The minimal interaction induced by the translation subgroup has a gap and possible relates to the strong fundamental interaction.

Alexander Braginsky (a.braginsky@mail.ru)
Uznyj Federal'nyj Universitet https://orcid.org/0000-0001-5626-0248

Research Article

Keywords: Low-symmetric State, Compensating Field, Distortion Tensor, Cooper Pair, Electron-phonon Interaction

Posted Date: February 26th, 2021

DOI: https://doi.org/10.21203/rs.3.rs-216905/v1

The minimal interaction induced by the translation subgroup has a gap and possible relates to the strong fundamental interaction.

Alexander Yakovlevich Braginsky
UFU, Institute of Physics, 344090, Rostov-on-Don, 194 Stachki.
e-mail: a.braginsky@mail.ru

Abstract

The paper investigates the low-symmetric state of the compensating field of the distortion tensor and proves that there is a gap in this state. It is shown that the distortion tensor is the compensating field of the minimal interaction induced by the translation subgroup. On the example of electron pairing in a Cooper pair it was proved that the distortion tensor is responsible for the electron-phonon interaction. In this paper, for the first time, an exact wave solution for sound pressure in a continuous medium is obtained from the equations of state for the distortion tensor. It is shown that the sound is described as "massive" wave of the distortion tensor, the spectrum of which has the minimal frequency, which corresponds to a gap. The presence of a gap in the low-symmetric state gives grounds to believe that the distortion tensor, as a compensating interaction field, describes a strong fundamental interaction. As it is known, the description of the gap in the strong fundamental interaction is declared a Millennium problem by the Clay Mathematical Institute (CMI).

Introduction

In 2000 at the International Mathematical Congress, the Millennium problem was formulated: the description of the gap of strong fundamental interaction. It is known that there is a gap in the strong fundamental interaction, but there is no description of it. The solution of this problem was assumed in the framework of the Yang-Mills model [1, 2]. However, in the description of the problem was stressed that to obtain a gap for Yang-Mills is impossible, since the non Abelian gauge group of symmetry leads to nonlinear self-interaction Yang-Mills in low-symmetric condition and the absence of wave solutions for the fields of Yang-Mills.

On the other hand, in [3] a model was constructed with a tensor compensating field of minimal interaction and an Abelian gauge group, which was induced by a commutative subgroup of spatial translations. Therefore, there is a reason to believe that in such a model a gap in the low-symmetric state will be obtained, since in Abelian models there is no self-action in the low-symmetric state. We must show that this description corresponds to a strong fundamental interaction.

The minimal interaction in the field theory is understood as the interaction, which is given by the extended derivative with the compensating interaction field [4]. The construction of a minimal interaction in the form of the extended derivative with a compensating interaction field was firstly proposed by Pauli for an electromagnetic field [5]. The existence of the extended derivative in electrodynamics was justified by the invariance of the lagrangian with respect to the local Abelian gauge group of symmetry. The gauge symmetry of the lagrangian [4, 5] has been associated with a quantum mechanical description using a complex wave function that has the uncertainty in the phase.

On the other hand, in [3] the lagrangian gauge group was induced by a local irreducible representation of a subgroup of spatial translations. In [6] it was proved that the charge in the elongated derivative of the minimal interaction induced by the translation subgroup [3] is the wave vector or quantum pulse, and the compensating field is the distortion tensor: A_{ij}.

As it is known, the distortion tensor was first defined in the theory of elasticity [7]. It specifies the dislocation density and is defined up to the gradient of the displacement vector. Since in [6] it was shown that the distortion tensor A_{ij} like an interaction field, as well as the stress tensor, which conjugated to it, exists not only in a solid, but wherever there is momentum and spatial translations.

Thus, the distortion tensor is not always related to the density of dislocations in a solid. Rather, on the contrary, under certain conditions it does indeed describe dislocations in a solid [6], but, in general, it is responsible for the interaction. Quantization of the Burgers vector in a solid is analogous to quantization of the magnetic flux in Abrikosov vortices [8]. The magnetic flux, as it is known, is not always quantized.

The law of conservation of momentum is associated with spatial translational symmetry. Therefore, in this model, the quantum momentum is realized as a coefficient in front of the compensating field: the distortion tensor A_{ij} in the extended derivative [6], as well as an electric charge for the electromagnetic interaction.

In [9] it was shown that the attraction of equal and opposite-directed quantum momenta leads to the pairing of electrons in the superconducting state [10]. In [9] it was proved that when the coherence length of a Cooper pair of electrons is less than a micrometer, the attraction of opposite and equal quantum momenta of electrons becomes greater than their natural electrical repulsion. The existence of Cooper pairs, in fact, is proof of the attraction of quantum momenta as charges.

In [10] this interaction of electrons was called electron-phonon interaction due to the fact that acoustic phonons were observed in the formation of Cooper pairs. Therefore, in the theory of BCS, electron pairing was associated with phonons, hence the name electron-phonon interaction. So we have the following knowledge.

