Supplementary materials for "Skyrmion crystal formation and magnetic field"

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(Dated: February 10, 2021)

SKYRMION SPIN-PROFILE

Encouraged by the well defined stripe width observed in Fig. 1 and the good profile of isolated circular skyrmion found in Ref. [1], we try to fit that grey and white strips in the presence of a magnetic field by $\Theta(x) = 2\arctan\left[\frac{\sinh(L_1/2w_1)}{\sinh(|x|/w_1)}\right]$ $(m_z < 0)$ and $\Theta(x) = 2\arctan\left[\frac{\sinh(|x|/w_2)}{\sinh(L_2/2w_2)}\right]$ $(m_z > 0)$, respectively, with $|x| \leq L_i/2$. Θ is the polar angle of the magnetization at position x and $x = 0$ is the centre of a stripe where $m_z = -1, 1$ respectively. L_i and w_i $(i = 1, 2)$ are respectively the stripe width and skyrmion wall thickness of white $(i = 1)$ and grey $(i = 2)$ stripes. Fig. S1(a) demonstrates the excellence of this approximate spin profile for those stripe skyrmions labelled by the green ⓝ in Fig. 1(a3-c3) of the main text (with model parameters of $A = 0.4$pJ/m, $D = -0.33$mJ/m^2, $K = 0.02$MJ/m^3, and $M_s = 0.15$MA/m). The $y-$axis is m_z and $x = 0$ is the stripe centre where $m_z = -1$. Symbols are numerical data and the solid curve is the fit of $\cos\Theta(x) = -\left(\frac{\sinh^2(L_1/2w_1)-\sinh^2(x/w_1)}{\sinh^2(L_1/2w_1)+\sinh^2(x/w_1)}\right)$ for $-L_1/2 < x < L_1/2$ and $\cos\Theta(x) = \frac{\sinh^2(L_2/2w_2)-\sinh^2[(x-L_1/2-L_2/2)/w_2]}{\sinh^2(L_2/2w_2)+\sinh^2[(x-L_1/2-L_2/2)/w_2]}$ for $L_1/2 < x < L_1/2 + L_2$ with $L_1 = L_2 = 8.1nm$ and $w_1 = w_2 = 2.7nm$. All data from different stripes fall onto the same curve demonstrate that stripes, building blocks of pattern, are identical. Fig. S1(b) demonstrates stripes falling onto the profile with $L_1 = 6.05$nm, $L_2 = 12.10$nm, and $w_1 = w_2 = 2.63$nm under a 0.4T magnetic field. Meanwhile, Fig. S1(c) show that spins along a line connecting two nearby skyrmion center fall also on our profile with $L_1 = 8.70$nm, $L_2 = 7.18$nm, and $w_1 = w_2 = 2.32$nm.

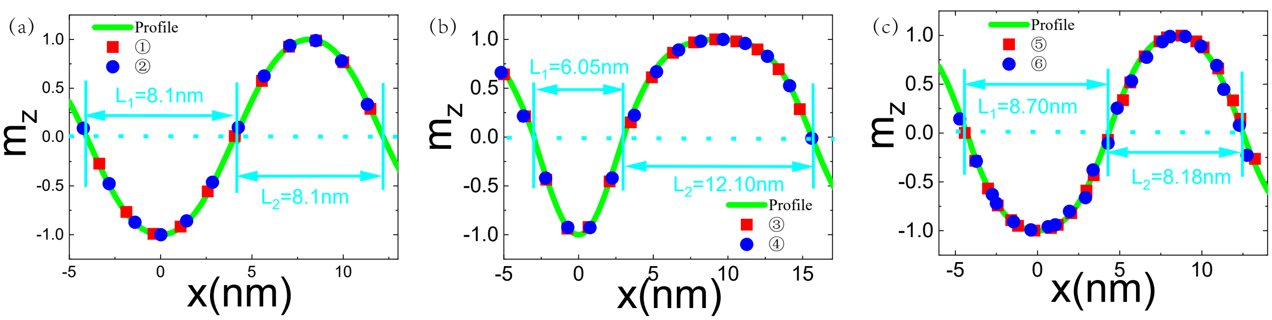

FIG. S1. The red points and blue points in (a), (b) and (c) are z component of the spins on the red line of Fig. 1(a1), (c1) and (c3). The green line are the fit to our approximate profile. We use the cyan text to show the width of different part of stripe width and skyrmion diameter.

