## Appendix Proof

### Proof of Lemma 1

Taking first and second derivatives of ,we get ,, therefore,  is concave in . Solving, we can obtain a maximum value . Thus, the optimal profit is . Similarly, we can obtain downstream manufacturer’s optimal production quantity is , optimal profit is .

### Proof of Proposition 1

Depending on  or , we propose two sub-optimization problems (Sub-problem 1 and Sub-problem 2). The optimal solution for the original problem is thus the maximum of these two sub-problems.

**Sub-problem 1**: When , upstream manufacturer’s profit is:





The unconstraint solution is . Substituting this unconstraint solution into the constraint identifies a cutoff value of . Due to ，is concave in. Thus, when , the unconstraint solution is optimal for sub-problem 1, and upstream manufacturer’s optimal production quantity and profit are,. When , upstream manufacturer’s optimal production quantity and profit are ,.

**Sub-problem 2**：When , upstream manufacturer’s profit is:





We solve this sub-problem similar to Sub-problem 1. When , upstream manufacturer’s optimal production quantity and profit are . When , upstream manufacturer’s optimal production quantity and profit are , .

We compare the two sub-problems and derive the optimal solution for the original problem.

(1) When , the optimal production quantity is , the optimal profit is ;

(2) When , we need to compare  and . Because , the optimal production quantity is ; the optimal profit is .

(3) When , the optimal production quantity is , the optimal profit is .

Therefore, Proposition 1 is proved

### Proof of Proposition 2

According to Proposition 1, we propose three sub-optimization problems (Sub-problem 1, Sub-problem 2 and Sub-problem 3). The optimal solution for the original problem is thus the maximum of these three sub-problems.

**Sub-problem 1**: When , The plant’s profit function is , which is increasing in . Due to ，Then the optimal transfer price and profit are, .

**Sub-problem 2**: When , The plant’s profit function is  which is increasing in . Due to . Hence, the optimal transfer price and profit are ,.

**Sub-problem 3**：When , thus,,The plant’s profit function is:



The unconstraint solution is . Substituting this unconstraint solution into the constraint identifies a cutoff value of . Due to , is concave in . Thus, when , the unconstraint solution is optimal for Sub-problem 3, and then the optimal transfer price and profit are,. When , the optimal transfer price and profit are ,.

We compare the three sub-problems and derive the optimal solution for the original problem.

(1) When, the optimal transfer price is , the optimal profit is；

(2) when ，we need to compare  and . Because , the optimal production quantity is , the optimal profit is .

(3) When , the optimal transfer price is , the optimal profit is .

Therefore, Proposition 2 is proved.

### Proof of Corollary 1

According to the processing capacity, we can obtain three cases.

**Case 1:** When ****, ,;

**Case 2：**When,,；

**Case 3：**When , . Because of  and ，hence, . Due to . In summary, Corollary1 is proved**.**

### Proof of Proposition 3

Depending on the supply of by-product, we propose two sub-optimization problems (Sub-problem 1 and Sub-problem 2).

**Sub-problem 1:** When , according to the table 1,the processing capacity constraint is . Under this condition, downstream manufacturer’s profit is:





The unconstraint solution is . Substituting this unconstraint solution into the constraint identifies a cutoff value of . Due to， is concave in . Let , thus when , the unconstraint solution is optimal for Sub-problem 1, and then downstream manufacturer’s optimal production quantity and profit are ,.  to ensure downstream manufacturer’s is positive. When ，downstream manufacturer’s optimal production quantity and profit are , .

**Sub-problem 2:** When , according to the table 1,the processing capacity constraint is ，Under this condition, downstream manufacturer’s profit is:





We solve this sub-problem similar to Sub-problem 1. Let ,When , downstream manufacturer’s optimal production quantity and profit are  . When ，downstream manufacturer’s optimal production quantity and profit are , 

Therefore, Proposition 3 is proved.

### Proof of Corollary 2

Obviously, in scenario L, ,; in scenario H, , .

, and , thus.

Hence, the production quantity and profit under different conditions are shown as follow in Fig A-1. We propose two sub-optimization problems (Sub-problem 1and Sub-problem 2).



Fig A-1: production quantity and profit under different conditions

**Sub-** **problem 1:** **Production quantity**

According to the price of raw material, we can obtain three cases.

Case 1: When , we need to compare with . However, .

Case 2: When ,we need to compare with. Let ，and the domain is.is decreasing in the domain, and the maximum value ，thus 。

Case 3: When, we need to compare with, ，thus .

Combine case 1,2 and 3, when ,; when ****,.

**Sub-** **problem 2: Profit**

According to the price of raw material, we can obtain three cases.

Case 1: when , we need to compare with., thus .

Case 2：When , we need to compare  with . Let



and the domain is，and . The first and second derivatives of  are ,. Hence  decreasing in the domain，and the maximum value.

Hence, 

Case 3：When , we need to compare  with . Let

and the domain is . The first and second derivatives of  are , . In addition, when , . Therefore,  decreasing in the domain，and the maximum value , thus .

Combine case 1,2 and 3,.

In summary, Corollary 2 is proved.

### Proof of Corollary 3

**Sub-** **problem 1: Scenario L**

According to the price of raw material, we can obtain two cases.

Case 1: When ,, and 

Case 2: When , let, and the domain is.  is decreasing in the domain，and the maximum value , hence . Let ，and the domain is  The first and second derivatives of  are ,. Therefore,  is decreasing in the domain，and the maximum value, hence，

Combine case 1,2 and 3,when,, ; when ,,.

**Sub-** **problem 2**：**Scenario H**

Similar to the Sub- problem 1, we can get the similar conclusions.

Therefore, Corollary 3 is proved.

### Proof of Proposition 4

**Sub-problem 1：**when , the derivatives with respect to:

,,,,,,,

**Sub-problem 2：**when , the derivatives with respect to:

,,,,,,,

**Sub-problem 3：**when , the derivatives with respect to

,,,,,,,

In summary, Proposition 4 is proved.

### Proof of Corollary 4

According to the processing capacity, we can obtain three cases

**Case 1:** When , ，because of and , hence .

**Case 2：**When , ，because of and , the positive or negative of  is indefinite. Let is the threshold, when , .

**Case 3:** When , . Similar to case 2, the positive or negative of  is indefinite., . Thus,  is the threshold.

In summary, Corollary 4 is proved.

### Proof of Corollary 5

**Sub-** **problem 1: Scenario L**

According to the price of raw material, we can obtain two cases.

Case 1: When, . According to the Corollary 2, we have . Therefore, .

Case 2：When, . According to the Corollary 2, we have . the positive or negative of  is indefinite. Let is the threshold. When ,.

Combine case 1 and case 2, when ,; when ,.

**Sub-** **problem 2: Scenario H**

Similar to the Sub-problem 1, we can get the similar conclusions.is the threshold, when,; when , .

Therefore, Corollary 5 is proved.

### Proof of Corollary 6

**Sub-** **problem 1: proof of (1)**

Case 1: According to **Corollary 1**, , . According to **Corollary 4**, when ,, so the when .

Case 2: Similar to the Sub- problem 1, we can get the similar conclusions.

**Sub-** **problem 2: proof of (2)**

According to **Proposition 1**, when , , which *K* is not included. The value of  will not change with *K*. So the value of ,  and  will not change with *K* either,  stays the same while *K* increase.

Therefore, Corollary 6 is proved.