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Focused beam self-cleaning during laser filamentation

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Abstract

The beam self-cleaning phenomenon is theoretically predicted by the two-dimensional nonlinear Schrödinger equation, which describes self-focusing, and is observed in the case of femtosecond laser filamentation in the collimated regime of propagation. However, the impact of external focusing on the self-cleaning has not been investigated so far. In this paper we systematically study this impact in a wide range of focusing conditions. We show that the energy range, in which self-cleaning can be observed, shrinks monotonically with the numerical aperture growth at some point vanishing at all.

Keywords: filamentation, beam self-cleaning, self-focusing, numerical aperture
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1 Introduction

Femtosecond laser filamentation is a process of high-power ultrashort laser pulse propagation in a nonlinear medium, featuring formation of extended plasma and light channels with the length of the latter greatly exceeding the corresponding Rayleigh length \([1, 2]\). Since the first observation in 1995 \([3]\), a number of outstanding properties making filaments a prominent radiation source for a variety of applications has been revealed \([4, 5]\). Indeed, intrinsically nonlinear nature of the filamentation phenomenon manifests in spectrum enrichment and supercontinuum generation in frequency domain \([6, 7]\), pulse self-shortening in time \([8, 9]\) and beam mode improvement also known as mode self-cleaning \([10–12]\). In this paper we discuss the latter property, which is of importance for such filament applications as a white-light laser \([4, 13]\) and pulse self-compression \([14]\). Mathematically, the self-cleaning phenomenon is explained by the existence of the self-similar symmetric attractor solution of the two-dimensional (2D) nonlinear Schrödinger equation (NLSE) \([15]\) that is its fundamental feature. For self-focusing in a Kerr medium this solution is given by the Townes mode, which corresponds to the exact balance of self-focusing and diffraction \([16]\). In \([17]\) the collapse of a distorted beam to the symmetric Townes mode was firstly demonstrated experimentally (for the case without any significant impact of plasma formation or any other process that can arrest the collapse). The authors of that paper note, that the higher initial deviations of the beam from the Townes mode are, the closer one needs to approach the collapse point to observe self-cleaning. Plasma formation being inevitable in a real filamentation process along with the filamentation dynamic nature, when the preceding pulse slices affect the propagation of the subsequent ones, make substantial corrections to this analysis. Thus, the first self-cleaning observation in filamentation was related only to the filament supercontinuum \([10]\). The further analysis showed even more complicated behavior: the red-shifted supercontinuum components propagating on the pulse front were perfectly self-cleaned, while the central wavelength along with the blue-shifted components still had notable inhomogeneities caused by the refraction on plasma \([12]\). Moreover, at pulse powers much higher than the self-focusing critical power \((P_{cr})\), self-cleaning was breached due to multifilamentation \([11, 12]\). Therefore, the pulse powers, at which self-cleaning takes place, appear to be bounded from below by approximately \(P_{cr}\) and from above by the multifilamentation threshold.

All the aforementioned papers examined self-cleaning either in collimated \([11, 12, 17]\) or in extremely close to collimated \([10]\) geometry. However, the multifilamentation threshold strongly depends on focusing conditions, decreasing with the increase of a numerical aperture (NA) \([18]\). Furthermore, the spectral composition of the filament radiation is also determined by the NA to a very large extent with the dominance of Stokes components at loose focusing and anti-Stokes components at tight focusing \([19]\). These facts suggest that self-cleaning process should depend on focusing conditions. However, to the best of our knowledge, no research on this question has been published
so far. Our paper aims to fill this gap by providing a systematic study of self-cleaning dependence on focusing conditions.

