterion
  
  $$R'_1: \, d_i(t) < v_d = \min\left(v_i + 1, v_{\text{max}}^{(f\text{ or }s)}\right).$$

- Safety criteria:
  
  $$R'_2: \, d_o(t) > d_i(t).$$
  $$R'_3: \, d_b(t) \geq v_{\text{max}}^{(f\text{ or }s)}.$$

Here, $d_b(t)$ ($d_o(t)$) denote number of free cells behind (in front) of the vehicle $i$ in the other lane (see Fig. 1a), while $v_d$ is the desired speed of the vehicle $i$ at instant $t$.

Even all those criteria are met the process of the lane changing is stochastic, therefore the vehicles change their lane according to the probability $ch$. Here, the lane-changing probability is defined according to the type of the vehicle (slow or fast) and to the lane (lane 1 or lane 2).

In this paper, only the symmetric case is adopted which means that there is no preference in a lane over another lane or the fast vehicle has a higher probability of lane-changing. The $ch$ is the probability of lane-changing for both types of vehicles from lane 1 (lane 2) to lane 2 (lane 1).

In reality, the traffic flow is affected by the bottleneck. In our model we considered two types of bottlenecks namely; moving and fixed. The slow vehicles are considered as a moving bottleneck. Hence, in order to mimic the fixed bottlenecks in our model, we consider a $LB$ sites in the lattice that present an obstacle where the both lanes are reduced to a single one as shown in Fig. 1(a).

2.3 Probability of rear-end collisions

As the fixed bottleneck reduces the road capacity, the interactions between vehicles increase which can leads to the occurrence of car accidents. To calculate the probability of rear-end collisions, we used the following conditions proposed by Boccara et al.

(1) : $d_i(t) \leq v_{\text{max}}^{(f\text{ or }s)}$: where the number of empty sites in front of the $i$-th vehicle is less than the maximum speed of vehicles (fast or slow).

(2) : $v_{i+1}(t) > 0$: the vehicle ahead is moving.
\[ (3) \colon v_i(t+1) = 0; \text{ the vehicle ahead stops in the next iteration.} \]

These dangerous situations are computed and considered as the signal of the occurrence of a rear-end collision with a probability \( p' \).

### 2.4 Probability of lane-changing collisions

On the other side, in the two lanes traffic the collision can be induced by the unsafe lane-changing of the vehicles (i.e., when the vehicles do not respect the safety criteria of the lane-changing). In this propose our investigation takes into consideration this type of collision. Therefore, we will consider two types of collision namely; rear-end and lane-changing collision.

We should note that the accident does not occur really in our system instead of that we have just calculated the probability of an accident to occur. The probability of the lane-changing collision to occur is computed if the following rules are fulfilled:

- Incentive criterion
- \( d_o(t) > d_i(t) \).
- \( d_b(t) < v_e(t + 1) \).

Here \( v_e \) is the expected speed of the coming vehicle of the destined lane (i.e the hoping lane for the vehicle that want to improve its speed by lane-changing, here this vehicle can collide with the coming vehicle in the hopping lane).

If the above conditions are achieved, an accident can occur with a probability \( p'' \). Here, the probability of accident is defined as follows:

\[
P_{acc}^{R \text{ or } L} = \frac{1}{T} \sum_{t=t_0}^{t_0+T} n_i(t)
\]

With:

- \( P_{acc}^{R \text{ or } L} \) refers to the probability of rear-end collision (lane-changing collision).
- \( N \) : is the total number of vehicles (In the case of \( P_{acc}^{R} \), \( N \) denotes the number of vehicles in the same lane).
- \( t_0 \) : is the time after which the calculation is carried out.
- \( T \) : is the calculation time.

\( n_i(t) = 1 \) if all the conditions of the accident are met, otherwise \( n_i(t) = 0 \).

### 2.5 Revised lane-changing rules at the zone \( I_z \)

In reality, the vehicles should not remain impounded in front of the bottleneck. Rather, they must change the lane if they have any opportunity to avoid being delayed because of the bottleneck. Near the bottleneck, when vehicles enter the zone \( I_z \) (see Fig. 1(b)), they can change
the lane with the bottleneck to the other one with the probability \( p_c \) \((p_c \neq ch)\). Consequently, a set of special lane-changing rules are revised:

- \( i \in I_z \), \( ch_{lane2} = 0 \) and \( ch_{lane1} = 1 \).
- \( v_e(t) < d_o(t) \) and \( p_c > \text{rand}() \). Where \( v_e(t) \neq 0 \), \( \text{rand}() \) stands for a random number between 0 and 1.

