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Article

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Progressive and asymmetric active and passive mechanisms of shallow tunnels under the effect of seismic forces

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Abstract: To investigate the effect of the seismic force on asymmetric failure mechanisms of shallow tunnels, progressive and asymmetric active and passive failure mechanisms of shallow tunnels were discussed in this paper. In each of these failure mechanisms, the lower stratum was inclined, and there were four different detaching curves. Based on the nonlinear yield criterion and the upper bound theorem of limit analysis, the expressions of detaching curves for describing the active and passive failure mechanisms of shallow circular tunnels were first derived by the variational method. Then the equations for calculating the range and weight of the failure blocks were obtained. After sensitivity analysis of parameters, the range and weight of the failure blocks under different parameters were computed. According to the results of this study, the rock stratum inclination has a decisive effect on the symmetry of both active and passive failure mechanisms of shallow circular tunnels. Besides, the seismic force is an important factor affecting the stability of the tunnel roof.

Key words: Asymmetric failure mechanism; Shallow tunnels; Seismic force; Nonlinear failure criterion

1. Introduction

Reasonable research on tunnel stability facilitates the optimization of tunnel design, and the stability of shallow tunnels deserves special attention. When a shallow tunnel is surrounded by weak rocks,
insufficient supporting pressure or improper support will lead to collapse (active failure), while excessive
supporting pressure will result in a blowout (passive failure)\textsuperscript{[1]}. Current studies on the stability of shallow
tunnels have made satisfactory progress, especially on the evaluation of the face stability\textsuperscript{[2–5]}. Davis et al.
\textsuperscript{[3]} evaluated the stability of shallow tunnels in a newly created failure mode by means of the upper and
lower limit theorems of limit analysis. They acquired the upper and lower limit stability solutions of
collapse under undrained conditions. Leca et al. \textsuperscript{[6]} analyzed safety factors for the collapse and blowout of
the tunnel face in sandy soil using the limit analysis approach, and sought three upper bound solutions
based on the motion mechanism of rigid blocks. In order to determine collapse pressure, Mollon et al. \textsuperscript{[7,}
\textsuperscript{8]} constructed a three-dimensional multi-block failure mechanism of the tunnel face using spatial
discretization technique within the framework of limit analysis. This method outperformed other
approaches in improving the accuracy of stability solutions. To evaluate the stability of the shallow tunnel
face, a combined method of the kinematics of limit analysis and a toric envelope in the velocity field was
adopted by Zhang et al. \textsuperscript{[4]} to calculate the face pressure during collapsing and blowing out. Finally, the
formulas for calculating the face pressure during collapsing and blowing out were proposed.

Tunnel roof failure has also been extensively studied. For instance, Atkinson et al. \textsuperscript{[2]} explored tunnel
stability through theories and experiments, and obtained the theoretical solutions of collapse pressure of
the tunnel roof in the framework of upper and lower limit theorems of limit analysis. Similarly, in Fraldi’s
research, the roof collapse of deep tunnels was examined by the nonlinear Hoek-Brown criterion and the
variational method based on the theorem of limit analysis \textsuperscript{[9]}. The collapsing range under different
parameters was also analyzed. Inspired by Fraldi’s research, Qin et al. \textsuperscript{[10]} investigated the progressive
two- and three-dimensional collapse mechanisms of deep tunnels and drew the three-dimensional shape
of the collapsing block. Their study results revealed that different strata would lead to progressive collapse
of tunnels. In the study of Yang et al., \textsuperscript{[11]} the nonlinear Hoek-Brown criterion was used to discuss the
collapse mechanism of shallow tunnels. Zhang et al. \cite{1} further reported the active and passive failure mechanisms of shallow tunnels in nonassociative soil with different water table. They deduced the formulas for calculating the failure range by using the nonlinear Mohr-Coulomb criterion. In addition, Yang et al. \cite{12} analyzed the progressive collapse of shallow tunnels in layered rocks. The tunnel collapse mentioned above is symmetrical, but in practice, it is asymmetric because the rock stratum is usually inclined \cite{13-15}. Some studies have elucidated the effect of rock stratum inclination on the symmetry of the collapsing block in tunnels. For example, Gao et al. \cite{16} established a three-dimensional physical model of coal tunnels with an inclined stratum based on the similar model theory. The asymmetric collapse process of coal tunnels was demonstrated by experiment, and the asymmetric support scheme was proposed to improve the stability of coal tunnels. Lyu et al. \cite{13} made an investigation on the asymmetric and progressive collapse of deep tunnels based on the upper bound theorem of limit analysis and nonlinear Hoek-Brown criterion. According to their results, the layered rockmass inclination had a crucial effect on the symmetry of the collapsing block in deep tunnels. The asymmetric and progressive collapse of shallow circular and rectangular tunnels in inclined strata was also analyzed by Lyu et al. \cite{14}. They found that the collapsing block was symmetrical only when the strata were horizontal. Therefore, asymmetric active and passive failure (collapse and blowout) mechanisms in tunnels require further research, in order to optimize the design of tunnels.

The purpose of this paper is to explore the asymmetric collapse of shallow tunnels based on the generalized nonlinear yield criterion and the effect of seismic forces on the tunnel collapse. Firstly, active and passive failure mechanisms were established for a single inclined stratum. The collapsing curves and upper bound solutions of the collapsing range were then obtained by the variational method within the framework of the upper bound theorem of limit analysis. Afterwards, the variation of the failure range with parameters was analyzed. The analytic results showed that the vertical seismic coefficient $k_v$ had a
significant effect on the range and weight of asymmetric failure blocks. This research provides a theoretical reference for optimizing the design of tunnels and improving the stability of the tunnel roof.

2. Theoretical basis

2.1 Generalized nonlinear yield criterion

Accumulating experimental data suggest that geotechnical materials often abide by the nonlinear strength criterion \cite{17, 18}. Inspired by the research of Baker \cite{18}, a generalized nonlinear strength criterion was introduced to estimate the strength of geotechnical materials in this paper. The criterion can be expressed as

\[
\tau_n = P_a A \left( \frac{\sigma_n}{P_a} + T \right)^n
\]

where \(P_a\) denotes atmospheric pressure; \(A, n\) and \(T\) are dimensionless parameters characterizing the strength of geotechnical materials; \(\sigma_n\) indicates the normal stress \cite{18, 19}. The linear Mohr-Coulomb and Griffith strength criteria are special cases of Eq. (1), which is another equivalent expression of the nonlinear Hoek-Brown strength criterion \cite{18}. Therefore, Eq.(1) is a generalized nonlinear strength criterion which can be widely used to evaluate the strength of geotechnical materials, especially in the composite strata composed of soil and rock.

