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Performance Analysis of Computational Intelligence Correction

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Abstract Kalman filter (KF) is a widely used navigation algorithm, especially for precise positioning applications. However, the exact filter parameters must be defined a priori to use standard Kalman filters for coping with low error values. But for the dynamic system model, the covariance of process noise is a priori entirely undefined, which results in difficulties and challenges in the implementation of the conventional Kalman filter. Kalman Filter with recursive covariance estimation applied to solve those complicated functional issues, which can also be used in many other applications involving Kalaman filtering technology, a modified Kalman filter called MKF-RCE. While this is a better approach, KF with SAR tuned covariance has been proposed to resolve the problem of estimation for the dynamic model. The data collected at (x: 706970.9093 m, y: 6035941.0226 m, z: 1930009.5821 m) used to illustrate the performance analysis of KF with recursive covariance and KF with computational intelligence correction by means of SAR (Search and Rescue) tuned covariance, when the covariance matrices of process and measurement noises are completely unknown in advance.

Keywords Standard Kalman Filter · Modified Kalman Filter · Recursive Covariance Estimation · Search and Rescue Optimization.

1 Introduction

In 1960 R.E. Kalman published his most popular work outlining a recursive approach to the non-linear filtering problem [4,5]. Since then, owing in large

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part to the advantages of recursive computation, the Kalman filter has been the subject of extensive research and implementation, especially in the field of autonomus or aided navigation [6]. Precise position estimation is the big challenge for any existing or upcoming navigation solutions [17], [18]. For several tracking and data prediction challenges [14], Kalman filter has long been used as the perfect solution. Prediction and updation are the two important steps involved in Kalman filter agorithm [2]. It is the primary module for the Kalman Filter's measurement Update and State Update [19]. Suppose the state of x_k to be predicted is regulated by the following dynamics (1).

$$\begin{cases} x_k = \phi_{k,k-1} x_{k-1} + \gamma_{k-1} w_{k-1} \\ y_k = H_k x_k + v_k \end{cases} \tag{1}$$

Where $\phi_{k,k-1}$ is the state transition matrix, H_k is the observation matrix, w_k is the process noise, v_k is the measurement noise, γ_k is the noise input matrix and y_k is the measurement at t_k. The state and measurement update estimates are obtained by the following equations (2). The $\hat{x}_{k,k-1}$ is predicted value of $\hat{x}_k$.

$$\begin{cases} \hat{x}_{k,k-1} = \phi_{k,k-1} \hat{x}_{k-1} \\ \hat{x} = \hat{x}_{k,k-1} + K_k (y_k - H_k \hat{x}_{k,k-1}) \\ P_{k,k-1} = \phi_{k,k-1} P_{k-1} \phi^T_{k,k-1} + \gamma_{k-1} Q_{k-1} \gamma^T_{k-1} \\ K_k = P_{k,k-1} H^T_k (H_k P_{k,k-1} H^T_k + R_k)^{-1} \\ P_k = (I - K_k H_k) P_{k,k-1} \end{cases} \tag{2}$$

The covariances of process and measurement noise under static conditions are considered as $Q = E[w_k w_k^T]$ and $R = E[v_k v_k^T]$ [20,21]. The mean square eror is given by $E[e_k e_k^T]$, that is equivalent to P_k. This expansion carried as given below (3).

$$P_k = E[e_k e_k^T] = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \tag{3}$$

Covariance is the model parameter used to achieve low error values[12]. The next section explains how recursive covariance estimation enhanced positioning efficiency relative to the standard Kalman filter.

2 Modified Kalman Filter with Recursive Covariance Estimation (MKF-RCE)

In the presence of large process uncertainty and measurement noise covariance matrices, the MKF-RCE algorithm is the efficient approach to the state estimation problem[7, 22]. The following section introduces the statistical study of MKF-RCE.

$$y_k = x_k + v_k \tag{4}$$

v_k is the measurement noise at the k^{th} step. Chosen a random variable ζ is that $E[\zeta_k \zeta_{k-p}^T] = 0$ where $p \geq k_m$ and T is the sample period. From these samples the covariance of chosen random variable is calculated as given below (5).

$$E[\zeta_k \zeta_{k-i}^T] = \lim_{k \to \infty} \sum_{j=1}^{k} \zeta_k \zeta_{j-1}^T \tag{5}$$

