Effect of Rented Warehouses for Deteriorating Items Under Stock Dependent Demand, Partial Backlogging

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Research Article

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Abstract:
In the area of two warehouses-based inventory models, various study has been carried out, authors presented their models under different parameters and environmental conditions. In most of the study, it can be seen that one warehouse assumed as own warehouse and other assumed as rented. Due to different facility and conditions both the warehouses have different rate of demand and deterioration along with different storage capacity. Shilpi et al. [2019], has developed an inventory model by considering both the warehouses as ranted. It was also assumed in the study that both warehouses having the same capacity. In addition, it has been also assumed that the items in the second ranted warehouse start decaying after some time whereas the items in the first warehouse used first to meet the demand of the customers.

After critically reviewed the model by Shilpi et al. [2019], it can be observed that the storage techniques of items need to be further explored, as model used the items from first warehouse to satisfy the demand of the customers wares the items whose deterioration about to start has been stored in the second warehouse. This concept of storage has to be change, as such items in the second warehouse totally deteriorated till their turn comes up. Keeping this fact in mind, we have developed an inventory model based on two different ranted warehouses for deteriorated items whose rate of deterioration is not so much high but after a very short of time these items start decaying. According to the current model, the demand of the customers has been satisfied from the first warehouse, also to avoid deterioration in the second warehouse, we have to transfer such items whose deterioration about to start from second warehouse to first ranted warehouse so that we can avoid the deterioration of the items. In the current model we have assumed both the warehouses as rented which is suitable for the manufacturer whose product demand is very high and space is not much enough. It is obvious that the demand for the items in the stock increases due to stock up to a certain level. In the model the demand for the first warehouse is assumed as stock dependent. When we have extra stock the second warehouse used for the products and we have also assumed that the demand has becomes constant. The model is also instigated under sensitivity analysis and a brief numerical example with real data has been illustrated.

Keywords: warehouses; stock-based demand, partial backlogging, deterioration.

1. Introduction

In the past few years, the concept of space restriction is also introduced as manufactures always wish to purchase more and more items. The first study based on two warehouses was developed by Sarma (1987) in which the rate
of deterioration was considered as a constant in both the warehouse. After some time, the concept of constant deterioration was ignored and new models were developed by using more parameters, in the same way, Pakkala and Achary (1994) introduced the concept of bulk release pattern and developed a two-warehouse inventory model for decaying items. Few more study based on the constant demand was developed such as, Yang (2004) provided a two-storage inventory model with constant demand and shortages under inflationary environment. Based on inflation, Wee et. al. (2005) presented a two-warehouse inventory model under a constant demand rate and variable deterioration. After some more research papers the concept of taking constant demand was no longer more applicable as it is not feasible to have a constant demand all the time, by considering the stock level dependent demand, Zhou and Yang (2005) developed a stock level dependent demand model under quantity-based transportation cost without shortage. Where, Lee and Hsu (2009) presented an updated form of the study which was being developed by Zhou and Yang (2005) by introducing an inventory model under two-warehouse by taking stock dependent demand for deteriorating items where shortages of items are also assumed. In all previous study made by various researches, the demand was mainly assumed as constant and some time it was considered as time dependent. It is not correct that the demand fixed as constant always, in case of the items with high rate of deteriorated products the concept of constant demand is not correct. Based on newsvendor-type products, Wang et al. (2010) developed a two-warehouse inventory model for the deteriorated items where two ordering opportunities has been carried out. By considering the condition-based delay in payment, Liang and Zhou (2011) presented a model on two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment.

A new approach of volume flexibility was introduced by Sanjay et. al. (2013), where, a production model for different demands under volume flexibiity being developed. The space related to extra inventory was a big issue to handle as if the inventory not being stored properly, it effects the costs as well the profit, in the same way a model by Tayal et al. (2014a) being developed with space restriction for the deteriorating items in which the demand was assumed as stock dependent. Again, by introducing the investment in preservation technology to improve the life of the inventory stored items Tayal et al. (2014b) introduced a model of two warehouse for deteriorating items was developed. It was also suggested by the model developed by Tayal et al. (2014b) that even if we pay extra for the preservation of items but it helps us to reduce the product’s exist rate of deterioration.

