

# Pairwise Interact-and-Imitate Dynamics

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## ABSTRACT

This paper introduces and studies a class of evolutionary dynamics — Pairwise Interact-and-Imitate Dynamics (PIID) — in which agents are matched in pairs, engage in a symmetric game, and imitate the opponent with a probability that depends on the difference in their payoffs. We provide a condition on the underlying game, named supremacy, and show that the population state in which all agents play the supreme strategy is globally asymptotically stable. We extend the framework to allow for payoff uncertainty, and check the robustness of our results to the introduction of some heterogeneity in the revision protocol followed by agents. Finally, we show that PIID can allow the survival of strictly dominated strategies, leads to the emergence of inefficient conventions in social dilemmas, and makes assortment ineffective in promoting cooperation.

## Introduction

In evolutionary game-theoretic models, it is standard practice to assume that agents make decisions according to short-sighted adaptive rules. These include avoidance of strategies which performed poorly in the past, best response to the empirical distribution of opponents' strategies, and imitation of successful peers<sup>1,2</sup>. The last has been shown to be common in both human and animal groups, and is generally recognized as a cognitively parsimonious social heuristic<sup>3-5</sup>.

Distinctions among imitative rules, in turn, can be made as to what drives behavior, who is imitated, and how much information is needed for choices to be made. For example, rules of the kind 'copy the first person you see' make actions only depend on their popularity, whereas other rules make decisions a function of observed payoffs. The target of comparisons may consist of either a single agent or a (possibly large) group of individuals, and information requirements can range from rather minimal to extremely high levels, yielding a taxonomy of imitative revision protocols<sup>6-8</sup>. The less demanding classes require agents to know the payoff of a single individual, either oneself or someone else, as in the case where agents change strategy depending on the difference between their payoff and a certain aspiration level<sup>9</sup>. Next come those revision protocols

24 which require individuals to observe both their own payoff and the payoff of another agent; this class includes  
25 the family of rules considered in this paper. Finally, at the opposite end of the taxonomy are those protocols  
26 which require agents to know either the average payoff of all strategies in the population or the information  
27 necessary to compute average payoffs<sup>10</sup>.

28 An important, but easily overlooked, aspect of imitative dynamics is the relationship between the structure  
29 of interactions and agents' reference groups. The interaction structure specifies how agents are matched,  
30 e.g., in a uniform random manner or according to other criteria. For example, in games played on graphs,  
31 interactions depend on the underlying topology of relations<sup>11,12</sup>; non-uniform random matching rules include  
32 type- and action-assortative rules, which reflect the tendency of individuals to interact with people who are  
33 similar to themselves in terms of observed characteristics and behavior<sup>13-16</sup>. An agent's reference group  
34 consists of those individuals whom that agent observes and takes as a reference for comparison purposes.  
35 Interaction structure and reference groups are frequently treated as separate entities: whenever an agent  
36 receives a revision opportunity, she randomly selects another individual, observes this individual's strategy,  
37 and switches to it with a probability that depends on relative payoffs<sup>17-19</sup>. A possible form of separation  
38 between interaction structure and reference groups is obtained in case of global random interactions and local  
39 comparisons based on observation of a fixed set of individuals<sup>20,21</sup>. This is most reasonable in the case of  
40 games against nature or when agents cannot observe their opponents' payoffs. However, cases also exist  
41 in which the decoupling of interaction structure and reference groups does not hold. Often people can only  
42 observe, and act upon, the behavior of those with whom they interact. This idea is recurrent in the literature  
43 on games on networks (even if there are exceptions<sup>22</sup>) where typically agents play with and imitate their  
44 immediate neighbors<sup>23-25</sup>, or indirect neighbors<sup>26,27</sup>. The interplay between the structure of interactions and  
45 agents' reference groups may help understand the result that local interactions can favor the evolution of  
46 cooperation in the case of death-birth and imitative processes but not in the case of birth-death processes,  
47 which make defection emerge in the long run<sup>28</sup>. This finding can be seen as originating from differences in  
48 the degree of overlap between interaction structure and reference groups. Indeed, while in all cases agents  
49 only interact with their nearest neighbors, the target of comparisons differs from case to case — in birth-death  
50 processes, the set of nearest neighbors; in death-birth processes, the set of next-nearest neighbors; in imitative  
51 processes, a mixture of nearest and next-nearest neighbors. This can also be interpreted in terms of the  
52 inclusive fitness theory perspective, focusing on the interplay between genetic relatedness (a measure of  
53 assortment) and local competition brought about by limited dispersal (local imitation), in the presence of  
54 overlapping generations.

55 Building on this insight, this paper introduces and studies a class of evolutionary dynamics in which  
56 interaction structure and reference groups overlap, that is, where those whom one interacts with are also those  
57 with whom she compares herself. When given a revision opportunity, an agent playing strategy  $i$  against an  
58 opponent playing strategy  $j$  will switch to  $j$  with positive probability whenever the payoff from  $j$  against  $i$  is

59 greater than the payoff from  $i$  against  $j$ . We name this protocol *Pairwise Interact-and-Imitate*.

60 Pairwise Interact-and-Imitate protocol is closely related to the local replicator dynamics<sup>29</sup>, where indi-  
61 viduals are uniformly matched at random in groups of size  $N$  to play pairwise games and imitate each other  
62 depending on payoff differences: for  $N = 2$  a Pairwise Interact-and-Imitate is obtained for the special case of  
63 uniform random matching (while we consider more general interaction structures). Earlier works that relate  
64 most closely to our own include the pairwise comparative model of traffic dynamics, where changes from  
65 one route to another occur at a frequency that depends upon differences in traveling costs<sup>30</sup>. Another related  
66 revision protocol is one where agents imitate a randomly selected individual with a probability proportional  
67 to how much better the latter performed than them<sup>17</sup>. It is known in the literature that the set of rest points  
68 of the resulting imitative dynamics comprises the Nash equilibria of the underlying game and the states that  
69 would be a Nash equilibrium if the strategies unused at those states were removed from the game<sup>7</sup>. A possible  
70 variant concerns protocols that are direct instead of imitative, meaning that revising agents sample candidate  
71 strategies directly rather than by observing someone else's behavior<sup>6,7</sup>. The dynamics resulting from pairwise  
72 interactions with more informationally demanding imitative rules are also studied, for instance in a setup in  
73 which agents know others' payoffs, the probabilities with which individuals can meet, and the distribution of  
74 strategies in the population<sup>31</sup>. Another possibility is when agents evaluate alternative strategies by computing  
75 their average payoffs in a randomly sampled reference group, and then choose whichever strategy yielded  
76 the highest average payoff<sup>32</sup>. The imitate-the-best protocol, which prescribes copying the strategy chosen  
77 by the most successful individual, has been shown to be nearly unbeatable in a wide variety of symmetric  
78 two-player games<sup>33</sup>. The implications of the imitate-the-best rule have also been discussed in oligopoly  
79 settings<sup>34</sup>. Both imitate-the-best and imitate-the-best-average behaviors have been studied experimentally in  
80 the Cournot game<sup>35</sup>.