2.2 Upper bound theorem of limit analysis

The upper bound theorem of limit analysis applied in this paper was proposed by Chen \cite{20} in 1975 to investigate the stability of tunnels. This theorem maintains that the rate of dissipated internal work remains greater than the rate of work done by external forces in any kinematically admissible velocity field \cite{13, 20}. Some hypotheses need to be made when the upper bound theorem is applied to the stability analysis of tunnels. In this work, it was supposed that the failure blocks of tunnels were a rigid body that did not deform, and the geotechnical materials complied with the associated flow rule \cite{20}. 
3. Asymmetric failure mechanisms

The supporting force plays a key role in the failure mechanism of tunnels, according to previous research \cite{1, 4}. An extremely small supporting force cannot maintain the stability of the roof in shallow tunnels, leading to collapse (active failure). An excessively large supporting force will bring about blowouts (passive failure) of surrounding rocks above the roof in shallow tunnels. Therefore, it is necessary to establish the active and passive failure (collapse and blowout) mechanisms of shallow tunnels (Fig. 1 and Fig. 2). In shallow tunnels, both active and passive failure is prone to extend to the ground surface, while surcharge loads $\sigma_s$ are often distributed on the horizontal ground surface \cite{21}. In these failure mechanisms, only the lower surrounding rocks are considered inclined. The outline of each failure
mechanism is composed of four curves, which are expressed as $Z_1(x)$, $Z_2(x)$, $Z_3(x)$ and $Z_4(x)$, respectively.

The dip angle of the layered interface is represented by $\alpha$, and $q$ represents the supporting force acting on shallow tunnels. It is noteworthy that these failure mechanisms are asymmetric with respect to the z-axis.

$\{A_1,n_1,T_1,\gamma_1\}$ and $\{A_2,n_2,T_2,\gamma_2\}$ are the strength parameters of the surrounding rock of the upper and lower strata, respectively.

4. Limit analysis of active and passive failure mechanisms

4.1 Limit analysis of the active failure mechanism

Previous studies \cite{1, 22} reveal that the rate of dissipated internal work $D$ down by internal forces at any point on the detaching curves of failure blocks is represented as

$$ D = \left[-(1-n^{-1}) \cdot (nA \cdot Z'(x))^\frac{1}{n} + T\right] \cdot P_a \cdot v/\sqrt{1+Z'(x)^2} $$

(2)

The rate of dissipated work done by internal forces acting on the detaching curves of the active failure blocks in Fig. 1 can be calculated by

$$ P_{D12} = \int_{L_1} D_1 v \cdot \sqrt{1+Z_1'(x)^2} dx + \int_{L_2} D_2 v \cdot \sqrt{1+Z_2'(x)^2} dx $$

(3)

$$ = \int_{L_1} \left[-(1-n^{-1}) \cdot (nA_1 \cdot Z_1'(x))^{\frac{1}{n}} + T_1\right] \cdot P_a \cdot v dx + \int_{L_2} \left[-(1-n^{-1}) \cdot (nA_2 \cdot Z_2'(x))^{\frac{1}{n}} + T_2\right] \cdot P_a \cdot v dx $$

$$ P_{D34} = \int_{L_3} D_3 v \cdot \sqrt{1+Z_3'(x)^2} dx + \int_{L_4} D_4 v \cdot \sqrt{1+Z_4'(x)^2} dx $$

(4)

$$ = \int_{L_3} \left[-(1-n^{-1}) \cdot (nA_3 \cdot Z_3'(x))^{\frac{1}{n}} + T_3\right] \cdot P_a \cdot v dx + \int_{L_4} \left[-(1-n^{-1}) \cdot (nA_4 \cdot Z_4'(x))^{\frac{1}{n}} + T_4\right] \cdot P_a \cdot v dx $$

where $P_{D12}$ and $P_{D34}$ denote the rates of dissipated internal work at the right and left sides of the failure block in the active failure mechanism, respectively; $D_1$, $D_2$, $D_3$ and $D_4$ are the dissipated power at any point on the detaching curves $Z_1(x)$, $Z_2(x)$, $Z_3(x)$ and $Z_4(x)$, respectively; $Z_1'(x)$, $Z_2'(x)$, $Z_3'(x)$ and $Z_4'(x)$ are the first derivatives of $Z_1(x)$, $Z_2(x)$, $Z_3(x)$ and $Z_4(x)$, respectively; $L_1$, $L_2$ and $L_3$ are marked as the widths of upper and lower failure blocks, respectively.
In the active failure mechanism (Fig. 1), the external forces include weight, the supporting force, uniform surcharge pressure \( \sigma_s \) and the seismic force of the failure block \([23]\). The rates of work done by weight at the right and left sides of the failure block are defined as:

\[
P_{y12} = -\int_{L_1}^{L_2} \gamma L_1 Z_1(x) \, dx + \gamma L_1 Z_1(L_4-L_2)v + \frac{1}{2} \gamma L_1 Z_1(L_4-L_2) + H_2 - H_1 - \sqrt{R^2 - L_1^2}L_2v
\]

\[
- \left[ \gamma L_1 Z_1(L_4)v - \int_{L_1}^{L_2} r J(x) \, dx \right] - \int_{L_1}^{L_2} \gamma L_1 Z_1(x) \, dx + \frac{1}{2} \gamma L_1 Z_1(L_4) + H_1) L_2v
\]

\[
P_{y34} = -\int_{L_1}^{L_2} \gamma L_1 Z_1(x) \, dx + \gamma L_1 Z_1(L_4-L_2)v + \frac{1}{2} \gamma L_1 Z_1(L_4-L_2) + H_2 - H_1 - \sqrt{R^2 - L_1^2}L_2v
\]

\[
- \left[ \gamma L_1 Z_1(L_4)v - \int_{L_1}^{L_2} r J(x) \, dx \right] - \int_{L_1}^{L_2} \gamma L_1 Z_1(x) \, dx + \frac{1}{2} \gamma L_1 Z_1(L_4) + H_1) L_2v
\]

where \( P_{y12} \) and \( P_{y34} \) indicate the rate of work done by weight at the right and left sides of the failure block, respectively; \( H_1 \) implies the vertical height from the intersection of z-axis and the layered interface to the ground surface, and \( H_2 \) is the vertical height from the center of a circular tunnel to the ground surface. The contour of the tunnel roof in Fig. 1 is

\[
J(x) = -\sqrt{R^2 - x^2} + H_2
\]

where \( R \) intends the radius of a circular tunnel. The rates of work done by the supporting force at the right and left sides of the failure block are defined as:

\[
P_{q1} = Rqv \cdot \arcsin \frac{L_1}{R} \cdot \cos \pi
\]

\[
P_{q2} = Rqv \cdot \arcsin \frac{L_2}{R} \cdot \cos \pi
\]

where \( P_{q1} \) and \( P_{q2} \) denote the rates of work done by the supporting force along the bottom of the right and left sides of the failure block, respectively. The rates of work done by the uniform pressure distributed on the ground surface are expressed as:

\[
P_{s1} = \sigma_s L_3v
\]

\[
P_{s2} = \sigma_s L_4v
\]

where \( P_{s1} \) and \( P_{s2} \) represent the rates of work done by the uniform pressure at the right and left sides of the failure block, respectively.