By considering trade credits on inventory model, Chen et al. (2014) developed a comprehensive model lot-sizing decisions for deteriorating items with two warehouses, in this model the order-size-dependent trade credits were assumed. Similar model has been developed by Khurana et al. (2015) presented in which inflationary environment for the deteriorated product with stock dependent demand under partial backlogging was assumed. In the mid of 2015, a new concept of assuming secondary location in the problem of two warehouses inventory models was introduced which was very useful for the items whose demands are seasonal. The parameters such as perishable products with trade credit also improve the profit of the inventory as we have to pay latter in the trade credit policy. Taking the same assumption, a model was developed by Tayal et al. (2015a) in which it has been assumed that the items are transferred from primary market to any secondary location to reduce the cost of storing the items up to the next season. A model with replenishment of the items was assumed by Lu et al. (2016) and developed optimal dynamic pricing and replenishment policy for perishable items with inventory-level-dependent demand. By considering the same policy, a model based on trade credit policy being developed by Lu et al. (2016) where an inventory model for perishable products with trade credit period and investment in preservation technology was considered. By considering demand as a function of time. By considering time dependent holding cost with exponential demand rate, an inventory model by Tayal et al. (2015b) presented in which non-instantaneous deteriorating item was assumed.

In the above model the inflationary conditions were not covered, which play a major important role in the inventory management. Jaggi et al. (2016) and Das (2017) presented a model based on replenishment policy for non-instantaneous deteriorating items in two storage facilities under inflationary conditions. By using a different approach on two warehouse-based model, Lin and Wang (2018) presented two-stage stochastic optimization techniques for warehouse configuration and inventory policy of deteriorating items. Based on multi-trip location-transportation problems a model by Moreno et al. (2018) has been developed where in this model an effective two-stage stochastic multi-trip location-transportation model with social concerns in relief supply chains was developed.

In this model the advance payment scheme was not covered although with the almost same assumptions, Jonas (2019), developed an inventory model in which we can pay in advance so that we can have the inventory items at the earliest also, in the study made by Khan et.al. (2019) a two-warehouse inventory model for the decaying items and partial backlogging was assumed. The model developed by Rafique (2019) considered the concept of warehouse inventory model for deteriorating items of a supply chain with nonlinear demand function and
permissible delay in payment under inflation. The trade credits policy was not covered in the above studies where the trade credits policy can also increase the profit of any model, Jonas (2019) considered the concept of trade credits in the inventory model and presented optimizing theory on a two-warehouse system where shortage backordering and trade credit was taken into consideration. A model in which the demand assumed as stock dependent and the effect of the inflation introduced first time was developed by Sanjay et. al. (2020). A model by Ghiami and Beullens (2020) has been presented the continuous resupply policy for deteriorating items with stock-dependent observable demand in a two-warehouse and two-echelon supply chain. In the most literatures presented by various researches, the demand was assumed as a single parameter dependent but it is also essential to take it two or more variable dependent which can improve the total profit. It is also seen that the holding cost related to holding the inventory of the model, is also assumed as constant but it is also important to assumed it variable. By considering all these aspects, first time, a model based on two parameters demand was considered by Tayal et al. (2021). In this study the demand was considered as stock and price of the items with quantity dependent trade credit and variable holding cost under partial backlogging. One more facility over warehouse was developed by Xu et al. (2021), in warehouse selection facility, we can select the mode of the warehouse which was very useful for the inventory models. In this study an inventory model for non-perishable items with partial backlogging and trapezoidal-type demand was assumed. In the same direction by making some more realistic changes in the parameters like preservation technology, Yang (2021) developed a partially backlogged two-warehouse EOQ model with non-instantaneous deteriorating items, where the demand depending on the expiration date with limited storage capacity. Recently, a two-warehouse based model with two variables selling and time dependent demand and variable holding cost-based inventory control model was developed by Jiang et al. (2022). The space restriction was also a problem faced by hole sealer as well as manufacturer, if proper space for the deteriorated items was not handle, the cost on deterioration incenses. When the items stored in the warehouses their preservation becomes important, Das (2022) developed a model in which two-warehouse supply chain model considered and a new approach preservation technology was implemented, the model has been assumed under stochastic demand with shortages. In most of the model discussed above, the demand was considered single parameters dependent, the concept of the multi variable demand also come to the light, even a multi variable demand can also increase the profit as it can handle a model in different dimension. In the same way, Dai and Zheng (2022) presented an optimizing two multi-echelon inventory model where the demand was considered with price and stock dependent, also the model was developed for the perishable products. A case study by Paam et al. (2022) was also presented, in this study a multi-warehouse, multi-product inventory control model for agri-fresh products was considered.