Firstly, the quantum momentum behaves as a charge of minimal interaction [6,9]. It sets the minimal interaction as the extended derivative, analogous to the electric charge for the electromagnetic fundamental interaction [4, 5].

Secondly, the compensating field of this interaction is the distortion tensor: A_{ij}, which was firstly introduced in the theory of elasticity [7] to describe plastic deformations with dislocations, as a generalization of the strain tensor at the destruction of a continuous elastic medium.

Thirdly, according to the BCS theory [10], sound waves are associated with this interaction.

The question is what fundamental interaction defines the quantum momentum and the tensor of distortion.

The paper shows that this interaction has a gap in the low-symmetric state. Therefore, it is likely that it corresponds to a strong fundamental interaction, which, as it is well known from the CMI Millennium problem, has a gap in the low-symmetric state.

1. The problem statement.

Heuristically, it is clear that at small distances, for small wavelengths of particles, a quantum momentum, as a charge, describes a very strong interaction, since the magnitude of the quantum momentum depends directly on the wavelength. From where it becomes clear why the smaller an elementary particle is the more energy is required to destroy it. Therefore, for the study of elementary particles large hadron colliders are built which allow to achieve very high energies.

Note that the charge in the Yang-Mills theory is fixed and does not depend on the characteristic sizes of elementary particles. Therefore, the Yang-Mills theory, in principle, can't answer the question why the smaller the particle, the more energy is required to destroy it. After all, if you follow the usual logic, then everything should be the opposite, the smaller the particle, the easier it is to destroy it.

Obviously, just as it was possible to describe the attraction of electrons in the Cooper pair [9], it is possible to describe the attraction of two protons with equal and opposite-directed quantum momenta. Therefore, we can expect that with the help of this formalism it will be possible to describe the attraction of protons and neutrons in the nucleus of an atom as the attraction of their quantum momenta. As you know, initially the strong fundamental interaction was found in the nucleus of an atom. In this case, the problem of describing the attraction of nucleons in the nucleus of an atom in general is three-dimensional and not one-dimensional, as for paired electrons, with a total quantum momentum equal to zero. However, the solution of this problem: the attraction of nucleons in the nucleus of an atom is beyond the scope of this article.

The plan to prove the fact that the quantum momentum can be the charge of the strong fundamental interaction will be based on the description of the gap of the strong fundamental interaction, declared as a Millennium problem in 2000.

As is known, a gap is a characteristic of a low-symmetric state in which the gauge or gradient symmetry of the equations of state for the compensating interaction field is broken. The characteristic feature of the gap is the minimal frequency of the waves of the compensating interaction field in the low-symmetric state. Therefore, it is necessary to prove that the waves of the distortion tensor have a minimal frequency in the low-symmetric state, when the gradient symmetry of the equations of state is broken.

In the future, we will call waves with minimal frequency "massive", by analogy with the solutions of the Klein-Gordon equation for a massive scalar field [4].

It is known that the Yang-Mills fields have self-action in the low-symmetric state, and, in case of broken gauge symmetry, their equations of the state are nonlinear. Therefore, they do not have wave solutions and can't describe the gap.

In connection with the Millennium problem, a natural question arises, what interactions have a gap in the low-symmetric state?

Firstly, it is the electromagnetic interaction. It is known that the gap exists in a low-symmetric state of the electromagnetic field, which occurs when the calibration or gradient symmetry of the Maxwell equations is broken [11].

In case of broken gauge symmetry of the electromagnetic field the superconducting state is formed, which are described the London equations $A_i = -\delta'^2 j_i$ [11, 12], here A_i - electromagnetic potential, j_i - current density, δ' - depth of magnetic field penetration in the superconductor.

As you know, London was able to describe the Meissner effect, the ejection of a magnetic field from a superconductor. It is obvious that when the London equations are performed, the gradient symmetry of the Maxwell equations is broken. In this case, electromagnetic waves in the superconducting state satisfy the inhomogeneous d'Alembert equation, which has wave solutions with a minimal frequency.

The minimal frequency is related to the depth of penetration of the magnetic field into the superconductor by the simple relation: $\omega'_0\delta' = c'$, where c' is the velocity of electromagnetic waves in the superconductor. Thus, the minimal frequency of electromagnetic waves in the superconducting state is an indicator of broken gauge symmetry in the low-symmetric state of the electromagnetic field.

In this example, physical interaction models containing a gap or a minimal frequency in a low-symmetric compensating field state end in the field theory.

Note that the term of gauge symmetry is commonly used in field theory along with gradient symmetry. In field theory, it is assumed that the abstract gauge symmetry of the lagrangian sets the minimal interaction [2, 4, 5]. Since in [3] it was possible to construct the expanded derivative induced by an ordinary translation subgroup, then, in our opinion, there is no reason to use an abstract gauge group of internal symmetries of the lagrangian. In p. 3. [13] it was shown that the Abelian symmetry calibration group for electromagnetic interaction can be interpreted as a local irreducible representation of the time translation subgroup. It is not necessary to postulate that the electromagnetic potential changes sign during inversion of time, as it is done in the field theory [2, 4].

