A 2-order Additive Fuzzy Measure Identification Method Based on Hesitant Fuzzy Linguistic Interaction Degree and Its Application in Credit Assessment

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A 2-order Additive Fuzzy Measure Identification Method Based on Hesitant Fuzzy Linguistic Interaction Degree and Its Application in Credit Assessment

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Abstract. In order to solve the problem that it is difficult to quantitatively evaluate the interactivity between attributes in the identification process of 2-order additive fuzzy measure, this work uses the hesitant fuzzy linguistic term set (HFLTS) to describe and deal with the interactivity between attributes. Firstly, the interactivity between attributes is defined by the supermodular game theory, and a linguistic term set is then established to characterize the interactivity between attributes. Secondly, under the linguistic term set, according to the above definition, the experts employ the linguistic expressions generated by the context-free grammar to evaluate the interactivity between attributes, and the opinions of all experts are then aggregated by using the defined hesitant fuzzy linguistic weighted power average operator (HFLWPA). Thirdly, based on the standard Euclidean distance formula of the hesitant fuzzy linguistic elements (HFLEs), the hesitant fuzzy linguistic interaction degree (HFLID) between attributes is defined and calculated by constructing a piecewise function. Finally, a 2-order additive fuzzy measure identification method based on HFLID is further proposed. Based on the proposed method, using the Choquet fuzzy integral as nonlinear integration operator, a multi-attribute decision making (MADM) process is presented. Taking the credit assessment of the big data listed companies in China as an application example, the feasibility and effectiveness of the proposed method is verified by the analysis results of application example.

Keywords: interactivity between attributes; hesitant fuzzy linguistic term set; 2-order additive fuzzy measure; Choquet fuzzy integral; multi-attribute decision making; credit assessment.

Declarations

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1 Introduction

In the process of multi-attribute decision making (MADM), due to the influence of the interaction among attributes, such as complementarity and repeatability, the classical weighted arithmetic mean method is often invalid [1].

Aiming at this problem, in order to flexibly describe and deal with any interaction among attributes, Sugeno [2] proposed the concept of fuzzy measure and fuzzy integral. Since then, the research on fuzzy measures had become increasingly rich, and gradually formed the theory of fuzzy measures [3, 4]. However, in the practical application process, when there are n attributes, the general fuzzy measure usually needs to determine 2^n-2 parameters [4, 5]. This complexity greatly limits its practical application ability. Thereafter, in order to face with the complexity of discrete fuzzy measures, Grabisch [6] proposed the concept of k-order additive fuzzy measure, including usual additive measures and fuzzy measures. Every discrete fuzzy measure is a k-order additive fuzzy measure for a unique k. The k-order additive fuzzy measures cover all fuzzy measures with any complexity from classical additive measure (k=1) to general fuzzy measure (k=n). Among them, the 2-order additive fuzzy measure not only merely needs to determine $n(n+1)/2$ parameters, but also merely involves the relative importance of attributes and the interaction between attributes, which well solves the contradiction between complexity and performance ability, so it has been widely used [7].

However, due to the difficulty to explain and understand the interaction between attributes [8] and the uncertainty of decision-makers' cognition, the decision-makers often can't give accurate quantitative evaluation on the interactivity between attributes, which is often fuzzy and hesitant. The existing 2-order additive fuzzy measure identification methods mainly use the subjective methods [9-15] and the objective methods [7, 16-17] to describe and deal with the interactivity between attributes. Compared with the objective methods, the subjective methods are more explanatory, so they have been applied more widely. However, the existing subjective methods still cannot reflect the fuzziness and hesitation of decision-makers in evaluation on the interactivity between attributes.

In [18], Rodríguez et al. introduced the concept of a hesitant fuzzy linguistic term set (HFLTS) to provide a linguistic and computational basis to increase the flexibility and richness of linguistic elicitation based on the fuzzy linguistic approach and the use of context-free grammar to support the elicitation of linguistic information by experts in hesitant situations under qualitative settings. Hesitant fuzzy linguistic information is based on the linguistic expressions given by people, so it is closer to people's thinking and cognition, and can flexibly and comprehensively reflect the real preferences of decision-makers [19]. HFLTS provides a new and powerful tool to characterize experts' qualitative decision information, so it has been successfully applied in the field of uncertain MADM [20-23].

Based on this observation, in view of the problem that it is difficult to quantitatively evaluate the interactivity between attributes, the present work uses the HFLTS to describe and deal with the interactivity between attributes. As a result, a 2-order additive fuzzy measure identification method based on hesitant fuzzy linguistic interaction degree (HFLID) is proposed. Based on the proposed method, using the Choquet fuzzy integral as nonlinear integration operator [24, 25], a MADM process is further presented.

This paper is structured as follows: Section 2 introduces the preparatory knowledge employed in this work. Section 3 proposes the 2-order additive fuzzy measure identification method based on HFLID. Section 4 presents the MADM process based on the proposed method. Section 5 describes the application example analysis results. Section 6 discusses the results obtained and Section 7 concludes this paper.

2 Preparatory knowledge

This section introduces the related definitions of HFLTS, 2-order additive fuzzy measure and Choquet fuzzy integral reported in the literatures. This is the basis of Section 3 and Section 4.

2.1 *Related definitions of HFLTS*

Definition 1 [26]: Let linguistic variable $x_i \in X$ ($i = 1,2,\cdots,N$) be fixed and $S = \{s_\beta | \beta = -\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a linguistic term set. Let H_S be a hesitant fuzzy linguistic term set (HFLTS) on *X*, its mathematical expression is

$$H_S = \{< x_i, h_S(x_i) > | x_i \in X\}$$

where $h_S(x_i)$ is a set of some values in the linguistic term set *S* and can be expressed as $h_S(x_i) = \{s_{\varphi_l}(x_i) | s_{\varphi_l}(x_i) \in S, l = 1,2,\cdots,L\}$ with *L* being the number of linguistic terms in $h_S(x_i)$. $h_S(x_i)$ denotes the possible degrees of linguistic variable x_i to the linguistic term set *S*. For convenience, $h_S(x_i)$ is called the hesitant fuzzy linguistic element (HFLE) and H_S is the set of all HFLEs.

Definition 2 [18, 19]: Let G_H be a context-free grammar, and $S = \{s_\beta | \beta = -\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a linguistic term set. The elements of $G_H = (V_N, V_T, I, P)$ are defined as follows:

V_N = {primary term, composite term, unary relation, binary relation, conjunction}; V_T = {"less than", "more than", "at least", "at most", "between", "and", "$s_{-\tau}$", $\cdots$, "s_{-1}", "s_0", "s_1", $\cdots$, "s_τ"}; $I \in V_N$; P = { I refers to the primary term or composite term; the primary term refers to "$s_{-\tau}$", $\cdots$, "s_{-1}", "s_0", "s_1", $\cdots$, "s_τ"; the composite term refers to unary relation + primary term, or binary relation + primary term + conjunction + primary term; the unary relation refers to "less than" or "more than" or "at least" or "at most"; the binary relation refers to "between"; the conjunction refers to "and"}.

Definition 3 [18, 19]: Let E_{G_H} be a function that transforms linguistic expressions ll, which are obtained by G_H, into HFLTS H_S, where $S = \{s_\beta | \beta = -\tau,\cdots,-1,0,1,\cdots,\tau\}$ is the linguistic term set that is used by G_H:

$$E_{G_H} : ll \to H_S .$$

The linguistic expressions ll that are generated by G_H will be transformed into HFLTS H_S in different ways according to their meaning:

1) $E_{G_H}(s_t) = \{s_t | s_t \in S\}$;

2) $E_{G_H}(\text{at most } s_m) = \{s_t | s_t \in S, \text{ and } s_t \le s_m\}$;

3) $E_{G_H}(\text{less than } s_m) = \{s_t | s_t \in S, \text{ and } s_t < s_m\}$;

4) $E_{G_H}(\text{at least } s_m) = \{s_t | s_t \in S, \text{ and } s_t \ge s_m\}$;

5) $E_{G_H}(\text{more than } s_m) = \{s_t | s_t \in S, \text{ and } s_t > s_m\}$;

6) $E_{G_H}(\text{between } s_m \text{ and } s_n) = \{s_t | s_t \in S, \text{ and } s_m \le s_t \le s_n\}$.