In our previous research [2], we use the profile to find parameter dependence of stripe width. The width is proportion to the product of A/D and a pre-factor of $f(\kappa)$ with $\kappa = \frac{\pi^2 D^2}{16AK}$. Fig. S2 shows the curve of $f(\kappa)$. The value of $f(\kappa)$ is very close to 6.23 when $\kappa \gg 1$. In our simulations, $\kappa = 8.396$.

MAXIMUM SKYRMION DENSITY

To substantiate our claim that there is a maximal skyrmion number density for a given film, we consider the films of the same size and same material parameters as those in the main text. To find the approximate maximal number of skyrmions that can maintain its metastability, we simply add more 2nm nucleation domains in the initial configuration of a regular lattice. The system always settles to a metastable state in which the number of skyrmions is the same as the number of initial nucleation domains as long as the the number is less than 500. If the number is bigger than 700, the final number of skyrmions in stable states would be less than the initial number of nucleation domains. Occasionally, we obtain metastable states containing about 700 skyrmions in a triangular lattice, corresponding to skyrmion density of 17,500/μm^2. Fig. S3 shows an SkX of 567 skyrmions when the initial configuration is 625

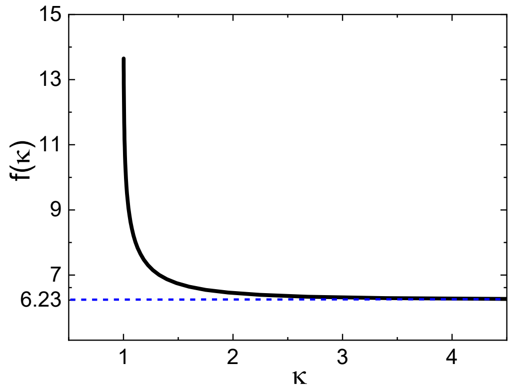

FIG. S2. The function $f(\kappa)$ is approximate 6.23 for $\kappa \gg 1$.

nucleation domains of 2nm in radius arranged in square lattice. The detail of how the final metastable states depends on the initial number of nucleation domains and their arrangement, as well as the boundary conditions, deserves a more careful study.

TRUE TEMPERATURE IN MUMAX3

According to Ref. [3], the temperature used in OOMMF simulator may not be the true temperature when cell size is not the true lattice constant. One can still use cells with different size in a simulation if the assumed temperature is rescaled by a/a_{cell}, where a is the lattice constant of the material and a_{cell} is the cell size used in simulations. The similar problem exists also in MuMax3 [4]. To reveal the problem, we consider a ferromagnetic model with the same model parameter as those used in the main text except $D = 0$, i.e. $A = 0.4$pJ/m, $K = 0.02$MJ/m^3 and $M_s = 0.15$MA/m. Assume 0.38nm be the lattice constant of a material described by this group of model parameters, we first solve stochastic LLG equation for a sample of 30×30×10 cells by MuMax3 with cell size of 0.38nm. Initially, all spins align along the z-direction. Fig. S4 shows the time-dependence of magnetization at different temperature T. M is normalized by M_s. It shows that M reaches its thermal equilibrium value within 0.5ns. Fig. S4(b) plots the $M - T$ curve (the black curve) where M is averaged over 0.25ns in the thermal equilibrium state. It shows a characteristic behaviour of classical magnetic phase transition at the Curie temperature of $T_c = 33$K. When we use cell size of 1nm and 2nm to simulate the stochastic LLG equation, the time-averaged M undergoes a similar transition at different assumed temperatures of 93K and 175K. However, if we rescale the assumed temperature by 0.38/1 and 0.38/2, respectively, two simulated curves (the red curve for 1nm cells and the blue for 2nm cells) are perfectly overlapped with the results of 0.38nm cells as shown in Fig. S4(b). Our simulated $M - T$ curves follow $M \sim (1 - T/T_c)^\beta$ near the Curie temperature and $1 - M/M_s \sim T^b$ near $T = 0$K. β is very close to the critical exponent of 0.693 of 3D Heisenberg model [5] as shown in Fig. S4(c) of $\ln(M/M_s)$-$\ln|1 - T/T_c|$ plot although the small system is used and a small anisotropy is included. b is very close to 1.5 from the theory of demagnetization of magnon excitation as shown in Fig. S4(e) of $\ln|1 - M/M_s|$-$\ln T/T_c$ plot although a small band gap exists in the spin wave. We have also studied how $M - T$ curve changes with the sample layer number from 1 to 12. Fig. S4(e) shows that M-T curves do not change above 8 layers.