81 The purpose of our paper is twofold. First, we introduce the Pairwise Interact-and-Imitate revision protocol  
82 and study the resulting dynamics in symmetric games. We give a condition on the stage game, named  
83 supremacy, and show that the population state in which all agents choose the supreme strategy is globally  
84 asymptotically stable. Roughly speaking, a strategy is supreme if it always yields a payoff higher than the  
85 payoff received by an opponent playing a different strategy. We then generalize the framework to allow for  
86 payoff uncertainty, we check the robustness of our results to the introduction of some heterogeneity in revision  
87 protocols, and we show that PIID can allow the survival of strictly dominated strategies. Second, we apply the  
88 revision protocol to social dilemmas, showing that PIID causes the emergence of inefficient conventions and  
89 makes assortment ineffective in facilitating cooperation.

## 90 Results

### 91 The model

92 Consider a unit-mass population of agents who repeatedly interact in pairs to play a symmetric stage game.  
93 The set of strategies available to each agent is finite and denoted by  $S \equiv \{1, \dots, n\}$ . A population state is a  
94 vector  $x \in X \equiv \{x \in \mathbb{R}_+^n : \sum_{i \in S} x_i = 1\}$ , with  $x_i$  the fraction of the population playing strategy  $i \in S$ . Payoffs  
95 are described by a function  $F : S \times S \rightarrow \mathbb{R}$ , where  $F(i, j)$  is the payoff received by an agent playing strategy  $i$   
96 when the opponent plays strategy  $j$ . As a shorthand, we refer to an undirected pair of individuals, one playing  
97  $i$  and the other playing  $j$ , as an  $ij$  pair. The set of all possible pairs is denoted by  $\mathcal{P}$ .

98 The interaction structure is modeled as a function  $p : X \times \mathcal{P} \rightarrow [0, 1/2]$  subject to  $\sum_{ij \in \mathcal{P}} p_{ij}(x) = 1/2$   
99 (since the mass of pairs is half the mass of agents), with  $p_{ij}(x)$  indicating the mass of  $ij$  pairs formed in state  $x$ .  
100 Note that the mass of  $ij$  pairs can never exceed  $\min\{x_i, x_j\}$ , that is,  $p_{ij}(x) \leq \min\{x_i, x_j\}$  for all  $x$ . We assume  
101 that  $p$  is continuous in  $X$  and that if  $x_i > 0$  and  $x_j > 0$ , then  $p_{ij}(x) > 0$  — meaning that if in a certain state  
102 strategies  $i$  and  $j$  are played by someone, then the probability of an  $ij$  pair being formed is strictly positive. In  
103 the case of uniform random matching,  $p_{ii} = x_i^2/2$  and  $p_{ij} = x_i x_j$  for any  $i$  and  $j \neq i$ .

104 The revision protocol is modeled as a function  $\phi : X \times \mathcal{P} \rightarrow [-1, 1]$ , where  $\phi_{ij}(x) \in [-1, 1]$  is the  
105 probability that an  $ij$  pair will turn into an  $ii$  pair minus the probability that it will turn into a  $jj$  pair,  
106 conditional on the population state being  $x$  and an  $ij$  pair being formed. We assume that  $\phi$  is continuous in  $X$ ,  
107 and note that  $\phi_{ii} = 0$  for all  $i \in S$ . Our main assumption on the revision protocol is the following, which is met,  
108 among others, by pairwise proportional imitative and imitate-if-better rules<sup>17</sup>.

109 **Assumption 1.** For every  $x \in X$ ,  $\phi_{ij}(x) > 0$  if  $F(i, j) > F(j, i)$ .

110 In what follows we consider a dynamical system in continuous time with state space  $X$ , characterized by  
111 the following equation of motion.

**Definition 1** (Pairwise Interact-and-Imitate Dynamics – PIID). For every  $x \in X$  and every  $i \in S$ :

$$\dot{x}_i = \sum_{j \in S} p_{ij}(x) \phi_{ij}(x). \quad (1)$$

### 112 Main findings

#### 113 Global asymptotic convergence

114 In any purely imitative dynamics, if  $x_i(t) = 0$ , then  $x_i(t') = 0$  for every  $t' > t$ . This implies that we cannot  
115 hope for global asymptotic convergence in a strict sense. Thus, to assess convergence towards a certain state  $x$   
116 in a meaningful way, we restrict our attention to those states where all strategies that have positive frequency  
117 in  $x$  have positive frequency as well. We denote by  $X_x$  the set of states whose support contains the support of  $x$ .

118 **Definition 2** (Supremacy). Strategy  $i \in S$  is supreme if  $F(i, j) > F(j, i)$  for every  $j \in S \setminus \{i\}$ .

119 Equivalently, a strategy  $i$  is supreme if the relative payoff from playing  $i$  against any other strategy<sup>33</sup> is positive,  
 120 i.e., if  $F(i, j) - F(j, i) > 0$  for all  $j \neq i$ . We note that, under PIID, the notion of supremacy is closely related to  
 121 that of asymmetry<sup>36,37</sup>, in that  $F(i, j) > F(j, i)$  implies that agents can only switch from strategy  $j$  to strategy  
 122  $i$ .

123 **Proposition 1.** *If  $i \in S$  is a supreme strategy, then state  $x^* \equiv \{x \in X : x_i = 1\}$  is globally asymptotically stable  
 124 for the dynamical system with state space  $X_{x^*}$  and PIID as equation of motion.*

### 125 **Relation to replicator dynamics**

126 To further characterize the dynamics induced by the pairwise interact-and-imitate protocol, we make two  
 127 additional assumptions. First, matching is uniformly random, meaning that everyone in the population has the  
 128 same probability of interacting with everyone else. Second, the probability that an agent has to imitate the  
 129 opponent is proportional to the difference in their payoffs if the opponent's payoff exceeds her own, and is  
 130 zero otherwise.

