Nonlinear dynamics of boring process considering unstable region of CNTs-reinforced composite cutter

Jinfeng Zhang (✉ zhangjf@sdust.edu.cn)  
Shandong University of Science and Technology  
https://orcid.org/0000-0002-6336-7323

Zhong Wang  
Shandong University of Science and Technology

Chao Feng  
Shandong University of Science and Technology

Junlei Jia  
Shandong University of Science and Technology

Zhenfang Tong  
Shandong University of Science and Technology

Yongsheng Ren  
Shandong University of Science and Technology

Peisi Zhong  
Shandong University of Science and Technology

Xiaolong Cao  
Shandong University of Science and Technology

Research Article

Keywords: CNTs-reinforced composite, Nonlinear chatter, Viscoelastic damping, Process damping, Unstable region

Posted Date: November 29th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-2286501/v1

License: ☋  This work is licensed under a Creative Commons Attribution 4.0 International License.  
Read Full License
Nonlinear dynamics of boring process considering unstable region of CNTs-reinforced composite cutter

Jinfeng Zhang\textsuperscript{a,d,*}, Zhong Wang\textsuperscript{a}, Chao Feng\textsuperscript{b,*}, Junlei Jia\textsuperscript{a}, Zhenfang Tong\textsuperscript{a}, Peisi Zhong\textsuperscript{a,d}, Xionglong Cao\textsuperscript{c}

\textsuperscript{a}College of Mechanical and Electronic Engineering, Shandong University of Science and Technology, Qingdao, China
\textsuperscript{b}Engineering and Training Center, Shandong University of Science and Technology, Qingdao, China
\textsuperscript{c}College of Ocean Science and Engineering, Shandong University of Science and Technology, Qingdao, China
\textsuperscript{d}Advanced Manufacturing Technology Center, Shandong University of Science and Technology, Qingdao, China.

*Corresponding author.

E-mail addresses: zhangjf@sdust.edu.cn (J. Zhang), haifeng246@163.com (C. Feng).

Abstract: The nonlinear dynamic analysis of rotating composite boring round bar containing carbon nanotubes (CNTs) in cutting system was investigated. Firstly, simplify the boring bar to the model of non-extendable rotary cantilever structure. Both the Halpin-Tsai model and micro-mechanical layering theory were used to predict the material properties of the boring round bar. Then the equations for the composite boring bar based on Euler Bernoulli shaft theory, including von Karman geometric nonlinearity are derived. The nonlinear dynamic model of cutting system including periodic regenerative chatter cutting force, periodic control force, viscoelastic and process damping is established by using Hamilton principle. The analytical solution of the steady-state response of the cutting system was subsequently obtained by the Galerkin approximation and the perturbation method of multiple time scales. Finally, the influences of carbon nanotube-related parameters, fiber volume fraction, fiber orientations, stacking sequences, damping coefficient and geometry properties of the cutter edge on the stability of the cutting system are evaluated. The obtained results show that the incorporation of CNTs has significant effect on the dynamic behavior of the cutting process. Increasing the process damping and changing the cutter edge and bar's cross section can improve stability of the cutting process. Furthermore, the unstable cutting region is sensitive to multi-valued properties generated by jumping.

Keywords: CNTs-reinforced composite; Nonlinear chatter; Viscoelastic damping; Process damping; Unstable region.

1. Introduction

Boring bar with a high length-diameter ratio tend to chatter. Chatter is a self-excited vibration, which is the result of the interaction between the dynamics of the machine structure and the cutting process. The regenerative chatter is well-known dynamic instability mechanisms in cutting processes\cite{1}. Unnecessary chattering vibration can lead to the generation of large dynamic cutting forces, excessive tool wear, and workpiece surface finish that does not meet accuracy requirements. If the dynamic stiffness of the flexible parts of the machine structure is increased, the chatter resistance of the machining process can be improved. The boring bar is most lower dynamic stiffness in the machine structures. Dynamic stiffness is directly dependent on static stiffness and damping ration\cite{2}. Therefore, the static stiffness and damping are two important parameters for enhancing stability of the rotational boring bar.

In recent decades, various new techniques and methods by increasing the dynamic stiffness have been developed to improve the resistance to chattering in various machining processes\cite{3}. The principal factors improving the cutting stability as shown in Fig. 1. It is trying to reduce the vibration hazards to minimum or to limit tolerable level. The against chatter can be classified as passive\cite{4-6} and active (and semi-active)\cite{7-9}, the former being a one-time prior design and the latter using feedback control and automatic control. However, the cost of active and semi-active vibration damping techniques is too high, so their application is very slower in practice. In contrast, the passive techniques are simpler and more effective and are mainly applied in slender boring bar where the geometry of the holes does not permit the increase of the diameter. One of the most promising ways are also confirmed at our work combines materials of high stiffness and damping.
In the past few years, the development of advanced composite materials has provided a new way to suppress cutting chatter. Many attempts have been tried to increase the dynamics by using the composite materials have high static stiffness and damping compared with conventional materials such as steel. The superior performance of fiber-reinforced composites has been proven and is widely used in aerospace applications [10, 11]. The composite boring bar was firstly proposed to increase cutting stability [12]. Then, a carbon fiber epoxy composite boring bar was designed and manufactured, the test results verified that the stability of boring bar was dependent on the the dynamic stiffness [13]. In addition, because CNTs have elastic modulus of over 1TPa and tensile strength of over 150GPa, they are harder and stronger than steel, while at the same time being three to five times lighter than steel [14]. Therefore, CNTs are more attractive. It has been found that multiscale composites consisting of high strength and modulus fibers, CNTs, and polymeric matrices can often be significantly lighter in weight and have the same or better properties than metallic materials [15, 16]. The macroscopic properties of carbon nanotube-reinforced multiscale composites usually exhibit nonlinearity, which includes viscoelastic behavior of the polymer matrix composite. Therefore, carbon nanotube-reinforced multiscale composites have higher specific stiffness and damping than conventional metal materials. If carbon nanotube-reinforced multiscale composites are used, both the dynamic stiffness and damping can be improved. This is very beneficial to the stability of high-speed machining of deep holes.

Since linear theory does not fully explain the dynamic behavior of the cutting process, the nonlinear modeling has been devoted to better understanding the practical cutting test. Hanna and Tobias [17] first proposed a time-delay nonlinear model incorporating secondary and tertiary structural stiffness and cutting forces and derived the conditions for steady-state chattering. Naren Deshpande et al [18] studied a single-degree-of-freedom turning model based on nonlinear regenerative chattering, considering nonlinear dynamics in zero-dimensional phase space. Moradi et al [19] studied the system dynamics in the case of main and super-harmonic resonance during milling considering both regenerative chattering, structural nonlinearity, tool wear and process damping effects. G. Stépán et al [20] studied state-of-the-art cutting model for the study of high-speed milling processes, discussing the local and global nonlinear behavior of the most typical excited delayed oscillators. Jalili et al [21] investigated the frequency response of milling system under primary resonance, sub-harmonic and super-harmonic resonance using three-dimensional nonlinear dynamics model of the milling process including structural and cutting force nonlinearities, gyroscopic moments and rotational inertia using the multi-scale approach. L. Vela-Martinez et al [22] proposed weakly nonlinear model with square and cubic terms for both structural stiffness and regenerative terms, analyzed the structural stiffness and cutting forces for regenerative chattering, represented the dynamical system in standard form, constructed approximate solutions, and established the instability conditions due to sub-critical Hopf bifurcation based on the eigenvalues of the system.
So far, the proposed nonlinear models have been adequately explained considered effects, such as intrinsic factors including cutting forces and gyroscopic. However, it seems that no nonlinear dynamics models for cutting systems with composite materials as boring bar containing process damping forces have been studied.