Where $i = 1, 2, 3, ..., k_m$. However, for online parameters estimating the above method of measurement is not effective since all samples from the vector cannot be obtained at any time.For real- time calculation it is also optimal to apply the recursive version of the above estimation algorithm. It can be obtained that by taking estimated covariance matrix from samples upto n and new sample $n+1$, that is $\hat{e}_n[\zeta_k \zeta_{k-i}^T]$ and with zero mean (7). Here ζ_k obtained from measurements as given in (6).

$$\zeta_k = y_k - 2y_{k-1} + y_{k-2} \tag{6}$$

$$E[\zeta_k] = 0 \tag{7}$$

$$E[\zeta_k \zeta_k] = T^2 Q + 6R \tag{8}$$

$$E[\zeta_k \zeta_{k-1}] = -4R \tag{9}$$

From these (8) and (9) Q_k,R_k values can be estimated. Those need to be substitute in standard kalman filter mathematical expressions.

2.1 Algorithm 1 - MKF-RCE

Initialization: P_0 the initial state of error covariance matrix
$\hat{e}_0(\zeta_k \zeta_k) = 0$,$\hat{e}_0(\zeta_k \zeta_{k-1}) = 0$,$\hat{Q}_0 = 0$,$\hat{R}_0 = 0$
Input:measurement sequence $[y_k]$
Output: state estimate $\hat{x}_k$, error covariance matrix P_k
1 : for $k = 1ton$ do
2 : Calculate ζ_k
3 : Update estimated covariance matrix
4 : Calculate the estimated $\hat{Q}_k$ and $\hat{R}_k$
5 : $\hat{x}_{k,k-1} = H\hat{x}_{k-1}$
6 : $\hat{P}_{k,k-1} = HP_{k-1}H^T + \hat{Q}_k$
7 : $\hat{K}_k = \hat{P}_{k,k-1}(\hat{P}_{k,k-1} + \hat{R}_k)^{-1}$
8 : $\hat{x}_k = \hat{x}_{k,k-1} + \hat{K}_k(y_k - \hat{x}_{k,k-1})$
9 : $P_k = (I - \hat{K}_k)\hat{P}_{k,k-1}$
10 : end for
11 : Return $\hat{Q}_k$,$\hat{R}_k$,$\hat{x}_k$,$\hat{R}_k$
Algorithm 1 can be used to deal with the state estimation problem when covariances of process and measurement noise are completely unknown. The below 2 depicts how error values reduced with MKF-RCE algorithm than standard Kalman filter.

3 Search and Rescue Optimization (SAR)

SAR is a metaheuristic algorithm inspired by explorations behavior during search and rescue operations. The performance of SAR is proved better than classical optimization techniques [13]. Real world engineering problems are difficult to resolve using existing optimization techniques, there gradient information is required [8]. Hence, a prominent optimization, which does not depend on gradient data is required to find the solution for constrained engineering problems. SAR is a such technique which does not need gradient information. Many creatures use different strategies while searching their goals [9]. rescue operation is one type of group explorations and search is a systematic operation. Hold clue and abandoned clue are two important types of clues in search and rescue operations by human beings. Every optimization consists a fitness function to achieve optimal solutions [3,10,11]. In SAR mathematical model, human's position is the solution and clue is the fitness. While finding main clues, some clues are left. Those left clues are stored in a memory matrix (M) and position matrix consists of human's positions, both the matrices dimensions are equal. The found clues are stored in clue matrix (C). The clue matrix is given as in (10).

$$C = \begin{pmatrix} X \\ M \end{pmatrix} \tag{10}$$

The main two control parameters of SAR algorithm are Social Effect (SE) and Maximum Unsuccessful Search Number (MU). In social phase, one random clue has to be assumed and need to check for clues around the positions. In individual phase of SAR, humans search for clues around their current positions. In boundary control state, the solutions obtained in both social and individual phases has to be located in solution space, if they are out of limits, they should be changed. Like this in each iteration the group members have to check their positions with previous positions. If the position is greater than previous position, the previous position is stored in memory matrix, oterwise it is not updated.

$$M_k = \begin{cases} X_i & \text{if } f(X_i') > f(X_i) \\ M_k & \text{otherwise} \end{cases} \tag{11}$$

Where, M_k is the position of k^{th} stored clue, and X_i is the previous position.