<table>
<thead>
<tr>
<th>Authors</th>
<th>Stock Dependent Demand</th>
<th>Selling Price Dependent Demand</th>
<th>Time-Dependent Demand</th>
<th>Time-Dependent Deterioration</th>
<th>Production based on Demand</th>
<th>Partial Backlogging</th>
<th>Variable Holding Cost</th>
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<td>NO</td>
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<tr>
<td>Yang (2012)</td>
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The present model has been developed according to the present market situation and present environmental conditions.

- The present situation after covid-19 the market is very unlicensed therefore the shortage and overstock both can affect the cost and profit as well, keeping these factors in the mind, current model work on the maximum utilization of the inventory.
- We have assumed both the warehouses as ranted, which is a new approach toward the manufactures space restriction issues. Also, in the past study related to the warehouse problems, demand was considered as stock dependent.
- We have considered a different strategy for the rented warehouses, we have considered that when we have to store the items in the warehouse then obviously, we have to store the items in the warehouse whose deterioration rate is high as compare with the items whose deterioration rate is low for. It can avoid the unwanted deterioration and hence we can improve the profit.
- The demand rate of the items in both warehouses assumed is assumed stock dependent up to a certain level only after that it is constant.

It is also assumed that the deterioration rate is different in both the warehouse as both the warehouses have different storage facility. The shortages are also entertained in the models and treated as partially backlogged when we are out of stock. The aim of this study is to find out the optimal total cost and shortage period in the case when both the warehouses are rented and product is deteriorating in nature. The model is also discussed with a suitable numerical example and the sensitivity analysis is also carried out.

### 2 Assumptions and Notations

#### 2.1 Assumptions

Present model has been developed under the following assumptions:

1. The model is developed under both rented warehouses.
2. For the proper running of the inventory, we have assumed that the demand in the first warehouse is stock based, whereas in the second warehouse the demand is assumed as constant throughout.
3. In the current study we have assumed that there is no deterioration takes place in the second(reserve) warehouse.
4. In the first warehouse, a time depend rate of deterioration assumed.
5. The fixed capacity of the warehouses assumed W units and capacity is assumed higher than the ordered quantity.
6. LIFO policy introduced for the storage of the goods.
7. For the proper formulation of the study, we have named the warehouse as $W_1$ and $W_2$ also, it is assumed that the stock is transferred to warehouse $W_2$ only after filling the warehouse $W_1$.
8. The inventory holding cost per unit time in $W_2$ is higher than those in $W_1$.
9. The shortages are allowed only in $W_1$.
10. The period when the items are not available in the stock are assumed partially backlogged with a constant rate of $\delta$.

#### 2.3 Notations

$W$ : Denote the capacity of the warehouse.
$t_1$: Used to represent the time at which the level of the inventory in warehouse $W_2$ becomes zero.

$t_2$: Used to represent the time at which the level of the inventory in warehouse $W_1$ becomes zero.