In this work, the nonlinear dynamics of the cutting process of rotating composite boring bar is studied. Nonlinear equations of the composite boring bar embedded CNTs are obtained based on the Halpin-Tsai model and micro-mechanical layering theory in section 2. In fact, combination effects of internal and external damping are considered in section 2.2.1. The equations for the composite boring bar based on the kinetic and the potential energy including von Karman geometric nonlinearity are derived using Hamilton principle in section 2.2.2. To analyze the nonlinear cutting stability, the equations are discretized by Galerkin method and then the multi-scale solution is applied in section 2.3 and 2.4. The correctness of the proposed dynamic model is validated in section 3. Then the effects of nonlinear dynamic response of the weight percentage, aspect ratio of CNTs and volume fraction, stacking sequences of fibers and processing parameters are explored in section 4. The unstable cutting region is discussed and analyzed in section 5. Finally, the following specific conclusions are listed in section 6.

2. Theoretical formulation of composite boring bar

![Fig. 2 Structure of the rotating boring bar](image)

The investigated rotating boring bar can be obtained by winding several layers of embedded fibers and CNTs on the mandrel, and each layer has orthogonal and anisotropic mechanical properties, as shown in Fig. 2. Where (1, 2, 3) are orthogonal and anisotropic axes. 1 is the fiber direction, 2 is the transversal direction to the fibers in the ply, 3 is the perpendicular direction to the layer, and $\theta$ is the fiber orientational angle with respect to the x axis. The boring round bar is clamped at one end using three-jaw self-centering chuck and free end with single cutter at the another. It is assumed that the boring bar can only vibrate in the direction perpendicular to the machined surface.

2.1 Modelling multi-scale composite
Several hybrid composite models have been proposed to represent the material properties of CNT-reinforced composite. The effective material properties of multiscale hybrid nanocomposites can be predicted according to combination of Halpin-Tsai[23] and micro-mechanical schemes[24, 25]. The final properties of CNT-reinforced composite are orthogonal and anisotropic and can be expressed as:

\[ E_{1h} = V_F E_F^1 + V_{MNC} E_{MNC} \]

\[ \frac{1}{E_{22h}} = \frac{V_F}{E_F^{22}} + \frac{V_{MNC} V_{MNC}^2 E_{MNC}}{V_F E_F^{22} + V_{MNC} E_{MNC}} \]

\[ \frac{1}{G_{12h}} = \frac{V_F^{12}}{G_{12}} + \frac{V_{MNC}}{G_{MNC}} \]

\[ V_{12h} = V_F V_F^1 + V_{MNC} V_{MNC} \]

\[ \rho_h = V_F \rho_F + V_{MNC} \rho_{MNC} \]

where \( E, G, v \) and \( \rho \) represent Young's modulus, shear modulus, Poisson's ratio and mass density, respectively. In addition, the subscripts \( F \) and \( MNC \) denote the fiber and CNTs corresponding to the isotropic nanocomposite matrix, respectively. Obviously, \( V_F \) and \( V_{MNC} \) are the volume fractions of the fiber and nanocomposite matrix, respectively. Moreover, the relationship of volume fractions between the fibers and the nanocomposite matrix can also be correlated with each other and expressed by[23]:

\[ V_F + V_{MNC} = 1 \]

According to the Halpin-Tsai model, the tensile modulus of the nanocomposite matrix can be expressed as follows[23]:

\[ E_{MNC} = \frac{E_{MER}^{CN}}{8} \left\{ 5 \left( 1 + 2 \beta_{dd} V_{CN}^{CN} \right) + \frac{1 + 2 \left( \frac{l_{CN}^{CN}}{d_{CN}^{CN}} \right) \beta_{dd} V_{CN}^{CN}}{1 - \beta_{dd} V_{CN}^{CN}} \right\} \]

\[ \beta_{dd} = \frac{E_{11}^{CN}}{E_{MER}^{CN}} - \left( \frac{d_{CN}^{CN}}{4 l_{CN}^{CN}} \right) \]

\[ \beta_{dd} = \frac{E_{11}^{CN}}{E_{MER}^{CN}} + \left( \frac{l_{CN}^{CN}}{2 d_{CN}^{CN}} \right) \]

\[ \beta_{dd} = \frac{E_{11}^{CN}}{E_{MER}^{CN}} - \left( \frac{d_{CN}^{CN}}{4 l_{CN}^{CN}} \right) \]

Where \( E_{CN}^{CN}, l_{CN}^{CN}, d_{CN}^{CN} \) and \( f_{CN}^{CN} \) respectively are the Young's modulus, thickness, outer diameter and length of carbon nanotubes. In addition, \( E_{MER}^{CN} \) and \( V_{MER}^{CN} \) represent the Young's modulus and volume fraction of the epoxy resin matrix, respectively.

The volume fraction of carbon nanotubes can be defined as[23]:

\[ V_{CN} = \frac{W_{CN}}{w_{CN} + \left( \rho_{CN} / \rho_{MER} \right) \left( \rho_{CN} / \rho_{MER} \right) W_{CN}} \]
where $w_{CN}$ is the mass fraction of carbon nanotubes, $\rho_{CN}$ and $\rho_{MER}$ are the densities of carbon nanotubes and epoxy resin matrix, respectively. It is worth mentioning that the volume fractions of carbon nanotubes and epoxy resin matrix can be correlated with each other as,

$$V_{CN} + V_{MER} = 1$$  \hfill (11.)

The Poisson's ratio, density and shear modulus of the nanocomposite matrix can be expressed as,

$$\nu_{MNC} = \nu_{MER}$$  \hfill (12.)

$$\rho_{MNC} = V_{CN}\rho_{CN} + V_{MER}\rho_{MER}$$  \hfill (13.)

$$G_{MNC} = \frac{E_{MNC}}{2(1+\nu_{MNC})}$$  \hfill (14.)

Where, the $\nu_{MER}$ denotes the Poisson's ratio of the epoxy resin matrix. Due to the small amount of CNTs, the Poisson's ratio of the carbon nanotube is assumed to be the same value as that of the epoxy resin, 0.33[24-26]. It is also assumed that the CNTs-fiber-matrix bonding and dispersion in matrix are perfect.