2 Experimental setup

Experiments were conducted using a Ti:Sa laser system delivering pulses with central wavelength of 750 nm, 90 fs pulse duration, 10 Hz repetition rate and 3 mm FW1/eM beam diameter. The pulse energy was adjusted using a diffraction attenuator placed after the pulse compressor. The pulse energy was measured after the attenuator using Ophir piroelectric detector. The energy root mean square (RMS) instability was 5-10%. After the attenuator, the beam was focused by either a spherical reflecting mirror or a lens. A set of 10 focusing elements with focal lengths from 5 to 350 cm was used allowing numerical aperture ranging for about 2 orders of magnitude. In this paper, we define numerical aperture as $NA = r/f$, where $r$ is the beam radius at $1/e$ level from its maximum, $f$ — the lens focal distance, in accordance with paper [19]. We also studied the collimated regime for a reference and data processing procedure control.

![Fig. 1 a) Measurement setup. 1,2 — silica wedges, 3 — CCD camera. b) Beam profile at the focal length $f = 20$ cm and pulse energy $E = 0.9$ mJ at the distance of 40 cm from the lens. c) Cross-section of the beam profile along the dotted line (see panel b).](image)

The scheme of the measurement setup is presented in fig. 1. The radiation from the filament was reflected twice from the silica wedges 1 and 2 the radiation power. After the wedge 2 the radiation was directed to CCD camera Spiricon SP620U (element 3 in fig. 1) with additional attenuation by the neutral density filters placed in front of the CCD. The distance between the focusing element and the CCD was maintained equal to $2f$, where $f$ is the lens or spherical mirror focal length. The only exception was $f = 350$ cm, where, due to the room dimensions limitations, the distance was taken equal to $1.5f$.

3 Data processing

It is widely known that filament transverse structure consists of an intense core and a reservoir. The latter contains the most of the initial pulse energy [20] that is not affected by self-cleaning [10, 17]. The reservoir also contains radiation components refracted from the plasma on the central and anti-Stokes
wavelengths [21]. This radiation forms ring-like structures which are particularly apparent at tight focusing. For instance, a typical beam profile, obtained at the focusing \( f = 20 \text{ cm} \) and the pulse energy \( E = 0.9 \text{ mJ} \) in our experiment is shown in fig. 1(b). The rings with an amplitude of about 1/3 of the maximum are clearly seen. That is why we had to refuse the standard beam quality analysis procedure consisting in \( M^2 \) measurements.[22] Indeed, to calculate \( M^2 \), one need to focus the beam accumulating all the reservoir energy with its ring-like structures. It will strongly affect the \( M^2 \) value making it impossible to distinguish the intense but relatively low energetic core from the high energy background reservoir.

However, it is possible to separate the self-cleaned part of the radiation from the poor-quality pedestal. It was suggested in [12] to employ a nonlinear parametric process for this purpose. Another possible solution is to separate the good-quality central portion of the radiation with a diaphragm in a far field. To simplify the experiment, in this paper we decided not to use a real diaphragm, but to separate the intense central part of the beam programatically during the data processing. For this purpose, after background was subtracted from each image and the low-scale noises were filtered, the data was cut at the level of 0.5 from the image maximum. Only the values lying in connected regions above this threshold were used. It allowed us to cut off the part of the reservoir radiation lying outside the filament core. Then, the data in the selected areas was processed using two different procedures.

In the first one, it was approximated by a symmetric 2D Gaussian. The Gaussian width, amplitude, and center position were treated as approximation parameters. To characterize quantitatively the correspondence of the data to the fitting model, the Spearman’s rank correlation coefficient \( \rho \) was used:[23]

\[
\rho = \frac{\text{cov}(u, v)}{\sigma_u \sigma_v}.
\]

Here \( \rho \) is the Spearman’s coefficient, \( u \) and \( v \) are the ranks for the experimental and approximation data (taken at the same points), correspondingly, \( \text{cov}(u, v) \) — covariance of the \( u \) and \( v \), \( \sigma_u \) and \( \sigma_v \) — standard deviations for the \( u \) and \( v \). The Spearman’s coefficient constitutes a better alternative to test the correlation between the variables in the case of nonlinear dependence between them, rather than the Pearson’s coefficient which is commonly used to analyze linear dependent variables.[24] The closer to a symmetric Gaussian the measured beam profile is, the higher Spearman’s coefficient we should obtain.