It is noted that in the active mechanism, the velocity of the failure block is vertical downward, so only the effect of the vertical seismic force on the collapse block is taken into account in this paper \([24-26]\).
where \( k_v \) is the vertical seismic coefficient. The negative vertical seismic coefficient is assumed to be upward by the quasi-static method, and the vertical seismic coefficient varies between -0.3~0.3 \([27, 28]\).

To solve the failure range and obtain the expressions of detaching curves, two objective functions, \( \Gamma_1[Z(x), Z'(x), x] \) and \( \Gamma_2[Z(x), Z'(x), x] \), which are composed of the rates of work done by internal and external forces, are established, respectively.

\[
\Gamma_1[Z(x), Z'(x), x] = P_{d12} - P_{y12} - P_{q1} - P_{s1} - P_{k12}
\]

\[
= \int_{l_0}^{l} U_1[Z_1(x), Z_1'(x), x] dx + \int_{l_0}^{l} U_2[Z_2(x), Z_2'(x), x] dx - (1 - k_v) \gamma_1 Z_1 (L_1)(L_1 - L_2)v
\]

\[
- \frac{1}{2} (1 - k_v) \gamma_1 \left( Z_1 (L_1) - Z_1 (L_2) + H_2 - H_1 - \sqrt{R^2 - L_2^2} \right) v + (1 - k_v) \left[ \gamma_1 L_1 Z_1 (L_1) v - \int_{0}^{l_1} J (x) v dx \right]
\]

\[
- \frac{1}{2} (1 - k_v) \gamma_1 \left( Z_2 (L_2) + H_1 \right) L_2 v - Rqv \cdot \arcsin \frac{L_2}{R} \cdot \cos \pi - \sigma_3 L_2 v
\]

\[
\Gamma_2[Z(x), Z'(x), x] = P_{d34} - P_{y34} - P_{q2} - P_{s2} - P_{k34}
\]

\[
= \int_{l_0}^{l} U_3[Z_3(x), Z_3'(x), x] dx + \int_{l_0}^{l} U_4[Z_4(x), Z_4'(x), x] dx - (1 - k_v) \gamma_2 Z_3 (L_3)(L_3 - L_4)v
\]

\[
- \frac{1}{2} (1 - k_v) \gamma_2 \left( Z_3 (L_3) - Z_3 (L_4) + H_4 - H_3 - \sqrt{R^2 - L_4^2} \right) L_3 v + (1 - k_v) \left[ \gamma_2 L_3 Z_3 (L_3) v - \int_{0}^{l_3} \gamma_2 J (x) v dx \right]
\]

\[
- \frac{1}{2} (1 - k_v) \gamma_2 \left( Z_4 (L_4) + H_3 \right) L_4 v - Rqv \cdot \arcsin \frac{L_4}{R} \cdot \cos \pi - \sigma_4 L_4 v
\]

where \( U_1[Z_1(x), Z_1'(x), x], U_2[Z_2(x), Z_2'(x), x], U_3[Z_3(x), Z_3'(x), x] \) and \( U_4[Z_4(x), Z_4'(x), x] \) are the functionals determining two objective functions. These functionals can be expressed as

\[
U_1[Z_1(x), Z_1'(x), x] = \left[ - \left( 1 - n_1^{-1} \right) \cdot \left( n_1 A_1 \cdot Z_1' (x) \right)^{1/n_1} + T_1 \right] \cdot P_a + (1 - k_v) \gamma_1 Z_1 (x)
\]

\[
U_2[Z_2(x), Z_2'(x), x] = \left[ - \left( 1 - n_2^{-1} \right) \cdot \left( n_2 A_2 \cdot Z_2' (x) \right)^{1/n_2} + T_2 \right] \cdot P_a + (1 - k_v) \gamma_2 Z_2 (x)
\]

\[
U_3[Z_3(x), Z_3'(x), x] = \left[ - \left( 1 - n_3^{-1} \right) \cdot \left( n_3 A_3 \cdot Z_3' (x) \right)^{1/n_3} + T_1 \right] \cdot P_a + (1 - k_v) \gamma_1 Z_3 (x)
\]

\[
U_4[Z_4(x), Z_4'(x), x] = \left[ - \left( 1 - n_4^{-1} \right) \cdot \left( n_4 A_4 \cdot Z_4' (x) \right)^{1/n_4} + T_2 \right] \cdot P_a + (1 - k_v) \gamma_2 Z_4 (x)
\]

On the basis of the variational principle, Euler’s equations corresponding to each of the four functionals are acquired:
\[ \delta U_1 [Z_1(x), Z_1'(x), x] = 0 \Rightarrow \frac{\partial U_1}{\partial Z_1(x)} - \frac{\partial}{\partial x} \left[ \frac{\partial U_1}{\partial Z_1'(x)} \right] = 0 \]  
(20)

\[ \delta U_2 [Z_2(x), Z_2'(x), x] = 0 \Rightarrow \frac{\partial U_2}{\partial Z_2(x)} - \frac{\partial}{\partial x} \left[ \frac{\partial U_2}{\partial Z_2'(x)} \right] = 0 \]  
(21)

\[ \delta U_3 [Z_3(x), Z_3'(x), x] = 0 \Rightarrow \frac{\partial U_3}{\partial Z_3(x)} - \frac{\partial}{\partial x} \left[ \frac{\partial U_3}{\partial Z_3'(x)} \right] = 0 \]  
(22)

\[ \delta U_4 [Z_4(x), Z_4'(x), x] = 0 \Rightarrow \frac{\partial U_4}{\partial Z_4(x)} - \frac{\partial}{\partial x} \left[ \frac{\partial U_4}{\partial Z_4'(x)} \right] = 0 \]  
(23)

Substituting Eq.(16) into Eq.(20), Eq.(20) can be expressed as

\[ \left[ -(1 - n_1)^{-1} \cdot (n_1 A_1)^{\frac{1}{n_1}} \cdot Z_1'(x) \right]^{\frac{2n_1-1}{n_1}} \cdot Z_1''(x) \cdot P_a + (1 - k_c) \gamma_1 = 0 \]  
(24)