Definition 4 [27]: Let $S=\{s_\beta \mid \beta=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a set of linguistic terms, $h_S=\{s_{\eta_l} \mid s_{\eta_l}\in S, l=1,2,\cdots,L\}$ is a HFLE defined on *S*, then the mean value of h_S is defined as

$$\mu(h_S)=\frac{1}{L}\sum_{l=1}^{L}\eta_l$$

and the variance of h_S is defined as

$$\nu(h_S)=\frac{1}{L}\sum_{l=1}^{L}(\eta_l-\mu(h_S))^2$$

Then, the binary relationship between $h_S^i=\{s_{\eta_l^i} \mid s_{\eta_l^i}\in S, l=1,2,\cdots,L\}$ and $h_S^j=\{s_{\eta_l^j} \mid s_{\eta_l^j}\in S, l=1,2,\cdots,L\}$ is defined as follows:

1) If $\mu(h_S^i)>\mu(h_S^j)$, then $h_S^i>h_S^j$;

2) If $\mu(h_S^i)=\mu(h_S^j)$, when $\nu(h_S^i)=\nu(h_S^j)$, then $h_S^i=h_S^j$; when $\nu(h_S^i)>\nu(h_S^j)$, then $h_S^i<h_S^j$; when $\nu(h_S^i)<\nu(h_S^j)$, then $h_S^i>h_S^j$.

For convenience of calculation, referring to [28], the h_S with fewer elements can be expanded by adding element $s_{\eta_l}=\zeta s_{\eta_l}^{+}\oplus(1-\zeta)s_{\eta_l}^{-}$ until meeting the need of calculation. Where $s_{\eta_l}^{+}$ and $s_{\eta_l}^{-}$ is the largest and smallest element in h_S respectively, and $\zeta\in[0,1]$ is the adjustment parameter. Without loss of generality, $\zeta=0.5$ is usually taken.

Definition 5 [29]: Let $S=\{s_\beta \mid \beta=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a set of linguistic terms, $h_S^i=\{s_{\eta_l^i} \mid s_{\eta_l^i}\in S, l=1,2,\cdots,L\}$ and $h_S^j=\{s_{\eta_l^j} \mid s_{\eta_l^j}\in S, l=1,2,\cdots,L\}$ are two HFLEs defined on *S*, if

$$d(h_S^i,h_S^j)=\sqrt{\frac{1}{L}\sum_{l=1}^{L}\left(\frac{\eta_l^i-\eta_l^j}{2\tau}\right)^2} \tag{1}$$

then $d(h_S^i,h_S^j)$ is called the standard Euclidean distance between h_S^i and h_S^j.

Definition 6 [29]: Let $S=\{s_\beta \mid \beta=-\tau,\cdots,-1,0,1,\cdots,\tau\}$ be a set of linguistic terms, $h_S^1,h_S^2,\cdots,h_S^n$ are *n* HFLEs defined on *S*. Let $\mathrm{HFLPA}:\Theta^n\to\Theta$, if

$$\mathrm{HFLPA}(h_S^1,h_S^2,\cdots,h_S^n)=\oplus_{i=1}^{n}\frac{1+T(h_S^i)}{\sum_{i=1}^{n}(1+T(h_S^i))}h_S^i \tag{2}$$

then HFLPA is called the hesitant fuzzy linguistic power average operator. Where $T(h_S^i)=\sum_{j=1,j\neq i}^{n}\sup(h_S^i,h_S^j)$, the support function $\sup(h_S^i,h_S^j)$ represents the support degree of h_S^i and h_S^j, it satisfies the following three conditions:

1) $\sup(h_S^i,h_S^j)\in[0,1]$;

2) $\sup(h_S^i,h_S^j)=\sup(h_S^j,h_S^i)$;

3) If $d(h_S^i,h_S^j)<d(h_S^s,h_S^t)$, then $\sup(h_S^i,h_S^j)>\sup(h_S^s,h_S^t)$.

2.2 *Related definitions of 2-order additive fuzzy measure and Choquet fuzzy integral*

Definition 7 [2]: Let $X=\{x_1,x_2,\cdots,x_n\}$ be a set of attributes, let $X^*=\{1,2,\cdots,n\}$ be a set of subscripts of attributes.

$P(X)$ is the power set of *X*, if the set function $g: P(X) \to [0,1]$ satisfies the following two conditions:

1) $g(\varnothing) = 0$, $g(X) = 1$;

2) If $K \in P(X)$, $T \in P(X)$, $K \subseteq T$, then $g(K) \le g(T)$; then *g* is called a fuzzy measure on $P(X)$.

Grabisch [6] proposed the *k*-order additive fuzzy measure based on pseudo-Boolean function and Möbius transformation. On this basis, the 2-order additive fuzzy measure is then defined as

$$g(K) = \sum_{i \in K^*} m_i + \sum_{\{i,j\} \subset K^*} m_{ij}, \forall K \subseteq X \tag{3}$$

where m_i is the Möbius transformation coefficient of x_i ($i = 1, 2, \cdots, n$), which is an overall importance; m_{ij} is the Möbius transformation coefficient of $\{x_i, x_j\}$ ($i, j = 1, 2, \cdots, n$; $i \ne j$), which represents the extent of interaction between x_i and x_j.

Definition 8 [13]: Let $X = \{x_1, x_2, \cdots, x_n\}$ be a set of attributes, $W = \{w_1, w_2, \cdots, w_n\}$ is the weight set of *X*, the Möbius transformation coefficients of x_i and $\{x_i, x_j\}$ are respectively

$$\begin{cases} m_i = {w_i}/{P} \\ m_{ij} = {\xi_{ij} w_i w_j}/{P} \end{cases}, \ i, j = 1, 2, \cdots, n; i \ne j \tag{4}$$

where $P = \sum_{i \in X^*} w_i + \sum_{\{i,j\} \subset X^*} \xi_{ij} w_i w_j$ is the sum of the importance of all x_i and $\{x_i, x_j\}$, ξ_{ij} is the interaction degree between x_i and x_j, $\xi_{ij} \in [-1, 1]$.

Definition 9 [30]: Let *f* be a nonnegative function defined on *X*, *F* is a σ-algebra composed of subsets of *X* (when *X* is finite, *F* is the power set $P(X)$ of *X*), *g* is a fuzzy measure defined on *F*, then the Choquet fuzzy integral of function *f* on set *X* for fuzzy measure *g* is defined as

$$(c)\int f dg = \int_0^\infty g(F_\alpha) d\alpha$$

where $F_\alpha = \{x \mid f(x) \ge \alpha, x \in X\}$, $\alpha \in [0, \infty]$; $\int_0^\infty g(F_\alpha) d\alpha$ is the Riemann integral.

When *X* is a finite set, the elements in *X* are rearranged as $\{x_{(1)}, x_{(2)}, \cdots, x_{(n)}\}$, which makes $f(x_{(1)}) \le f(x_{(2)}) \le \cdots \le f(x_{(n)})$. Let $H = (c)\int f dg$, then the Choquet fuzzy integral has the following simplified formula:

$$H = (c)\int f dg = \sum_{i=1}^{n} \left[f(x_{(i)}) - f(x_{(i-1)}) \right] g(X_{(i)}) \tag{5}$$

where $X_{(i)} = \{x_{(i)}, x_{(i+1)}, \cdots, x_{(n)}\}$, $(i) = (1), (2), \cdots, (n)$; $f(x_{(0)}) = 0$.

3 The proposed method

This section uses the HFLTS to describe and deal with the interactivity between attributes, and then proposes a 2-order additive fuzzy measure identification method based on HFLID. In addition, the correctness of the proposed method is proved theoretically.