The beam ellipticity was the second quality we tested in our analysis. In this case, we approximated the data by 2D Gaussian with arbitrary (and independent from each other) axes lengths. The cross section of this surface by a horizontal plane (Gaussian value \( I(x, y) = \text{const} \)), is an ellipse. An eccentricity of such a section at level \( 1/e \) from maximum was calculated to quantitatively
characterize the beam ellipticity.

\[ \varepsilon = \sqrt{1 - \left( \frac{\sigma_x}{\sigma_y} \right)^2} \]  

Here \( \varepsilon \) is the eccentricity, \( \sigma_x \) and \( \sigma_y \) are the ellipse small and long half-axes, correspondingly.

For each given experimental conditions (focusing, pulse energy), a sample of 16 beam profiles was measured. For each of them the Spearman’s coefficient \( \rho \) and the beam eccentricity \( \varepsilon \) were calculated, and then averaged throughout the sample. The resulting uncertainties were taken as the standard deviations of \( \rho \) and \( \varepsilon \).

It is important to note, that for practical applications it is more useful to achieve beam self-cleaning in a broad spectral range rather than in a selected narrowband one. That is why we did not undertake spectral selection of the detected radiation, as it used to be done in the previous studies on mode self-cleaning in collimated filaments\[10, 12\].

4 Results and discussion

First of all, we examined the case of collimated filamentation. This allowed us to validate the data processing technique and provided a reference for focused geometry analysis. Fig. 2a demonstrates the transverse fluence distribution of the laser beam at the output of the laser system. This distribution, along with the other beam profiles provided in our paper, are normalized to their maximum. After the laser pulse propagation along the distance of 13 m from the laser output (fig. 2b) at energy corresponding to approximately self-focusing critical power \( P_{\text{cr}} \) (which was not enough to collapse the laser beam along this distance), the beam ellipticity becomes more pronounced due to the initial beam astigmatism. This astigmatism is introduced by the off-axis telescope build of two spherical mirrors being a part of the laser system. At pulse energy of 6.2 mJ (that corresponds to about \( 20P_{\text{cr}} \) for our laser system radiation) the beam collapse due to the self-focusing takes place before the registration system location, an intense and symmetric maximum being formed in the beam center (fig. 2c). So, here we observe a clear manifestation of the self-cleaning phenomenon. However, the pedestal corresponding to the energy reservoir is clearly observed here as well at level of about 0.2 from the maximum.

We measured the Spearman’s rank correlation coefficients and the eccentricities of the beam intense part according to the procedure described above at about 20 various pulse energies. The results are presented in fig. 2d. For better trends visibility, the graph also depicts the results of data averaging using the running mean with the window size of three points, with further smoothing by a quadratic spline (black dashed and red dotted curves in fig. 2d). The Spearman’s coefficient exhibits monotonic growth, while the beam eccentricity — monotonic decrease. This behavior can be explained as follows. With
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energy growth the point of collapse (nonlinear focus) is moving from infinity (for $P = P_{cr}$, $E \approx 0.3 \text{ mJ}$) towards the laser output, the nonlinear focus coming closer to our registration system. The effect of self-cleaning starts to be notable before the collapse point [17]. This effect is gradually increasing with the nonlinear focus approaching; moreover, the contrast between the self-cleaned peak and the reservoir should also increase leading to better separation of the maximum from the reservoir in our algorithm. Finally, the nonlinear focus moves to the area in front of the registration system. This leads to a perfect self-cleaning and highly symmetric mode (see fig. 2c and the corresponding point at fig. 2d). However, it is important to note, that even small fluctuations of the laser pulse energy in the collimated regime cause significant shifts of the nonlinear focus position. The nonlinear focus “jumps” forward and backward appearing before and after the registration setup. This results in drastic changes of the size, ellipticity and quality of the beam intense part that is expressed in large Spearman’s coefficient and eccentricity dispersions (see the points between 4 and 6 mJ in fig. 2d). We did not reach multifilamentation regime and the corresponding beam break-up, reported in previous papers[11, 12] due to limitations of laser pulse energy and the propagation path length (13 m path appeared to be not enough for multifilamentation to develop even at $20P_{cr}$).