SUPPLEMENTARY VIDEOS

We have recorded the evolutions of those process described in Fig. 3 of the main text. In all videos below, magnetization distribution is the average of 8 layers. The color [color bar shown in Fig. 3(a)] encodes the azimuthal angle of magnetization in skyrmions and the black denotes the background.

Movie-a shows stability of metastable helical state at 20K and at the optimal field of 0.3T. The initial state has 10 stripe skyrmions. The skyrmion number has only increased by 1 to 11 skyrmions after 30ns although the thermal equilibrium state is an SkX of about 140 skyrmions.

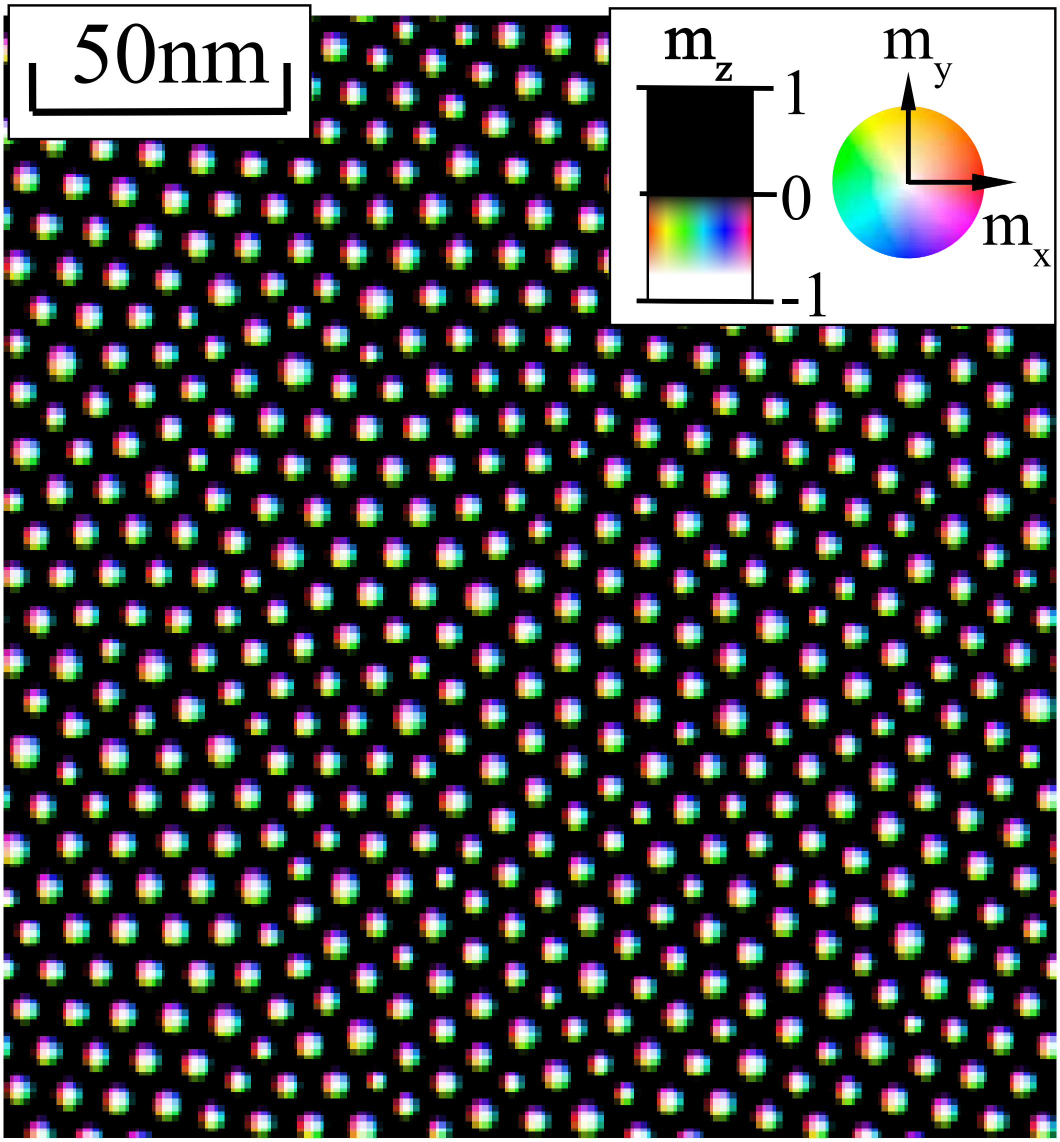

FIG. S3. An SkX of 567 skyrmions when 625 nucleation domains of 2nm in radius are initially arranged in square lattice.

Movie-b shows how a helical state reaches the thermal equilibrium of an SkX at 29K and at the optimal field of 0.3T. There are 10 stripe skyrmions in the initial state. It transforms into the thermal equilibrium state of an SkX of 132 skyrmions after 30ns.

Movie-c shows how an SkX with too many (150) skyrmions, metastable and out of equilibrium, reaches the thermal equilibrium state of a different SkX of 136 skyrmions at 29K and at the optimal field of 0.3T.

Movie-d shows how an SkX of 150 skyrmions reaches the thermal equilibrium state of a helical state during the zero-filed cooling process from 30K to 0K. The SkX becomes helical states within 1ns at 30K. The system has 7 skyrmions with mixture of circular skyrmions, stripe skyrmions and ramified stripe skyrmions.

Movie-e shows how an SkX of 150 skyrmions reaches the thermal equilibrium of a helical state during the zero-filed warming process from 0K to 30K. The SkX does not change much at relative low temperature. It starts to transform into helical state above 22K. The final state after 30ns has 4 skyrmions of mixture of one circular skyrmion and 3 ramified stripe skyrmions.

Movie-f shows how an initial ferromagnetic states, far from its equilibrium phase at 29K and at the optimal field of