Let $A = (A_1, A_2, \cdots, A_m)$ be a finite set of alternatives, and $C = (C_1, C_2, \cdots, C_n)$ be a set of attributes to compare the alternatives, let $C^* = \{1, 2, \cdots, n\}$ be a set of subscripts of attributes. The weight vector of attributes is $W_C = (w_1, w_2, \cdots, w_n)$, where $w_i \in [0,1]$, and $\sum_{i=1}^{n} w_i = 1$. Let $D = (D_1, D_2, \cdots, D_t)$ be a set of experts, the weight vector of experts is $W_D=(w_1,w_2,\cdots,w_t)$, where $w_p\in[0,1]$, and $\sum_{p=1}^{t}w_p=1$. Using the HFLTS, the identification process of 2-order additive fuzzy measure is as follows:

Step 1: Establish a linguistic term set S to characterize the interactivity between C_i and C_j ($i\neq j$).

According to the supermodular game theory [31], the interactivity between attributes is defined as follows:

Definition 10: Let any two attributes C_i and C_j ($i\neq j$) in attribute set C have partial order relation, the supremum $C_i\vee C_j$ and the infimum $C_i\wedge C_j$ are in C, then C is called a sub-lattice [31]. Let f be a real-valued function defined on the sub-lattice C, $C\subseteq R^n$. For $\forall C_i,C_j\in C$, when $f(C_i\vee C_j)+f(C_i\wedge C_j)>f(C_i)+f(C_j)$, f is a supermodular function [31], then it is said that there is complementarity between C_i and C_j ($i\neq j$); when $f(C_i\vee C_j)+f(C_i\wedge C_j)<f(C_i)+f(C_j)$, f is a submodular function [31], then it is said that there is repeatability between C_i and C_j ($i\neq j$); particularly, when $f(C_i\vee C_j)+f(C_i\wedge C_j)=f(C_i)+f(C_j)$, then it is said that there is independence between C_i and C_j ($i\neq j$).

According to Definition 10, a linguistic term set $S=\left\{s_\beta\left|\beta=-\tau,\cdots,-1,0,1,\cdots,\tau\right.\right\}$ is established to characterize the interactivity between C_i and C_j ($i\neq j$). Where $s_1,s_2,\cdots,s_\tau$ are the linguistic terms describing complementarity, the larger β is, the stronger the complementarity is; $s_{-\tau},s_{-\tau+1},\cdots,s_{-1}$ are the linguistic terms describing repeatability, the smaller β is, the stronger the repeatability is; s_0 is then a linguistic term describing independence, at this point, $\beta=0$.

Step 2: Calculate the individual evaluation result of the expert D_p ($p=1,2,\cdots,t$) on the interactivity between C_i and C_j ($i\neq j$).

Under the linguistic term set S, according to Definition 10, every expert employs the linguistic expressions ll generated by the context-free grammar G_H (see Definition 2) to evaluate the interactivity between C_i and C_j ($i\neq j$) (C_n^2 pairs in total). Through the transformation function $E_{G_H}:ll\rightarrow H_S^{ij}$ (see Definition 3), the linguistic expressions ll are further transformed into the HLFTS H_S^{ij}.

Let $H_S^{ij(p)}$ be the HFLTS of the expert D_p $(p=1,2,\cdots,t)$, $h_S^{ij(p)}=\left\{s_{\eta_l^{ij(p)}}\left|s_{\eta_l^{ij(p)}}\in S,l=1,2,\cdots,L\right.\right\}$ is the HFLE in $H_S^{ij(p)}$ (C_n^2 pairs in total), thus, the individual evaluation result $h_S^{ij(p)}$ of the expert D_p ($p=1,2,\cdots,t$) on the interactivity between C_i and C_j ($i\neq j$) is then given.

Step 3: Calculate the group evaluation result of t experts on the interactivity between C_i and C_j ($i\neq j$).

Based on Definition 6, considering the weights of experts, the hesitant fuzzy linguistic weighted power average operator (HFLWPA) is defined as follows:

Definition 11: Let $S=\left\{s_\beta\left|\beta=-\tau,\cdots,-1,0,1,\cdots,\tau\right.\right\}$ be a set of linguistic terms, $h_S^{ij(1)},h_S^{ij(2)},\cdots,h_S^{ij(t)}$ are t HFLEs defined on S. The weight vector of t HFLEs is $(w_1,w_2,\cdots,w_t)$, where $w_p\in[0,1]$, and $\sum_{p=1}^{t}w_p=1$. Let $\mathrm{HFLWPA}:\Theta^t\rightarrow\Theta$, if

$$\mathrm{HFLWPA}(h_S^{ij(1)},h_S^{ij(2)},\cdots,h_S^{ij(t)})=\oplus_{p=1}^{t}\frac{w_p(1+T(h_S^{ij(p)}))}{\sum_{p=1}^{t}w_p(1+T(h_S^{ij(p)}))}h_S^{ij(p)} \tag{6}$$

then HFLWPA is called the hesitant fuzzy linguistic weighted power average operator (when $w_p=1/t$, HFLWPA degenerates to HFLPA [29]). Where $T(h_S^{ij(p)})=\sum_{q=1,q\neq p}^{t}\sup(h_S^{ij(p)},h_S^{ij(q)})$, the support function $\sup(h_S^{ij(p)},h_S^{ij(q)})$

represents the support degree of $h_S^{ij(p)}$ and $h_S^{ij(q)}$, it satisfies the following three conditions:

1) $\sup(h_S^{ij(p)}, h_S^{ij(q)}) \in [0,1]$;

2) $\sup(h_S^{ij(p)}, h_S^{ij(q)}) = \sup(h_S^{ij(q)}, h_S^{ij(p)})$;

3) If $d(h_S^{ij(p)}, h_S^{ij(q)}) < d(h_S^{ij(r)}, h_S^{ij(s)})$, then $\sup(h_S^{ij(p)}, h_S^{ij(q)}) > \sup(h_S^{ij(r)}, h_S^{ij(s)})$.

In [32], Yager defined different support functions, and different support functions lead to different degrees of support. In this paper, we take the support function as follows: $\sup(h_S^{ij(p)}, h_S^{ij(q)}) = 1 - d(h_S^{ij(p)}, h_S^{ij(q)})$, where $d(h_S^{ij(p)}, h_S^{ij(q)})$ is the standard Euclidean distance between $h_S^{ij(p)}$ and $h_S^{ij(q)}$.

According to Definition 4, after expanding those $h_S^{ij(p)}$ ($p = 1, 2, \cdots, t$) with fewer elements, using the HFLWPA to aggregate the individual evaluation results of t experts, the group evaluation result of t experts on the interactivity between C_i and C_j ($i \neq j$) is then obtained as $h_S^{ij} = \left\{ s_{\eta_l^{ij}} \middle| s_{\eta_l^{ij}} \in S, l = 1, 2, \cdots, L \right\}$.

Step 4: Determine the HFLID $H(\tilde{\xi}_{ij})$ between C_i and C_j ($i \neq j$).