Thus, one can get away from abstract local gauge groups of internal lagrangian symmetries and to construct an expanded derivative, use local representations of the global subgroup of space-time translations. In this case, the equations of state for the compensating fields will have gradient symmetry which is also sometimes called gauge symmetry.

In the future, the name gradient symmetry will be used in relation to equations of state, and the name gauge symmetry will be used in relation to abstract models of field theory not related to space-time symmetry, for example for Yang-Mills fields.

As noted above, there is a very important aspect in the theory of BCS [10]. In it, electron pairing was associated with electron-phonon interaction. In [9] it was shown that lattice oscillations have no relation to the attraction of electrons. The attraction of electrons is due to the interaction of their quantum momenta with each other using the distortion tensor, which is the compensating interaction field in the expanded derivative as well as the electromagnetic potential.

The observed acoustic waves in the formation of Cooper pairs give reason to believe that the waves of the distortion tensor in a continuous elastic medium describe the sound. After all, the distortion tensor was originally introduced in the theory of elasticity as a generalization of the strain tensor [7].

It is obvious that the sound exists in a continuous medium with density: ρ . Therefore, it is necessary to investigate the equations of state for the compensating field: the distortion tensor, in a continuous medium, and make sure that they describe the sound waves.

In this paper, for the first time, an exact wave solution for sound pressure in a continuous elastic medium is obtained in the form: $p = p_0 \exp(-i\omega(t - x/c))$, where c is the speed of the sound, and p_0 is the amplitude of the pressure in the sound wave. This solution is derived from the equation of state for the distortion tensor in a continuous medium when the momentum is proportional to the velocity field: $p_i = \rho \upsilon_i$. The equations of state for the distortion tensor contain a quadratic d'Alembert operator due to the gradient invariance of the equations of state derived from the action minimum.

As it is known, in gases and liquids sound waves are described by Euler's equations of hydrodynamics [14]. However, this description does isn't true.

Firstly, the Euler's equations are nonlinear and have no wave solutions. This is easily seen, if we substitute the wave solution to the equations of Euler. Therefore, in order to obtain wave solutions the Euler's equation is linearized. I. e., neglect nonlinear terms in the Euler equations. Obviously, this cannot be done, since the nonlinear potential term in Euler's equations is responsible for the kinetic energy of the continuum medium in Bernoulli's equation. Therefore, it is responsible for the kinetic energy of the mechanical wave.

Secondly, the linearized Euler equations do not contain the quadratic operator d'Alembert, as the Euler equation is linear in derivatives. Therefore, to obtain wave equations with the d'Alembert operator, velocity and pressure are usually defined as derivatives of the scalar potential function φ : $\upsilon_i = \partial\varphi / \partial x_i$, $p = -\rho\partial\varphi / \partial t$ [14]. However, the wave solutions obtained in this way contain a linear dependence of the pressure from the frequency: $p = \omega\rho \operatorname{Re}(i\varphi_0 \exp(-i\omega(t - x/c)))$, see pp. 351-354 [14].

It is obvious that these solutions, in general, do not correspond to sound waves, since in sound waves the frequency does not depend on the pressure. This is well known and used in extracting sound from all musical instruments. For example, "forte" and "piano" makes sense "louder" and "quieter". It is known that the frequency of sound does not change when pressing the same keys in a piano with different pressures.

Since in a continuous elastic medium the velocity field υ_i and the distortion tensor A_{ij} are proportional to the conjugate observed fields: the momentum p_i and the stress tensor σ_{ij} accordingly, then the equations of the state for compensating fields υ_i, A_{ij} [6] in a continuous elastic medium are inhomogeneous d'Alembert equations. Therefore, for the sound in a continuous elastic medium, not ordinary mechanical waves are responsible, but "massive" waves of the distortion tensor, which have a spectrum with a minimal frequency.

The continuous medium for the distortion tensor is a low-symmetric state in which the gradient symmetry of the state equations is broken when the distortion tensor becomes observable and proportional to the stress tensor. In fact, this relation can be seen as a generalization of Hooke's law for the distortion tensor, which holds exactly in a continuous elastic medium. After all, when the continuous medium ceases to be elastic, the distortion tensor ceases to be proportional to the stress tensor, and describes plastic deformations with dislocations [7]. In this case, the solid elastic medium is destroyed, and the distortion tensor justifies its name as the destruction tensor.

Thus, the theory where charge is the quantum momentum, and compensating field is the distortion tensor [6] describes a gap in a low-symmetric state in the continuous elastic medium. As you know, the Clay Mathematical Institute (CMI) declared the description of the gap of the strong fundamental interaction-the problem of the Millennium. Therefore, there is a reason to believe that the "massive" waves of the distortion tensor in a continuous elastic medium describe the gap of the strong fundamental interaction and the quantum momentum as the charge, sets the strong fundamental interaction.