Through integral calculation, the formulas of \( Z_1(x) \) and its first derivative \( Z_1'(x) \) can be obtained from Eq.(20):

\[ Z_1'(x) = n_1^{-1} A_1^{\frac{1}{n_1}} \left( \frac{1 - k_c}{P_a} \right)^{\frac{1-n_1}{n_1}} \cdot \left[ x + \frac{c_1}{(1 - k_c) \gamma_1} \right]^{\frac{1-n_1}{n_1}} \]  
(25)

\[ Z_1(x) = A_1^{\frac{1}{n_1}} \left( \frac{1 - k_c}{P_a} \right)^{\frac{1-n_1}{n_1}} \cdot \left[ x + \frac{c_1}{(1 - k_c) \gamma_1} \right]^{\frac{1}{n_1}} + d_1 \]  
(26)

where \( c_1 \) is an unsolved constant, and \( d_1 \) is an integration constant. Similarly, by integral calculation and combining Eqs.(17)-(19) with Eqs.(21)-(23), we have the formulas of \( Z_2(x) \), \( Z_3(x) \) and \( Z_4(x) \) and their first derivatives \( Z_2'(x) \), \( Z_3'(x) \) and \( Z_4'(x) \):

\[ Z_2'(x) = n_2^{-1} A_2^{\frac{1}{n_2}} \left( \frac{1 - k_c}{P_a} \right)^{\frac{1-n_2}{n_2}} \cdot \left[ x + \frac{c_2}{(1 - k_c) \gamma_2} \right]^{\frac{1-n_2}{n_2}} \]  
(27)

\[ Z_3'(x) = n_1^{-1} A_1^{\frac{1}{n_1}} \left( \frac{1 - k_c}{P_a} \right)^{\frac{1-n_1}{n_1}} \cdot \left[ x + \frac{c_3}{(1 - k_c) \gamma_1} \right]^{\frac{1-n_1}{n_1}} \]  
(28)

\[ Z_4'(x) = n_2^{-1} A_2^{\frac{1}{n_2}} \left( \frac{1 - k_c}{P_a} \right)^{\frac{1-n_2}{n_2}} \cdot \left[ x + \frac{c_4}{(1 - k_c) \gamma_2} \right]^{\frac{1-n_2}{n_2}} \]  
(29)

\[ Z_2(x) = A_2^{\frac{1}{n_2}} \left( \frac{1 - k_c}{P_a} \right)^{\frac{1-n_2}{n_2}} \cdot \left[ x + \frac{c_2}{(1 - k_c) \gamma_2} \right]^{\frac{1}{n_1}} + d_2 \]  
(30)
where \( c_2, c_3 \) and \( c_4 \) are unknown constants that need to be further determined; \( d_2, d_3 \) and \( d_4 \) are the integral constants of \( Z_2(x) \), \( Z_3(x) \) and \( Z_4(x) \), respectively. Because there is no shear stress on the ground surface in Fig.1, the following constraint is given: \(^{[14, 29]}\)

\[
\tau_{xz}(x = L_3, z = 0) = \tau_{xz}(x = L_6, z = 0) = 0
\] (33)

The expressions of constants \( c_2 \) and \( c_4 \) can be obtained by introducing Eqs.(27) and (29) into Eq.(33), respectively.

\[
c_2 = -\gamma_2 (1 - k_v) L_3
\] (34)
\[
c_4 = -\gamma_2 (1 - k_v) L_6
\] (35)

The values of the same first derivative should be shared at the intersection of upper and lower failure blocks, so we have:

\[
Z_1'(L_2) = Z_2'(L_2)
\] (36)
\[
Z_3'(L_5) = Z_4'(L_5)
\] (37)

To facilitate the calculation, we assume that constants \( e_1 \) and \( e_2 \) can be written as.

\[
e_1 = n_1^{-1} A_1 \left[ \frac{1 - k_v}{P_a} \right]^{1/n_1}
\] (38)
\[
e_2 = n_2^{-1} A_2 \left[ \frac{1 - k_v}{P_a} \right]^{1/n_2}
\] (39)

Plugging Eq.(25) and Eqs.(27)-(29) into Eqs.(36) and (37), constants \( c_1 \) and \( c_3 \) are expressed as:

\[
c_1 = (1 - k_v) \gamma_1 \left\{ \frac{e_2}{e_1} (L_2 - L_3) \left[ \frac{1 - k_v}{P_a} \right]^{1/n_2} - L_2 \right\}
\] (40)
The geometric conditions in the active failure mechanism (Fig. 1) are as follows:

\[ Z_1 (L_2) = Z_2 (L_2) = H_1 - L_2 \cdot \tan \alpha \]  \hspace{1cm} (42)

\[ Z_1 (L_3) = Z_4 (L_3) = H_1 + L_3 \cdot \tan \alpha \]  \hspace{1cm} (43)

By bringing Eq.(26) and Eqs.(30)-(32) into Eqs.(42) and (43), the expressions of integral constants \( d_1, d_2, d_3 \) and \( d_4 \) are represented as:

\[ d_1 = H_1 - L_2 \cdot \tan \alpha - A_1 \left( \frac{1 - k_v}{P_a} \right) \frac{1}{n_1} \cdot \left[ L_2 + \frac{c_1}{(1 - k_v) \gamma_1} \right] \frac{1}{n_1} \]  \hspace{1cm} (44)

\[ d_2 = H_1 - L_2 \cdot \tan \alpha - A_2 \left( \frac{1 - k_v}{P_a} \right) \frac{1}{n_2} \cdot \left[ L_2 + \frac{c_2}{(1 - k_v) \gamma_2} \right] \frac{1}{n_2} \]  \hspace{1cm} (45)

\[ d_3 = H_1 + L_3 \cdot \tan \alpha - A_1 \left( \frac{1 - k_v}{P_a} \right) \frac{1}{n_1} \cdot \left[ L_3 + \frac{c_3}{(1 - k_v) \gamma_1} \right] \frac{1}{n_1} \]  \hspace{1cm} (46)

\[ d_4 = H_1 + L_3 \cdot \tan \alpha - A_2 \left( \frac{1 - k_v}{P_a} \right) \frac{1}{n_2} \cdot \left[ L_3 + \frac{c_4}{(1 - k_v) \gamma_2} \right] \frac{1}{n_2} \]  \hspace{1cm} (47)

According to the additional geometric conditions in Fig. 1, we have:

\[ Z_1 (L_4) = H_2 - \sqrt{R^2 - L_4^2} = A_1 \left( \frac{1 - k_v}{P_a} \right) \frac{1}{n_1} \cdot \left[ L_4 + \frac{c_1}{(1 - k_v) \gamma_1} \right] \frac{1}{n_1} + d_1 \]  \hspace{1cm} (48)

\[ Z_3 (L_4) = H_2 - \sqrt{R^2 - L_4^2} = A_2 \left( \frac{1 - k_v}{P_a} \right) \frac{1}{n_2} \cdot \left[ L_4 + \frac{c_3}{(1 - k_v) \gamma_1} \right] \frac{1}{n_2} + d_3 \]  \hspace{1cm} (49)

\[ Z_2 (L_3) = Z_4 (L_3) = 0 \]  \hspace{1cm} (50)