### 2.2 Equations of motion

#### 2.2.1 Internal and external damping model

The displacement field of any point in the boring round bar along the $x$-axis and $z$-axis using the Euler-Bernoulli beam theory can be expressed by $u(x, z, t)$ and $w(x, z, t)$, respectively as follow[27, 28],

$$u(x, z, t) = u_0(x, t) - z \frac{\partial w_0(x, t)}{\partial x}$$

$$w(x, z, t) = w_0(x, t)$$  \hfill (15)

where $u_0(x, t)$ is the displacement of the neutral axis along the $x$-direction and $w_0(x, t)$ is the deflection through the $z$-axis. The strain-displacement relation of the boring bar based on the von Karman deformation assumption is acquired as follows,

$$\varepsilon_x = \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 - z \frac{\partial^2 w_0}{\partial x^2}$$  \hfill (16)

The stress-strain relation for a composite boring bar including the viscoelastic internal damping is written as[29],

$$\sigma_{xx} = \bar{Q}_{11} \varepsilon_x + c_i \bar{Q}_{11} \ddot{c_x}$$  \hfill (17)

where, $\bar{Q}_{11}$ is the material stiffness matrix and $c_i$ is the internal damping coefficient, which are expressed in Appendix A.

For external damping, it is assumed that the process damping has greatly effect on chatter dampened of composite boring bar. Altintas[30] proposed a new analytical model to predict the process damping and obtained an equivalent external damping coefficient $c_{eq}$ as follows,

$$c_{eq} = \int_{\frac{V}{\lambda}}^{\frac{3\lambda}{2}} F_{PD, r}(x) \cos\left(\frac{2\pi x}{\lambda}\right) dx$$  \hfill (18)

where, $F_{PD, r}$, $V$, $A_0$, $\lambda$ respectively represents the process damping force, the cutting speed, the amplitude of
chatter, and the wavelength of the vibration left on workpiece surface as shown in Fig. 3

![Fig. 3 Regenerative chattering and discrete process damping calculation points](image)

When the cutting edge element penetrate into the wavy workpiece material and the resulting plowing force due to rubbing between the edge radius \( R_e \) and the material machined or plowed are produced. Since the process damping are induced from the dynamic indentation of the cutting edge element, which improves the chatter stability, so the static indentation should be removed. That is, the process damping term is represented by subtracting the blue from red areas in Fig. 3.

2.2.2 Hamilton Principle

The kinetic energy of the rotating composite boring bar including the translatory, rotary inertia and gyroscopic effects can be written as [31]:

\[
T = \frac{1}{2} \int_0^L \left[ \rho A (\ddot{u}^2 + \dot{w}^2) + \rho I \left( \frac{\partial^2 \ddot{u}}{\partial x^2} + \left( \frac{\partial \dot{w}}{\partial x} \right)^2 \right) - 4\Omega \rho \psi_x \dot{\psi}_z \right] dx
\]

where, \( L \) is the length of the composite boring bar. \( u \) and \( w \) denote respectively the displacement of the neutral axis along the \( x \)- and \( z \)-axis directions; \( I \) is the moment of inertia of the cross-section, \( \psi_x \) and \( \psi_z \) represent respectively the angles of rotation of the cross-section around the \( x \)- and \( z \)-axes, defined as: \( \psi_x = \frac{\partial u}{\partial x}, \psi_z = \frac{\partial w}{\partial x} \).

The variation of the potential energy of the rotating boring bar can be expressed as:

\[
U = \int_0^L \int_A \sigma_{xx} e_{xx} dAdx
\]

here we assume that the inertia in the plane is negligible, and the distributed axial force is zero. Based to the Euler-Bernoulli shaft theory, Combine Eq. (16) and replacing Eq. (17) into Eq.(20) yields the following result.

\[
U = \int_0^L \int_A \left( \ddot{\psi}_x \dot{e}_x^2 + c_1 \ddot{\psi}_x e_{xx} \right) dAdx
\]

Here, we assumed that the boring round bar subjected to a work virtual done due to both the process damping and cutting forces. The virtual dissipative work can be represented as follows respectively.

The expression of the process damping dissipation force of the rotating boring bar is:
The virtual of work of the cutting force can be developed in the following form,
\[ \delta W_E = \frac{1}{2} \int_0^L c_{eq} (u \delta u + \dot{w} \delta w) dx \]  
(22)

The virtual of work can be developed in the following form,
\[ \delta F = \int_0^L q_z \delta w dx \]  
(23)

where \( q_z \) is uniformly distributed force acting on boring round bar.

Using the above expressions for kinetic energy (19), potential energy (21), virtual work of process damping dissipation and cutting force, the governing differential equations are obtained by Hamilton's principle as follows,
\[ \int_T^t \delta (T-U+F+W_E^E) dt = 0 \]  
(24)

Here, based on von Karman geometric nonlinearity the parameter \( e \), which represents the strain along the centerline of the boring round bar is introduced and can be written as,
\[ e = \sqrt{(1 + u')^2 + w'^2} - 1 \]  
(25)

Assuming that the composite boring bar is inextensible, i.e., \( e = 0 \), it follows that
\[ u' = \sqrt{1 - w'^2} - 1 \approx -\frac{1}{2} w'^2 \]  
(26)

Therefore, for the composite boring bar that is simplified to cantilever shaft and contains viscoelastic model, the vibration control equation can be expressed as follow,
\[ m \frac{\ddot{w}^2}{c_\tau^2} - I_0 \left( \frac{\ddot{w}^2}{c_\tau^2} + 2 \Omega \frac{\ddot{w} \dot{w}}{c_\tau^2} \right) - c_{eq} \frac{\partial w}{\partial t} + \tilde{D}_{11} \frac{\partial^4 w}{\partial x^4} - c_{D_{11}} \frac{\partial^4 w}{\partial x^4} = 0 \]  
(27)

where \( I_0 \) denotes the mass moment of inertia, \( m \) denotes the mass per unit length of composite boring bar, \( \tilde{A}_{11} \) and \( \tilde{D}_{11} \) are respectively the extensional and flexural stiffnesses coefficients, which are defined as[31]:
\[ I_0 = \rho I = \frac{\pi}{4} \sum_{k=1}^{K} \rho_k \left( z_k^4 - z_{k-1}^4 \right) \]  
(28)

\[ m = \rho A = \pi \rho \left( r_N^2 - r_1^2 \right) \]  
(29)

\[ \tilde{A}_{11} = \frac{\pi}{2} \sum_{k=1}^{N} \tilde{q}_1 \left( r_{k+1}^2 - r_k^2 \right) \]  
(30)

\[ \tilde{D}_{11} = \frac{\pi}{4} \sum_{k=1}^{N} \tilde{q}_1 \left( r_{k+1}^4 - r_k^4 \right) \]  
(31)