The same experimental technique was applied to study the focused propagation geometry. As an example, the results obtained at $f = 1 \text{ m}$ are presented in fig. 3. At low pulse energies the laser beam is slightly elliptical and has a multitude of low-scale inhomogeneities (fig. 3a). With the increase of energy, self-cleaning comes into play forming a symmetric maximum (fig. 3b). However, further growth of the pulse energy leads to the formation of the second maximum, and the self-cleaning ceases. These changes are clearly seen in fig. 3d, where the Spearman’s coefficient and the eccentricity dependencies on the

![Fig. 2](image_url)  
Fluence distribution profiles of the collimated beam at the output of the laser system (a) and at a distance of 13 m after the laser system for pulse energy of 0.4 mJ (b) and 6.2 mJ (c). Spearman’s correlation coefficient (black) and two-dimensional Gaussian eccentricity (red) for the collimated beam at a distance of 13 m after the laser system output as a function of the laser pulse energy (d).
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Fig. 3 Fluence distribution profiles of a focused beam (focal length 1 m) at a distance of 2.0 m from the focusing mirror at a pulse energy of 0.1 mJ (a), 2 mJ (b), and 3.5 mJ (c). Spearman’s correlation coefficient and eccentricity of the two-dimensional Gaussian for the specified beam as a function of the laser pulse energy.

... pulse energy are plotted. Starting at the value of about 0.85 at $E < 0.3$ mJ, the Spearman’s coefficient sharply increases to approximately 0.96 at $E = 0.8$ mJ. Then it gradually decreases to the initial level at $W \approx 3.0–3.5$ mJ, at which an additional filament emerges. The eccentricity demonstrates similar dynamics but with the reverse order of the growth and decline. It is interesting to note, that the Spearman’s coefficient reaches its maximum, as well as the eccentricity — its minimum, not at the energy, corresponding to $P_{cr}$, but when the energy is several times higher (0.8–1.0 mJ). This stems from the fact, that for the pulse peak power $P \approx P_{cr}$, only a very small fraction of the pulse (the slices near its maximum) can undergo self-focusing. The vast bulk of the pulse energy in this case cannot contribute to the self-cleaned mode. At higher pulse energies, the fraction of the pulse energy in the slices with power $P > P_{cr}$ increases making the self-cleaning more pronounced.

Nevertheless, the further increase of the pulse energy leads to the gradual decrease in the Spearman’s coefficient and to the corresponding creeping increase in the eccentricity. The reason for this behavior can lie in plasma channel extending with the growth of the pulse energy. The longer the plasma region, the stronger it influences the propagation of the pulse trailing part. Moreover, at values much higher than the self-focusing power, the initial intensity inhomogeneities can affect the resulting plasma distribution (and, therefore, the propagation of the trailing slices of the pulse).[12]

Our results demonstrate, that the transition between a single self-cleaned filament and the absence of self-cleaning due to the beam break-up do not have threshold character. The transition to a “pure” multifilament regime occurs rather gradually.