131 Let

$$132 \quad F(i, x) := \sum_j x_j F(i, j),$$

$$133 \quad F(x, i) := \sum_j x_j F(j, i), \text{ and}$$

$$134 \quad F(x, x) := \sum_i \sum_j x_i x_j F(i, j).$$

Under these assumptions, at any point in time, the motion of  $x_i$  is described by:

$$\begin{aligned} \dot{x}_i &= \sum_{j \neq i} x_j x_i [F(i, j) - F(j, i)] = x_i \sum_j x_j [F(i, j) - F(j, i)] \\ &= x_i [F(i, x) - F(x, i)], \end{aligned} \tag{2}$$

135 which is a modified replicator equation. According to (2), for every strategy  $i$  chosen by one or more agents  
 136 in the population, the rate of growth of the fraction of  $i$ -players,  $\dot{x}_i/x_i$ , equals the difference between the  
 137 expected payoff from playing  $i$  in state  $x$  and the average payoff received by those who are matched against an  
 138 agent playing  $i$ . In contrast, under the standard replicator dynamics<sup>38</sup>, the fraction of agents playing  $i$  varies  
 139 depending on the excess payoff of  $i$  with respect to the current average payoff in the whole population, i.e.,  
 140  $\dot{x}_i = x_i [F(i, x) - F(x, x)]$ .

141 A noteworthy feature of the replicator dynamics is that it is always payoff monotone: for any  $i, j \in S$ , the  
 142 proportions of agents playing  $i$  and  $j$  grow at rates that are ordered in the same way as the expected payoffs  
 143 from the two strategies<sup>39</sup>. In the case of PIID, this result fails. To see this, simply consider any symmetric  $2 \times 2$   
 144 game where  $F(i, j) > F(j, i)$  but  $F(j, x) > F(i, x)$  for some  $x \in X$ , meaning that  $i$  is the supreme strategy but  
 145  $j$  yields a higher expected payoff in state  $x$ . If strategies are updated according to the interact-and-imitate  
 146 protocol, this game only admits switches from  $i$  to  $j$ , therefore violating payoff monotonicity. This can have  
 147 important consequences, including the survival of pure strategies that are strictly dominated.

148 **Survival of strictly dominated strategies**

149 An important topic in evolutionary game theory is to what extent does support exist for the idea that strictly  
150 dominated strategies will not be played. It has been shown that if a strategy  $i$  does not survive the iterated  
151 elimination of pure strategies strictly dominated by other pure strategies, then the fraction of the population  
152 playing  $i$  converges to zero in all payoff monotone dynamics<sup>40,41</sup>. This result does not apply in our case, as  
153 PIID is not payoff monotone.

154 More precisely, under PIID, a strictly dominated strategy may be supreme and, therefore, not only survive  
155 but even end up being adopted by the whole population. This suggests that, from an evolutionary perspective,  
156 support for the elimination of dominated strategies may be weaker than is often thought. Our result contributes  
157 to the literature studying conditions under which evolutionary dynamics fail to eliminate strictly dominated  
158 strategies in some games, considering a case which was not previously studied by the literature<sup>42</sup>.

159 To see that a strictly dominated strategy may be supreme, consider the simple example shown in Figure 1.  
160 Here each agent has a strictly dominant strategy to play  $A$ ; however, since the payoff from playing  $B$  against  $A$   
161 exceeds that from playing  $A$  against  $B$ , strategy  $B$  is supreme. Thus, by Proposition 1, the population state in  
162 which all agents choose  $B$  is globally asymptotically stable.

	$A$	$B$
$A$	4	2
$B$	3	1

**Figure 1.** A game where the supreme strategy is strictly dominated.

163 **Applications**

164 Having obtained general results for the class of finite symmetric games, we now restrict the discussion  
165 to the evolution of behavior in social dilemmas. We show that, under the conditions of Proposition 1,  
166 inefficient conventions emerge in the Prisoner's Dilemma, Stag Hunt, Minimum Effort, and Hawk-Dove  
167 games. Furthermore, these results hold both without and with the assumption that agents interact assortatively.

168 **Ineffectiveness of assortment**

169 Consider the  $2 \times 2$  game represented in Figure 2. If  $c > a > d > b$ , then mutual cooperation is Pareto superior  
170 to mutual defection, but agents have a dominant strategy to defect. The resulting stage game is the Prisoner's  
171 Dilemma, whose unique Nash equilibrium is  $(B, B)$ . Moreover, since  $F(B, A) > F(A, B)$ ,  $B$  is the supreme

172 strategy and the population state in which all agents defect is globally asymptotically stable.

173 We stress that defection emerges in the long run for every matching rule satisfying our assumptions, and  
174 therefore also in the case of assortative interactions. Assortment reflects the tendency of similar people to  
175 clump together, and can play an important role in the evolution of cooperation<sup>43–46</sup>. Intuitively, when agents  
176 meet assortatively, the risk of cooperating in a social dilemma may be offset by a higher probability of playing  
177 against other cooperators. However, under PIID, this is not the case; the decision whether to adopt a strategy  
178 or not is independent of expected payoffs, and like-with-like interactions have no effect other than reducing  
179 the frequency of switches from  $A$  to  $B$ .

	$A$	$B$
$A$	$a$	$b$
$B$	$c$	$d$

**Figure 2.** A  $2 \times 2$  stage game.

### 180 ***Emergence of the maximin convention***

181 If  $a > c > b$ ,  $a > d$ , and  $d > b$ , then the game in Figure 2 becomes a Stag Hunt game, which contrasts risky  
182 cooperation and safe individualism. The payoffs are such that both  $(A,A)$  and  $(B,B)$  are strict Nash equilibria,  
183 that  $(A,A)$  is Pareto superior to  $(B,B)$ , and that  $B$  is the maximin strategy, i.e., the strategy which maximizes  
184 the minimum payoff an agent could possibly receive. We also assume that  $a + c \neq c + d$ , so that one of  $A$  and  
185  $B$  is risk dominant<sup>47</sup>. If  $a + b > c + d$ , then  $A$  (Stag) is both payoff and risk dominant. When the opposite  
186 inequality holds, the risk dominant strategy is  $B$  (Hare).