Through combining Eqs.(30), (32) and (50), the integral constants \( d_2 \) and \( d_4 \) are equal to 0.

\[ d_2 = d_4 = 0 \]  \hspace{1cm} (51)

Then, the expressions of functionals can be expressed as:

\[ U_1 [Z_1(x), Z_2'(x), x] = \frac{1}{n_1} \cdot A_1 \left( \frac{1 - k_v}{P_a} \right) \frac{1}{n_1} \cdot \left[ x + \frac{c_1}{(1 - k_v) \gamma_1} \right] \frac{1}{n_1} + T_1 \cdot P_a + (1 - k_v) \gamma_1 d_1 \]  \hspace{1cm} (52)
Finally, let $\Gamma_1[Z(x), Z'(x), x] = \Gamma_2[Z(x), Z'(x), x] = 0$, and we can acquire the equations for calculating the failure range of shallow tunnels:

\[
\frac{1}{1+n_1} \cdot A_1^{1-n_1} \left[ \frac{(1-k_v)\gamma_1}{P_a} \right]^{1-n_1} \left[ L_4 + \frac{c_1}{(1-k_v)\gamma_1} \right]^{1+n_1} - \left( L_5 + \frac{c_2}{(1-k_v)\gamma_1} \right)^{1+n_1} P_a + T_1P_a(L_4-L_5) + (1-k_v)\gamma_dL_4-L_5 \right]
\nu\left[ L_2 + \frac{c_1}{(1-k_v)\gamma_2} \right]^{1+n_2} \left( L_5 + \frac{c_2}{(1-k_v)\gamma_2} \right)^{1+n_2} P_a + T_2P_a(L_2-L_5)
\nu\left( L_3 - Z_1(L_4) - Z_2(L_5) + H_2 - H_1 - \sqrt{R^2-L_4^2} \right)\nu\left( L_2 - \frac{1}{2}(1-k_v)\gamma_2(Z_1(L_2) + H_1)\nu L_2 + (1-k_v)\gamma_1Z_1(L_4) - \gamma_1\nu\left( -\frac{L_4}{2}\sqrt{R^2-L_4^2} - \frac{R^2}{2} \arcsin \frac{L_4}{R} + H_4L_4 \right) - Rq \cdot \arcsin \frac{L_4}{R} \cdot \cos \pi - \sigma L_4 = 0
\n\frac{1}{1+n_1} \cdot A_1^{1-n_1} \left[ \frac{(1-k_v)\gamma_1}{P_a} \right]^{1-n_1} \left[ L_4 + \frac{c_1}{(1-k_v)\gamma_1} \right]^{1+n_1} - \left( L_5 + \frac{c_2}{(1-k_v)\gamma_1} \right)^{1+n_1} P_a + T_1P_a(L_4-L_5) + (1-k_v)\gamma_dL_4-L_5 \right]
\nu\left[ L_2 + \frac{c_1}{(1-k_v)\gamma_2} \right]^{1+n_2} \left( L_5 + \frac{c_2}{(1-k_v)\gamma_2} \right)^{1+n_2} P_a + T_2P_a(L_2-L_5)
\nu\left( L_3 - Z_1(L_4) - Z_2(L_5) + H_2 - H_1 - \sqrt{R^2-L_4^2} \right)\nu\left( L_2 - \frac{1}{2}(1-k_v)\gamma_2(Z_1(L_2) + H_1)\nu L_2 + (1-k_v)\gamma_1Z_1(L_4) - \gamma_1\nu\left( -\frac{L_4}{2}\sqrt{R^2-L_4^2} - \frac{R^2}{2} \arcsin \frac{L_4}{R} + H_4L_4 \right) - Rq \cdot \arcsin \frac{L_4}{R} \cdot \cos \pi - \sigma L_4 = 0
\]

Combining Eqs.(48), (49), (51) with Eqs.(56) and (57), the upper bound solutions of the failure range are obtained by MATLAB software. The upper bound solutions are taken into Eq.(58), which is derived from Eqs.(5) and (6), and represents the rate of work down by weight of the failure block.
\[ W_1 = \gamma_1 \frac{n_1}{1+n_1} \cdot A_1^{-\frac{1}{n_1}} \cdot \left[ \frac{(1-k_i)\gamma_i}{P_u} \right]^{1-n_1} \cdot \left[ L_2 + \frac{c_i}{(1-k_i)\gamma_i} \right]^{1+n_1} - \left[ L_1 + \frac{c_i}{(1-k_i)\gamma_i} \right]^{1+n_1} + \gamma_1 Z_1(L_1) (L_1 - L_2) \]

\[ + \frac{1}{2} \gamma_1 \left( Z_1(L_1) - Z_1(L_2) + H_2 - H_1 - \sqrt{R^2 - L_2^2} \right) L_2 - \left( \gamma_1 L_1 Z_1(L_1) - \gamma_1 \left( \frac{L_1}{2} \sqrt{R^2 - L_2^2} - \frac{R^2}{2} \arcsin \frac{L_1}{R} + H_1 L_1 \right) \right) \]

\[ W_2 = \gamma_2 \frac{n_2}{1+n_2} \cdot A_2^{-\frac{1}{n_2}} \cdot \left[ \frac{(1-k_i)\gamma_i}{P_u} \right]^{1-n_2} \cdot \left[ L_2 + \frac{c_i}{(1-k_i)\gamma_i} \right]^{1+n_2} - \left[ L_1 + \frac{c_i}{(1-k_i)\gamma_i} \right]^{1+n_2} + \gamma_2 Z_2(L_2) (L_2 - L_1) \]

\[ + \frac{1}{2} \gamma_2 \left( Z_2(L_2) - Z_2(L_1) + H_2 - H_1 - \sqrt{R^2 - L_1^2} \right) L_2 - \left( \gamma_2 L_2 Z_2(L_2) - \gamma_2 \left( \frac{L_2}{2} \sqrt{R^2 - L_1^2} - \frac{R^2}{2} \arcsin \frac{L_2}{R} + H_1 L_2 \right) \right) \]

\[ W_{12} = W_1 + W_2 \]

where \( W_1 \) and \( W_2 \) denote the weights of the right and left sides of the failure block, respectively; \( W_{12} \) is the total weight.