We denote \( i \) the position of vehicle on the lane 1 at time \( t \), \( p_c \) the new lane-change probability and \( ch_{lane2}(ch_{lane1}) \) is the probability of lane-changing for both types of vehicles from lane 2(1) to lane 1(2).

3. Results and discussion

In this section we will study the effect of the bottleneck on the traffic flow and the probability of car accidents in both lanes. We consider a two-lane road, where each lane is composed of 1000 sites. The braking probability for all vehicles is \( p = 0.2 \) and the lane-changing probability is \( ch=1 \). The probability for a dangerous situation (i.e., the all condition of an accident to occur are met) that can cause a rear-end collision (lane-changing collision) is sited as \( p' = 0.01 \) \((p'' = 0.01)\). The traffic is heterogenous that means the existence of two kinds of vehicles that are distinguished by their maximum speed; \( v_{max}^f = 5 \) for fast vehicles and \( v_{max}^s = 3 \) for slow ones.

The simulations of the system model are carried out under the periodic boundary condition which means the number of vehicles in the lattices is conserved. The fraction of slow (fast) vehicles is considered as \( f_s = 0.2 \) \((f_f = 1-f_s = 0.8)\). The systems run for 60000 time-steps. The calculation is done for the last 10000-time steps with 100 independent simulations.

3.1 Without a revised lane-changing rules at \( I_z \)

Traffic flow can be affected by two types of defects, slow moving vehicles that are considered as moving defect and the fixed bottleneck zone which is considered as the static defect.

As we know bottlenecks increase the slow platoons and condense free space in front of them likewise the Bose-Einstein condensation in the low temperature. The static defect could induce an emergent phenomenon which is the jam caused by the deceleration of vehicles that reduce the traffic flow locally and can also cause a serious problem in traffic, such as the accident. Subsequently, in our computational study, both types of accidents (rear-end collision and the collision due to unsafe lane-changing) will be studied. During the rest of this paper and
for simplification we refer to the moving bottleneck (fixed bottleneck) by the slow vehicles (bottleneck).

Subsequently, in our computational study, the mixture of both defects is considered (slow vehicles and bottleneck) in the same system. As a first step let us see the throughput in both lanes with the presence of bottleneck and slow vehicles. Fig. 2 shows the fundamental diagram for different values of bottleneck length $L_B$. For the lane 1, where the bottleneck is sited (See Fig. 2.a), the current $J$ increases rapidly as the density $\rho$ increases until it reaches a maximum value, then it starts decreasing slightly. For high density, the current starts decreasing rapidly until its value becomes zero. For lane 2 (See Fig. 2.b), the current augments with the density until a maximum value then it drops. Here, the increase of $L_B$ reduces (increase) the current in lane 1 (lane 2).

As we know the interaction between the both lanes are induced by the lane-changing of vehicles. Therefore, to understand the variation of the throughput in both lanes it is better to illustrate the lane-changing frequency where it is defined by the frequency of vehicles that change from lane1(lane2) to lane2(lane1). Figure 3 depicts the lane-changing frequency as a function of the density for different values of $L_B$. The curves of lane-changing frequency show similar features, it reveals that there is a rapid increase in the lane-changing frequency until its maximum value then starts decreasing gradually, beyond $\rho =0.75$ it becomes zero. The effect of lane-changing is only presented in the density range 0 to 0.75. Another important observation is that the lane changing frequency for the case of short length $L_B$ is the highest. We can explain that by the decrease of the distance between vehicles when the length of the bottleneck is increased, therefore the task of finding an adequate gap to change the lane becomes not obvious which reduces the lane-changing frequency. For the case of $\rho > 0.75$, the lane-changing becomes impossible, therefore, the current is determined only by the longitudinal interaction. In this case, for lane 1 the value of the current becomes zero because of the obstruction of the bottleneck, the vehicles get trapped in this lane (See Fig. 3.a). However, for lane 2, lane-changing is not possible for all vehicles, therefore the order of vehicles reminds the same, and for those range of densities, the slow and fast vehicles were forced to reduce their speed. The value of the current remains the same for lane 2 even if the size of the bottleneck in the other lane is reduced (See Fig. 3.b).