187 Since  $F(B,A) > F(A,B)$ ,  $B$  is supreme independently of whether or not it is risk dominant to cooperate.  
188 This can result in large inefficiencies because, in the long run, the process will converge to the state in which  
189 all agents play the riskless strategy regardless of how rewarding social coordination is. As in the case of the  
190 Prisoner's Dilemma, this holds for all matching rules satisfying our assumptions.

### 191 ***Evolution of effort exertion***

In a minimum effort stage game, agents simultaneously choose a strategy  $i$ , usually interpreted as a costly  
effort level, from a finite subset  $S$  of  $\mathbb{R}$ . An agent's payoff depends on her own effort and on the minimum  
effort in the pair:

$$F(i, j) = \alpha \min\{i, j\} - \beta i,$$

192 where  $\beta > 0$  and  $\alpha > \beta$  are the cost of and the benefit from effort, respectively. The best response to a  
193 choice of  $j$  by the opponent is to choose  $j$  as well, and coordinating on any common effort level gives a Nash  
194 equilibrium. Nash outcomes can be Pareto-ranked, with the highest-effort equilibrium being the best possible  
195 outcome for all agents. Thus, choosing a high  $i$  is rationalizable and potentially rewarding but may also result  
196 in a waste of effort.

197 Under PIID, any  $i > j$  implies  $\phi_{ij} < 0$  by Assumption 1, meaning that agents will tend to imitate the  
198 opponent when the opponent's effort is lower than their own. The supreme strategy is therefore to exert as  
199 little effort as possible, and the population state in which all agents choose the minimum effort level is the  
200 unique globally asymptotically stable state.

### 201 ***Emergence of aggressive behaviors***

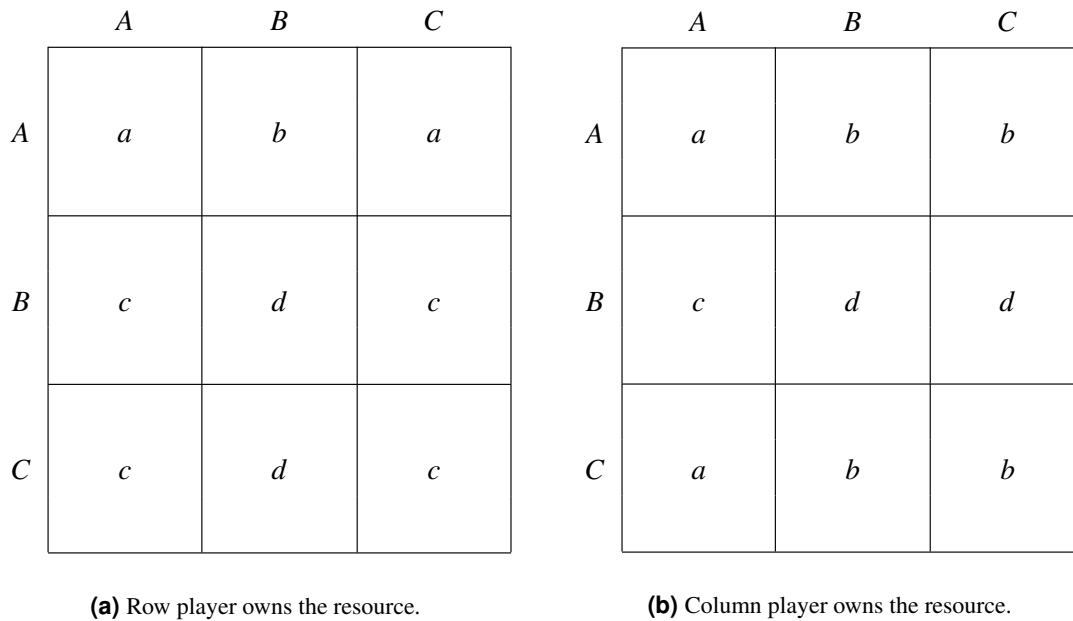
202 Consider again the payoff matrix shown in Figure 2. If  $c > a > b > d$ , then the stage game is a Hawk-Dove  
203 game, which is often used to model the evolution of aggressive and sharing behaviors. Interactions can be  
204 framed as fights over a contested resource. When two Doves (who play  $A$ ) meet, they share the resource  
205 equally, whereas two Hawks (who play  $B$ ) engage in a brawl and suffer a cost. Also, when a Dove meets a  
206 Hawk, the latter takes the entire prize. Again we have that  $F(A, B) < F(B, A)$ , implying that  $B$  is the supreme  
207 strategy and that All Hawk is the sole asymptotically stable state.

208 The inefficiency that characterizes the  $(B, B)$  equilibrium in the Hawk-Dove game arises from the cost  
209 that Hawks impose on one another. This can be viewed as stemming from the fact that neither agent owns  
210 the resource prior to the interaction or cares about property. A way to overcome this problem may be to  
211 introduce a strategy associated with respect for ownership rights, the Bourgeois, who behaves as a Dove or  
212 Hawk depending on whether or not the opponent owns the resource<sup>48</sup>. If we make the standard assumption  
213 that each member of a pair has a probability of 1/2 to be an owner, then in all interactions where a Bourgeois  
214 is involved there is a 50 percent chance that she will behave hawkishly (i.e., fight for control over the resource)  
215 and a 50 percent chance that she will act as a Dove.

216 Let  $R$  and  $C$  denote the agent chosen as row and column player, respectively, and let  $\omega_R$  and  $\omega_C$  be the  
217 states of the world in which  $R$  and  $C$  owns the resource. The payoffs for the resulting Hawk-Dove-Bourgeois  
218 game are shown in Figure 3. If agents behave as expected payoff maximizers, then All Bourgeois can be  
219 singled out as the unique asymptotically stable state. Under PIID, this is not so; depending on who owns  
220 the resource, an agent playing  $C$  against an opponent playing  $B$  may either fight or avoid conflict and let  
221 the opponent have the prize. It is easy to see that  $F(C, B | \omega_R) = F(B, C | \omega_C) = d$ , meaning that the payoff  
222 from playing  $C$  against  $B$ , conditional on owning the resource, equals the payoff from playing  $B$  against  
223  $C$  conditional on not being an owner. In contrast, the payoff from playing  $C$  against  $B$ , conditional on not  
224 owning the resource, is always worse than that of the opponent, i.e.,  $F(C, B | \omega_C) = b < c = F(B, C | \omega_R)$ .  
225 Thus, in every state of the world,  $B$  (Hawk) yields a payoff that is greater or equal to that from  $C$  (Bourgeois).