This conclusion is based on the fact that until now the gap has been described only in the low-symmetric state of the electromagnetic field [11], and for the strong fundamental interaction the gap has not been described until now. At the same time, there are no and can't be other gaps associated with the symmetry of space-time.

Indeed, since the expanded derivative of the minimal interaction [6] is associated with a subgroup of spatial translations, and the expanded derivative of the electromagnetic interaction is associated with temporal translations [13], there can be no other gaps, based on the translational symmetry of space-time.

It is easy to see that there are no other nonequivalent local irreducible representations other than those of the translation subgroup suitable for constructing an expanded derivative of minimal interaction.

Assumptions that fundamental interactions in nature can be induced by abstract local gauge groups of internal lagrangian symmetries unrelated to space-time symmetry are, in our opinion, not substantiated and unlikely.

And now let's give a mathematical justification for the above.

2. The equation of state for the compensating field of interaction: υ_i, A_{ij}.

In the article [6] the minimal interaction in the form of an expanded derivative was recorded:

$$D_j\psi_{\vec{k}} = \left(\frac{\partial}{\partial x_j} - i\sum_p \kappa_p A_{pj}\right)\psi_{\vec{k}} . \tag{1}$$

Where the wave vector κ_p is the coefficient before the compensating field A_{pj} is the distortion tensor [6]. Here $\psi_{\vec{k}}$ is the wave function (or order parameter [3]) which is transformed by the local irreducible representation of the translation subgroup: $\hat{a}_q \psi_{\vec{k}} = \exp(i\delta_{pq} k_p a_q)\psi_{\vec{k}}$, where $k_p = k_p(x_j)$, and A_{pj} is transformed by:

$$\hat{a}_q(\kappa_p A_{pj}) = \kappa_p A_{pj} + \delta_{pq}\,\partial(k_p a_q)/\partial x_j \,. \tag{2}$$

Then the extended derivative (1) is the eigenfunction of the translation operator:

$\hat{a}_q(D_j \psi_{\vec{k}}) = \exp(i\delta_{pq} k_p a_q) D_j \psi_{\vec{k}}$.

Similarly, the velocity field υ_i compensates for the time derivative $\psi_{\vec{k}}$:

$$D_0 \psi_{\vec{k}} = \left(\frac{\partial}{\partial t} - i\sum_n \kappa_n \upsilon_n\right)\psi_{\vec{k}}\,, \tag{3}$$

$$\hat{a}_q(\kappa_i \upsilon_i) = \kappa_i \upsilon_i + \delta_{iq}\,\partial(k_i a_q)/\partial t \,. \tag{4}$$

The equations of state for the compensating interaction field: υ_i, A_{ij}, or the 4-distortion tensor [15, 16] induced by the quantum momentum as the charge [6,9] have the form:

$$p_i = -\frac{\gamma}{c^2}\frac{\partial \varepsilon_{ij}}{\partial x_j}\,, \tag{5}$$

$$\sigma_{ij} = \gamma\, e_{jkp}\frac{\partial \rho_{ip}}{\partial x_k} - \frac{\gamma}{c^2}\frac{\partial \varepsilon_{ij}}{\partial t}\,, \tag{6}$$

where

$$\varepsilon_{ij} = -\frac{\partial \upsilon_i}{\partial x_j} + \frac{\partial A_{ij}}{\partial t} \tag{7}$$

the centrally symmetric tension of the compensating fields υ_i, A_{ij}, and

$$\rho_{ip} = -e_{pkn}\frac{\partial A_{in}}{\partial x_k}\,, \tag{8}$$

vortex tension of the distortion tensor A_{ij}. Here γ - the dimensional coefficient, e_{jkn} - the anti-symmetric Levi-Civita tensor.

These equations of state (5, 6) can be obtained by variation of the lagrangian: $\delta L/\delta\upsilon_i = 0$, $\delta L/\delta A_{ij} = 0$, for the 4-distortion tensor υ_i, A_{ij} [15, 16]:

$$L = p_i \upsilon_i - \sigma_{ij} A_{ij} + \frac{\gamma}{2}\left(\frac{1}{c^2}\varepsilon_{ij}\varepsilon_{ij} - \rho_{ij}\rho_{ij}\right), \tag{9}$$

where $p_i = \partial L/\partial\upsilon_i$, $\sigma_{ij} = -\partial L/\partial A_{ij}$. This conclusion is analogous to the conclusion of Maxwell's equations from the lagrangian of the electromagnetic field in the field theory [17].

The lagrangian (9) is a consequence of the lagrangian $L = L(\psi_{\vec{k}}, \upsilon_i, A_{ij})$, where the momentum p_i and the stress tensor σ_{ij} are the first integrals dependent on the fields $\psi_{\vec{k}}, \upsilon_i, A_{ij}$, according to E. Noether's theorem [9]. However, when the momentum p_i and stress tensor σ_{ij} are external sources for the fields υ_i, A_{ij}, the lagrangian has the form (9).