$T$: Used to represent cycle of the replenishment.

$\theta$: Used to represent deterioration in the warehouse $W_2$.

$a, b$: Used to represent the demand parameters.

$0$: Used to represent the cost of ordering cost per cycle.

$h_1$: Denote the cost of holding the items per unit in warehouse $W_1$.

$h_2$: Denote the cost of holding the items per unit in warehouse $W_2$.

$c_h$: Denote the shortage cost per unit.

$l$: Denote the cost being paid by lost sale per unit.

$d$: Represent the cost due to deterioration per unit in warehouse $W_2$.

$I_{w_1}(t)$: Represent the inventory level at any time $t$ in warehouse $W_1$.

$I_{w_2}(t)$: Represent the inventory level at any time $t$ in warehouse $W_2$.

$S$: Denote the cost due to ordering quantity.

### 3.0 Mathematical Formulation

The current model is being developed for the deteriorated items stored in two rented warehouses, where the demand of the items is assumed as stock dependent. We have also assumed that the items those are having a short life are stored in the $W_2$ warehouse because we have to clear such items first. And the rate of deterioration in the $W_1$ warehouse is not assumed which is most important aspect of the study. Now initially, in the beginning of the cycle $S$ units of inventory received out of which $W$ units of the items are kept in warehouse $W_1$ and the remaining ($S-W$) units are kept in warehouse $W_2$. For proper running of the inventory the LIFO policy is used.

As we have stored the low life items in the warehouse $W_2$ so we have used the stock of this warehouse, now between the time period $[0, t_1]$ the stock level in warehouse $W_2$ is decreases because of the effect of demand of the item and deterioration. At the time $t_1$ the stock level becomes 0 in the $W_2$, now as we have assumed 0 deterioration in the second warehouse $W_1$ during the time period $[0, t_1]$ so full stock of this warehouse is available for the customer’s demand. Now, to satisfy the customer’s demand we have to start using stock from the warehouse $W_1$ during $[t_1, t_2]$. Now the stock start depletes in the warehouse $W_1$ due to combined effect of demand and deterioration. At the point $t_2$ entire stock is completed and shortages start at this point.

The graphical behaviours of the model are shown in Figure 1.

![Fig.-1 Graphical behaviour of the inventory with respect to time.](image-url)
Case-1: For the warehouse $W_1$

The differential equations representing the behaviour in the warehouse $W_1$ are given as follow:

\[
\frac{dI_{W_1}(t)}{dt} = W, \quad 0 \leq t \leq t_1
\]

\[
\frac{dI_{W_1}(t)}{dt} = -K_1tI_{W_1}(t) - (a + b(I_{W_1}(t))), \quad t_1 \leq t \leq t_2
\]

\[
\frac{dI_{W_1}(t)}{dt} = -a, \quad t_2 \leq t \leq T
\]

With the help of the boundary conditions, we have the following solutions of the equation (1), (2) and (3)

\[
I_{W_1}(0) = W
\]

\[
I_{W_1}(t_2) = W
\]

\[
I_{W_1}(t) = W, \quad 0 \leq t \leq t_1
\]

\[
I_{W_1}(t) = ae^{-bt-rac{K_1}{2}(t_2-t)^2} + \frac{K_1}{6}(t_2^3 - t_1^3), \quad t_1 \leq t \leq t_2
\]

\[
I_{W_1}(t) = a(t_2 - t), \quad t_2 \leq t \leq T
\]

Case-2: For the warehouse $W_2$

The differential equations representing the behaviour in the warehouse $W_2$ are given as follow:

\[
\frac{dI_{W_2}(t)}{dt} = -K_2tI_{W_2}(t) - (a + b(I_{W_2}(t))), \quad 0 \leq t \leq t_1
\]

Under the boundary condition $I_{W_2}(t_1) = 0$

By considering the boundary condition the solution of the equation (8) is given by

\[
I_{W_2}(t) = [e^{(t_1-t)K_2} - 1] \left(\frac{a+bW}{K_2}\right), \quad 0 \leq t \leq t_1
\]