226 Moreover, since  $F(B,A) > F(A,B)$  in both states of the world, strategy  $B$  is weakly supreme by Definition 4,  
 227 and play unfolds as an escalation of hawkishness and fights.



**Figure 3.** The Hawk-Dove-Bourgeois game.

## 228 Discussion

229 We have studied a novel class of evolutionary dynamics, named Pairwise Interact-and-Imitate Dynamics, in  
 230 which agents choose whether or not to change strategy by comparing their payoff with that of their opponent.  
 231 Our main result is that, under PIID, if there exists a supreme strategy (that is, a strategy that always yields a  
 232 payoff higher than the payoff received by an opponent playing a different strategy), then the state in which the  
 233 whole population chooses the supreme strategy is globally asymptotically stable. Importantly, the supreme  
 234 strategy may be a dominated strategy, and the profile of strategies played in the asymptotically stable state  
 235 may not be a Nash equilibrium.

236 Under PIID, externalities have an important role. Whenever a strategy, say  $i$ , generates an increase  
 237 (decrease) in the payoff of an opponent playing a different strategy, say  $j$ , it is more (less) likely that  $i$  will be  
 238 updated in favor of  $j$ . Moreover, this is so regardless of the payoff received when using the same strategy as  
 239 the opponent. Thus, ceteris paribus, strategies that impose negative (positive) externalities are more (less)  
 240 likely to be selected by evolution, possibly leading to inefficient outcomes.

241 However, it is worth noting that PIID does not necessarily lead to inferior outcomes with respect to other  
 242 evolutionary dynamics. The simple example of Figure 4 shows this. For instance, under Pairwise Proportional  
 243 Imitation<sup>7,17</sup>, if the fraction of agents playing  $A$  is sufficiently large, then the system moves towards the  
 244 state where the whole population plays  $A$  — which, however, is Pareto dominated by everyone playing  $B$ .

245 Conversely, under PIID, the system always moves towards the state where everyone plays  $B$  (since  $B$  is  
246 supreme). This result holds in general, i.e., even without the additional assumptions required to represent  
247 evolution by means of the modified replicator equation.

	$A$	$B$
$A$	2	0
$B$	1	3

**Figure 4.** A game where PIID selects an efficient outcome.

248 Overall, our findings provide a case for why individual behaviors may direct evolution towards outcomes  
249 that do not meet Pareto efficiency and strategy dominance criteria. Rather, our dynamics depend on which  
250 strategy, if any, is supreme, i.e., it systematically outperforms other strategies when these are chosen by  
251 one's opponents. This implies that the outcome of evolution can be either very undesirable or very desirable,  
252 depending on how large the payoff from the supreme strategy is when this strategy is chosen by everyone in  
253 the population. Moreover, since the structure of interactions plays no role in determining which strategy is  
254 supreme, the long-run equilibrium selected by PIID is not affected by the presence of institutions and other  
255 factors that influence how agents interact, such as those generating assortment.

256 We have shown that when applied to study the evolution of behavior, PIID leads to clear-cut and sometimes  
257 surprising results in a large variety of games. However, not all classes of games are suited to our revision  
258 protocol; in this paper we have only considered symmetric games, leaving aside those cases where agents  
259 can choose among different strategies or have different payoff functions. When agents' strategy sets differ  
260 from one another, it does not seem very reasonable to assume that choices are updated according to a pairwise  
261 imitative rule based on payoff differentials. However, we believe that an imitative protocol like ours may  
262 still be applied in a meaningful way to those cases in which agents have the same strategy set but differ in  
263 some other respect. For instance, in a setting where agents differ in wealth, a poor individual may be driven  
264 to imitate the strategy chosen by a rich individual earning a high payoff, even if this is due to differences in  
265 wealth rather than in strategy.

266 An extension of the model developed here would be to consider the case of a finite population of agents.  
267 This would make comparisons with some of the literature easier<sup>29</sup>, but would come at the cost of hindering  
268 the analysis when introducing payoff uncertainty and studying how PIID relates to the replicator dynamics.  
269 Another extension would concern moving from two-player to  $n$ -player symmetric games, which would require

270 defining the class of Groupwise Interact-and-Imitate Dynamics, beyond adjusting the notion of supremacy to  
 271 consider the relative performance of a strategy towards profiles of others' strategies.

272 Finally, a question that may be worthy of further investigation is how the dynamics will behave when  
 273 no supreme strategy exists. To answer, one may define a binary relation  $\succ$  such that  $i \succ j$  if and only if  
 274  $F(i, j) \geq F(j, i)$ . One may then define  $\succ^*$  as the transitive closure of  $\succ$ , and let  $\mathcal{S} := \{i \in S : i \succ^* j \forall j \in S\}$   
 275 be set of supremal strategies. Our conjecture is that, under PIID, all strategies that do not belong to  $\mathcal{S}$  will  
 276 die out independently of the structure of interactions; however, the precise characterization of limit sets may  
 277 depend on details of the payoff structure, the interaction structure, and the revision protocol.

## 278 Methods

### 279 Lyapunov's method

To prove Proposition 1 we use Lyapunov's second method for global stability. We want to show that  
 $f : X_{x^*} \rightarrow [0, 1)$ , with

$$f(x) = 1 - x_i,$$

is a strict Lyapunov function. It is easy to see that  $f$  is of class  $C^1$ , that  $f(x) > 0$  for every  $x \in X_{x^*} \setminus \{x^*\}$ , and  
 that  $f(x^*) = 0$ . We are left to show that (i)  $\dot{f}(x^*) = 0$  and (ii)  $\dot{f}(x) < 0$  for every  $x \in X_{x^*} \setminus \{x^*\}$ . Taking the  
 time derivative of  $f$  and using Definition 1, we can write:

$$\dot{f}(x) = \frac{\partial f(x)}{\partial x_i} \dot{x}_i = - \sum_{j \in S} p_{ij}(x) \phi_{ij}(x).$$

280 We observe that  $p_{ij}(x^*) = 0$  for every  $j \in S$ , which implies  $\dot{f}(x^*) = 0$ . For every  $x \in X_{x^*} \setminus \{x^*\}$ , there  
 281 exists  $k \neq i$  such that  $x_k > 0$ . Since  $x_i > 0$  and  $x_k > 0$ , we therefore have that  $p_{ik}(x) > 0$ . Moreover, since  
 282  $F(i, j) > F(j, i)$  for every  $j \in S \setminus \{i\}$ , we have that  $\phi_{ik}(x) > 0$  by Assumption 1. To see that  $\dot{f}(x) < 0$  for every  
 283  $x \in X_{x^*} \setminus \{x^*\}$ , we finally note that  $p_{ij}(x)$  is always non-negative and that  $\phi_{ij}(x)$  is positive for all  $j \in S \setminus \{i\}$   
 284 by the definition of supremacy and Assumption 1.