To get a better insight into the effect of the bottleneck on the traffic characteristics, we present the space-time configuration in Fig. 4. For low densities, in the lane 1 the traffic flow is
controlled by the fast vehicles. However, the slow vehicles can be seen only in lane 2. The vehicles try to move with their desired speed, as the vehicles approach the bottleneck, they change the lane. As we know in this density ranges the number of free spaces is large, this makes the slow vehicles satisfied and confined in the lane 2. Nevertheless, for the fast vehicles, the situation is different they change the lane in order to increase their speed because of the bottleneck and the slow vehicles. At first, they are obligated to change the lane 1 to skip from the hindrance caused by the bottleneck. In the lane 2, fast vehicles change the lane because of the slow-moving vehicles that control this lane.

As the density increases, the queue of stopped vehicles increases and the situation in both lanes is worsen. The jam formed behind the bottleneck in lane 1 increases and spread over all this lane. In this case, the lane-changing from lane 1 to lane 2 is reduced because the safety criterion is never fulfilled. On the other hand, the bottleneck blocks the vehicles in lane 1 thus increase the free space upstream of the bottleneck in the same lane. In this case, if the length of the bottleneck $L_B$ increases, the density of vehicles is reduced locally as corresponding to lane 2. This can be explained by the impossibility of lane-changing in the region of the bottleneck (i.e., vehicles in lane 2 cannot move to lane 1 because of the bottleneck), vehicles in lane 2 moves as a cluster with the same speed and when they reach the last sites of the bottleneck vehicles can move to the lane 1 where the free space exists (because of the condensation of the free space caused by the bottleneck) which improve the traffic situation in lane 2 even for high densities.

As we saw from the spacetime configuration the bottleneck plays an important role to defines the traffic flow in both lanes. On one hand, the bottleneck condenses the free space in front of it in both lanes because the hindrance of traffic in lane 1 increases the free space, which is useful for lane-changing for vehicles of lane 2, which reduces the number of vehicles in lane 2 as seen in Fig. 3.b. On the other hand, the queue of stopped vehicles increases as the density increases. The bottleneck is considered as a local defect in the two-lane model, the distribution of both defects (bottleneck and slow vehicles) on road can also cause serious problem in traffic, such as the accident.

Hereafter, both types of accidents (rear-end collision and the collision due to unsafe lane-changing) will be studied.

Let’s start with the rear-end collision probability (see Fig. 5). For extremely low densities, $P_{acc}^R$ equals zero for both lanes, because all vehicles can move with their desired speeds even with
the existence of the bottleneck. However, as the density increases, small perturbations in both lanes induced by the bottleneck appears which reduce the speed of vehicles, as a result, the stopped vehicles emerge which increase the probability of rear-end collision. The car accident probability reaches a maximum then starts decalin. In that case, the number of stopped vehicles in both lanes become higher. In lane 1, for high density, the rear-end collision probability becomes zero because of the deadlock situation caused by the bottleneck and the impossibility of lane changing (see Fig. 5.a). In lane 2 as figure 5.b shown, for low and high density the probability of vehicles rear-end collision is independent of the bottleneck length here the longitudinal interaction between vehicles controls the traffic situation. For the intermediate densities, car accident probability decreases as the length of the bottleneck increases. In order to understand this feature, let’s see the frequency of speed in both lanes for \( L_B = 10 \) and \( L_B = 200 \) (see Fig. 6). For the lane 1 (See Fig. 6.a), the range of the specter of speed is wide for the case of \( L_B = 10 \) which means that even the presence of stopped vehicles they don’t control the system. In this case, the heterogeneities of speed in lane 1 for \( L_B = 10 \) increase the rear-end collision. On the other hand, for \( L_B = 200 \) even the probability of stopped vehicles is higher than that in the case of \( L_B = 10 \) (see Fig. 7.a), however, the length of the bottleneck reduces the fluctuation of speed vehicles which reduces the probability of rear-end collision. For lane 2 (See Fig. 6.b), the situation is slightly different in both terms (qualitatively and quantitatively), here, the specter of speed is heterogenous for both cases, and the bottleneck is not presented in this lane, the increase of the stopped vehicles increases the probability of vehicles accident (see Fig. 7.b).