4.2 Limit analysis of the passive failure mechanism

As shown in Eqs.(61) and (62), the rate of dissipated internal work of the potential failure block of the tunnel roof in the passive mechanism (Fig.2) differs from that in the active mechanism (Fig.1).

\[ P_{\text{dis}} = \int_i^o Dy \cdot \sqrt{1 + Z_i'(x)^2} \, dx + \int_i^o Dy \cdot \sqrt{1 + Z_i'(x)^2} \, dx \]

\[ = \int_i^o \left[ - \left( 1 - n_i^{-1} \right) \left( n_i A_i \cdot Z_i'(x) \right)^{1/n_i} + T_i \right] \cdot P_u \cdot vdx + \int_i^o \left[ - \left( 1 - n_i^{-1} \right) \left( n_i A_i \cdot Z_i'(x) \right)^{1/n_i} + T_i \right] \cdot P_u \cdot vdx \]

\[ \left( 1 - n_i^{-1} \right) \left( n_i A_i \cdot Z_i'(x) \right)^{1/n_i} + T_i \right] \cdot P_u \cdot vdx \]

In the passive mechanism, the rates of work at the right and left sides of the failure block are expressed as:

\[ P_{\gamma 1} = \int_i^o \gamma_1 Z_i(x) \, vdx - \int_i^o rF(x) \, vdx - \frac{1}{2} \gamma_1 (-Z_1(L_2) + H_1) L_2 v + \frac{1}{2} \gamma_2 (-Z_1(L_2) + H_1) L_2 v - \int_i^o \gamma_2 Z_2(x) \, vdx \]
\[ P_{y_{34}} = -\int_{0}^{L_y} \gamma_y Z_3(x) vdx - \int_{0}^{L_y} \gamma_y F(x) vdx - \frac{1}{2} \gamma_z \left(-Z_3(L_y) + H_1\right) L_y v + \frac{1}{2} \gamma_z \left(-Z_3(L_y) + H_1\right) L_y v - \int_{0}^{L_y} \gamma_z Z_4(x) vdx \]  

(64)

The contour of the tunnel roof in Fig. 2 is described as:

\[ F(x) = \sqrt{R^2 - x^2} - H_2 \]  

(65)

The rates of work done by the vertical seismic force at the right and left sides of the passive failure block are represented as:

\[ P_{k_{12}} = k_v P_{y_{12}} \]  

(66)

\[ P_{k_{34}} = k_v P_{y_{34}} \]  

(67)

In the passive mechanism (Fig. 2), the rates of work done by the supporting force at the right and left sides of the passive failure block can be defined as:

\[ P_{q_1} = Rqv \cdot \arcsin \frac{L_1}{R} \]  

(68)

\[ P_{q_2} = Rqv \cdot \arcsin \frac{L_2}{R} \]  

(69)

The rates of work done by the uniform pressure distributed on the ground surface are:

\[ P_{s_1} = \sigma_L v \cdot \cos \pi \]  

(70)

\[ P_{s_2} = \sigma_L v \cdot \cos \pi \]  

(71)

According to the dissipation energy and the rate of work done by external forces calculated above, the objective equations in the passive mechanism can be constructed as:

\[ \Gamma_1 \left[ Z(x), Z'(x), x \right] = P_{D_{12}} - P_{y_{12}} - P_{q_1} - P_{s_1} - P_{k_{12}} \]

\[ = \int_{L_y}^{L_y} U_1 \left[ Z_1(x), Z_1'(x), x \right] vdF(x) dx + \int_{L_y}^{L_y} U_2 \left[ Z_2(x), Z_2'(x), x \right] vdx + \left(1 - k_y\right) \int_{0}^{L_y} F(x) vdx + \frac{1}{2} \gamma_z \left(1 - k_y\right) \left(-Z_3(L_y) + H_1\right) L_y v - Rqv \cdot \arcsin \frac{L_2}{R} + \sigma L_2 v \]

\[ \Gamma_2 \left[ Z(x), Z'(x), x \right] = P_{D_{34}} - P_{y_{34}} - P_{q_2} - P_{s_2} - P_{k_{34}} \]

\[ = \int_{L_y}^{L_y} U_3 \left[ Z_3(x), Z_3'(x), x \right] vdF(x) dx + \int_{L_y}^{L_y} U_4 \left[ Z_4(x), Z_4'(x), x \right] vdx + \left(1 - k_y\right) \int_{0}^{L_y} F(x) vdx + \frac{1}{2} \gamma_z \left(1 - k_y\right) \left(-Z_3(L_y) + H_1\right) L_y v - Rqv \cdot \arcsin \frac{L_2}{R} + \sigma L_2 v \]  

(72)

(73)

Similar to Section 4.1, the expressions of the functions and their first derivatives are derived by the variational method:
The constraints in Fig. 2 are:

\[ \tau_{xz} (x = L_3, z = 0) = \tau_{xz} (x = L_6, z = 0) = 0 \]  
\[ Z_1 \cdot (L_2) = Z_2 \cdot (L_2) \]  
\[ Z_3 \cdot (L_3) = Z_4 \cdot (L_3) \]

Introducing Eqs.(74)-(77) into Eqs.(82)-(84), constants \( i_1, i_2, i_3 \) and \( i_4 \) are shown as:

\[ i_2 = \gamma_2 (1-k_y) L_3 \]  
\[ i_4 = \gamma_2 (1-k_y) L_6 \]  

\[ i_1 = (1-k_y) \gamma_1 \cdot \left\{ \frac{e_2}{e_1} (-L_2 + L_3)^{1-n_1/n_2} + L_2 \right\} \]  
\[ i_3 = (1-k_y) \gamma_1 \cdot \left\{ \frac{e_2}{e_1} (-L_3 + L_6)^{1-n_1/n_2} + L_3 \right\} \]
The geometric conditions in Fig. 2 can be represented by:

\[ Z_1(L_2) = Z_2(L_2) = -H_1 + L_2 \cdot \tan \alpha \]  
\[ Z_3(L_3) = Z_4(L_3) = -H_1 - L_3 \cdot \tan \alpha \]  

