APPENDIX

Analytic formula for strike is also a least squares solution

In order to develop a least squares solution for equation 15, we need to take the derivative of with respect to and then setting it to zero. However, this is not necessary. Let us consider for the moment that in equation 15. Swift’s formula can be obtained minimizing or maximizing . Now then, the derivative of is the same as the derivative of, except that the elements of the phase tensor are real. Now substitute the corresponding elements of the phase tensor in equation 11 as . The results is

This is the least squares solution for the strike in terms of the elements of the phase tensor. However, there is nothing new about this formula. Consider the equivalent form

Or in more familiar terms

It can be recognized that the ratios in the numerator and the denominator are, according to equation 7, and, respectively. The result is the familiar trigonometric identity for the double angle

This means that adapting equation 11 to the phase tensor doesn’t add to the original estimation of. It simply replaces the estimation of 2 by that of 4. In other words, in the case of 2D structures the analytic formula for strike is a least squares solution. This means that Bahr’s (1988) analytic formula is also a least squares solution. For the general case when it is not necessary to replace the elements of in equation 15 and proceed with the algebra as before. A shortcut is possible considering that the effect of the skew in equation 15 is to rotate through an extra angle. The negative sign is because of the transpose of the rotation matrix. The clue for the shortcut is that the skew is invariant under rotation of coordinates (Caldwell et al., 2004). This means that in the derivatives for the minimization the factor is a constant. The solution is then the same as given by equation A3 but shifting the angle from. Thus we can replace in equation A3 by. The result is

Again, the least square solution doesn’t add anything to the original estimation of. It simply replaces the estimation of 2 by that of 4. This means that the analytic formula for strike derived from the phase tensor is also a least squares solution.