4.0 Calculation of the total inventory cost those are associated with the current models

4.1 Cost associated to the ordering cost of the items per cycle: We have to made order at the stating of each cycle hence

Ordering Cost: $O$

4.2 Cost associated with holding of the items.

4.2.1 For the warehouse $W_1$

As we have used the items from the warehouse $W_2$ hence the inventory items in warehouse $W_1$ for the whole time period of positive inventory, hence the holding cost is given by

\[
H.C_{W_1} = h_1 \left[ \int_{t_1}^{t_1} I_{W_1}(t) dt + \int_{t_1}^{t_2} I_{W_2}(t) dt \right] = h_1 \left[ \int_{t_1}^{t_1} a(t_2 - t) dt + \int_{t_1}^{t_2} [e^{(t_1-t)K_2} - 1] \left(\frac{a+bW}{K_2}\right) dt \right]
\]

\[
H.C_{W_1} = h_1 \left[ (a(t_2 - t_1) - \frac{t_1^2}{2}) + (a + bW)t_2 e^{t_1K_2} \right]
\]

4.2.2 For the warehouse $W_2$

It can be seen from the graphical representation of the model that the stock in warehouse $W_2$ is positive only the duration of $[0, t_1]$, so holding cost during this period in warehouse $W_2$ will be

\[
H.C_{W_2} = h_2[\int_{t_1}^{t_1} I_{W_2}(t) dt] = h_2[\int_{t_1}^{t_1} I_{W_2}(t) dt] = h_2 \left[ \frac{a+bW}{K_2} \right](1 - e^{t_1K_2}) - t_1
\]

4.3 Shortage cost: Shortage cost per cycle can be calculated as follow;

\[
SC = c_h \int_{t_2}^{T} a \ dt = a c_h(T - t_2)
\]
4.4 Lost sale cost

When at time $t_2$ we are out of stock and shortages occurs then it has been assumed that a fix fraction of occurring demand is turn into the backlogging and remaining fraction of the demand lost, so the lost sale cost can be calculated as:

$$\text{LCS} = m \int_{t_2}^{T} (1 - \theta) a \, dt = a m (1 - \theta) (T - t_2)$$

(14)

4.5 Costs related to the deterioration in the warehouse

4.5.1 Cost in warehouse $W_1$

To calculate the units deteriorated we have to find the difference between the initial stock level and total demand which occurring in warehouse $W_1$. But in the current study we have not assumed any deterioration takes place as we have stored such items in the warehouse $W_2$.

4.5.2 Cost in warehouse $W_2$

$$\text{D.C} = d \left( I_{W_2}(0) - \int_{0}^{t_1} a + bW \, dt \right) = d \left( e^{(t_1K_2)} - \frac{(a+bW)}{K_2} - \int_{0}^{t_1} (a + bW) \, dt \right)$$

$$= d \frac{(a+bW)}{K_2} \left( e^{(t_1K_2)} - 1 - (t_1K_2) \right)$$

(15)

5.0 Total average cost of the system: The total average cost involved in the model is the sum of all associated costs:

$$\text{T.A.C.} = \frac{1}{T} [\text{O.C.} + \text{H. C.} + \text{S.C.} + \text{L.S.C.} + \text{D.C.}]$$

Now, we can easily find the minimum value of $t_1$, $t_2$ and $T$ by using any maximization procedure.

6.0 Numerical example: Following data has been taken to validated the model

The capacity of the warehouse ($W$) = 2,000 units, Demand parameters ($a = 40$ units, $b = 0.01$), $h_1 = 0.5$ rs/unit, $h_2 = 0.8$ rs/units, $\theta = 0.6$, $K = 0.02$, $d = 14$ rs/unit, $s = 5$ rs/unit, $l = 7$ rs/unit, $O = 400$ rs/order. The output of the model and the optimal value of $t_1$, $t_2$, $T$ and T.A.C. are as follow $t_1 = 850.552$ days, $t_2 = 950.659$ days, $T = 1,276.15$ days, T.A.C. = Rs. 89,783

7.0 Sensitivity analysis: The effect of each parameter as a sensitivity analysis is also carried out taking one at a time.