The transformation of the following dimensionless parameters and variables are introduced as[32]:
\[
\ddot{x} = \frac{x}{L}, \quad \ddot{w} = \frac{w}{h}, \quad \ddot{\eta} = \frac{h}{L}, \quad \ddot{A}_{11} = \frac{\ddot{A}_{11}}{A_{10}}, \quad \ddot{I} = \frac{t}{L} \sqrt{\frac{A_{10}}{m} = \omega_h t},
\]
\[
\ddot{D}_{11} = \frac{\ddot{D}_{11}}{L^2 A_{10}}, \quad \ddot{q}(\xi, \tau) = \frac{q_r(x, t)L^2}{hA_{10}}, \quad \ddot{C} = \frac{Lc_{eq}}{\sqrt{mA_{10}}, \quad \ddot{I}_0 = \frac{I_0}{mL^2}, \quad \ddot{\Omega} = \frac{\Omega h}{L}
\]

where \( A_{110} \) denotes the value of \( \ddot{A}_{11} \) for the isotropic composite boring bar and over-tild sign above dimensionless element are ignored for convenience of expression, the differential governing Eq. (27) can be rewritten in dimensionless form as follows,
\[
\frac{\partial^2 w}{\partial t^2} - \ddot{C} \frac{\partial w}{\partial t} - \ddot{I}_0 \frac{\partial^2 \tilde{w}}{\partial \xi^2} - 2\ddot{\Omega} \ddot{I}_0 \frac{\partial \tilde{w}}{\partial \xi} \frac{\partial^2 \tilde{w}}{\partial \xi^2} + D_{11} \frac{\partial^4 \tilde{w}}{\partial \xi^4} + c_i \ddot{D}_{11} \frac{\partial^4 \tilde{w}}{\partial \xi^4} - A_i \ddot{\eta} \frac{\partial^2 \tilde{w}}{\partial \xi^2} \int_0^1 \frac{\partial \tilde{w}}{\partial \xi} d\xi - c_i A_i \ddot{\eta} \frac{\partial^2 \tilde{w}}{\partial \xi^2} \int_0^1 \left( \frac{\partial \tilde{w}}{\partial \xi} \right)^2 d\xi - q(x, \tau) = 0
\]

Note that the horizontal bars above the variables \( x \) and \( t \) in the equation have been removed for simplicity.

### 2.3 Modal Expansion

The first mode of the chatter vibration is greatly considered for the transverse displacement \( w \), which is expressed as the product of spatial coordinate function and a function of time. It is assumed that the nonlinear nature of \( w \) is included in the temporal part of the solution,
\[
w(x, t) = \phi_1(x)W(t)
\]

where \( W(t) \) is the unknown function of time and \( \phi_1(x) \) denotes the vibration function of the rotating boring bar, which needs to satisfy the boundary conditions for clamped-free composite boring bar.
\[
\phi_1(x) = \cos \beta_1 x - \cosh \beta_1 x - \frac{\cos \beta_1 L}{\sin \beta_1 L + \sinh \beta_1 L} \left( \sin \beta_1 x - \sinh \beta_1 x \right)
\]

For the cantilever boring bar, \( \phi_1(x) \) must also satisfy the following boundary conditions,
\[
\phi_1(0) = \phi_1'(0) = \phi_1''(L) = \phi_1'''(L) = 0
\]

The natural frequencies of the boring bar can be obtained by:
\[
\omega_i = (\beta_i L)^2 \sqrt{\frac{EI}{\rho A L}}, i = 1, 2, 3, \ldots
\]

In the actually boring process, the value of chattering frequency is usually around the first-order natural frequency of the boring system, so which is considered and taken as \( \beta_1 L = 1.8751 \).

By considering the uniformly distribution of regenerative chatter forces and time periodic stationary force, \( q_z \) in Eq. (27) can be reproduced as follows,
\[
q_z = (\Delta F(t) + F \cos \omega t) \delta(x - L) = [K_i b((w(L, t) - w(L, t - \tau)) + F \cos \omega t)] \delta(x - L)
\]

where, \( \delta \) is the Dirac function. \( F \) indicates the amplitude of the periodic component of force causing forced vibration, \( \Delta F(t) \) denotes the dynamic part related to the regenerative effect at the end of boring bar \( x=L \), \( K_i \)
represents the cutting force coefficient, \( b \) denotes the depth of cut, \( w(t) \) denotes the dynamic cutting thickness, \( \tau \) stand for the regenerative delay responding to the rotation period of the tool teeth \((\tau=2\pi/N\Omega)\), \( N \) is the number of tool teeth, and \( \Omega \) is the rotational speed of boring bar [19, 33].

Combine Eq. (38) and replacing the mentioned Eq.(34) into Eq.(33), multiplying by \( \Phi_1(x) \) before and after the resulting equation and integrating over the domain, and yields the following nonlinear ordinary differential equation for composite boring bar.

\[
B_1 \dot{W} + B_2 \ddot{W} + B_3 \dot{W}^2 + B_4 \dot{W} + B_5 \dot{W}^3 + B_6 \dot{W}^3 \dot{W} = K \beta B \tau (W - W_r) - B_4 F \cos \omega t
\]  

(39)

In Eq.(39):

\[
B_1 = \tilde{T} \int_0^L \phi(x) \phi^*(x) dx - \int_0^L \phi^2(x) dx, B_2 = \tilde{C} \int_0^L \phi^2(x) dx - c_i \tilde{D}_i \int_0^L \phi(x) \phi^{(4)}(x) dx,
\]

\[
B_3 = 2 \tilde{G}_i \int_0^L \phi(x) \phi^\prime(x) \phi^\prime(x) dx, B_4 = - \tilde{D}_i \int_0^L \phi(x) \phi^{(4)}(x) dx,
\]

\[
B_5 = \frac{1}{2} \tilde{T}_i \int_0^L \phi(x) \phi^\prime(x) dx \int_0^L (\phi^\prime(x))^2 dx, B_6 = c_i \tilde{D}_i \int_0^L \phi(x) \phi^{(4)}(x) dx \int_0^L (\phi^\prime(x))^2 dx,
\]

\[
B_7 = \phi_i^2(L), B_8 = \phi_i(L), W = W(t), W_r = W(t - \tau)
\]  

(40)

Let:

\[
A_0 = \frac{B_2}{B_1}, A_1 = \frac{B_3}{B_1}, A_2 = \frac{B_4}{B_1}, A_3 = \frac{B_5}{B_1},
\]

\[
A_4 = \frac{B_6}{B_1}, A_5 = \frac{B_7}{B_1}, W_c = K \beta, \tilde{F} = F \frac{B_8}{B_1}
\]  

(41)

Therefore, Eq. (39) can be rewritten and yield the following equation,

\[
\dot{W} + A_0 \ddot{W} + A_1 \dot{W}^2 + A_2 \dot{W} + A_3 \dot{W}^3 + A_4 \dot{W}^3 \dot{W} = W_c A_5 (W - W_r) - \tilde{F} \cos \omega t
\]  

(42)