From (5, 6) by direct differentiation follows the continuity equation.This is the law of conservation of momentum in differential form:

$$\frac{\partial \sigma_{ij}}{\partial x_j} = \frac{\partial p_i}{\partial t} . \tag{10}$$

To obtain wave equations with the d'Alembert operator, as in the field theory [17], we use the pseudo Lorentz calibration condition [15, 16] for fields υ_i, A_{ij}.

$$\frac{\partial A_{ij}}{\partial x_j} = c^{-2} \frac{\partial \upsilon_i}{\partial t} . \tag{11}$$

Substituting (7, 8) in (5, 6), taking into account the condition (11), we obtain the equations of state in the form:

$$\frac{c^2}{\gamma} p_i = (\frac{\partial^2}{\partial x_n \partial x_n} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \upsilon_i , \tag{12}$$

$$\frac{1}{\gamma} \sigma_{ij} = (\frac{\partial^2}{\partial x_n \partial x_n} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) A_{ij} . \tag{13}$$

The equations of state (12, 13) for interaction fields: υ_i, A_{ij}, contain the d'Alembert operator. This is a consequence of the gradient invariance of the equations of the state (5-8). It follows from (12, 13) that the source of the interaction fields: υ_i, A_{ij}, are conjugate observable fields: p_i and σ_{ij}. According to the continuity equation (10), the stress tensor σ_{ij} is the momentum flow with a minus sign: $\sigma_{ij} = -p_i v_j$, where v_j is the flow rate. Thus, the equations (12, 13) are similar to Maxwell's equations [17], where the source of the electromagnetic field is the charge and the current density.

3. "Massive" waves of the distortion tensor in a continuous elastic medium.

As it is known, in the isotropic continuous medium with density ρ, there is the directly proportional relationship between momentum and velocity: $p_i = \rho \upsilon_i$ (the difference between a quantum momentum and the conventional momentum is shown below, in paragraph 5). Then the equations of state (12, 13) for the fields υ_i, A_{ij} in a continuous medium will have the form of the inhomogeneous wave equation:

$$\frac{\rho c^2}{\gamma} \upsilon_i = (\frac{\partial^2}{\partial x_n \partial x_n} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \upsilon_i , \tag{14}$$

$$\frac{\rho c^2}{\gamma} A_{ij} = (\frac{\partial^2}{\partial x_n \partial x_n} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) A_{ij} . \tag{15}$$

Indeed, equation (14) is obtained by direct substitution $p_i = \rho \upsilon_i$ in (12). It follows from the known relation between momentum and velocity in a continuous medium with density ρ.

Equation (15) is obtained from the relation $p_i = \rho \upsilon_i$ and two equations: the continuity equation (10) and the gauge condition (11), which also has the form of a continuity equation. It follows from the continuity equation (10) that the stress tensor can be represented as the momentum flow with a minus sign $\sigma_{ij} = -p_i v_j$. Similarly, it follows from equation (11) that the distortion tensor A_{ij} can be represented as the flow of the velocity field with a minus sign and a coefficient $1/c^2$: $A_{ij} = -c^{-2}\upsilon_i v_j$. Since in a continuous medium the velocity field coincides with the flow velocity: $\upsilon_i = v_j$, we obtain: $\sigma_{ij} = -\rho\upsilon_i\upsilon_j$, $A_{ij} = -c^{-2}\upsilon_i\upsilon_j$. It follows that in a continuous medium with density ρ:

$$\sigma_{ij} = \rho c^2 A_{ij}. \qquad (16)$$

This means that there is elasticity in a continuous medium, since the distortion tensor A_{ij} was originally introduced into elasticity theory as a generalization of the strain tensor [7]. From the relation: $\sigma_{ij} = \rho c^2 A_{ij}$, which has the form of Hooke's law for the distortion tensor, follows equation (15). Here the coefficient: $K = \rho c^2$, in fact, characterizes the elasticity of the continuous medium.

Thus, twelve equations of state (12, 13), for the compensating field of interaction: υ_i, A_{ij}, in a continuous isotropic medium turned into the same inhomogeneous d'Aalembert equation with the same spectrum. Let us say at once that this phenomenon is associated with the violation of the gradient symmetry of the equations of state (12, 13) and with the phase transition to the state of a continuous elastic medium, which will be discussed later in paragraph 5.

We call the equations (14,15) the "massive" wave equation due to the fact that it is similar to the Klein-Gordon equation for a massive scalar field [4], and due to the fact that the spectrum with the minimal frequency in the equations (14,15) is given by the density of the continuous medium ρ not equal to zero.