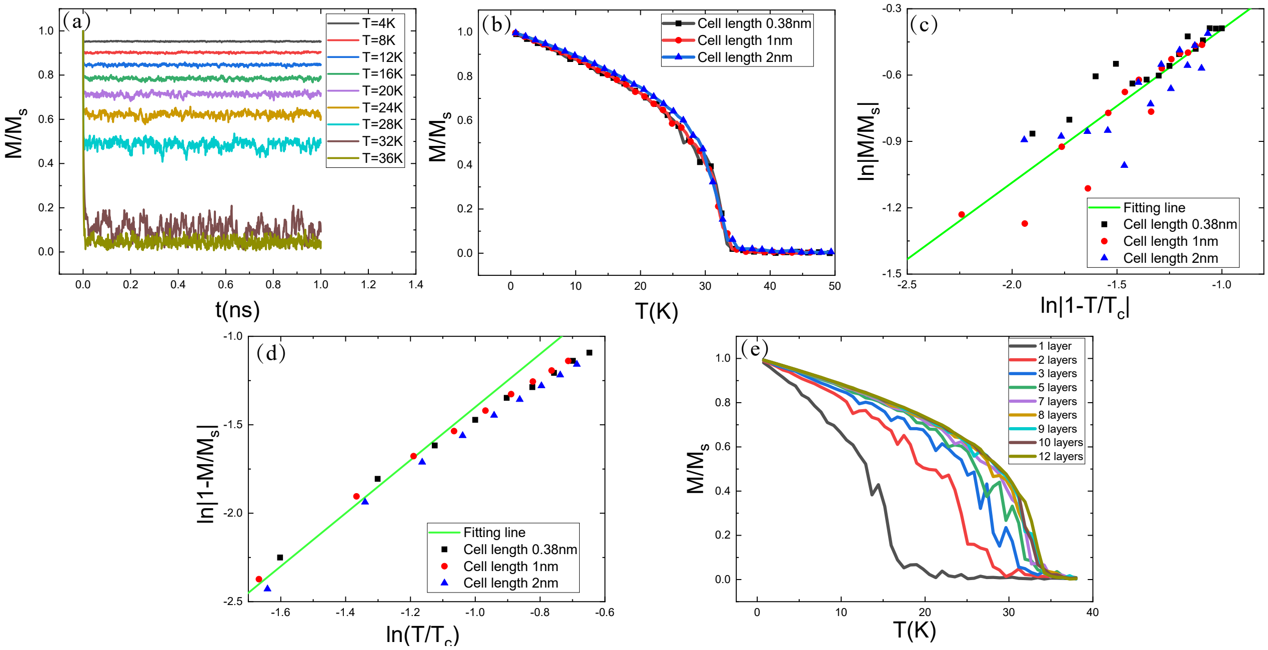

FIG. S4. (a) The magnetization M/M_s of a sample of $30 \times 30 \times 10$ cells with cell length 0.38nm at 4K(the black line), 8K(the red line), 12K(the blue line), 16K(the green line), 20K(the purple line), 24K(the yellow line), 28K(the cyan line), 32K(the brown line) and 36K(the atrovirens line) temperature. The system reaches it thermal equilibrium magnetization within 0.5ns. (b) The temperature dependence of M/M_s of a sample when the cell size in MuMax3 is 0.38nm (the black line), 1nm (the red line) and 2nm (the blue line). The temperature for cell size of 1nm and 2nm are rescaled by 0.38/1 and 0.38/2, repsectively, while the temperature for 0.38nm cell size is the same as the assumed temperature in MuMax3. (c)$\ln|M/M_s|$ vs. $\ln(1-T/T_c)$. The black, red, and blue dots are from cell size of 0.38nm, 1nm, and 2nm, respectively. The green line is function $M/M_s = 1.45(1-T/T_c)^{0.6930}$. (d)$\ln|1-M/M_s|$ vs. $\ln(T/T_c)$. The black, red, and blue dots are from cell size of 0.38nm, 1nm, and 2nm, respectively. The green line is function $|1-M/M_s| = 1.02(T/T_c)^{3/2}$. (e) M/M_s vs. T for film layer number: 1 layer (the black line), 2 layers (the red line), 3 layers (the blue line), 5 layers (the green line), 7 layers (the purple line), 8 layers (the yellow line), 9 layers (the cyan line), 10 layers (the brown line) and 12 layers (the atrovirens line). The Curie temperature of samples of 8, 10, 12 layers are almost the same.

0.3T, transforms into the thermal equilibrium state of an SkX of 133 skyrmions. To visualise how skyrmions develop from the thermally generated nucleation centres one by one. The video contains two sections. The first 0.5ns is a faster process and the first group of skyrmions are generated within order of 10ps. The second section of about 9.5ns is a slower process in which skyrmions are developed with in nanosecond time scale. The final state after 10ns is a SkX of 133 skyrmions.

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