In order to get better insight into the microscopic analysis, the probability of vehicles accidents is plotted as a function of the lattice site \( i \) (see Fig. 8). For lane 1, \( P_{acc}^R \) decreases as we approach the bottleneck, this means that near the bottleneck the queue of stopped vehicles becomes stable (see Fig. 9). The disturbance of the bottleneck causes stops and go far from it as seen in the spacetime configuration and increase the probability to find a stopped vehicle near of it (see Fig. 9(a)), thus reduce the vehicles accident near the bottleneck. For lane 2 (see Fig. 9(b)), as one approach to the bottleneck the lane-changing frequency decreases which reduces the density and the probability to find a stopped vehicle in the adjacent sites of the bottleneck, thereby reducing the accident probability. As we saw, the bottleneck affects the vehicle's accident in both lanes, the rear-end collisions depend strongly on the bottleneck length. Now let’s study the collision due to the unsafe lane-changing. Fig. 10 shows the probability of vehicles accident due to the unsafe lane changing \( P_{acc}^L \). As we can see the probability of vehicles collision increases as the density increases, reaching a maximum then starts to decrease until
reaching zero, here, the probability of vehicles collision shows similar features like the lane-changing frequency. The flexibility of vehicles for changing lanes is affected by the length of the bottleneck. For a short length $L_B$, the vehicles change the lane more often because the probability to find an adequate space is higher than the case of the large bottleneck length, thus, increase the number of critical cases that can induce the vehicles collision. To understand the features of $P_{acc}$, we depict the space-time diagrams (see Fig. 11) which show us typically the position of the accidents resulting from the unsafe lane-changing for two values of densities: $\rho = 0.1$ and $\rho = 0.6$. For small density in lane 1, most accidents occur directly before the bottleneck (See Fig. 11(a)). Far from the bottleneck, all vehicles can change lanes safely because of the large space in both lanes. As the vehicles approach the bottleneck, the scenario is different for slow vehicles, whose change to lane 2 in order to enhance their speed due to the fixed and moving bottleneck. while fast ones will not always find sufficient large gaps in front of it, hence they remain trapped in lane 1 and increases the risk of an unsafe lane-changing collisions. Subsequently, immediately after the bottleneck on lane 2 as fig. 11.a shown, a free space is created, which provides an opportunity for some vehicles to change their lane, leading to an increase in $P_{acc}$. In addition, at intermediate density (i.e., $\rho =0.6$) the situation where the fast vehicle is still hindered occurs more frequently than at small densities. The lane-changing rules are not always satisfied, which decreases the collision due to unsafe lane changing (See Fig. 11.b). As a result, we conclude that the impact of bottleneck is obvious at small density as spacetime shows. In contrast, at intermediate density the accidents are distributed all over the two lanes, here the bottleneck has not had a big effect on the occurrence of lane-changing collisions. Still, the main factor is the density of vehicles.

### 3.2 With a revised lane-changing rules at $I_c$

In this section, we take a small value of $I_c$ (i.e., $I_c =10$) to make sure that the vehicles are very approached to the bottleneck, and we will compare the current $J$ in two lanes with and without a revised lane-changing rules as a function of density $\rho$. As Fig. 12.a depicts that the current $J$ in the lane 1 after revised lane-changing is rapidly raised to the maximum value at the small densities as a compared with the flux in the case without a revised lane-changing rules, then $J$ decreased curvedly in the medium and large densities, reached zero at full density (i.e., $\rho=1$). We note that the effect of revised lane-changing rules is prominent as it reduces the current $J$ at intermediate density, while at large densities increase it. In addition, we note that $J$ does not vanish after the density $\rho=0.85$ compared to the flux without a revised lane-changing rules. On
the other hand, in lane 2, from the fundamental diagram for $\rho<0.9$ (see Fig 12.b) the flux increases rapidly with the density showing a no-linear relationship in the case with a revised lane-changing rules compared to the flux without it. Thereafter, the current $J$ reaches a maximum at $\rho=0.7$. afterwards decreased to value of zero at $\rho=1.0$. Please note that in a medium density the revised lane-changing rules decreases the flux $J$, while in a high density the currents are similar.