Indeed, the equations (14, 15) have solutions: : $\upsilon_i = \upsilon_{i0}\exp(i\vec{k}\vec{x} - i\omega t)$, $A_{ij} = A_{ij0}\exp(i\vec{k}\vec{x} - i\omega t)$, with spectrum: $\omega = c\sqrt{\vec{k}^2 + c^{-2}\omega_0^2}$ and minimal frequency:

$$\omega_0 = c^2\sqrt{\rho/\gamma}. \qquad (17)$$

The frequency ω_0 is also called the zero frequency, below which there can be no wave solutions for the equations of state (14, 15). Knowing the minimal frequency: $\omega_0 = c^2\sqrt{\rho/\gamma}$, you can calculate a dimensional constant $\gamma = \rho c^4/\omega_0^2$, whence its physical meaning follows.

We show that the waves of the 4-distortion tensor describe the sound. In the future, we will omit the 4-distortion tensor, and use the expression distortion tensor to denote a pair of fields: υ_i, A_{ij}.

The equations of state (5-8) by construction have gradient invariance:

$$A_{ij} \to A_{ij} + \partial u_i/\partial x_j,\ \upsilon_i \to \upsilon_i + \partial u_i/\partial t, \qquad (18)$$

where u_i is the displacement vector [7]. The displacement vector u_i satisfies the wave equation in a continuous medium. This follows from the pseudo Lorentz condition (11) when substituting a gradient transformation (18):

$$\frac{\partial^2 u_i}{\partial x_n \partial x_n} = \frac{1}{c^2}\frac{\partial^2 u_i}{\partial t^2} . \tag{19}$$

Thus, (19) describes mechanical waves in a continuous medium, and equations (14, 15) describe sound in a continuous medium. But sound, or distortion tensor waves, are not mechanical waves, since the spectrum of sound waves has a minimal frequency and is different from the spectrum of mechanical waves.

Indeed, mechanical waves can occur at any low frequency, and sound waves exist only when their frequency is higher than the minimal frequency. In this case, sound waves in a continuous medium coincide with mechanical waves. Since there can be only one displacement and one velocity in a continuous medium.

This, at first glance, unexpected effect is well known on the example of the low-symmetric state of the electromagnetic field, when the gradient symmetry of Maxwell's equations is broken: $A_i = -\delta^2 j_i$ [11, 12]. In the superconducting state, the charge waves and the electromagnetic field propagate together, with the electromagnetic field waves having a gap or minimal frequency. And charge waves can propagate with any arbitrarily small given frequency.

4. The exact wave solution for the sound pressure in gases and liquids.

Let's consider the propagation of waves of the distortion tensor in the gas or liquid. In gases and liquids the stress tensor has a symmetric diagonal form and depends on the pressure: $\sigma_{ij} = -\delta_{ij} p$. Then $A_{ij} = \beta\sigma_{ij}$, where $\beta = 1/\rho c^2$ is the compressibility of the continuous medium. Hence, the distortion tensor in gases and liquids has a symmetric form: $A_{ij} = -\beta\delta_{ij} p$.

Note also that here the density ρ is a constant. In this model ρ is the constant equilibrium density of the continuous medium, and the density in the sound wave from the coordinates, by construction, is set to field A_{ij} as an independent variable describing deformation of continuous medium. The expression $A_{ij} = -\beta\delta_{ij} p$ means that the distortion tensor is diagonal A_{ij} and its diagonal elements $A_{ij} = -\delta_{ij} A$ are proportional to the pressure: $\rho c^2 A = p$ or $A = \beta p$. The value determines A the deviation of the density from the equilibrium value: $\rho' = \rho A$. As is known, the deviation of pressure and density in the sound wave are related by the ratio $p = c^2\rho'$ [14].

Substitute $A_{ij} = -\beta\delta_{ij} p$ in (15), then get:

$$\frac{\rho c^2}{\gamma} p = (\frac{\partial^2}{\partial x_n \partial x_n} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}) p . \tag{20}$$

Equation (20) has an exact wave solution for pressure: $p = p_0 \exp(i\vec{k}\vec{x} - i\omega t)$.

Equation (20) has never been taken into account when describing a continuous elastic medium before, since the equations of state for the distortion tensor (5, 6), as an independent variable [6], describing the physical state, were not taken into account before.

Thus, for the first time an exact wave solution for sound pressure in a continuous elastic medium with density ρ (20) is obtained from the equation of state (6,13,15) without any approximations. These solutions are accurate and do not contain the dependence of the pressure amplitude on the frequency, as the solutions obtained from the Euler equations of hydrodynamics, see pp. 351-354 [14].

On the other hand, the spectrum of wave solutions of equation (20) contains a minimal frequency below which there are no sound waves. At first glance, this is an unexpected result. To make sure that the wave solutions (20) describe the sound, we calculate the energy density of the sound waves for the solutions obtained from equation (20).

According to lagrangian (9), the energy density of sound waves consists of the sum of the mechanical energy density given by the first two terms of the lagrangian and the energy density of the distortion tensor tensions.

The energy density of mechanical waves is known and equal, see $E = \beta p^2$ [14].