### Table 1

<table>
<thead>
<tr>
<th>% Variation in W</th>
<th>W</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$T$</th>
<th>T.A.C</th>
</tr>
</thead>
<tbody>
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<td>−20%</td>
<td>1,175</td>
<td>1,068.06</td>
<td>1,212.06</td>
<td>1,269.76</td>
<td>96,860.4</td>
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<td>1,088.25</td>
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<td>1,042.78</td>
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<td>1,009.64</td>
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<td>1,053.95</td>
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7.1 Comparison of the model with previous study: In this section we have compared the current study with the study those are already done in the two warehouses-based models. Based on the data available we have compared the total average cost with respect to the warehouse capacity.

Figure – 2, represent the graphical representation of total average cost with respect to the warehouse capacity for the current model and figure – 2(a), represent the graphical representation of total average cost with respect to the warehouse capacity for the with the model by model by Tayal et al. (2019)

Figure 2.0 T.A.C. vs. Warehouse Capacity

Figure (a) T.A.C. vs. W
### Analysis of Sensitivity for the demand parameter ‘a’

<table>
<thead>
<tr>
<th>% Variation in W</th>
<th>a</th>
<th>t₁</th>
<th>v</th>
<th>T</th>
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<td>1,098.06</td>
<td>1,122.06</td>
<td>1,359.76</td>
<td>97,860.4</td>
</tr>
<tr>
<td>-15%</td>
<td>52.5</td>
<td>1,054.5</td>
<td>1,080</td>
<td>1,305.76</td>
<td>95,923.5</td>
</tr>
<tr>
<td>-10%</td>
<td>55</td>
<td>1,015.78</td>
<td>1,042.78</td>
<td>1,257.77</td>
<td>94,276.3</td>
</tr>
<tr>
<td>-5%</td>
<td>57.5</td>
<td>981.144</td>
<td>1,009.64</td>
<td>1,214.84</td>
<td>92,874.8</td>
</tr>
<tr>
<td>0%</td>
<td>60</td>
<td>949.978</td>
<td>979.978</td>
<td>1,176.21</td>
<td>91,682.7</td>
</tr>
<tr>
<td>5%</td>
<td>62.5</td>
<td>921.785</td>
<td>953.285</td>
<td>1,141.27</td>
<td>90,667.9</td>
</tr>
<tr>
<td>10%</td>
<td>65</td>
<td>896.161</td>
<td>929.161</td>
<td>1,109.51</td>
<td>89,808.9</td>
</tr>
<tr>
<td>15%</td>
<td>67.5</td>
<td>872.77</td>
<td>907.27</td>
<td>1,080.52</td>
<td>89,084.7</td>
</tr>
<tr>
<td>20%</td>
<td>70</td>
<td>851.333</td>
<td>887.333</td>
<td>1,053.95</td>
<td>88,478.8</td>
</tr>
</tbody>
</table>