3.1 Algorithm 2 - SAR OPTIMIZATION

1. Initialization: population of 2N solutions, $[P_j^{min}, P_j^{max}], \quad j = 1, 2, ..., D$.
2. find P_{best} by sorting solutions in descending order.
3. Position matrix(x) =First half of sorted solutions, Memory matrix (M) =Others
4. Unsuccessful search number $U_i = 0$, where $i = 1, ..., N$.
5.While (not satisfied) do
6. for $i = 1toN$

7. Update C
8. if $r < 0.5$
9. Apply social phase
10. Else
11. Apply individual phase
12.End if
13. Boundary control
14. Update Memory
15. if $U > MU$ & X_i is fit
16. X_i into search space
17. $U = 0$
18. Else $U > MU$ & X_i is not fit
19. $U = 0$
20. End If
21. Restart
22. End
23. find current position and update P_{best}
24. End while
25. Return P_{best}

4 Kalman Filter with SAR Tuned Covariance

Kalman filter is a navigation solution used to estimate the anonymous state of a dynamic system and to suppress noise existing in aviation control systems [16]. However, the effect is more susceptible to parameters of KF, whose choice purely based on previous experience of an operator. In this paper the parameter tuning using SAR is presented. And condition number of observation matrix is also considered as performance metric by checking condition number is reaching unity or not.

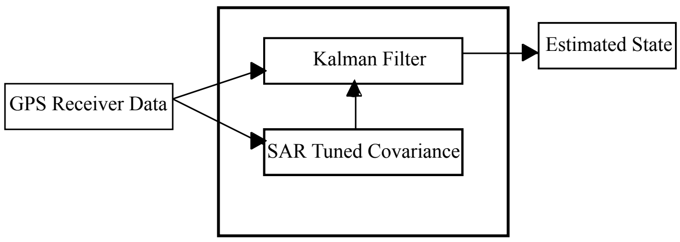

Fig. 1 Kalman Filter with SAR Tuned Covariance.

The fitness function (12) for SAR optimization to achieve optimal covariances of process and measurement noise is

$$J = \| P \| + \| Q \| + \| R \| \tag{12}$$

Table 1 Statistical Accuracy Measures

2D position Accuracy Measures	3D Position Accuracy Measures
Distance root mean squared error (DRMS) = $\sqrt{(\sigma_x^2) + (\sigma_y^2)}$	Mean radial spherical error (MRSE) = $\sqrt{(\sigma_x^2) + (\sigma_y^2) + (\sigma_z^2)}$
Circular error probability (CEP) = $0.56\sigma_x + 0.62\sigma_y$	Spherica error probability (SEP) = $0.51(\sigma_x + \sigma_y + \sigma_z)$

For the error analysis of state estimation using SAR tuned covariance the following expressions can be used.
Infinity norm of the forward error

$$\| x_k - \hat{x}_k \|_\infty \tag{13}$$

Relative forward error of state estimate

$$\| x_k - \hat{x}_k \|_\infty / \| x_k \|_\infty \tag{14}$$

Relative backward error of state estimate

$$\| z - Hx_k \|_\infty / \| z \|_\infty \tag{15}$$

Where z is the measurement residual. Error magnification factor

$$\frac{\| x_k - \hat{x}_k \|_\infty / \| x_k \|_\infty}{\| z - Hx_k \|_\infty / \| z \|_\infty} \tag{16}$$

Condition number of observation matrix (H) is

$$\| H \| . \| H^{-1} \| \tag{17}$$

The proposed Kalman Filter with SAR tuned covariance improves the positioning performance by updating the covariances of process and measurement noises using SAR optimization and by setting objectives as condition number and error maginification factor criteria. The presented work is depicted in 1. The performance analysis is based on $2D$ position accuracy measures and $3D$ accuracy measures [1], which are listed in 1.