2.4 Multi-scale solution

The method of multiple time scales is used to study the nonlinearity of Eq.(42). A small parameter \( \varepsilon \) is introduced in the following form

\[
\dot{W} + \varepsilon A_0 \ddot{W} + \varepsilon A_1 \dot{W}^2 + \varepsilon A_2 \dot{W} + \varepsilon A_3 \dot{W}^3 + \varepsilon A_4 \dot{W}^3 \dot{W} = \varepsilon W_c A_5 (W - W_r) - \varepsilon \tilde{F} \cos \omega t
\]  

(43)

According to reference [19], by simplifying the time-delay term \( W - W_r \) in Eq. (43) through the Laplace transform and combined the Pade approximation \( (e^{-s\tau}) \approx 1 - s\tau \), Rearranging the terms in Eq.(43) yields the following Eq. (44),

\[
\dot{W} + \varepsilon (A_0 - W_c A_5 \tau) \ddot{W} + \varepsilon A_1 \dot{W}^2 + A_2 \dot{W} + \varepsilon A_3 \dot{W}^3 + \varepsilon A_4 \dot{W}^3 \dot{W} = - \varepsilon \tilde{F} \cos \omega t
\]  

(44)

In order to obtain the solvability condition, its time dependence will be exhibited. For this purpose, the detuning parameter is introduced as follow,

\[
\Omega = \omega_L + \varepsilon \sigma
\]  

(45)

where, \( \sigma \) represent proximities between the rotational speed \( \Omega \) and the natural frequency \( \omega_L \).

The approximate solution for small amplitudes that vary weakly with time can be represented by an expansion of different time scales in the following form,
\[ W = W_0(T_0, T_1) + \varepsilon W_1(T_0, T_1) + \ldots \]  
(46)

Where, \( T_0 = t, T_1 = \varepsilon t \). \( T_0 \) and \( T_1 \) are the fast and slow scales respectively.

Therefore, the derivative with respect to \( t \) becomes an expansion of the partial derivative with respect to the \( T_n \) according to following expression,

\[
\dot{W} = D_0 W_0 + \varepsilon (D_1 W_0 + D_0 W_1) + \ldots \\
\ddot{W} = D_0^2 W_0 + \varepsilon (2D_1 D_0 W_0 + D_0^2 W_1) + \ldots 
\]
(47)

Where, \( D_i^j = \frac{\partial^j}{\partial T_i^j} \), \( j = 0, 1, 2 \ldots n \).

Bring Eq. (46) and Eq. (47) into Eq.(44) and equating the coefficients of \( \varepsilon^0 \) and \( \varepsilon^1 \) to zero respectively, and the corresponding expression are obtained as follows,

\[ O(\varepsilon^0) : D_0^2 W_0 + A_2 W_0 = 0 \]  
(48)

\[
O(\varepsilon^1) : D_0^2 W_1 + (A_0 - W_1 A_2) D_0 W_0 + A_2 W_1 = -2D_1 D_0 W_0 \\
- A_4 (D_0 W_0)^2 - A_5 W_0^3 - A_6 (D_0 W_0) W_0^2 - \tilde{F} \cos \omega t 
\]
(49)

The solution of Eq.(48) is sought in the form,

\[ W_0 = A(T_1) e^{j\omega T_0} + cc \]  
(50)

where \( A \) is the unknown function of \( T_1 \) and \( cc \) denotes the complex conjugate term of the former. The solvability condition is derived by substituting Eq.(50) into Eq.(49) and setting all the resonant terms to zero to avoid secular terms in the solution as follow.,

\[ 2 j \omega \dot{A} + (A_0 - W_1 A_2) j \omega A + (3A_3 + j \omega A_4 - 2A_6 \omega_1^2) A^2 + \frac{\tilde{F}}{2} e^{j\sigma T_0} = 0 \]  
(51)

The polar forms are introduced to solve Eq. (51), which is written as,

\[ A(T_1) = \frac{1}{2} a(T_1) e^{j\psi(T_1)} \]
(52)

where the amplitude \( a(T_1) \) and the phase angle \( \psi(T_1) \) are real functions of \( T_1 \) respectively.

Substituting Eq.(52) into Eq.(51) and let \( \sigma T_1 - \psi = \Gamma \), then that can be written as,

\[ 2 j \omega \dot{a} + j a (\sigma - \Gamma') + (A_0 - W_1 A_2) j \omega a + (3A_3 + j \omega A_4 - 2A_6 \omega_1^2) - \frac{1}{4} a^3 + \frac{\tilde{F}}{2} e^{j\gamma} = 0 \]  
(53)

Separating the real and imaginary parts for Eq.(53) to obtain a dynamical system that controls the evolution of the amplitude and phase of the response.
\[-2\omega_{1}^{2}a(\sigma - \Gamma') + p_{1}a^{3} = \tilde{F}\cos \Gamma\]
\[2\omega_{1}^{2}a' + (A_{y} - W_{x}A_{z})\omega_{1}a + p_{2}a^{3} = \tilde{F}\sin \Gamma\]

(54)

Where, \( p_{1} = \frac{1}{4}(3A_{z} - 2A_{y}A_{z}), p_{2} = \frac{1}{4}A_{z}A_{x} \).

For steady-state response, it is taken as \( a' = \Gamma' = 0 \), and the trigonometric function angles in Eq. (54) are eliminated, then the detuning parameter equation containing the frequency response can be obtained as follows,

\[
\sigma = \frac{p_{1}a^{3} \pm \sqrt{\tilde{F}^{2} - [(A_{y} - W_{x}A_{z})\omega_{1}a + p_{2}a^{3}]^{2}}}{2\omega_{1}a}
\]

(55)

3. Validation of the model

In order to validate the correctness of the proposed dynamic model for the composite boring round bar CNTs embedded, the first natural frequencies and the dynamic response of system are compared with those available references. The fundamental frequency of the four-layer equal thickness composite cantilever boring bar obtained from the proposed in the work are compared with the results of X.Q He and Y.S Ren et al as shown in Table 1 and Table 2.

The differences exist between the results are defined by: Difference=100%[(Present-Ref.)/ Ref.]. The geometric parameters and material properties of composite boring bar are \( E_{11} = 144.8 \text{GPa}, E_{22} = 9.65 \text{GPa}, v_{12} = 0.3, \rho = 1389.23 \text{kg/m}^{3}, d = 0.03 \text{m} \).

The obtained results using the developed dynamic model in this work are in good agreement with those obtained from Y.S Ren et al. In fact, the kinetic energy and potential energy of the rotating composite boring bar including regenerative and forced vibration are considered in this work that are different from dynamic theories of X.Q He and D.H. Yao. So a slightly differences for the case of stacking sequence while close to the same for the case of single fibers orientation angles of plies.