According to the established linguistic term set S (see step 1), constructing a piecewise function based on Definition 5, the hesitant fuzzy linguistic interaction degree (HFLID) between attributes is defined as follows:

Definition 12: Let $S = \left\{ s_\beta \middle| \beta = -\tau, \cdots, -1, 0, 1, \cdots, \tau \right\}$ be a linguistic term set characterizing the interactivity between C_i and C_j ($i \neq j$), $h_S^{ij} = \left\{ s_{\eta_l^{ij}} \middle| s_{\eta_l^{ij}} \in S, l = 1, 2, \cdots, L \right\}$ is the group evaluation result of t experts on the interactivity between C_i and C_j ($i \neq j$). Let $S^+ = \left\{ s_\beta \middle| \beta = 0, 1, \cdots, \tau \right\}$ be a subset of S ($S^+ \subset S$), when $s_{\eta_l^{ij}} \in S^+$, $h_S^{ij} = h_{S^+}^{ij}$; let $S^- = \left\{ s_\beta \middle| \beta = -\tau, \cdots, -1, 0 \right\}$ be another subset of S ($S^- \subset S$), when $s_{\eta_l^{ij}} \in S^-$, $h_S^{ij} = h_{S^-}^{ij}$. Let $h_S^0 = \left\{ s_0 \middle| s_0 \in S \right\}$, $h_{S^+}^0 = \left\{ s_0 \middle| s_0 \in S^+ \right\}$, $h_{S^-}^0 = \left\{ s_0 \middle| s_0 \in S^- \right\}$, obviously, $h_S^0 = h_{S^+}^0 = h_{S^-}^0$. Based on Definition 5, a piecewise function is constructed as

$$H(\tilde{\xi}_{ij}) = \begin{cases} +d(h_{S^+}^{ij}, h_{S^+}^0) = +\sqrt{\dfrac{1}{L}\displaystyle\sum_{l=1}^{L}\left(\dfrac{\eta_l^{ij}}{\tau}\right)^2} & s_{\eta_l^{ij}} \in S^+, l = 1, 2, \cdots, L \\ -d(h_{S^-}^{ij}, h_{S^-}^0) = -\sqrt{\dfrac{1}{L}\displaystyle\sum_{l=1}^{L}\left(\dfrac{\eta_l^{ij}}{\tau}\right)^2} & s_{\eta_l^{ij}} \in S^-, l = 1, 2, \cdots, L \end{cases} \tag{7}$$

where $H(\tilde{\xi}_{ij})$ is called the hesitant fuzzy linguistic interaction degree (HFLID) between C_i and C_j ($i \neq j$), obviously, $H(\tilde{\xi}_{ij}) \in [-1, 1]$. If there is complementarity between C_i and C_j ($i \neq j$), then $H(\tilde{\xi}_{ij}) > 0$, and the larger $H(\tilde{\xi}_{ij})$ is, the stronger complementarity is. If there is repeatability between C_i and C_j ($i \neq j$), then $H(\tilde{\xi}_{ij}) < 0$, and the smaller $H(\tilde{\xi}_{ij})$ is, the stronger repeatability is. If C_i and C_j ($i \neq j$) are independent of each other, then $H(\tilde{\xi}_{ij}) = 0$.

Therefore, according to Definition 12, the HFLID $H(\tilde{\xi}_{ij})$ between C_i and C_j ($i \neq j$) can be determined.

Step 5: Calculate the Möbius transformation coefficients m_i and m_{ij} of attributes.

According to the attribute weight w_i and the HFLID $H(\tilde{\xi}_{ij})$ between C_i and C_j ($i \neq j$), using Eq. (4), the Möbius transformation coefficients m_i and m_{ij} of attributes can be calculated as

$$\begin{cases} m_i = {w_i}/{P} \\ m_{ij} = {H(\tilde{\xi}_{ij}) w_i w_j}/{P} \end{cases}, \quad i, j = 1, 2, \cdots, n; i \neq j \tag{8}$$

where $P = \sum_{i \in C^*} w_i + \sum_{\{i,j\} \subset C^*} H(\tilde{\xi}_{ij}) w_i w_j$ is the sum of the importance of all C_i and $\{C_i, C_j\}$ (*i≠j*).

Step 6: Identify the 2-order additive fuzzy measure g_K.

According to the Möbius transformation coefficients m_i and m_{ij} of attributes, using Eq. (3), the 2-order additive fuzzy measure g_K can be identified as

$$g_K = g(K) = \sum_{i \in K^*} m_i + \sum_{\{i,j\} \subset K^*} m_{ij}, \forall K \subseteq C \tag{9}$$

Theorem 1: The fuzzy measure identified by steps 1 to 6 is a 2-order additive fuzzy measure.

To prove that the fuzzy measure identified by steps 1 to 6 is a 2-order additive fuzzy measure, it is only necessary to prove that the determined Möbius transformation coefficients satisfy the following constrained conditions [6]:

1) $m(\varnothing) = 0$;

2) $m_i \geq 0$, $\forall i \in C^*$;

3) $\sum_{i \in C^*} m_i + \sum_{\{i,j\} \subset C^*} m_{ij} = 1$;

4) $m_i + \sum_{j \in K^* \setminus \{i\}} m_{ij} \geq 0$, $\forall K \subset C$.

Proof:

1) $m(\varnothing) = 0$, obviously holds.

2) Because $H(\tilde{\xi}_{ij}) = H(\tilde{\xi}_{ji})$, and $\sum_{i=1}^{n} w_i = 1$, *P* can be further written as

$$P = 1 + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1, i \neq j}^{n} H(\tilde{\xi}_{ij}) w_i w_j = 1 + \frac{1}{2} \sum_{i=1}^{n} \left[w_i \sum_{j=1, i \neq j}^{n} H(\tilde{\xi}_{ij}) w_j \right]$$

Since $H(\tilde{\xi}_{ij}) \in [-1, 1]$, and $w_j \in [0, 1]$, we have

$$-w_j \leq H(\tilde{\xi}_{ij}) w_j \leq w_j, \quad i \neq j$$

Sum the two sides of the above inequality to *j*, we obtain

$$-(1 - w_i) \leq \sum_{j=1, i \neq j}^{n} H(\tilde{\xi}_{ij}) w_j \leq 1 - w_i$$

Multiply both sides of the above inequality by w_i, we can get

$$-(w_i - w_i^2) \leq w_i \sum_{j=1, i \neq j}^{n} H(\tilde{\xi}_{ij}) w_j \leq w_i - w_i^2$$

Sum the two sides of the above inequality to *i*, the following inequality can be given

$$-(1 - \sum_{i=1}^{n} w_i^2) \leq \sum_{i=1}^{n} \left[w_i \sum_{j=1, i \neq j}^{n} H(\tilde{\xi}_{ij}) w_j \right] \leq 1 - \sum_{i=1}^{n} w_i^2$$

Therefore, we have

$$\frac{1}{2} + \frac{1}{2} \sum_{i=1}^{n} w_i^2 \leq 1 + \frac{1}{2} \sum_{i=1}^{n} \left[w_i \sum_{j=1, i \neq j}^{n} H(\tilde{\xi}_{ij}) w_j \right] \leq \frac{3}{2} - \frac{1}{2} \sum_{i=1}^{n} w_i^2$$

That is to say

$$0 < \frac{1}{2} + \frac{1}{2}\sum_{i=1}^{n} w_i^2 \le P$$

Because $P > 0$, and $w_i \ge 0$, we can get $m_i = {w_i}/{P} \ge 0$.

3) $\sum_{i \in C^*} m_i + \sum_{\{i,j\} \subset C^*} m_{ij} = \sum_{i \in C^*} {w_i}/{P} + \sum_{\{i,j\} \subset C^*} {H(\tilde{\xi}_{ij}) w_i w_j}/{P} = \frac{1}{P}\left[\sum_{i \in C^*} w_i + \sum_{\{i,j\} \subset C^*} H(\tilde{\xi}_{ij}) w_i w_j\right] = 1$, obviously holds.

4) $m_i + \sum_{j \in K^* \setminus \{i\}} m_{ij} = \frac{w_i}{P} + \sum_{j \in K^* \setminus \{i\}} \frac{H(\tilde{\xi}_{ij}) w_i w_j}{P} = \frac{w_i}{P}\left[1 + \sum_{j \in K^* \setminus \{i\}} H(\tilde{\xi}_{ij}) w_j\right]$.

Since $H(\tilde{\xi}_{ij}) \in [-1,1]$, and $w_j \in [0,1]$, we have $-(1-w_i) \le \sum_{j \in K^* \setminus \{i\}} H(\tilde{\xi}_{ij}) w_j \le 1 - w_i$. Thus, the following inequality can be given

$$\frac{w_i}{P}\left[1 + \sum_{j \in K^* \setminus \{i\}} H(\tilde{\xi}_{ij}) w_j\right] \ge \frac{w_i}{P}\left[1 - (1 - w_i)\right] \ge \frac{w_i^2}{P} \ge 0$$

Hence, we get $m_i + \sum_{j \in K^* \setminus \{i\}} m_{ij} \ge 0$, $\forall K \subset C$. Q.E.D.