5 Results

To illustrate the performance analysis, data collected by GPS receiver located at Andhra University, Visakhapatnam is used. The position estimation is done over a period of 23hr 56 mins(2640 epochs) with initial reference location (x:1208443.5605, y:5966808.3895, z:1897080.4657m).The collected data is sampled at an interval of 30 sec. The receiver located in coastal area of Andhra Pradesh is estimated using conventional Kalman filter, MKF-RCE and Kalman filter with SAR tuned covariance. The positional errors in $x-$,$y-$

and $z-$ directions over a day with existing approaches along with proposed technique are shown in 2. The optimized parameter values of covariances of process and measurement noise are $P_0 = 0.1154$,$R = 0.9415$ and $Q = 0.0053$. The precision of positioning of implemented algorithm in terms of 2D and 3D position accuracy measures. It is observed that SAR tuned KF has DRMS of 10.98m, CEP of 8.83m ,MRSE of 11.36m and SEP of 8.98m.The statistical accuracy measures of standard kalman filter, MKF-RCE algorithm and proposed SAR tuned Kalman filter are listed in 2. It is observed that there is a large difference betweeen mean values in $Y-$direction compared to existing kalman filter and MKF-RCE approaches.The average error difference between standard KF and MKF-RCE are 2.46m,4.35m and 1.45m in $x-$,$y-$ and $z-$ directions respectively. Error difference between KF and SAR-KF are 9.88m,17.44m and 5.80m in $x-$,$y-$ and $z-$ directions respectively.Error difference between MKF-RCE and SAR-KF are 7.41m,13.08m and 4.35m in $x-$,$y-$ and $z-$ directions respectively. It is observed that from the accuracy measures and position error values the proposed Kalman filtering approach with SAR tuned covariance is giving the more accurate values than MKF-RCE approach.

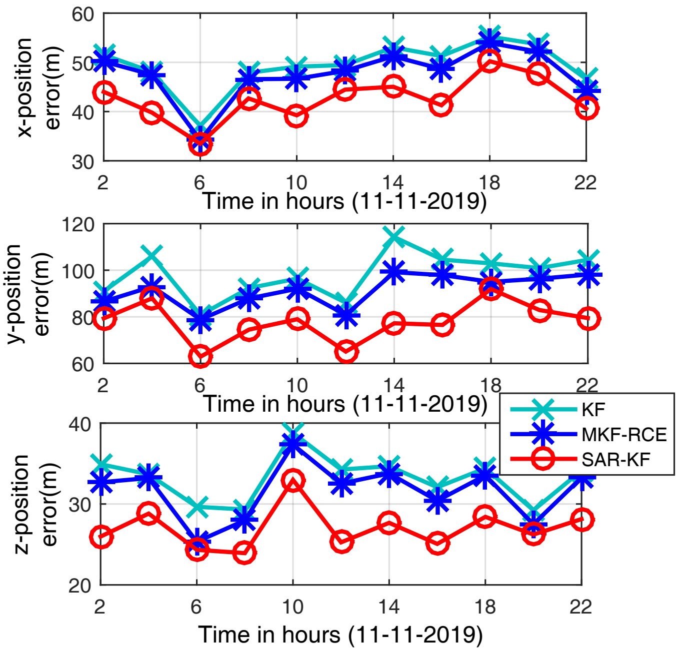

Fig. 2 Positioning performance in $x-$direction (DGPS Receiver located at southern region of Indian subcontinent (Lat/Lon: $17.72^0N/83.32^0E$)).

Table 2 Statistical Accuracy Meaures Analysis

Statistical Accuracy Measures	Standard Kalman Filter	MKF-RCE	SAR Tuned KF
x_{mean}	49.36	47.59	42.31
y_{mean}	98.15	92.87	72.04
z_{mean}	33.19	31.59	26.29
$x_{deviation}$	6.73	5.24	4.87
$y_{deviation}$	12.53	11.42	9.84
$z_{deviation}$	5.94	3.45	2.91
$DRMS$	18.78	12.57	10.98
CEP	14.64	10.02	8.83
$MRSE$	19.20	13.04	11.36
SEP	15.41	10.26	8.98

6 Conclusion

The standard KF, MKF-RCE and proposed SAR tuned KF performances are assessed interms of precision and accuracy. The DGPS receiver located at Andhra University, Andhra Pradesh,(Lat/lon: $17.72^0N/83.32^0E$) data used in the analysis. In the proposed technique, the parameters of Kalman filter are optimized by search and rescue optimization technique. The suggested SAR tuned KF was used in position estimation and its performance is evaluated with various SAM. Results depicting that SAR-KF converges in less time and has an accuracy difference of 1.19 meters circular error probability when compared to MKF-RCE technique. It is observed that SAR-KF has high accuracy and faster convergence rate than conventional Kalman filter over southern region of Indian subcontinent.

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