Table 1
Comparison of the fundamental frequency \( \omega_{1} = \frac{1}{2}E_{11}L^{2}\sqrt{\rho/(4E_{22}d^{2})} \) for a four-layer \((0^\circ/90^\circ/90^\circ/0^\circ)\) CNTFPC boring bar. (\( E_{11} = 144.8 \text{GPa}, E_{22} = 9.65 \text{GPa}, v_{12} = 0.3, \rho = 1389.23 \text{kg/m}^{3}, d = 0.03 \text{m} \)).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_{1} ) (rad/s)</td>
<td>0.1859</td>
<td>0.2072</td>
<td>0.2246</td>
<td>0.2582</td>
</tr>
<tr>
<td>Different (%)</td>
<td>38.89%</td>
<td>24.61%</td>
<td>14.96%</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 2
Comparison of the fundamental frequency \( \omega_{1} = \frac{1}{2}E_{11}L^{2}\sqrt{\rho/(4E_{22}d^{2})} \) for a four-layer \((\theta^\circ/\theta^\circ/\theta^\circ/\theta^\circ)\) CNTFPC boring bar. (\( E_{11} = 144.8 \text{GPa}, E_{22} = 9.65 \text{GPa}, v_{12} = 0.3, \rho = 1389.23 \text{kg/m}^{3}, d = 0.03 \text{m} \)).

<table>
<thead>
<tr>
<th>Source</th>
<th>Type</th>
<th>0°</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
</tr>
</thead>
<tbody>
<tr>
<td>X.Q. He[14]</td>
<td>0.1859</td>
<td>0.1734</td>
<td>0.1394</td>
<td>0.0929</td>
<td></td>
</tr>
<tr>
<td>D.H. Yao[34]</td>
<td>0.2072</td>
<td>0.1986</td>
<td>0.1762</td>
<td>0.1493</td>
<td></td>
</tr>
<tr>
<td>Y.S Ren[35]</td>
<td>0.2246</td>
<td>0.2096</td>
<td>0.1687</td>
<td>0.1129</td>
<td></td>
</tr>
<tr>
<td>Present</td>
<td>0.2582</td>
<td>0.2409</td>
<td>0.1938</td>
<td>0.1295</td>
<td></td>
</tr>
<tr>
<td>Different (%)</td>
<td>14.96%</td>
<td>14.93%</td>
<td>14.88%</td>
<td>14.70%</td>
<td></td>
</tr>
</tbody>
</table>
Therefore, the nonlinear vibration response of composite boring bar is further compared with research results of Y.S Ren et al as shown in Fig. 4. It can be found that the results are in acceptable agreement with those of reference[35] and concluded that the proposed dynamic theories are more adequate for the dynamic nonlinear analysis of composite boring bar considering effect of CNTs.

4. Numerical results and analysis

After the present formulation are validated, the effects with respect to nonlinear dynamic response of the weight percentage, aspect ratio of CNTs and volume fraction, stacking sequences of fibers and processing parameters are explored for composite boring bar. CNTs as reinforcements are assumed to be uniformly distributed in the composite boring bar. Typical geometric parameters, material properties are summarized in Table 3, Table 4 and Table 5.

Table 3
Geometric parameters of boring bar

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>0.03</td>
<td>m</td>
<td>Radius of outer circle of cross section</td>
</tr>
<tr>
<td>d1</td>
<td>0.02</td>
<td>m</td>
<td>Cross-section inner circle radius</td>
</tr>
<tr>
<td>L</td>
<td>0.3</td>
<td>m</td>
<td>Length of composite shank</td>
</tr>
<tr>
<td>F</td>
<td>10000</td>
<td>Pa</td>
<td>Periodic excitation force amplitude</td>
</tr>
<tr>
<td>σ</td>
<td>750</td>
<td>-</td>
<td>Detuning parameter</td>
</tr>
</tbody>
</table>
Table 4
Materials properties of CNTs[25]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{CN}$</td>
<td>0.34</td>
<td>nm</td>
<td>Thickness of carbon nanotubes</td>
</tr>
<tr>
<td>$d_{CN}$</td>
<td>1.4</td>
<td>nm</td>
<td>Outer diameter of carbon nanotubes</td>
</tr>
<tr>
<td>$l_{CN}$</td>
<td>25</td>
<td>μm</td>
<td>Length of carbon nanotubes</td>
</tr>
<tr>
<td>$w_{CN}$</td>
<td>5</td>
<td>%</td>
<td>Weight percentage of carbon nanotubes</td>
</tr>
<tr>
<td>$\rho_{CN}$</td>
<td>1350</td>
<td>kg/m$^3$</td>
<td>Mass density of carbon nanotubes</td>
</tr>
<tr>
<td>$E_{ij}$</td>
<td>640</td>
<td>Gpa</td>
<td>Young’s modulus of carbon nanotubes</td>
</tr>
<tr>
<td>$v_{ij}$</td>
<td>0.33</td>
<td></td>
<td>Poisson’s ratio of carbon nanotubes</td>
</tr>
</tbody>
</table>

Multi-walled carbon nanotubes (MWCNTs)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{CN}$</td>
<td>0.34</td>
<td>nm</td>
<td>Thickness of carbon nanotubes</td>
</tr>
<tr>
<td>$d_{CN}$</td>
<td>20</td>
<td>nm</td>
<td>Outer diameter of carbon nanotubes</td>
</tr>
<tr>
<td>$l_{CN}$</td>
<td>50</td>
<td>μm</td>
<td>Length of carbon nanotubes</td>
</tr>
<tr>
<td>$w_{CN}$</td>
<td>0</td>
<td>%</td>
<td>Weight percentage of carbon nanotubes</td>
</tr>
<tr>
<td>$\rho_{CN}$</td>
<td>1350</td>
<td>kg/m$^3$</td>
<td>Mass density of carbon nanotubes</td>
</tr>
<tr>
<td>$E_{ij}$</td>
<td>400</td>
<td>Gpa</td>
<td>Young’s modulus of carbon nanotubes</td>
</tr>
<tr>
<td>$v_{ij}$</td>
<td>0.33</td>
<td></td>
<td>Poisson’s ratio of carbon nanotubes</td>
</tr>
<tr>
<td>$\rho_{MER}$</td>
<td>1200</td>
<td>kg/m$^3$</td>
<td>Mass density of epoxy resin matrix</td>
</tr>
<tr>
<td>$E_{MER}$</td>
<td>2.72</td>
<td>Gpa</td>
<td>Young’s modulus of epoxy resin matrix</td>
</tr>
<tr>
<td>$v_{MER}$</td>
<td>0.33</td>
<td></td>
<td>Poisson’s ratio of epoxy resin matrix</td>
</tr>
</tbody>
</table>

Table 5
Materials properties of epoxy resin matrix and E-Glass fibers[25]

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{MER}$</td>
<td>1200</td>
<td>kg/m$^3$</td>
<td>Mass density of epoxy resin matrix</td>
</tr>
<tr>
<td>$E_{MER}$</td>
<td>2.72</td>
<td>Gpa</td>
<td>Young’s modulus of epoxy resin matrix</td>
</tr>
<tr>
<td>$v_{MER}$</td>
<td>0.33</td>
<td></td>
<td>Poisson’s ratio of epoxy resin matrix</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{F}$</td>
<td>1200</td>
<td>kg/m$^3$</td>
<td>Mass density of E-Glass fibers</td>
</tr>
<tr>
<td>$E_{F}$</td>
<td>1169</td>
<td>GPa</td>
<td>Young’s modulus of E-Glass fibers</td>
</tr>
<tr>
<td>$v_{F}$</td>
<td>0.2</td>
<td></td>
<td>Poisson’s ratio of E-Glass fibers</td>
</tr>
<tr>
<td>$V_F$</td>
<td>0.3</td>
<td></td>
<td>Volume fraction of E-Glass fibers</td>
</tr>
</tbody>
</table>

4.1 Weight percentage Effects of CNTs

Whether composite boring bar containing CNTs or not is investigated for different stacking sequences of composite boring bar. The results obtained are listed as shown in Table 6.