### 285 Supremacy under uncertainty

286 So far we have only considered games in which payoffs are not subject to any uncertainty in their realization.  
 287 Here we extend our analysis by allowing for this possibility, which is relevant for a variety of applications  
 288 (e.g., the Hawk-Dove and Hawk-Dove-Bourgeois games).

289 Let  $\Omega$  be a finite set of states of the world and let  $q_\omega$  be the probability of state  $\omega \in \Omega$  occurring. We write  
 290  $F(i, j|\omega)$  to denote the payoff received by an agent playing strategy  $i$  against an opponent playing  $j$  when  
 291 state of the world  $\omega$  occurs. For every  $x \in X$  and every  $\omega \in \Omega$ , let  $\phi(x|\omega)$  be the  $n \times n$ -matrix with typical  
 292 element  $\phi_{ij}(x|\omega)$ , the latter being the net inflow from  $j$  to  $i$  when the population state is  $x$ , the state of the  
 293 world is  $\omega$ , and an  $ij$  pair is formed.

294 We replace Assumption 1 with the following.

295 **Assumption 2.** For every  $x \in X$  and every  $\omega \in \Omega$ ,  $\phi_{ij}(x|\omega) > 0$  if  $F(i, j|\omega) > F(j, i|\omega)$ .

We also assume that, at any point in time, the net population inflow from  $j$  to  $i$  is obtained by averaging  $\phi_{ij}$  over the set of states of the world, i.e.:

$$\phi_{ij}(x) = \sum_{\omega \in \Omega} q_{\omega} \phi_{ij}(x|\omega). \quad (3)$$

296 The convergence result of Proposition 1 can be extended to the case of uncertainty if we define supremacy  
297 as follows.

298 **Definition 3** (Supremacy under uncertainty). *In the presence of uncertainty, strategy  $i \in S$  is supreme if*  
299  *$F(i, j|\omega) > F(j, i|\omega)$  for every  $\omega \in \Omega$  and every  $j \in S \setminus \{i\}$ .*

300 To see that the result holds, note that if strategy  $i$  is supreme by Definition 3 and Assumption 2 holds, then  
301  $\phi_{ij}(x) > 0$  for every  $j \in S \setminus \{i\}$  and every  $x \in X$ . Moreover,  $\phi$  is continuous in  $X$  if  $\phi(\cdot, \omega)$  is continuous in  $X$   
302 for every  $\omega \in \Omega$ . By a reasoning analogous to that used in the proof of Proposition 1, we therefore have that  
303 state  $\{x \in X : x_i = 1\}$  is globally asymptotically stable for the dynamical system with state space  $X_{x^*}$  and PIID  
304 as equation of motion.

305 A less restrictive definition of supremacy under uncertainty is given below.

306 **Definition 4** (Weak supremacy under uncertainty). *In the presence of uncertainty, strategy  $i \in S$  is weakly*  
307 *supreme if  $F(i, j|\omega) \geq F(j, i|\omega)$  for every  $\omega \in \Omega$  and every  $j \in S \setminus \{i\}$ , and if for every  $j \in S \setminus \{i\}$  there exists*  
308  *$\hat{\omega} \in \Omega$  such that  $F(i, j|\hat{\omega}) > F(j, i|\hat{\omega})$ .*

309 Under the conditions of Definition 4, our convergence result holds if we strengthen Assumption 2 as  
310 follows.

311 **Assumption 3.** For every  $x \in X$  and every  $\omega \in \Omega$ ,  $\phi_{ij}(x|\omega) > 0$  if and only if  $F(i, j|\omega) > F(j, i|\omega)$ .

312 Here the ‘only if’ is required to deal with those states of the world where  $F(i, j|\omega) = F(j, i|\omega)$ , which are not  
313 ruled out by Definition 4.

314 An even weaker definition of supremacy can be given when focusing on a specific  $\phi$ . For instance, consider  
315 the case where the probability that agents have to imitate the opponent is proportional to the difference in their  
316 payoffs if the opponent’s payoff exceeds their own, and is zero otherwise. Under this protocol, letting the  
317 expected payoff from playing  $i$  against  $j$  be  $\mathbb{E}[F(i, j)] = \sum_{\omega \in \Omega} q_{\omega} F(i, j|\omega)$ , we can define the following.

318 **Definition 5** (Supremacy in expectation under uncertainty). *In the presence of uncertainty, strategy  $i \in S$  is*  
319 *supreme in expectation if  $\mathbb{E}[F(i, j)] > \mathbb{E}[F(j, i)]$  for every  $j \in S \setminus \{i\}$ .*

It can now be seen that:

$$\phi_{ij}(x) > 0 \Leftrightarrow \sum_{\omega \in \Omega} q_{\omega} [F(i, j|\omega) - F(j, i|\omega)] > 0 \Leftrightarrow \mathbb{E}[F(i, j)] > \mathbb{E}[F(j, i)],$$

320 meaning that the net population inflow from  $j$  to  $i$  is positive if and only if  $i$  is supreme in expectation. This  
321 suffices to replicate the result of Proposition 1.

## 322 Heterogeneous revision protocols

323 Although we believe the interact-and-imitate protocol to be reasonable in many circumstances, it may well be  
 324 the case that agents also rely on other revision protocols occasionally. If our results crucially hinged on the  
 325 assumption that agents always follow the interact-and-imitate rule, they would be of little interest.

326 Let the set of possible states of the world be  $\Omega = \{\omega_1, \omega_2\}$ . Suppose that agents follow the pairwise  
 327 interact-and-imitate protocol in state  $\omega_1$  and a different revision protocol in state  $\omega_2$ . Now let us define a  
 328 continuous function  $\rho : X \times \mathcal{P} \rightarrow [-1, 1]$ , where  $\rho_{ij}(x) \in [-1, 1]$  is the probability that an  $ij$  pair will turn into  
 329 an  $ii$  pair minus the probability that it will turn into a  $jj$  pair, conditional on the population state being  $x$ , an  $ij$   
 330 pair being formed, and the state of the world being  $\omega_2$ . Note that  $\rho$  may also reflect the fact that members of a  
 331 pair interact after, rather than before, having updated their strategies, in which case the interact-and-imitate  
 332 protocol cannot be applied.