As it can be seen from the graph fig. 3d, in the case of $f = 1$ m ($\text{NA}=1.5 \cdot 10^{-3}$) in our experimental conditions, the self-cleaning (detected by enhancement in Spearman’s coefficient or lower beam ellipticity) is observed
in energy range of approximately 0.7–3 mJ. The measurements and data processing, analogues to those described above, were conducted for another nine focusing conditions ranging from 0.05 to 3.50 m (NA = 4 \cdot 10^{-4} \div 3 \cdot 10^{-2}). The results are presented in fig. 4. The energy range, where the self-cleaning is observed, was found for each focusing. While the lower self-focusing energy limits, determined from the Spearman’s coefficient and the eccentricity, are the same with good accuracy, the upper limits often differ significantly. That is why the upper limits defined by the two methods are plotted separately (black triangles and white circles in fig. 4, correspondingly). The reason for this discrepancy is that the beam quality degradation due to the refraction on the filament-generated plasma does not necessary lead to the increase of the beam eccentricity. The Spearman’s coefficient, in turn, detects any types of deviations from an ideal symmetric Gaussian. For example, a flat-top beam pattern would result in Spearman’s coefficient decreasing, preserving the eccentricity at a constant value. However, appearing of a distinct second maximum (with intensity above 0.5 of the main maxima) is reflected by both parameters.

![Fig. 4](https://example.com/fig4.png)

**Fig. 4** Dependence of the laser pulse energy range, for which the beam self-cleaning occurs, on the numerical aperture (colored area).

The resulted area in $E(NA)$ plot, where the self-cleaning phenomenon was observed, is colored in fig. 4. The energy range, where the self-cleaning is observed, is contracting with the NA increase and, eventually, it shrinks almost to the point at $NA \approx 1.5 \cdot 10^{-2}$. The measurements were also conducted for $NA = 3 \cdot 10^{-2}$, where the self-cleaning was not observed at all.

The observed phenomenon can be explained by the impact of three factors. The first is the drastic growth of the plasma concentration with the NA increase, which reaches several orders of magnitude when the NA changes in the considered limits [25]. The consequent refractive index gradient growth reinforces the refraction of the pulse trailing slices. The second factor comes from the nonlinear to linear focusing regime transition with NA growth [19, 26].
In nonlinear focusing regime that takes place at relatively small NA, the beam focusing process is dominated by the Kerr nonlinearity. In this case, the propagation length before the focus is long enough, that the Kerr nonlinearity can collapse the beam as a whole. This results in high quality of the intense beam core. At greater NA, this regime gives way to linear focusing, at which the wavefront bending introduced by the focusing element plays the decisive role. In this case, the plasma formation is strongly affected by initial beam perturbations resulting in a strongly non-uniform plasma profile. That is why the second filament appears in this regime at lower energy than in the case of nonlinear focusing. The third factor affecting the self-cleaning is the spectral composition of filament radiation. In the case of loose focusing, the red-shifted components travelling in front of the pulse bear the most of its energy. In contrast, at tight focusing the blue-shifted components, which experience strong influence of the filament-generated plasma, start to prevail. A combined action of these three effects monotonically reduces the pulse energy range at which the self-cleaning is observed with the NA growth. Finally, at NA > 1.5 · 10⁻² the self-cleaning phenomenon completely disappears. The exact ranges of pulse energy and NA, at which the self-cleaning occurs, could certainly be affected by the initial beam quality, amplitude and size of perturbations and aberration it has. However the general trends observed in our experimental conditions, should have universal character.

5 Conclusions

In this research, we conducted a study of focusing conditions influence on the self-cleaning phenomenon. It is shown, that the energy range, at which self-cleaning is observed, decreases monotonically with the increase of numerical aperture. At NA greater than a certain value depending on the initial beam quality, the self-cleaning is absent at all. This result, which is, seemingly, in contrast with a universal nature of a Townes profile as an attractor solution of 2D NLSE, stems from the impact of plasma and complicated temporal dynamics of femtosecond laser pulse, which is not accounted by the simple NLSE model of self-focusing. The results obtained can be of interest for such applications of femtosecond filamentation as pulse self-compression or a femtosecond supercontinuum laser source, where the good quality of the output beam is essential.

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Data Availability: Due to confidentiality agreements between authors, data and/or its analysis during the current study is available from the corresponding author [D. V. Pushkarev via email: d-push@yandex.ru] on reasonable request to bona fide researchers.
Declarations

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