4 A MADM process based on the proposed method

Using the Choquet fuzzy integral as nonlinear integration operator, this section presents a MADM process based on the proposed method.

Step 1: Construct the normalized decision matrix.

According to the types of attributes (including positive type, negative type and neutral type), the decision matrix is normalized, and the normalized decision matrix is then constructed as

$$\tilde{X} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m1} & \cdots & \tilde{x}_{mn} \end{bmatrix}_{m \times n}$$

where $\tilde{x}_{ji}$ is the normalized attribute value of alternative A_j ($j = 1, 2, \cdots, m$) under attribute C_i ($i = 1, 2, \cdots, n$).

Step 2: Identify the 2-order additive fuzzy measure g_K.

Given the weight vector W_D of experts, the HFLID $H(\tilde{\xi}_{ij})$ between C_i and C_j ($i \neq j$) can be determined by step 1 to step 4 in Section 3. Given the weight vector W_C of attributes, the 2-order additive fuzzy measure g_K can be further identified by step 5 and step 6 in Section 3.

Step 3: Calculate the Choquet fuzzy integral values and the ranking of alternatives.

Reordering the normalized attribute value $\tilde{x}_{ji}$ ($i = 1, 2, \cdots, n$) of the alternative A_j ($j = 1, 2, \cdots, m$) from small to large, the $\tilde{x}_{j(i)}$ can be then obtained. Substituting the $\tilde{x}_{j(i)}$ and the 2-order additive fuzzy measure g_K into Eq. (5), the Choquet fuzzy integral value H_j of the alternative A_j can be calculated. Simultaneously, the ranking of alternatives can be given, where the larger H_j is, the better the alternative A_j is.

5 Application Example

This section takes the credit assessment of the big data listed companies in China as an application example to illustrate the feasibility and effectiveness of the proposed method (see Section 3).

5.1 Credit assessment index system and sample data

Considering the characteristics of big data enterprises [33], and following the principles of selecting indicators, such as scientificalness, objectivity, systematization, functionality, dynamics, relative independence, feasibility (or operability), comparability and so on, a credit assessment index system for big data enterprises was constructed, as shown in Table 1.

Table 1. Credit assessment index system for big data enterprises.

Primary Indicators	Secondary Indicators
Debt Paying Ability C_1	Current Ratio C_{11}; Quick Ratio C_{12}; Asset-liability Ratio C_{13}
Operational Capability C_2	Turnover Rate of Accounts Receivable C_{21}; Turnover Rate of Total Assets C_{22}; Inventory Turnover C_{23}
Profitability C_3	Profit Margin of Main Business C_{31}; Return on Equity C_{32}; Return on Total Assets C_{33}
Growth Capability C_4	Net Profit Growth Rate C_{41}; Growth Rate of Main Business C_{42}
Technological Innovation Capability C_5	Development Expenditure C_{51}; Growth Rate of Intangible Assets C_{52}; Number of Invention Patent Applications Announced C_{53}
Industry Growth C_6	Network Attention of Industry C_{61}; Industry Average Net Profit Growth Rate C_{62}

We selected the big data listed companies in the Growth Enterprise Market (GEM) in China – Wangsu Science & Technology Co., Ltd. (300017), Beijing Lanxum Technology Co., Ltd. (300010), and Wuhan Tianyu Information Industry Co., Ltd. (300205) to form a set of alternatives, denoted by $A=\{A_1, A_2, A_3\}$. Where the alternative A_1 and A_2 belong to the software service industry, and the alternative A_3 belongs to the electronic components industry. The sample data were the section data of 2016, and the original data were shown in Table 2. Where the original data of the Number of Invention Patent Applications Announced were from Tian Yan Cha website, the original data of the Network Attention of Industry were from Baidu Index website, and the rest of the original data were from East Money website.

Table 2. Original data.

Secondary Indicators (Unit)	A_1	A_2	A_3
C_{11} (---) ^{3}	6.73	1.67	2.48
C_{12} (---) ^{3}	6.49	1.38	1.76
C_{13} (%) ^{2}	15.08	28.52	33.54
C_{21} (time) ^{1}	5.48	2.81	3.40
C_{22} (time) ^{1}	0.73	0.37	0.97
C_{23} (time) ^{1}	17.32	2.78	2.88
C_{31} (%) ^{1}	28.07	15.94	0.28
C_{32} (%) ^{1}	17.08	5.21	1.79
C_{33} (%) ^{1}	20.52	5.83	0.27
C_{41} (%) ^{1}	50.52	130.25	12.38

C_{42} (%) [1]	51.67	84.10	9.71
C_{51} (10 thousand yuan) [1]	5605	9868	0
C_{52} (%) [1]	60.57	51.24	-6.73
C_{53} (piece) [1]	83	0	24
C_{61} (time) [1]	159 [4]	159 [4]	457 [4]
C_{62} (%) [1]	30.77	30.77	33.42

[1] The index belongs to positive type. [2] The index belongs to negative type. [3] The index belongs to neutral type. [4] The Network Attention of Industry is the overall daily average of Baidu Search Index with the industry name as the key word.

5.2 Process and results of credit assessment

Step 1: Construct the normalized decision matrix.

Using the algorithm given in [34], based on Table 2, the weight vector W_C of attributes was calculated as

$$W_C = (0.1702, 0.1708, 0.1810, 0.1208, 0.2261, 0.1312).$$

Combined with the algorithm given in [35], based on Table 2, the normalized decision matrix $\tilde{X}$ was further constructed as

$$\tilde{X} = \begin{bmatrix} 0.4709 & 0.8976 & 0.9151 & 0.8192 & 0.8949 & 0.7789 \\ 0.8662 & 0.6048 & 0.6148 & 0.9500 & 0.7786 & 0.7789 \\ 0.8147 & 0.7671 & 0.1140 & 0.6054 & 0.2868 & 0.9156 \end{bmatrix}.$$

Step 2: Determine the HFLID $H(\tilde{\xi}_{ij})$ between C_i and C_j ($i≠j$).

According to step 1 in Section 3, a linguistic term set S was established to characterize the interactivity between C_i and C_j ($i≠j$), where S={ s_{-4} = repeatability is extremely strong, s_{-3} = repeatability is very strong, s_{-2} = repeatability is strong, s_{-1} = repeatability is relatively strong, s_0 = independence, s_1 = complementarity is relatively strong, s_2 = complementarity is strong, s_3 = complementarity is very strong, s_4 = complementarity is extremely strong }.

In this paper, three experts were invited to analyze the six attributes in pairs respectively. According to step 2 in Section 3, under the linguistic term set S, according to Definition 10, every expert employed the linguistic expressions ll generated by the context-free grammar G_H to evaluate the interactivity between C_i and C_j ($i≠j$) (C_6^2 pairs in total). Through the transformation function $E_{G_H}: ll \to H_S^{ij}$, the linguistic expressions ll were further transformed into the HLFTS H_S^{ij}. Thus, the individual evaluation results $h_S^{ij(p)}$ $(p = 1,2,3)$ of three experts on the interactivity between C_i and C_j ($i≠j$) were then given, as shown in Table 3.

Adopting the cycle mutual evaluation method [36], the weight vector of experts was calculated as $W_D = (0.3976, 0.3012, 0.3012)$ (see Appendix A for full calculation principle and process).

According to step 3 in Section 3, after expanding those $h_S^{ij(p)}$ $(p = 1,2,3)$ with fewer elements, using the HFLWPA to aggregate the individual evaluation results of three experts, the group evaluation result of three experts on the interactivity between C_i and C_j ($i≠j$) was then obtained (see Table 3). According to step 4 in Section 3, with Eq. (7), the HFLID $H(\tilde{\xi}_{ij})$ between C_i and C_j ($i≠j$) was calculated, as shown in Table 3.