By substituting Eqs. (78)-(81) into Eqs. (89) and (90), integral constants \( k_1, k_2, k_3 \) and \( k_4 \) can be acquired:

\[ k_1 = -H_1 + L_2 \cdot \tan \alpha + A_1 \frac{1}{n_1} \left[ \frac{(1-k_v) \gamma_1}{P_a} \right]^{1-n_1} \left[ -L_2 + \frac{i_1}{(1-k_v) \gamma_1} \right]^{1} \]  
\[ k_2 = -H_1 + L_2 \cdot \tan \alpha + A_2 \frac{1}{n_2} \left[ \frac{(1-k_v) \gamma_2}{P_a} \right]^{1-n_2} \left[ -L_2 + \frac{i_2}{(1-k_v) \gamma_2} \right]^{1} \]  
\[ k_3 = -H_1 - L_3 \cdot \tan \alpha + A_1 \frac{1}{n_1} \left[ \frac{(1-k_v) \gamma_1}{P_a} \right]^{1-n_1} \left[ -L_3 + \frac{i_3}{(1-k_v) \gamma_1} \right]^{1} \]  
\[ k_4 = -H_1 - L_3 \cdot \tan \alpha + A_2 \frac{1}{n_2} \left[ \frac{(1-k_v) \gamma_2}{P_a} \right]^{1-n_2} \left[ -L_3 + \frac{i_4}{(1-k_v) \gamma_2} \right]^{1} \]

Based on Section 4.1, the integral constants in circular tunnels are:

\[ Z_1(L_4) = -H_2 + \sqrt{R^2 - L_4^2} = -A_1 \frac{1}{n_1} \left[ \frac{(1-k_v) \gamma_1}{P_a} \right]^{1-n_1} \left[ -L_4 + \frac{i_1}{(1-k_v) \gamma_1} \right]^{1} + k_1 \]  
\[ Z_3(L_4) = -H_2 + \sqrt{R^2 - L_4^2} = -A_1 \frac{1}{n_1} \left[ \frac{(1-k_v) \gamma_1}{P_a} \right]^{1-n_1} \left[ -L_4 + \frac{i_3}{(1-k_v) \gamma_1} \right]^{1} + k_3 \]

\[ Z_2(L_5) = Z_4(L_6) = 0 \]

Integral constants \( k_2 \) and \( k_4 \) are determined by Eqs. (79), (81) and (97).

\[ k_2 = k_4 = 0 \]

Then, the expressions of functionals in Eqs. (72) and (73) can be written as follows:

\[ U_1[Z_1(x), Z_1'(x), x] = \left\{ \frac{1}{n_1} \cdot A_1 \frac{1}{n_1} \left[ \frac{(1-k_v) \gamma_1}{P_a} \right]^{1-n_1} \left[ -x + \frac{i_1}{(1-k_v) \gamma_1} \right]^{1} + T_1 \right\} P_a + (1-k_v) \gamma k_1 \]  
\[ U_2[Z_2(x), Z_2'(x), x] = \left\{ \frac{1}{n_2} \cdot A_2 \frac{1}{n_2} \left[ \frac{(1-k_v) \gamma_2}{P_a} \right]^{1-n_2} \left[ -x + \frac{i_2}{(1-k_v) \gamma_2} \right]^{1} + T_2 \right\} P_a \]
Ultimately, the equations that can be used to solve the failure range of the passive mechanism are derived by the same way described in Section 4.1.

\[
U_3[Z_3(x), Z_3'(x), x] = \left\{ \frac{1}{n_1} A_{n_1}^{-\frac{1}{n_1}} \left[ \frac{(1-k_s)^{\gamma_1}}{P_a} \right]^{\frac{1}{n_1}} \left[ -x + \frac{i_1}{(1-k_s)^{\gamma_1}} \right]^{\frac{1}{n_1}} + T_1 \right\} P_a + (1-k_s)\gamma_1k_3 \tag{101}
\]

\[
U_4[Z_4(x), Z_4'(x), x] = \left\{ \frac{1}{n_2} A_{n_2}^{-\frac{1}{n_2}} \left[ \frac{(1-k_s)^{\gamma_2}}{P_a} \right]^{\frac{1}{n_2}} \left[ -x + \frac{i_2}{(1-k_s)^{\gamma_2}} \right]^{\frac{1}{n_2}} + T_2 \right\} P_a \tag{102}
\]

Eqs. (95), (96), (98), (103) and (104) form a system of equations for solving the failure range. The rates of work done by the weight of the failure block in the passive mechanism are

\[
W_i = \gamma_i \frac{n_i}{1+n_i} A_{n_i}^{-\frac{1}{n_i}} \left[ \frac{(1-k_s)^{\gamma_i}}{P_a} \right]^{\frac{1}{n_i}} \left[ -x + \frac{i_1}{(1-k_s)^{\gamma_i}} \right]^{\frac{1}{n_i}} \left[ -x + \frac{i_2}{(1-k_s)^{\gamma_i}} \right]^{\frac{1}{n_i}} + \frac{i_1}{(1-k_s)^{\gamma_i}} \left[ -x + \frac{i_2}{(1-k_s)^{\gamma_i}} \right]^{\frac{1}{n_i}} \tag{105}
\]
To examine the active and passive failure mechanisms of shallow tunnels with changing water table, Zhang et al. \cite{1} adopted the nonlinear yield criterion shown in Eq.(107), which is different from the yield criterion proposed by Baker \cite{30}.

\[
\tau_n = C_0 \left(1 + \frac{\sigma_n}{\sigma_t}\right)^{\frac{1}{m}} \tag{107}
\]

where \(m\) is the nonlinear coefficient; \(C_0\) denotes the initial cohesion; \(\sigma_t\) represents the axial tension; \(\sigma_n\) and \(\tau_n\) indicate the normal stress and shear stress, respectively \cite{31}. Eq.(1) can be converted to Eq.(107), when \(n = 1/m, C_0 = P_a A T^n\) and \(\sigma_t = P_a T\).

In order to verify the effectiveness of this method, it is assumed that the pore pressure coefficient \(r_u = 0\) in Ref.\cite{1}, the vertical seismic coefficient \(k_v = 0\) and the dip angle \(\alpha = 0^\circ\) and \(20^\circ\) in the present work, .

The calculation results are shown in Figs.3 and 4. Other parameters required for calculation are presented in Table 1.

### Table 1

<table>
<thead>
<tr>
<th>Parameters in Active and Passive Mechanisms</th>
<th>(T_1) (MPa)</th>
<th>(\gamma_1) (kN/m³)</th>
<th>(T_2) (MPa)</th>
<th>(\gamma_2) (kN/m³)</th>
<th>(q) (kPa)</th>
<th>(\sigma_n) (m)</th>
<th>(H_1) (m)</th>
<th>(H_2) (m)</th>
<th>(R) (m)</th>
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<td>Active Mechanism</td>
<td>0.01</td>
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<td>0.012</td>
<td>20</td>
<td>0</td>
<td>100</td>
<td>3</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Passive Mechanism</td>
<td>0.01</td>
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<td>0.012</td>
<td>20</td>
<td>500</td>
<td>100</td>
<td>3</td>
<td>10</td>
<td>6</td>
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</table>
Fig. 3. Comparison of the failure ranges in the active mechanism: (a) different values of $A_1(A_2)$; (b) different values of $n_1(n_2)$