Table 6
Effect of different stacking sequences and SWCNTS weight percentages $w_{CN}$ on the natural frequency of
boring bar.

<table>
<thead>
<tr>
<th>$w_{CN}$</th>
<th>Stacking sequence</th>
<th>[45^\circ/45^\circ/45^\circ/45^\circ]</th>
<th>[0^\circ/90^\circ/90^\circ/0^\circ]</th>
<th>[30^\circ/60^\circ/60^\circ/30^\circ]</th>
<th>[60^\circ/60^\circ/60^\circ/60^\circ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>0.1048</td>
<td>0.2087</td>
<td>0.1567</td>
<td>0.0534</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>0.1206</td>
<td>0.2404</td>
<td>0.1805</td>
<td>0.0612</td>
<td></td>
</tr>
<tr>
<td>X%</td>
<td>15.1%</td>
<td>15.2%</td>
<td>15.2%</td>
<td>14.6%</td>
<td></td>
</tr>
</tbody>
</table>

[X%]: $X\% = 100\% \frac{\omega_{L}^{15\%} - \omega_{L}^{0\%}}{\omega_{L}^{0\%}}$

It is found that the addition of small amount of SWCNTS (5%) to the composite boring bar can significantly increase the natural frequency for stacking sequences $[0^\circ/90^\circ/90^\circ/0^\circ]$ and $[30^\circ/60^\circ/60^\circ/30^\circ]$.

Table 7

<table>
<thead>
<tr>
<th>$\nu^F$</th>
<th>SWCNTs</th>
<th>MWCNTs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{CN}$</td>
<td>0%</td>
<td>2%</td>
</tr>
<tr>
<td>0.3</td>
<td>0.2087</td>
<td>0.2219</td>
</tr>
<tr>
<td>0.4</td>
<td>0.2374</td>
<td>0.2474</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2629</td>
<td>0.2705</td>
</tr>
</tbody>
</table>

The effect of different CNTs and their weight percentages with fiber volume fraction on the natural frequency of the composite boring bar are studied as shown in 7. It is observed that the fundamental natural frequency of the boring bar increases with increasing weight percentages of CNTs and is highly for the SWCNTs compared to the MWCNTs. In other words, the SWCNTs exhibit better resistance to bending vibration compared to MWCNTs. Also, the increase in fiber volume fraction leads to an increase in the natural frequency of the boring bar. However, it is expected that the effect of CNTs enhancement decreases at higher fiber volume fractions.

4.2 Functional parameters effects
Fig. 5 The effect of (a) internal damping coefficient, (b) stacking sequences, (c) nanotube aspect ratio, (d) weight percentage, (e) fiber volume fraction, and (f) amplitude of control force on the frequency response of boring bar.

The effect of different parameters on the frequency response are investigated as shown in Fig. 5. With increasing in the internal damping coefficient, the amplitude of the vibration decreases as shown Fig. 5(a). This is to be expected that the increase of damping can dissipate vibration energy of the cutting system. It is observed in Fig. 5(b) that the primary resonance amplitude increases with the increase of ply angles for different stacking sequences. These results due to the longitudinal elastic modulus $E_{11}$ is significantly larger than the transverse elastic modulus $E_{22}$ of composite bar, along with the equivalent bending stiffness ($EI$) of the boring bar decreases with the increase of ply angle.

The effects of aspect ratio and weight percentages of SWCNTs and fiber volume fraction on the frequency response curves are investigated respectively. The results shown that the stiffness of the cutting system can be improved by increasing the aspect ratio, weight percentages and the fiber volume fraction, while the amplitude of the frequency response will be reduced as shown in Fig. 5(c)-(e).

The effect of the amplitude of the periodic force on the frequency response is also shown in Fig. 5(f). It can be found that the curves represent more stronger nonlinearity with increasing of the periodic force. And the boring bar is more likely to be destabilized, this phenomenon is consistent with what is exhibited on forced vibration during machining. It is also pointed out that the unstable curves grows with the increasing of vibration response.

In order to further investigate vibration response with respective to the periodic force, the effects of detuning parameters, stacking sequences, aspect ratio, weight percentage of SWCNTs and fiber volume fraction are explored respectively as shown in Fig. 5.
It is noticed that the multi-valued and single-valued curves with respective to different detuned parameters $\sigma$ selected in Fig. 6(a). For multi-valued curves, it exists unstable region responding to the right part of the more stronger nonlinearity. In contrast, for single-valued curves, it has unstable region responding to the left part of the more stronger nonlinearity. It is also found that the vibration response $a$ increases slowly from zero with increasing of the periodic force $\bar{F}$, and subsequently $a$ will jump upwards spontaneously, after $a$ increases slightly. From physical point of view, the jumping phenomena corresponding to cutting unstable should be avoided. In order to investigate the nature of the unstable phenomena, the values of the tuning parameters $\sigma$ are taken as 750 in the rest of the graph in Fig. 6.

The unstable amplitude with respective to the stacking sequences $[30^\circ/60^\circ/60^\circ/30^\circ]$ jump more largely than those of the stacking sequences $[0^\circ/90^\circ/90^\circ/0^\circ]$ in Fig. 6(b). The results attribute to $0^\circ$ orientation ply angles at most outer layer of composite boring bar has more bending stiffness than those of $30^\circ$ orientation ply angles. This conclusion is consistent with results obtained in Table 6.

It can be clearly observed that the amplitude jumping region increases very slowly with the increase of the
aspect ratio and weight percentage of SWCNTs, which further indicates the SWCNTs as reinforcement of the matrix material has a greatly significant effect on degrading vibration of machining system. And also, the same phenomenon exists for the effect of fiber volume fraction shown in Fig. 6(e).

4.3 Damping parameters effect

According to the processing damping coefficient in the part 2, it notes that the effects of tool flank angle \( \alpha \), the cutting-edge radius \( R_e \), cutting speed \( V_c \) and chattering amplitude of surface machined \( A_0 \) are explored using the detuned parameter \( \sigma \) for dynamic response of cutting system.

![Fig. 7 The effect of (a) tool flank angle, (b) cutting-edge radius, (c) cutting speed, and (d) chattering amplitude on the frequency response of CNTFPC boring bar.](image)

With increasing of the flank angle in Fig. 7(a), the contact area between the flank face of the cutter and the workpiece machined decreases, and resulting in a decrease in friction force both surfaces above, and their dissipative energy would be reduced, therefore, the vibration amplitude would also not be attenuated.