Taking $H(\tilde{\xi}_{12})$ as an example, its calculation process was as follows:

1) Using Eq. (1), the standard Euclidean distance among $h_S^{ij(p)}$ $(p = 1,2,3)$ was calculated as

$$d(h_S^{12(1)}, h_S^{12(2)}) = 0,\ \ d(h_S^{12(1)}, h_S^{12(3)}) = 0.0807,\ \ d(h_S^{12(2)}, h_S^{12(3)}) = 0.0807.$$

According to Definition 11, their corresponding support degree was also obtained as

$$\sup(h_S^{12(1)}, h_S^{12(2)}) = 1,\ \ \sup(h_S^{12(1)}, h_S^{12(3)}) = 0.9193,\ \ \sup(h_S^{12(2)}, h_S^{12(3)}) = 0.9193.$$

Thus, we can get

$$T(h_S^{12(1)}) = 1.9193,\ \ T(h_S^{12(2)}) = 1.9193,\ \ T(h_S^{12(3)}) = 1.8386.$$

2) Using Eq. (6), the group evaluation result of three experts on the interactivity between C_1 and C_2 was calculated as

$$h_S^{12} = \{s_1, s_{1.1471}, s_{1.2941}\}.$$

3) With Eq. (7), the HFLID $H(\tilde{\xi}_{12})$ between C_1 and C_2 was then calculated as

$$H(\tilde{\xi}_{12}) = d(h_{S^+}^{12}, h_{S^+}^{0}) = \sqrt{\frac{1}{3}\left(\left(\frac{1}{4}\right)^2 + \left(\frac{1.1471}{4}\right)^2 + \left(\frac{1.2941}{4}\right)^2\right)} = 0.2883.$$

Table 3. Hesitant fuzzy linguistic interaction degrees between attributes.

Attributes C_i and C_j ($i \neq j$)	Individual Evaluation Results of Interactivity			Group Evaluation Results of Interactivity	Hesitant Fuzzy Linguistic Interaction Degrees
	Expert 1	Expert 2	Expert 3		
$\{C_1, C_2\}$	$\{s_1\}$	$\{s_1\}$	$\{s_1, s_2\}$	$\{s_1, s_{1.1471}, s_{1.2941}\}$	0.2883
$\{C_1, C_3\}$	$\{s_2, s_3, s_4\}$	$\{s_2\}$	$\{s_2\}$	$\{s_2, s_{2.3860}, s_{2.7721}\}$	0.6017
$\{C_1, C_4\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_1\}$	$\{s_1, s_{1.1966}, s_{1.3933}\}$	0.3018
$\{C_1, C_5\}$	$\{s_1\}$	$\{s_1\}$	$\{s_1, s_2\}$	$\{s_1, s_{1.1471}, s_{1.2941}\}$	0.2883
$\{C_1, C_6\}$	$\{s_1\}$	$\{s_0, s_1\}$	$\{s_0, s_1\}$	$\{s_{0.3933}, s_{0.6966}, s_1\}$	0.1848
$\{C_2, C_3\}$	$\{s_2, s_3\}$	$\{s_2, s_3\}$	$\{s_2\}$	$\{s_2, s_{2.3529}, s_{2.7059}\}$	0.5926
$\{C_2, C_4\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_0, s_1\}$	$\{s_{0.7093}, s_{1.2093}, s_{1.7093}\}$	0.3191
$\{C_2, C_5\}$	$\{s_1\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_1, s_{1.3034}, s_{1.6067}\}$	0.3317
$\{C_2, C_6\}$	$\{s_1\}$	$\{s_0, s_1\}$	$\{s_0, s_1\}$	$\{s_{0.3933}, s_{0.6966}, s_1\}$	0.1848
$\{C_3, C_4\}$	$\{s_2, s_3\}$	$\{s_2\}$	$\{s_2\}$	$\{s_2, s_{2.1966}, s_{2.3933}\}$	0.5506
$\{C_3, C_5\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_1, s_2\}$	$\{s_{1.2941}, s_{1.6471}, s_2\}$	0.4180
$\{C_3, C_6\}$	$\{s_1, s_2\}$	$\{s_1\}$	$\{s_2\}$	$\{s_{1.2981}, s_{1.5000}, s_{1.7019}\}$	0.3773
$\{C_4, C_5\}$	$\{s_1, s_2\}$	$\{s_1, s_2\}$	$\{s_2\}$	$\{s_{1.2941}, s_{1.6471}, s_2\}$	0.4180
$\{C_4, C_6\}$	$\{s_{-2}\}$	$\{s_{-2}, s_{-3}\}$	$\{s_{-2}\}$	$\{s_{-2}, s_{-2.1471}, s_{-2.2941}\}$	-0.5376
$\{C_5, C_6\}$	$\{s_1\}$	$\{s_0, s_1\}$	$\{s_1\}$	$\{s_{0.7059}, s_{0.8529}, s_1\}$	0.2153

Step 3: Calculate the Möbius transformation coefficients m_i and m_{ij} of attributes.

Based on the weight vector W_C of attributes and Table 3, using Eq. (8), the Möbius transformation coefficients m_i and m_{ij} of attributes were calculated, as shown in Table 4, where $P = 1.1378$. Taking m_{12} as an example, we had

$$m_{12} = (0.2883 \times 0.1702 \times 0.1708) / 1.1378 = 0.0074.$$

Table 4. Calculation results of Möbius transformation coefficients.

Möbius Transformation Coefficients	Coefficient Values	Möbius Transformation Coefficients	Coefficient Values	Möbius Transformation Coefficients	Coefficient Values
m_1	0.1496	m_{13}	0.0163	m_{26}	0.0037
m_2	0.1501	m_{14}	0.0055	m_{34}	0.0106
m_3	0.1591	m_{15}	0.0098	m_{35}	0.0150
m_4	0.1062	m_{16}	0.0036	m_{36}	0.0079
m_5	0.1987	m_{23}	0.0161	m_{45}	0.0100
m_6	0.1153	m_{24}	0.0058	m_{46}	-0.0075
m_{12}	0.0074	m_{25}	0.0113	m_{56}	0.0056

Step 4: Identify the 2-order additive fuzzy measure g_K.

Based on Table 4, using Eq. (9), the 2-order additive fuzzy measure g_K was calculated, as shown in Table 5. Taking $g_{\{1,2\}}$ as an example, we had

$$g_{\{1,2\}} = 0.1496 + 0.1501 + 0.0074 = 0.3071 .$$

Table 5. Calculation results of 2-order additive fuzzy measures.

K	g_K	*K*	g_K	*K*	g_K	*K*	g_K
{ø}	0.0000	{3, 4}	0.2758	{2, 3, 4}	0.4478	{1, 3, 4, 5}	0.6807
{1}	0.1496	{3, 5}	0.3728	{2, 3, 5}	0.5503	{1, 3, 4, 6}	0.5665
{2}	0.1501	{3, 6}	0.2823	{2, 3, 6}	0.4522	{1, 3, 5, 6}	0.6809
{3}	0.1591	{4, 5}	0.3149	{2, 4, 5}	0.4821	{1, 4, 5, 6}	0.5968
{4}	0.1062	{4, 6}	0.2140	{2, 4, 6}	0.3736	{2, 3, 4, 5}	0.6829
{5}	0.1987	{5, 6}	0.3196	{2, 5, 6}	0.4847	{2, 3, 4, 6}	0.5672
{6}	0.1153	{1, 2, 3}	0.4985	{3, 4, 5}	0.4996	{2, 3, 5, 6}	0.6828
{1, 2}	0.3071	{1, 2, 4}	0.4245	{3, 4, 6}	0.3915	{2, 4, 5, 6}	0.5992
{1, 3}	0.3250	{1, 2, 5}	0.5268	{3, 5, 6}	0.5016	{3, 4, 5, 6}	0.6209
{1, 4}	0.2612	{1, 2, 6}	0.4297	{4, 5, 6}	0.4284	{1, 2, 3, 4, 5}	0.8713
{1, 5}	0.3581	{1, 3, 4}	0.4472	{1, 2, 3, 4}	0.6265	{1, 2, 3, 4, 6}	0.7496
{1, 6}	0.2685	{1, 3, 5}	0.5485	{1, 2, 3, 5}	0.7333	{1, 2, 3, 5, 6}	0.8694
{2, 3}	0.3253	{1, 3, 6}	0.4518	{1, 2, 3, 6}	0.6291	{1, 2, 4, 5, 6}	0.7750
{2, 4}	0.2621	{1, 4, 5}	0.4797	{1, 2, 4, 5}	0.6542	{1, 3, 4, 5, 6}	0.8056
{2, 5}	0.3601	{1, 4, 6}	0.3727	{1, 2, 4, 6}	0.5396	{2, 3, 4, 5, 6}	0.8079
{2, 6}	0.2691	{1, 5, 6}	0.4826	{1, 2, 5, 6}	0.6551	{1, 2, 3, 4, 5, 6}	1.0000