**Figure 3.0 T.A.C. vs. a**
Table 3.0  
Analysis of Sensitivity for the demand parameter ‘b’

<table>
<thead>
<tr>
<th>% Variation in b</th>
<th>b</th>
<th>$t_1$</th>
<th>v</th>
<th>T</th>
<th>T.A.C</th>
</tr>
</thead>
<tbody>
<tr>
<td>–20%</td>
<td>0.017</td>
<td>1,098.06</td>
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<td>97,860.4</td>
</tr>
<tr>
<td>–15%</td>
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<td>1,054.5</td>
<td>1,080</td>
<td>1,305.76</td>
<td>95,923</td>
</tr>
<tr>
<td>–10%</td>
<td>0.019</td>
<td>1,015.78</td>
<td>1,042.78</td>
<td>1,257.77</td>
<td>94,276.3</td>
</tr>
<tr>
<td>–5%</td>
<td>0.020</td>
<td>981.144</td>
<td>1,009.64</td>
<td>1,214.84</td>
<td>92,874.8</td>
</tr>
<tr>
<td>0%</td>
<td>0.021</td>
<td>949.978</td>
<td>979.978</td>
<td>1,176.21</td>
<td>91,682</td>
</tr>
<tr>
<td>5%</td>
<td>0.022</td>
<td>921.785</td>
<td>953.285</td>
<td>1,141.27</td>
<td>90,667.9</td>
</tr>
<tr>
<td>10%</td>
<td>0.023</td>
<td>896.161</td>
<td>929.161</td>
<td>1,109.51</td>
<td>89,808.9</td>
</tr>
<tr>
<td>15%</td>
<td>0.024</td>
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<td>907.27</td>
<td>1,080.52</td>
<td>89,084.7</td>
</tr>
<tr>
<td>20%</td>
<td>0.025</td>
<td>851.333</td>
<td>887.333</td>
<td>1,053.95</td>
<td>88,478.8</td>
</tr>
</tbody>
</table>

Figure 4.0 T.A.C. vs. b
Table 4.0

<table>
<thead>
<tr>
<th>% Variation in $\theta$</th>
<th>$\theta$</th>
<th>$t_1$</th>
<th>$v$</th>
<th>$T$</th>
<th>T.A.C</th>
</tr>
</thead>
<tbody>
<tr>
<td>−20%</td>
<td>0.45</td>
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<td>1,359.76</td>
<td>97,860.4</td>
</tr>
<tr>
<td>−15%</td>
<td>0.48</td>
<td>1,054.5</td>
<td>1,080</td>
<td>1,305.76</td>
<td>95,923</td>
</tr>
<tr>
<td>−10%</td>
<td>0.51</td>
<td>1,015.78</td>
<td>1,042.78</td>
<td>1,257.77</td>
<td>94,276.3</td>
</tr>
<tr>
<td>−5%</td>
<td>0.54</td>
<td>981.144</td>
<td>1,009.64</td>
<td>1,214.84</td>
<td>92,874.8</td>
</tr>
<tr>
<td>0%</td>
<td>0.57</td>
<td>949.978</td>
<td>979.978</td>
<td>1,176.21</td>
<td>91,682</td>
</tr>
<tr>
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<td>0.60</td>
<td>921.785</td>
<td>953.285</td>
<td>1,141.27</td>
<td>90,667.9</td>
</tr>
<tr>
<td>10%</td>
<td>0.63</td>
<td>896.161</td>
<td>929.161</td>
<td>1,109.51</td>
<td>89,808.9</td>
</tr>
<tr>
<td>15%</td>
<td>0.66</td>
<td>872.77</td>
<td>907.27</td>
<td>1,080.52</td>
<td>89,084.7</td>
</tr>
<tr>
<td>20%</td>
<td>0.69</td>
<td>851.333</td>
<td>887.333</td>
<td>1,053.95</td>
<td>88,478.8</td>
</tr>
</tbody>
</table>

Figure 5.0 T.A.C. vs. $\theta$
Table 5.0  Analysis of Sensitivity for the backlogging parameter \( K \)