We define:

$$\hat{\phi}_{ij}(x) := (1 - \varepsilon)\phi_{ij}(x) + \varepsilon\rho_{ij}(x), \quad (4)$$

333 with  $\varepsilon \in (0, 1)$ . The equation of motion for this dynamical system is the following.

**Definition 6** (Quasi pairwise interact-and-imitate dynamics – QPIID). *For every  $x \in X$  and every  $i \in S$ :*

$$\dot{x}_i = \sum_{j \in S} p_{ij}(x) \hat{\phi}_{ij}(x). \quad (5)$$

334 Note that Assumption 1 concerns  $\phi$  only, and that we have no analogous assumption for  $\rho$ . This  
 335 notwithstanding, a convergence result in the spirit of Proposition 1 can be obtained if agents follow the  $\rho$   
 336 protocol rarely enough.

337 **Proposition 2.** *If  $i \in S$  is supreme, then there exists  $\bar{\varepsilon} > 0$  such that, for every  $\varepsilon \in (0, \bar{\varepsilon})$ , state  $x^* \equiv$   
 338  $\{x \in X : x_i = 1\}$  is globally asymptotically stable for the dynamical system with state space  $X_{x^*}$  and QPIID as  
 339 equation of motion.*

340 We want to show that if  $F(i, j) > F(j, i)$  for every  $j \in S \setminus \{i\}$ , then there exists  $\bar{\varepsilon} > 0$  such that, for every  
 341  $\varepsilon \in (0, \bar{\varepsilon})$ ,  $\hat{\phi}(x) > 0$  for every  $x \in X$ . Moreover, since both  $\phi$  and  $\rho$  are continuous in  $X$ ,  $\hat{\phi}$  is continuous in  $X$   
 342 as well. As a consequence, the statement in Proposition 2 can be proven by replicating the argument used in  
 343 the proof of Proposition 1.

We consider the worst case to have  $\hat{\phi}(x)$  positive. For every  $j \in S \setminus \{i\}$ , we have that  $\bar{\phi}_{ij} := \min_{x \in X} \phi_{ij}(x)$   
 exists, since  $\phi_{ij}$  is a continuous function on a compact set. Moreover, by continuity of  $\phi_{ij}$  and noting that  
 $\phi_{ij}(x) > 0$  due to  $F(i, j) > F(j, i)$  and Assumption 1, we also have that  $\bar{\phi}_{ij} > 0$ . We define  $\bar{\phi}_i := \min_{j \neq i} \bar{\phi}_{ij}$ ,  
 and note that  $\bar{\phi}_i > 0$ . Also, we define:

$$\bar{\varepsilon} := \frac{\bar{\phi}_i}{1 + \bar{\phi}_i}. \quad (6)$$

344 Since  $\rho_{ij}$  cannot be smaller than  $-1$ , if  $\varepsilon < \bar{\varepsilon}$  then we have that  $\hat{\phi}(x) > 0$  for every  $x \in X$ , as can be easily  
345 checked from (4).

346 We stress that the bound on  $\varepsilon$  given in (6) is independent of which  $\rho$  protocol is being considered. A  
347 more precise value for the bound can be obtained by making specific assumptions about the strategy revision  
348 process. For example, suppose that in state of the world  $\omega_1$ , agents imitate the opponent with unit probability  
349 1 whenever the latter receives a higher payoff than they do (so that  $\phi_{ij} = 1$  if  $F(i, j) > F(j, i)$ ). In this case,  
350 we can have the maximum possible amount of heterogeneity in revision protocols (i.e.,  $\bar{\varepsilon} = 1/2$ ) and still have  
351 global asymptotic stability of the state in which the whole population chooses the supreme strategy.

## 352 References

- 353 1. Fudenberg, D. & Levine, D. K. *The theory of learning in games* (MIT press, 1998).
- 354 2. Young, H. P. *Individual Strategy and Social Structure: An Evolutionary Theory of Institutions* (Princeton  
355 University Press, 1998).
- 356 3. Pingle, M. & Day, R. H. Modes of economizing behavior: Experimental evidence. *J. Econ. Behav. &*  
357 *Organ.* **29**, 191–209 (1996).
- 358 4. Rendell, L. *et al.* Why copy others? insights from the social learning strategies tournament. *Science* **328**,  
359 208–213 (2010).
- 360 5. Gigerenzer, G. & Gaissmaier, W. Heuristic decision making. *Annu. Rev. Psychol.* **62**, 451–482 (2011).
- 361 6. Sandholm, W. Pairwise comparison dynamics and evolutionary foundations for nash equilibrium. *Games*  
362 **1**, 3–17 (2010).
- 363 7. Sandholm, W. H. *Population Games and Evolutionary Dynamics* (MIT press, 2010).
- 364 8. Newton, J. Evolutionary game theory: a renaissance. *Games* **9**, 31 (2018).
- 365 9. Bjrönerstedt, J. & Weibull, J. W. Nash equilibrium and evolution by imitation. In Arrow, K., Colombatto,  
366 E., Perlman, M. & Schmidt, C. (eds.) *The Rational Foundations of Economic Behavior* (Macmillan,  
367 1996).
- 368 10. Robson, A. J. & Vega-Redondo, F. Efficient equilibrium selection in evolutionary games with random  
369 matching. *J. Econ. Theory* **70**, 65–92 (1996).
- 370 11. Szabó, G. & Fáth, G. Evolutionary games on graphs. *Phys. Reports* **446**, 97–216 (2007).
- 371 12. Jackson, M. O. & Zenou, Y. Games on networks. In Young, H. P. & Zamir, S. (eds.) *Handbook of Game*  
372 *Theory, Volume 4*, 95–164 (North Holland, 2014).
- 373 13. Alger, I. & Weibull, J. W. Homo moralis—preference evolution under incomplete information and  
374 assortative matching. *Econometrica* **81**, 2269–2302 (2013).