Step 5: Calculate the Choquet fuzzy integral values and the ranking of alternatives.

Take the alternative A_1 as an example.

According to step 3 in Section 4, reordering the normalized attribute value $\tilde{x}_{1i}$ ($i = 1, 2, \cdots, 6$) of alternative A_1 from small to large, we can get $\tilde{x}_{11} < \tilde{x}_{16} < \tilde{x}_{14} < \tilde{x}_{15} < \tilde{x}_{12} < \tilde{x}_{13}$, which can be denoted by

$$\tilde{x}_{1(1)} < \tilde{x}_{1(2)} < \tilde{x}_{1(3)} < \tilde{x}_{1(4)} < \tilde{x}_{1(5)} < \tilde{x}_{1(6)} .$$

Substituting the $\tilde{x}_{1(i)}$ and the 2-order additive fuzzy measure g_K into Eq. (5), the Choquet fuzzy integral value of the alternative A_1 was calculated as

$$\begin{aligned} H_1 &= (0.4709 - 0.0000) \times 1.0000 + (0.7789 - 0.4709) \times 0.8079 + (0.8192 - 0.7789) \times 0.6829 \\ &+ (0.8949 - 0.8192) \times 0.5503 + (0.8976 - 0.8949) \times 0.3253 + (0.9151 - 0.8976) \times 0.1591 = 0.7926 . \end{aligned}$$

Similarly, we can also obtain $H_2 = 0.7643$ and $H_3 = 0.5138$.

Since $H_1 > H_2 > H_3$, then the ranking of alternatives was $A_1 \succ A_2 \succ A_3$.

That is to say, the credit status of the alternative A_1 was relatively good, and the credit status of the alternative A_3 was relatively poor.

5.3 Comparative analysis

For comparison, we invited the above three experts to use the scoring method [13] to determine the interaction degrees between attributes, the individual scoring interaction degrees were then obtained, as shown in Table 6. Given the weight vector $W_D = (0.3976, 0.3012, 0.3012)$ of experts, using the weighted arithmetic mean method to aggregate the opinions of three experts, the group scoring interaction degrees were further obtained, as shown in Table 6.

Table 6. Scoring interaction degrees between attributes.

Attributes C_i and C_j ($i \neq j$)	Individual Scoring Interaction Degrees			Group Scoring Interaction Degrees
	Expert 1	Expert 2	Expert 3	
$\{C_1, C_2\}$	0.20	0.30	0.20	0.2301
$\{C_1, C_3\}$	0.30	0.50	0.50	0.4205
$\{C_1, C_4\}$	0.20	0.30	0.20	0.2301
$\{C_1, C_5\}$	0.20	0.30	0.20	0.2301
$\{C_1, C_6\}$	0.20	0.00	0.20	0.1398
$\{C_2, C_3\}$	0.30	0.50	0.50	0.4205
$\{C_2, C_4\}$	0.20	0.20	0.30	0.2301
$\{C_2, C_5\}$	0.20	0.20	0.30	0.2301
$\{C_2, C_6\}$	0.20	0.00	0.20	0.1398
$\{C_3, C_4\}$	0.30	0.50	0.30	0.3602
$\{C_3, C_5\}$	0.20	0.30	0.30	0.2602
$\{C_3, C_6\}$	0.20	0.30	0.30	0.2602
$\{C_4, C_5\}$	0.20	0.30	0.30	0.2602
$\{C_4, C_6\}$	-0.30	-0.50	-0.50	-0.4205
$\{C_5, C_6\}$	0.20	0.00	0.20	0.1398

In addition, we also invited the above three experts to use the diamond pairwise comparisons (DPC) method [9] to determine the interaction degrees between attributes, the individual DPC interaction degrees were then obtained, as shown in Table 7. Given the weight vector $W_D = (0.3976, 0.3012, 0.3012)$ of experts, using the weighted arithmetic mean method to aggregate the opinions of three experts, the group DPC interaction degrees were further obtained, as shown in Table 7.

Table 7. DPC interaction degrees between attributes.

Attributes C_i and C_j ($i \neq j$)	Individual DPC Interaction Degrees			Group DPC Interaction Degrees
	Expert 1	Expert 2	Expert 3	
$\{C_1, C_2\}$	0.25	0.35	0.25	0.2801
$\{C_1, C_3\}$	0.35	0.55	0.55	0.4705
$\{C_1, C_4\}$	0.25	0.35	0.25	0.2801
$\{C_1, C_5\}$	0.25	0.35	0.25	0.2801
$\{C_1, C_6\}$	0.25	0.00	0.25	0.1747
$\{C_2, C_3\}$	0.35	0.55	0.55	0.4705
$\{C_2, C_4\}$	0.25	0.25	0.35	0.2801
$\{C_2, C_5\}$	0.25	0.25	0.35	0.2801
$\{C_2, C_6\}$	0.25	0.00	0.25	0.1747
$\{C_3, C_4\}$	0.35	0.55	0.35	0.4102
$\{C_3, C_5\}$	0.25	0.35	0.35	0.3102
$\{C_3, C_6\}$	0.25	0.35	0.35	0.3102
$\{C_4, C_5\}$	0.25	0.35	0.35	0.3102
$\{C_4, C_6\}$	-0.35	-0.55	-0.55	-0.4705
$\{C_5, C_6\}$	0.25	0.00	0.25	0.1747

Furthermore, we replaced the hesitant fuzzy linguistic interaction degrees (see Table 3) in step 2 of Section 5.2 with the group scoring interaction degrees (see Table 6). Ceteris paribus, the Choquet fuzzy integral values of alternatives were calculated as

$$H_1' = 0.7946, \ H_2' = 0.7686, \ H_3' = 0.5244.$$

Since $H_1' > H_2' > H_3'$, then the ranking of alternatives was $A_1 \succ A_2 \succ A_3$.

Similarly, we also replaced the hesitant fuzzy linguistic interaction degrees (see Table 3) in step 2 of Section 5.2 with the group DPC interaction degrees (see Table 7). Ceteris paribus, the Choquet fuzzy integral values of alternatives were calculated as

$$H_1'' = 0.7931,\ H_2'' = 0.7670,\ H_3'' = 0.5211.$$

Since $H_1'' > H_2'' > H_3''$, then the ranking of alternatives was $A_1 \succ A_2 \succ A_3$.