<table>
<thead>
<tr>
<th>% Variation in ( K )</th>
<th>( K )</th>
<th>( t_1 )</th>
<th>( v )</th>
<th>( T )</th>
<th>T.A.C</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20%</td>
<td>0.0017</td>
<td>1,198.62</td>
<td>1,120.0</td>
<td>1,358.76</td>
<td>97,862.4</td>
</tr>
<tr>
<td>-15%</td>
<td>0.0018</td>
<td>1,044.52</td>
<td>1,080.55</td>
<td>1,325.76</td>
<td>95,923.2</td>
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<tr>
<td>-10%</td>
<td>0.0019</td>
<td>1,015.78</td>
<td>1,042.78</td>
<td>1,257.77</td>
<td>94,276.3</td>
</tr>
<tr>
<td>-5%</td>
<td>0.0020</td>
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<td>1,019.64</td>
<td>1,214.84</td>
<td>92,674.8</td>
</tr>
<tr>
<td>0%</td>
<td>0.0021</td>
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<td>969.978</td>
<td>1,166.21</td>
<td>91,582</td>
</tr>
<tr>
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<td>0.0022</td>
<td>931.785</td>
<td>943.285</td>
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<td>90,567.9</td>
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<tr>
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<td>919.161</td>
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<td>89,708.9</td>
</tr>
<tr>
<td>15%</td>
<td>0.0024</td>
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<td>1,070.52</td>
<td>89,094.7</td>
</tr>
<tr>
<td>20%</td>
<td>0.0025</td>
<td>841.333</td>
<td>887.333</td>
<td>1,043.95</td>
<td>88,378.8</td>
</tr>
</tbody>
</table>

Figure 6.0  T.A.C. vs. \( K \)
8.0 Observations: The model is examining through sensitivity analysis for the parameters, following main observations has been found.

(i) We can observe from the Table 1 that total average cost T.A.C decreases by making increment in warehouse capacity ‘W’, even the same increment also seen in the figure 2. But in figure 2(a) the total average cost T.A.C increases by making increment in warehouse capacity ‘W’. this is the main outcome of the study.

(ii) We can observe from the Table 2 that total average cost T.A.C and Q both found increased by increasing the demand parameter ‘a’ where there are no changes noted in the parameter $t_1$. From figure 2 and 2(a) it can be seen that

It is also noted from the Table 3 that the same changes are found for the demand parameter ‘b’

(iii) From Table 4, we can observe that when the rate of deterioration parameter ‘$\theta$’ increased its reverse changes are noted on $t_1$ and Q.

(iv) Table 5 shows that for increased values of backlogging parameters ‘$k$’ and ‘$\beta$’, its reverse effect is noted on T.A.C. and Q, which show that as we increased the backlogging time its directly affect the demand of the items and retailers can also purchase the items from the different market.

9.0 Future extension of the model

Current study focused on the two-warehouse based inventory control model where both the warehouse is assumed ranted and the items with low life are transferred to the warehouse whose items are used first. Also, there are some future scopes of the study like trade credits on the items can also be use. The model is further extended by applying Fuzzy algorithm.

10.0 Acknowledgment

The authors are grateful to the colleagues for their support and help.

11.0 Conflict of interest
There is no conflict of interest with any financial organization regarding the material discussed in the manuscript.

Conclusions

Present model being developed for the deteriorating items in which we have assumed both the warehouses as rented. It is observed the assumption of transferring the items whose deterioration rate is high in the second warehouse so that we can sale such items before the spoilage was a very good idea.

Following major high list of the study found;

- Model also suggested that as the capacity of the storage increased its means it can also increases the costs associated with the model.
- The current model also developed under different demand rate for the different warehouse where we have assumed that for the first warehouse, we have taken the demand rate as stock dependent where we have assumed it constant for the second warehouse which make this study more attractive and up to the mark as per the current market situation.
- From the table 1 to table 5 we have calculated the sensitivity analysis of the total average cost at each of the parameters we have used in the model and the clear views of the effect of the sensitivity carried out.
- As we have assumed that the shortages are allowed only in first warehouse which was partially backlogged so that the retailer can also purchase more items at the lowest cost from wholesaler and stocks it in two different warehouses.
- In the current model we have presented the updated form of the model being developed by Tayal et al. (2019) where in the model by Tayal et al. (2019) the items with low life were stored in the warehouse where we have to face the deterioration of the items.
- We have used the different concept as a result the current model play important role for the items with low life.

References