It is observed that an increase workpiece in the cutter-edge radius leads to a decrease in vibration amplitude in Fig. 7(b). The results attribute to the material penetrated by the cutter-edge is relatively large as the cutting-edge radius increases, and resulting in the higher process damping force.

It can also be indicated from Fig. 7(c) and Fig. 7(d) that both the larger cutting speed and the lower chattering amplitude are lead to the stability reduction of cutting system. The obtained results are in good agreement with those obtained using both theories and experiment from Altintas[30].

5 Discussion and analysis

5.1 Unstable boring region
The numerical solution of the vibration response for the composite boring bar responding to cutting system is shown in Fig. 8. The one initiate at left of vibration response, by increasing detuning parameters at point A, the amplitude $a$ will gradually increase until it reaches the peak point B, a jump downward from point B to point C. After that, $a$ slowly decreases to point D. The other the detuning parameter keeps increasing on the contrary, it decreases from point D to point E, during the amplitude $a$ increases and starts to jump upward from point E to point F. Then with the detuning parameter is further reduced and the amplitude $a$ decrease until it reaches point A. However, this jump phenomenon should be avoided in cutting process due to the jump region containing both stable parameters $p_1$ and $p_3$ and unstable parameter $p_2$, which means that the boring bar may be working in unstable cutting condition at identical detuning value $\sigma$. It can be seen that the range of stable cutting frequency becomes narrower as considering the nonlinear factor, which due to the cutting frequency needs to avoid not only the large amplitude but also the unstable cutting region. Therefore, in order to investigate the cutting frequency in unstable region, the detuning parameter is taken as 750.

5.2 Internal damping coefficients

In order to enlarge stability region or reduced the unstable region in the actual cutting process, it can be seen that the suitable parameters should be adjusted and selected, such as the larger internal damping coefficient and the aspect ratio, weight percentages of CNTs.
The ply angle is also an important influencing factor on stability with respective to rotational speed internal damping coefficient.

It can be observed that the amplitude changes from a multi-value curve to a single-value curve with the increase of the ply angle, and the primary resonance response changes from unstable case to stable. The decrease of speed and increase of internal damping within certain range about from 0° to 50° ply angles will make the unstable case end in advance, and the unstable cutting area would be become narrow. In other words, the larger the ply angle, the higher the response amplitude is. The stability of the system may not be improved due to the equivalent bending stiffness of the boring bar decreases with the increase of ply angle.

We also discusses the behavior of the geometric effects of CNTs on internal damping coefficient as shown in Fig. 11(a)-(d). It can be found that the amplitude of vibration decreased as the the geometric of CNTs increased from 1000 to 20000 and 0.5% to 5% corresponding to the aspect ratios and weight percentages respectively. The former with respect to internal damping coefficients was little lower than that of the latter in identical conditions.
5.3 Structural effects

The geometric effects of boring bar and edge of cutter such as the cross section, the flank angle, the edge radius is investigated on the steady-state amplitude with different values of parameters.

![Fig. 12 The effect of cross-section on boring bar](image)

(a) The effect of cross-section on boring bar (a) detuning parameter (b) forced load (c) rotational speed

The effects of variable and equal cross sections on the vibration response with respect to detuning parameter, forced load and rotational speed of boring bar are investigated as shown in Fig. 12. The response amplitude of the variable section boring bar is significantly reduced compared with those of the equal, while the nonlinearity is also degraded, and the unstable region of the system becomes more narrower. It can also be observed that the rotational speed with respect to the equal cross section bar appears to have an instability point at high cutting speed, which means that the variable bar can withstand more speed than those of the equal cross section bar.

We now discusses another behavior of the composite boring bar considering geometric case such as the flank angle, the edge radius as shown in Fig. 13.
The stacking sequences is taken to \([0^\circ/90^\circ/90^\circ/0^\circ]\) in the set of graphical results. It can be seen that except for the unstable curves in middle case, all the other stable curves are not identical with section 4.3. The results attributes to the frequency-response plots change lower in amplitude and narrower in detuning range. In other words, there will be a spontaneous downward jump for stable parameters from \(p_1\) and \(p_3\) to \(p'_1\) and \(p'_3\) respectively and downward jump for unstable parameter from \(p_2\) to \(p'_2\) as shown in Fig.14. That shows the unstable region of boring processing shrunk with either higher weight percentage or higher aspect ratio of CNTs. As it could be predicted, the reason is that the process damping force increasing with either the flank angle or the edge radius increasing respectively.

6. Conclusion

In this work, the analysis of the nonlinear dynamical behavior of the cutting system for boring bar containing CNTs considering forced vibration and regenerate chatter is presented. Several examples have been processed to validate this theory and to determine the influence of the weight percentage, aspect ratio of CNTs and volume fraction, stacking sequences of fibers and processing parameters on rotating composite boring bar dynamical characteristics. The following specific conclusions are obtained.

(1) Decreasing the amplitude of the forced load, increasing the internal damping coefficient, and the nanotube aspect ratio, SWCNTs weight percentage and fiber volume fraction can reduce the amplitude of the primary resonance response and improve the system stability. With the addition of carbon nanotubes, the natural frequency of the boring bar increases significantly. In addition, SWCNTs have more significant effect on the natural frequency of the system compared to the MWCNTs.

(2) The process damping force also has significant effect on the primary resonance amplitude. Increasing the
cutting-edge radius and the chattering amplitude and decreasing the cutting speed and tool flank angle will lead to increase in the resulting process damping force and improve the stability of the system.

(3) The stability of variable section boring bar is significantly higher than that of equal section. This is because the variable cross section increases the natural frequency of the bar, and improving the stability of the system.

Acknowledgments This work was financially supported by the National Natural Science Foundation of China (Grant no. 11672166) and the Natural Science Foundation of Shandong Province (Grant no. ZR202103070107)

Data availability The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Declarations
Conflict of interest The authors declare that they have no conflict of interest.

References
Appendix A

The transformed material constants of the kth layer of the CNTFPC boring bar is defined as

\[
\widetilde{Q}_{ij} = Q_{11} \cos^4 \theta^{(k)} + 2(Q_{12} + 2Q_{66}) \cos^2 \theta^{(k)} \sin^2 \theta^{(k)} + Q_{22} \sin^4 \theta^{(k)}
\]  

(A-1)

where \(\theta^{(k)}\) is the layered angle of each layer of material.

The expression for \(Q_{11}, Q_{12}, Q_{22}, Q_{66}\) is as follows:

\[
Q_{11} = \frac{E_{11}}{1-V_{12}V_{21}}, \quad Q_{22} = \frac{E_{22}}{1-V_{12}V_{21}}, \quad Q_{12} = \frac{E_{12}}{1-V_{12}V_{21}}, \quad Q_{66} = G_{12}
\]  

(A-2)

Where, \(V_{21} = \frac{V_{12}E_{22}}{E_{11}}\)