We further investigated the discrimination of the MADM method based on the hesitant fuzzy linguistic interaction degrees (hereinafter referred to as Method 1) for alternatives, as well as the MADM method based on the group scoring interaction degrees (hereinafter referred to as Method 2) and the MADM method based on the group DPC interaction degrees (hereinafter referred to as Method 3). Adopting the algorithm of discrimination given in [37] (see Appendix B for full calculation principle), the discrimination of Method 1 for alternatives was calculated as

$$\rho = \frac{0.7926 - 0.7643}{0.7926} + \frac{0.7926 - 0.5138}{0.7926} + \frac{0.7643 - 0.5138}{0.7643} = 0.7153.$$

Similarly, the discrimination of Method 2 for alternatives was calculated as

$$\rho' = \frac{0.7946 - 0.7686}{0.7946} + \frac{0.7946 - 0.5244}{0.7946} + \frac{0.7686 - 0.5244}{0.7686} = 0.6903.$$

Similarly, the discrimination of Method 3 for alternatives was calculated as

$$\rho'' = \frac{0.7931 - 0.7670}{0.7931} + \frac{0.7931 - 0.5211}{0.7931} + \frac{0.7670 - 0.5211}{0.7670} = 0.6965.$$

In summary, although the ranking results of Method 2 and Method 3 are consistent with that of Method 1, the discrimination of Method 1 for alternatives is higher than that of both Method 2 and Method 3. Which means that the decision-making effect of Method 1 is better than that of both Method 2 and Method 3.

6 Discussion

From the results and analysis of the previous section, we observed that Method 1 was able to obtain the higher discrimination value than Method 2 and Method 3 (Method 1 was 0.7153, Method 2 was 0.6903, and Method 3 was 0.6965), and the slightly lower Choquet fuzzy integral mean value than Method 2 and Method 3 (Method 1 was 0.6902, Method 2 was 0.6959, and Method 3 was 0.6938).

Compared with Method 2 and Method 3, Method 1 can obtain the higher discrimination value. Since the variance of interaction degrees determined by Method 1 (its value was equal to 0.0673) was higher than that of both Method 2 (its value was equal to 0.0355) and Method 3 (its value was equal to 0.0449). According to Eq. (4), the variances of m_i and m_{ij} increase with the increase of the variance of interaction degrees. From Eq. (3) and Eq. (5), we can further see that the variance of $g(K)$ and the variance of H also increase correspondingly. Thus, according to the algorithm of discrimination [37], the discrimination value becomes larger.

Compared with Method 2 and Method 3, Method 1 can obtain the slightly lower Choquet fuzzy integral mean value.

Since the average value of interaction degrees determined by Method 1 (its value was equal to 0.3023) was higher than that of both Method 2 (its value was equal to 0.2087) and Method 3 (its value was equal to 0.2490). According to Eq. (4), when the average value of interaction degrees increases, the mean value of P increases, meanwhile, the mean value of m_i decreases and the mean value of m_{ij} increases. However, because $|m_i| \gg |m_{ij}|$, from Eq. (3), we can further see that the mean value of $g(K)$ also decreases correspondingly. Thus, according to Eq. (5), the mean value of H becomes smaller.

Both the variance and the average value of interaction degrees determined by Method 1 were higher than that of both Method 2 and Method 3, this was closely related to the fact that the experts employed the linguistic expressions generated by the context-free grammar to evaluate the interactivity between attributes in Method 1, which can flexibly and comprehensively reflect their real opinions in hesitant situations under qualitative settings.

7 Conclusion

In this paper, the proposed method defines the interactivity between attributes by using the supermodular game theory, so that the interaction between attributes is easier to explain and understand, which lays a solid foundation for experts to qualitatively evaluate the interactivity between attributes. The proposed method allows the experts to qualitatively describe the interactivity between attributes by using linguistic expressions generated by the context-free grammar, when they are hesitant among multiple possible linguistic information, which can flexibly and comprehensively reflect their real opinions. Furthermore, the proposed method uses the defined HFLWPA to aggregate the opinions of all experts, which not only considers the weights of experts, but also considers the mutual support degree of opinions of experts, thereby ensuring the rationality of decision-making. In particular, the proposed method uses the standard Euclidean distance formula of HFLEs to define and calculate the HFLID between attributes, so the transformation from qualitative description to quantitative characterization is finally realized. As a result, using the HFLTS, this work successfully solves the problem that it is difficult to quantitatively evaluate the interactivity between attributes in the identification process of 2-order additive fuzzy measure.

The proposed method applies the HFLTS to determine the interaction degree between attributes in a set of attributes. Its calculation principle is easy to understand, and calculation process is relatively simple, thereby improving the simplicity and operability of decision-making. In addition, the proposed method can not only be used to identify the 2-order additive fuzzy measure, but also can be extended to identify the k-order additive fuzzy measure, which has high practical application value and broad application prospects.

This work proposed a 2-order additive fuzzy measure identification method based on HFLID. Obviously, on the one hand, compared with the objective methods describing and dealing with the interactivity between attributes (as in [7], [16] and [17]), the proposed method has subjectivity. On the other hand, compared with the subjective methods describing and dealing with the interactivity between attributes (as in [9], [10], [11], [12], [13], [14] and [15]), the proposed method has fuzziness and hesitation.

Future application example analysis will consider increasing the number of samples and experts, since increasing the number of samples can improve the persuasiveness of application example analysis results, and increasing the number of experts can improve the stability of interactivity evaluation. Furthermore, the established linguistic term set S contains only nine linguistic terms, which is relatively extensive. Therefore, it is necessary to add the linguistic terms in S in the future. In addition, Method 1 should be compared with other methods except Method 2 and Method 3, such as the proportional scaling method [11], multicriteria correlation preference information method [12], qualitative cross-impact analysis method [15], etc.

According to the supermodular game theory, Definition 10 gives the definition of interactivity between attributes. However, this definition is still relatively general. The detailed theoretical analysis of the connotation of interactivity between attributes needs to be completed in the future. It should be noticed that the subjective methods describing and dealing with the interactivity between attributes have a strong explanatory power, but a poor objectivity, and the objective methods describing and dealing with the interactivity between attributes have a poor explanatory power, but a strong objectivity. Thus, combining the results of subjective methods and objective methods well be one of the main research directions in the future.

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Appendix A

In step 2 of Section 5.2, the weight vector of experts was calculated by using the cycle mutual evaluation method [36]. Its calculation principle is as follows:

Suppose there are p experts, b_{ij} is the mutual evaluation weight given by expert i to expert j, then the mutual evaluation weight matrix is $B=(b_{ij})_{p\times p}$, where $0\le b_{ij}\le 1$, and $\sum_{i=1}^{p} b_{ij}=1$. Then the weight of expert j given by the group is $q_j = p^{-1}\sum_{i=1}^{p} b_{ij}$. Therefore, the final weights of experts are determined through the cycle evaluation. Let t be the number of mutual evaluations, when t=1, the weight vector of experts is $q^1=(q_1,q_2,\cdots,q_p)$; when t>1, the weight vector of experts is $q^t=q^{t-1}B$, until q^t converges to the stable value.

In this paper, the mutual evaluation weight matrix was given by three experts, it was

$$B=\begin{bmatrix} 0.50 & 0.25 & 0.25 \\ 0.38 & 0.31 & 0.31 \\ 0.28 & 0.36 & 0.36 \end{bmatrix}.$$

When t=1, the weight vector of experts was $q^1=(0.3866,0.3067,0.3067)$; when t=5, the weight vector of experts converged to the stable value $q^5=(0.3976,0.3012,0.3012)$. Therefore, the final weight vector of experts was $W_D=(0.3976,0.3012,0.3012)$.

Appendix B

In Section 5.3, the discrimination of the MADM method for alternatives was calculated by using the algorithm of discrimination [37]. Its calculation principle is as follows:

Suppose a decision model or algorithm evaluate the alternatives with decision coefficient α, the decision coefficient for alternative A_i is α_i, the decision coefficient for alternative A_j is α_j, and $\alpha_i > \alpha_j$, then the discrimination of the decision model or algorithm for alternatives A_i and A_j is defined as

$$\rho_{ij} = \frac{\alpha_i - \alpha_j}{\alpha_i} \times 100\%$$

Obviously, the larger the discrimination ρ_{ij} is, the better the decision-making effect of the decision model or algorithm for alternatives A_i and A_j is.

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