Fig. 4. Comparison of the failure ranges in the passive mechanism: (a) different values of $A_1(A_2)$; (b) different values of $n_1(n_2)$

It can be seen from Figs. 3 and 4 that when the dip angle of the rock stratum is equal to 0°, two different methods achieve the same failure range solutions in this paper and Ref. [1]. Moreover, the failure range is symmetrical with respect to the Z-axis. However, when the dip angle of the rock stratum is equal to 20°, the failure ranges of both active and passive mechanisms are asymmetric. The variation trend of failure range solutions with $A_1(A_2)$ and $n_1(n_2)$ values when the dip angle is 0° is the same as that when the dip angle is 20°. Therefore, the method proposed in the present work can be used to analyze the asymmetric failure mechanisms of shallow tunnels and effectively predict the failure range.
6. Analysis results

6.1 Active mechanism

Table 2

Upper Bound Solutions for the Lower Stratum under Varying Parameters in the Active Mechanism

<table>
<thead>
<tr>
<th>$A_1$</th>
<th>$n_1$</th>
<th>$T_1$ (MPa)</th>
<th>$L_1$ (m)</th>
<th>$L_2$ (m)</th>
<th>$L_3$ (m)</th>
<th>$L_4$ (m)</th>
<th>$L_5$ (m)</th>
<th>$L_6$ (m)</th>
<th>$W_1$ (kN)</th>
<th>$W_2$ (kN)</th>
<th>$W_{12}$ (kN)</th>
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($A_2=0.2$, $n_2=0.9$, $T_2=0.012$, $\gamma_2=20$kN/m$^3$, $R=6$m, $a=10^6$, $H_1=3$m, $H_2=10$m, $q=10$kPa, $\sigma_c=100$kPa, $k_c=-0.2$)

The upper bound solutions for the range and weight of the failure block are used to evaluate the influence of changing parameters on the active failure in circular tunnels. Analytical solutions under mechanical parameters of different strata are worked out (Tables 2 and 3). The width and weight of the failure block increase as mechanical parameters $A_1$, $T_1$, $A_2$ and $T_2$ raise, but decrease as mechanical parameters $\gamma_1$ and $n_2$ elevate. When the parameter $n_1$ enlarges from 0.5 to 0.9, the width and weight of the failure block increase. However, the range and weight of the failure block decrease when the Mohr envelope is linear ($n_1=1$). With the rising weight per unit volume $\gamma_2$ in the upper stratum, the width of the failure block increases while the weight decreases.
in the study of asymmetric active failure mechanisms in shallow tunnels.

Thus, the effect of the vertical seismic force should be taken into account and leads to the stability of the tunnel roof. Thus, the effect of the vertical seismic force should be taken into account.

Thus, the effect of the vertical seismic force should be taken into account.

Besides, the enlarging vertical seismic coefficient $k_v$ leads to the increasing width and weight of the failure block (Fig.6). However, a large width and weight are adverse to the stability of the tunnel roof. Thus, the effect of the vertical seismic force should be taken into account in the study of asymmetric active failure mechanisms in shallow tunnels.

### Table 3

Upper Bound Solutions for the Upper Stratum under Varying Parameters in the Active Mechanism

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| 0.2   | 0.9   | 0.012       | 24          | 1.44      | 1.23      | 0.67      | 1.35      | 1.22      | 0.57      | 95.88     | 93.40     | 189.28        

(A_1=0.2, B_1=0.8, T_3=0.01, \gamma_1=23kN/m^3, R=6m, \alpha=10^6, H_1=3m, H_2=10m, \sigma=10kPa, \sigma_v=100kPa, k_v=-0.2)
Fig. 5. Effects of the dip angle $\alpha$ on the asymmetric active mechanism: (a) width and (b) weight of the failure block.

Fig. 6. Effects of the vertical seismic coefficient $k_v$ on the asymmetric active mechanism: (a) width and (b) weight of the failure block.

6.2 Passive mechanism

Tables 4 and 5 summarize analytical solutions for upper and lower strata under different parameters, respectively. The width and weight of the failure block increase as parameters $A_1, n_1, T_1, A_2$ and $T_2$ increase and parameter $n_2$ decreases. Increasing the weight per unit volume will reduce the width and raise the weight of the passive failure block.

**Table 4**

Upper Bound Solutions for the Lower Stratum under Varying Parameters in the Passive Mechanism

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(A_2=0.2, n_2=0.9, T_2=0.012, γ_2=20kN/m³, R=6m, α=10°, H_1=3m, H_2=12m, q=500kPa, σ_σ=100kPa, k_σ=-0.2)
In Fig. 7, as the dip angle $\alpha$ enlarges, widths $L_1$, $L_2$ and $L_3$ of the passive failure block increase while widths $L_4$, $L_5$ and $L_6$ decrease. The weight of the passive failure block increases constantly with the increasing dip angle $\alpha$. Similarly, the passive failure mechanism is symmetrical with respect to the z-axis only if $\alpha = 0$. Therefore, the dip angle $\alpha$ has a notable effect not only on the symmetry of the passive failure mechanism, but also on the range and weight of the passive failure block. The width and weight of the failure block in the passive mechanism decrease with the increase of the vertical seismic coefficient $k_v$ (Fig. 8). This change pattern is dissimilar to that in the active mechanism.

Fig. 7. Effects of the dip angle $\alpha$ on the asymmetric passive mechanism: (a) width and (b) weight of the failure block.
7. Conclusions

When analyzing the stability of the tunnel roof, collapse (active mechanism) caused by insufficient supporting pressure and blowout (passive mechanism) resulting from excessive supporting pressure are two essential phenomena to be examined. The asymmetric active and passive failure mechanisms under the action of the vertical seismic force are described by four curves. Based on the generalized nonlinear yield criterion and the upper bound theorem of limit analysis, the expressions of detaching curves for describing both active and passive failure mechanisms and the equations for calculating the range and weight of the failure block are derived by the variational method. The upper bound solutions for the width and weight of the active and passive failure blocks under different parameters are discussed. It should be highlighted that the dip angle of the inclined stratum ($\alpha \neq 0$) has a decisive effect on the symmetry of the failure block in shallow circular tunnels. The active and passive failure blocks are symmetrical with respect to the $z$-axis only if $\alpha = 0$. With the rising vertical seismic coefficient $k_v$, the width and weight of the failure block increase in the active mechanism, but decrease in the passive mechanism. In this work, the asymmetric failure is more realistic than the traditional symmetrical failure, and the research on asymmetric failure mechanisms can provide better reference for improving practical tunnel roof stability in practice.

Declaration of Competing Interest

The authors declare that they have no known competing economic interests or personal relationships that may affect the work reported in this paper.
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Data availability statement

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.
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