The energy density of the strains of the distortion tensor has the form:

$E = \frac{\gamma}{2}(c^{-2}\varepsilon_{ij}\varepsilon_{ij} + \rho_{ij}\rho_{ij})$ and is equal to: $E = \beta p^2 \frac{\omega^2}{\omega_0^2}$.

Indeed, the centrally symmetric tension ε_{ij} (7) in a continuous medium has the form $\varepsilon_{ij} = -\frac{\partial \upsilon_i}{\partial x_j} - \beta\delta_{ij}\frac{\partial p}{\partial t}$. Substituting in this expression the wave solution for longitudinal waves, and considering that the amplitude of pressure and velocity in a sound wave are related as $p = \rho c \upsilon$ [14], which follows from the continuity equation (10), we see that in the sound wave centrally symmetric tension (7) identically equal to zero: $\varepsilon_{ij} = 0$. Substituting wave solutions into the expression: $E = \frac{\gamma}{2}\rho_{ij}\rho_{ij}$, and given that $\gamma = \rho c^4 / \omega_0^2$, we obtain an expression $E = \beta p^2 \frac{\omega^2}{\omega_0^2}$ for the energy density of the distortion tensor in a continuous elastic medium.

Thus, the energy density of the sound wave is equal to the sum of the mechanical energy and the energy density of the distortion tensor tensions:

$$E = \beta p^2 + \beta p^2 \frac{\omega^2}{\omega_0^2}. \tag{21}$$

The fact that the energy density of the sound wave (21) contains the energy density of mechanical vibrations of a continuous medium is the expected result. The expected result is the dependence of the energy density of the distortion tensor on the square of the frequency, since it is known that the energy density of electromagnetic waves, for example, depends on the frequency squared. This follows from the fact that the lagrangian (9) is a quadratic invariant function of the tensions of the distortion tensor (7, 8).

A nontrivial result is the dependence of the energy density of the tensions of the distortion tensor: ε_{ij}, ρ_{ij}, in a continuous medium, on the pressure. This is equivalent to the dependence of the energy density of electromagnetic waves on the current density, which is observed only in the superconducting state when the gradient symmetry of the equations of state is broken.

The dependence of the energy density of the distortion tensor on the pressure in a continuous elastic medium makes it possible to compare the mechanical energy density and the energy density of the tensions of the distortion tensor (21). Two conclusions follow from expression (21).

Firstly, the minimal frequency in the expression for energy density (21) cannot be zero. Indeed, in a continuous medium with density ρ, the minimal frequency (17) is always more than zero.

Secondly, in the frequency range above the minimal frequency, for example by an order of magnitude, the energy density of mechanical vibrations in the sound wave can be neglected (21). This fact is reflected in the diagram equal volume Fletcher-Manson (F-M) [18], on which the sound volume is measured in decibels.

Indeed, when the frequency of sound waves increases, for example, by two orders of magnitude from 20 Hz to 2 kHz, the energy density of sound waves increases by four orders of magnitude, and the energy density of mechanical vibrations $E = \beta p^2$ - doesn't change. This explains the increase in volume by four orders of magnitude, or 40 dB for the same sound pressure at the transition frequency from 20 Hz to 2 kHz, according to diagram of F-M from which it follows that the energy density of the waves of the distortion tensor (21) according to the volume in figure F-M. This removes the existing contradiction between the description of sound by mechanical waves and psychoacoustics, which is reflected in the F-M diagram. A more detailed study of the F-M diagram is beyond the scope of this article and is given in the article [19].

Thus, the energy density of the sound wave is equal to the sum of the energy density of the mechanical wave and the energy density of the distortion tensor (21). The minimal frequency (17) in the spectrum of the sound wave $\omega = c\sqrt{\vec{k}^2 + c^{-2}\omega_0^2}$ indicates that the sound is not mechanical vibrations of a continuous medium. Since the spectrum of mechanical vibrations has the form: $\omega = ck$. Mechanical vibrations are possible with any low frequency, and sound waves occur only when the frequency of the sound source is higher than the minimal frequency (14, 15, 20).

The difference between sound waves and mechanical vibrations in a continuous medium is the same as the difference between electromagnetic waves and charge density waves in a superconductor. The sound waves (distortion tensor waves) and the mechanical waves in a continuous medium, as the electromagnetic waves and the charge density waves in a superconductor, propagate together at the same speed. Sound and mechanical waves cannot be separated, just as it is impossible to separate momentum from velocity and the stress tensor from the distortion tensor in a continuous elastic medium.

Note that this description of the sound is free from the disadvantages of describing the sound using Euler's equations of hydrodynamics [14].

Firstly, the equation of state (14, 15, 20) contain quadratic operator d'Alembert, in contrast to the Euler equations. Therefore, the expressions for the amplitude of the sound pressure and the velocity of the continuum in the sound wave do not depend on the frequency.

