APPENDIX

Analytic formula for strike is also a least squares solution

In order to develop a least squares solution for equation 15, we need to take the derivative of $C\_{L\_{2}}\left(θ\right)$ with respect to $θ$ and then setting it to zero. However, this is not necessary. Let us consider for the moment that $β=0$ in equation 15. Swift’s formula can be obtained minimizing $\left|Z\_{xx}^{'}\right|^{2}+\left|Z\_{yy}^{'}\right|^{2}$ or maximizing$ \left|Z\_{xy}^{'}\right|^{2}+\left|Z\_{yx}^{'}\right|^{2}$ . Now then, the derivative of $\left|Z\_{xy}^{'}\right|^{2}+\left|Z\_{yx}^{'}\right|^{2}$ is the same as the derivative of$\left|Φ\_{12}^{'}\right|^{2}+\left|Φ\_{21}^{'}\right|^{2}$, except that the elements of the phase tensor are real. Now substitute the corresponding elements of the phase tensor in equation 11 as$ Z\_{xx}\leftrightarrow Φ\_{11}, Z\_{xy}\leftrightarrow Φ\_{12}, Z\_{yx}\leftrightarrow Φ\_{21}, Z\_{yy}\leftrightarrow Φ\_{22}$ . The results is

$$tan4θ=\frac{2\left[\left(Φ\_{22}-Φ\_{11}\right)(Φ\_{12}+Φ\_{21}\right]}{(Φ\_{12}+Φ\_{21})^{2}-(Φ\_{22}-Φ\_{11})^{2}} . (A1)$$

This is the least squares solution for the strike in terms of the elements of the phase tensor. However, there is nothing new about this formula. Consider the equivalent form

$$ tan4θ=\frac{2\frac{Φ\_{12}+Φ\_{21}}{Φ\_{22}-Φ\_{11}}}{\frac{(Φ\_{12}+Φ\_{21})^{2}}{(Φ\_{22}-Φ\_{11})^{2}}-1} . (A2)$$

Or in more familiar terms

$$ tan4θ=\frac{2\frac{Φ\_{12}+Φ\_{21}}{Φ\_{11}-Φ\_{22}}}{1-\frac{(Φ\_{12}+Φ\_{21})^{2}}{(Φ\_{11}-Φ\_{22})^{2}}} . (A1)$$

It can be recognized that the ratios in the numerator and the denominator are, according to equation 7,$ 2tan2α$ and$ 1-tan^{2}2α$, respectively. The result is the familiar trigonometric identity for the double angle

$$ tan4θ=\frac{2tan2α}{1-tan^{2}2α} . (A3)$$

This means that adapting equation 11 to the phase tensor doesn’t add to the original estimation of$ tan2α$. It simply replaces the estimation of 2$θ$ by that of 4$θ$. In other words, in the case of 2D structures the analytic formula for strike is a least squares solution. This means that Bahr’s (1988) analytic formula is also a least squares solution. For the general case when $β\ne 0$ it is not necessary to replace the elements of $ΦR^{T}\left(2β\right)$ in equation 15 and proceed with the algebra as before. A shortcut is possible considering that the effect of the skew in equation 15 is to rotate $Φ$ through an extra angle$ -2β$. The negative sign is because of the transpose of the rotation matrix. The clue for the shortcut is that the skew$ β$ is invariant under rotation of coordinates (Caldwell et al., 2004). This means that in the derivatives for the minimization the factor $R^{T}\left(2β\right)$ is a constant. The solution is then the same as given by equation A3 but shifting the angle from$ 2α to 2α-2β$. Thus we can replace in equation A3 $tan2α$ by$ tan2(α-β)$. The result is

$$ tan4θ=\frac{2tan2(α-β)}{1-tan^{2}2(α-β)} . (A4)$$

Again, the least square solution doesn’t add anything to the original estimation of$ tan2θ$. It simply replaces the estimation of 2$θ$ by that of 4$θ$. This means that the analytic formula for strike derived from the phase tensor is also a least squares solution.