To understand the features of the current $J$ after the revised lane-changing rules in the two lanes, we depict the space-time diagrams (see Fig. 13) for three densities (i.e., $\rho =0.05$, $\rho =0.3$, and $\rho = 0.9$), where the white color represented the empty space between vehicles, the yellow color designed the bottleneck position, the black color represents the fast vehicles while slow ones are represented by red color. At a small density (i.e., $\rho=0.05$), as we mentioned before, the vehicles move at their maximum speed and change the lane easily when the changing-lane conditions are fulfilled which makes the fast vehicles reach the zone $I_z$ quickly in transitory state, and trigger the new criteria of changing-lane mentioned above, this changes the curve of current $J$ and explains why the relationship between the flow and density is not a straight line at low density. On the other hand, the slow vehicles dominate the lane2. where after the bottleneck, we notice that some of fast vehicles, return back to lane1 in order to improve their speed, while slow vehicles do not need to change their lanes because they are moving at their maximum speed. At the medium densities (i.e., $\rho=0.3$), most of the fast vehicles remain trapped in the lane 1 due to the presence of the bottleneck and because they do not find sufficient conditions to change to lane 2, which gives rise to a huge number of fast vehicles before it, hence the width of the cluster of fast vehicles near the bottleneck is greater compared to the width in the case of small density. In addition, in lane2 we found that the slow vehicles are dominant and control the speed of all vehicles behind them. On the other side, after the bottleneck, it appeared that lane2 consisted of a mixture of fast and slow vehicles, while lane1 is empty of vehicles, which explains that the safety and incentive conditions don’t encourage any vehicle to change to the lane 1. At a high density (i.e., $\rho=0.9$), in lane 2 the distribution of vehicles is large, the cluster of fast ones undergoes the go and stop event, which makes the situation difficult for fast vehicles to change their actual lane, while in lane 1, and despite the fact that there are many stopped vehicles, the conditions in the real situation that we have provided cause some vehicles to change to lane 2, where we see in fig. 14 that the histogram of lane changing frequency of vehicles at the zone $I_z$ does not vanishes where it decays smoothly
in the density region $\rho \in [0.9, 1]$, this explains why the flux does not tend to zero in these densities.

Now, we will compare the rear-end collision probability $P_{acc}^R$ in two lanes before and after revised criteria of lane-changing against the density $\rho$. As shown in lane 1 (see Fig. 15.a) the rear-end collision start to occur from the low densities ($\rho = 0$) because the $p_c$, and rapidly increases until it reaches its maximum, after that, it decreases to vanishes for $\rho = 1$. While in the case without adding the new changing rules, the $P_{acc}^R$ remains constant but after ($\rho \approx 0.05$) increases sharply with increasing of $\rho$, thereafter, decays to zero at occupancy $\rho=0.85$. We should note that despite the fact that there are many stopped vehicles in lane 1 at high density (see Fig. 16.a), the conditions in a realistic situation that we have provided cause some vehicles to change from lane 1 to lane 2, in this case, the $P_{acc}^R$ does not vanish (See Fig. 14). As a result, the effect of the addition of revised lane-changing rules in two lanes is significant as it increases the $P_{acc}^R$ for all density regions.

In the other lane in the case without revised lane-changing rules (see Fig. 15.b), there is a critical density below which no vehicle accident can occur, which means that the vehicles move at the same average speed, and there is no stopping vehicle, as a result, the absence of rear-end accident. But beyond the critical densities $\rho=0.08$, the vehicular accident probability $P_{acc}^R$ trends up to the maximum then decreases with the additional increase of $\rho$ since the changing lane becomes more and more difficult (i.e., decrease of empty spaces). These results are easy to understand because the occurrence of the accident in lane 2 is directly related to the number of stopped vehicles. (See Fig. 16.b). On another hand, the impact of a revised lane-changing rules contributes to making accidents start from the beginning where we found for extremely low densities that the accidents start to occur from zero density. Afterward, we observe the same feature found in the case without a revised lane-changing rules. Please note that the revised changing-lane at zone $I_z$ increase the $P_{acc}^R$ for all densities below $\rho \approx 0.5$ whereas it reduces slightly in the interval $\rho= [0.75, 0.95]$ and remains similar in the rest of the densities. Figure 17 displays the lane-changing collision probability $P_{acc}^L$ as a function of $\rho$ for two cases (i.e., with and without a revised lane-changing rules). It is noted that only a moderate change of $P_{acc}^L$ can result from the new criteria. Where we found an enhancement of the unsafe lane changing collisions (i.e., minimize the $P_{acc}^L$).
4. Conclusion

In this paper, the bottleneck in a two lanes system was investigated. A heterogeneous system was considered (i.e., two types of vehicles slow and fast). The bottleneck was considered as a local defect in the two-lane model, the distribution of both defects (moving and fixed) can also cause a serious problem in traffic, such as an accident. The spacetime diagram helped us to understand the interaction between vehicles where we observed that for extremely low densities, vehicles order themselves as one lane is considered as a fast lane and the other lane was controlled by the slow vehicles. Furthermore, the rear-end and lane-changing collisions were investigated. We found that the length of the fixed bottlenecks increases the probability of both types of collision. In addition, we found that the fixed bottleneck has even less impact on the occurrence of lane-changing collisions. Additionally, in the lane where there is a bottleneck, the revised lane-changing rules decreases the flow at a small and a medium density and increases it at a large density. In contrast to the other lane, the revised lane-changing rules showed that the relation between flow and density is not a straight line for extremely low density, and reduced the current at intermediate density. Furthermore, the findings show that the revised lane-changing rules maximizes rear-end collisions and minimizes the risk of unsafe lane-changing collisions. This work can be considered an attempt to understand the interaction of vehicles in a two-lane system with the presence of two types of defects (moving and fixed).