- 375 **14.** Alger, I. & Weibull, J. W. Evolution and kantian morality. *Games Econ. Behav.* **98**, 56–67 (2016).
- 376 **15.** Bilancini, E., Boncinelli, L. & Wu, J. The interplay of cultural intolerance and action-assortativity for the  
377 emergence of cooperation and homophily. *Eur. Econ. Rev.* **102**, 1–18 (2018).
- 378 **16.** Newton, J. The preferences of homo moralis are unstable under evolving assortativity. *Int. J. Game*  
379 *Theory* **46**, 583–589 (2017).
- 380 **17.** Schlag, K. H. Why imitate, and if so, how? a boundedly rational approach to multi-armed bandits. *J.*  
381 *Econ. Theory* **78**, 130–156 (1998).
- 382 **18.** Alós-Ferrer, C. & Schlag, K. H. Imitation and learning. In Anand, P., Pattanaik, P. & Puppe, C. (eds.)  
383 *Handbook of Rational and Social Choice* (Oxford University Press, 2009).
- 384 **19.** Izquierdo, S. S. & Izquierdo, L. R. Stochastic approximation to understand simple simulation models. *J.*  
385 *Stat. Phys.* **151**, 254–276 (2013).
- 386 **20.** Khan, A. Coordination under global random interaction and local imitation. *Int. J. Game Theory* **43**,  
387 721–745 (2014).
- 388 **21.** Cui, Z. & Wang, R. Collaboration in networks with randomly chosen agents. *J. Econ. Behav. & Organ.*  
389 **129**, 129–141 (2016).
- 390 **22.** Ohtsuki, H., Pacheco, J. M. & Nowak, M. A. Evolutionary graph theory: Breaking the symmetry between  
391 interaction and replacement. *J. Theor. Biol.* **246**, 681–694 (2007).
- 392 **23.** Fosco, C. & Mengel, F. Cooperation through imitation and exclusion in networks. *J. Econ. Dyn. Control.*  
393 **35**, 641–658 (2011).
- 394 **24.** Tsakas, N. Imitating the most successful neighbor in social networks. *Rev. Netw. Econ.* **12**, 403–435  
395 (2014).
- 396 **25.** Cimini, G. Evolutionary network games: Equilibria from imitation and best response dynamics. *Com-*  
397 *plexity* **2017**, 1–14 (2017).
- 398 **26.** Alós-Ferrer, C. & Weidenholzer, S. Contagion and efficiency. *J. Econ. Theory* **143**, 251–274 (2008).
- 399 **27.** Alós-Ferrer, C. & Weidenholzer, S. Imitation and the role of information in overcoming coordination  
400 failures. *Games Econ. Behav.* **87**, 397–411 (2014).
- 401 **28.** Ohtsuki, H., Hauert, C., Lieberman, E. & Nowak, M. A. A simple rule for the evolution of cooperation  
402 on graphs and social networks. *Nature* **328**, 502–505 (2006).
- 403 **29.** Hilbe, C. Local replicator dynamics: a simple link between deterministic and stochastic models of  
404 evolutionary game theory. *Bull. Math. Biol.* **73**, 2068–2087 (2011).
- 405 **30.** Smith, M. J. The stability of a dynamic model of traffic assignment—an application of a method of  
406 lyapunov. *Transp. Sci.* **18**, 245–252 (1984).

- 407 **31.** Helbing, D. A mathematical model for behavioral changes by pair interactions. In Haag, G., Mueller,  
408 U. & Troitzsch, K. G. (eds.) *Economic Evolution and Demographic Change: Formal Models in Social*  
409 *Sciences*, 330–348 (Springer, 1992).
- 410 **32.** Ellison, G. & Fudenberg, D. Word-of-mouth communication and social learning. *Q. J. Econ.* **110**, 93–125  
411 (1995).
- 412 **33.** Duersch, P., Oechssler, J. & Schipper, B. C. Unbeatable imitation. *Games Econ. Behav.* **76**, 88–96 (2012).
- 413 **34.** Vega-Redondo, F. The evolution of walrasian behavior. *Econometrica* **65**, 375–384 (1997).
- 414 **35.** Apesteguia, J., Huck, S. & Oechssler, J. Imitation—theory and experimental evidence. *J. Econ. Theory*  
415 **136**, 217–235 (2007).
- 416 **36.** Peski, M. Generalized risk-dominance and asymmetric dynamics. *J. Econ. Theory* **145**, 216–248 (2010).
- 417 **37.** Newton, J. Conventions under heterogeneous choice rules (2019). Forthcoming in *Rev. Econ. Stud.*  
418 Available at SSRN: <https://ssrn.com/abstract=3383471>.
- 419 **38.** Taylor, P. D. & Jonker, L. B. Evolutionary stable strategies and game dynamics. *Math. Biosci.* **40**,  
420 145–156 (1978).
- 421 **39.** Weibull, J. W. *Evolutionary Game Theory* (MIT Press, 1995).
- 422 **40.** Nachbar, J. H. “evolutionary” selection dynamics in games: Convergence and limit properties. *Int. J.*  
423 *Game Theory* **19**, 59–89 (1990).
- 424 **41.** Samuelson, L. & Zhang, J. Evolutionary stability in asymmetric games. *J. Econ. Theory* **57**, 363–391  
425 (1992).
- 426 **42.** Hofbauer, J. & Sandholm, W. H. Survival of dominated strategies under evolutionary dynamics. *Theor.*  
427 *Econ.* **6**, 341–377 (2011).
- 428 **43.** Eshel, I. & Cavalli-Sforza, L. L. Assortment of encounters and evolution of cooperativeness. *Proc. Natl.*  
429 *Acad. Sci.* **79**, 1331–1335 (1982).
- 430 **44.** Bergstrom, T. C. The algebra of assortative encounters and the evolution of cooperation. *Int. Game*  
431 *Theory Rev.* **5**, 211–228 (2003).
- 432 **45.** Bergstrom, T. C. Measures of assortativity. *Biol. Theory* **8**, 133–141 (2013).
- 433 **46.** Allen, B. & Nowak, M. A. Games among relatives revisited. *J. Theor. Biol.* **378**, 103–116 (2015).
- 434 **47.** Harsanyi, J. C. & Selten, R. *A General Theory of Equilibrium Selection in Games* (MIT Press, 1988).
- 435 **48.** Maynard Smith, J. *Evolution and the Theory of Games* (Cambridge University Press, 1982).



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## 441 **Author contributions statement**

442 E.B., L.B., and N.C. contributed equally to setting the model, developing theoretical results and finding  
443 applications, as well as to writing the paper.

## 444 **Additional information**

445 The authors declare no competing interests.