References


Figure caption

**Fig. 1** Sketch of a symmetric model (a): before, (b) after adding the new criteria. The blue color represents the fast vehicles while the red color denotes the slow vehicles. The colored positions present the zone $I_z$.

**Fig. 2.** Fundamental diagrams for several length of the bottleneck where (a): lane1, (b): lane2.
Fig. 3. Lane change frequency against the total density of vehicles, where (a): from lane 1 to lane 2, (b): from lane 2 to lane 1.

Fig. 4. Spacetime configuration for several values of $L_B$: a) $L_B=10$, $\rho=0.05$ b) $L_B=200$, $\rho=0.05$, c) $L_B=10$, $\rho=0.5$ d) $L_B=200$, $\rho=0.5$, e) $L_B=10$, $\rho=0.65$, f) $L_B=200$, $\rho=0.65$ (The white color represents the free space while the black color denotes the fast vehicles and the red color represents the slow vehicles, hence, the yellow color denotes the bottleneck). The horizontal axis represents the space where the evolution of space is from left to the right, while the perpendicular axis represents time, where the evolution of time is from up to down.

Fig. 5 Rear end collision probability for several lengths of bottleneck in a) lane 1, b) lane 2.

Fig. 6 Speed frequency $P_v$ for $\rho=0.6$ a) lane 1, b) lane 2.

Fig. 7 Probability of stopped vehicles $P_S$ for several lengths of bottleneck for a) lane 1, b) lane 2.

Fig. 8 Profile of rear end collision probability as a function of the lattice site $i$ for $\rho=0.6$ a) lane 1, b) lane 2.

Fig. 9 Profile of stopped vehicles $P_0$ for $\rho=0.6$ a) lane 1, b) lane 2.

Fig. 10 Probability of vehicles collision due to unsafe lane-changing for several length of the bottleneck.

Fig. 11 Spacetime configuration for (a): $\rho=0.1$, (b): $\rho=0.6$ in two-lane traffic model with $L_B=10$ where the white color represents the free space while the black color represents the place of the lane-changing collision, hence, the orange color denotes the bottleneck. The horizontal axis represents the space where the evolution of space is from left to the right, while the perpendicular axis represents time, where the evolution of time is from up to down.

Fig. 12 The fundamental diagrams with and without revised lane-changing rules at the zone $I_z$, with $L_B=10$, $I_z=10$, and $p_c = 0.01$ in : a)Lane 1, b) Lane 2.

Fig. 13 Space time diagram after correction in a symmetric two-lane traffic model. For (a) $\rho = 0.05$ (b) $\rho = 0.3$ (c) $\rho = 0.9$. Where the white color is the free-space and the black color
corresponds to fast vehicles, red color corresponds to slow vehicles while the yellow color represents the bottleneck.

**Fig. 14** Histogram of lane-changing frequency against the density of vehicles in the zone $I_z$ with $p_c = 0.01$.

**Fig. 15** Rear-end collision probability $P_{acc}^R$ as a function of density $\rho$. in a) lane 1, b) lane 2, before and after revised criteria of lane-changing with $p_c = 0.01$.

**Fig. 16** Probability of stopped vehicles $P_S$ in two lanes before and after revised criteria of lane-changing at the zone $I_z$. With $p_c = 0.01$.

**Fig. 17** lane-changing collision against the density of vehicles before and after revised criteria of lane-changing at the zone $I_z$, with $p_c = 0.01$. 

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**Fig. 1**
Fig. 2

Fig. 3
Fig. 4

Lane 1

Lane 2

(f)

Fig. 5

(a)

(b)
Fig. 6

(a) 

(b)

Fig. 7

(a) 

(b)
Fig. 11
Fig. 12

(a) Lane 1

(b) Lane 2
Fig. 13

Lane 1

Lane 2

Fig. 14

Lane-changing frequency from lane 1 to lane 2
Fig. 15

(a)

(b)

Fig. 16
without a revised lane changing rules
with a revised lane changing rules

Fig. 17