Secondly, in order to obtain an accurate wave solution, it is not necessary to neglect the nonlinear terms in the equation of motion of a continuous medium [20]:

$$\rho\frac{\partial \upsilon_j}{\partial t} = -\frac{\partial p}{\partial x_j} - \rho\upsilon_i\frac{\partial \upsilon_i}{\partial x_j} - \frac{\upsilon_j}{c^2}\frac{\partial p}{\partial t} + \frac{\upsilon_i\upsilon_i}{c^2}\frac{\partial p}{\partial x_j} - \frac{\upsilon_j\upsilon_i}{c^2}\frac{\partial p}{\partial x_i}, \qquad (22)$$

since equation (22) for sound wave solutions is identical.

Indeed, the vortex force $f_j^V = \frac{\upsilon_i\upsilon_i}{c^2}\frac{\partial p}{\partial x_j} - \frac{\upsilon_j\upsilon_i}{c^2}\frac{\partial p}{\partial x_i}$ in (22) is identically zero for the longitudinal sound wave.

The centrally symmetric force $f_j^C = -\rho\upsilon_i\frac{\partial \upsilon_i}{\partial x_j} - \frac{\upsilon_j}{c^2}\frac{\partial p}{\partial t}$ is also annulled, since, as shown above, the centrally symmetric tension: $\varepsilon_{ij} = -\frac{\partial \upsilon_i}{\partial x_j} - \beta\delta_{ij}\frac{\partial p}{\partial t}$, is zero.

The continuity equation (10) for the gas or liquid is as follows: $\rho \frac{\partial \upsilon_j}{\partial t} = -\frac{\partial p}{\partial x_j}$. It is performed identically, since in the sound wave the pressure and velocity amplitudes are related by the ratio: $p = \rho c \upsilon$ [14].

Thus, the equations of motion of a continuous elastic medium (22) [20] are performed identically for a sound wave. The same cannot be said about Euler's equations of hydrodynamics, since in Euler's equations the potential force $-\rho \upsilon_i \, \partial \upsilon_i / \partial x_j$ isn't zero.

The nonlinear terms are not annulled in Euler equations because the Euler equations lack a force component $-c^{-2} \upsilon_j \, \partial p / \partial t$ (22). This force is related to the gradient invariance (18) of the centrally symmetric tension (7) in the highly symmetric state. Being in a low-symmetric state of a continuous elastic medium, where there is no gradient symmetry, it is impossible to justify a centrally symmetric force or tension (7), and, therefore, it is impossible to construct correct equations of motion.

A critique of the derivation of Euler's hydrodynamics equations is given in [21]. In [21] it is shown that Euler's equations are invalid and don't have a valid mathematical conclusion. Obviously, it is impossible to neglect the potential force $-\rho \upsilon_i \, \partial \upsilon_i / \partial x_j$ in Euler's equations, which is associated with a change in the kinetic energy of a continuous medium, which is clearly seen, for example, in Bernoulli's equation: $\rho \upsilon^2 / 2 + p = const$.

Note that although in a sound wave the vortex force in (22) is zero, the vortex tension ρ_{ij} is not zero. It is the vortex tension that gives the main contribution to the energy density of the sound wave $E = \frac{\gamma}{2} \rho_{ij} \rho_{ij}$ (21), which is reflected in the F-M diagram.

However, the description of the sound is not the aim of this article. A more detailed description of the sound and the F-M diagram is given in [19]. The aim of this paper is to describe the gap of minimal interaction induced by the translation subgroup [6, 9].

5. The gap as an indicator of the phase transition to a low-symmetric state.

As it is known, the gap is observed in the low-symmetric state of the compensating interaction field. Higgs was the first to describe the phase transition to the low-symmetric state of the compensating interaction field in 1964 [22]. He associated this phase transition with the breach of the gauge symmetry of the minimal interaction when the compensating field becomes observable. We show that the gap is a consequence of such a phase transition, and that it occurs when the gradient symmetry of the equations of state for the distortion tensor is broken.

The gap or minimal frequency characterizes the low-symmetric state of the compensating interaction field. The appearance of a gap of a strong fundamental interaction is associated with the appearance of a mass. Everyone knows the phrase Higgs Boson, which has already become well known. According to the expression (12) for the minimal frequency of the "massive" wave of the distortion tensor: $\omega_0 = c^2 \sqrt{\rho / \gamma}$, this is indeed the case in a continuous elastic medium. Since the minimal frequency or gap exists only in the presence of a non-zero density ρ of the continuous medium. However, in this article we will not study this very interesting question of the origin of the mass of a continuous medium in the low-symmetric state of the compensating field of the distortion tensor.

The fact is that the minimal frequency of the compensating field in the low-symmetric state has very important information. It is an indicator of the phase transition. In our opinion, this is the main function of the gap. We will dwell in more detail on this function of the gap or the minimal frequency of the waves of the compensating field in the low-symmetric state.

It is possible that this is not the case for the Higgs phase transition. Since the Higgs phase transition [22] did not obtain a minimal frequency for compensating Yang-Mills fields in the low-symmetric state, due to their self-action. Therefore, the gap of strong fundamental interaction is still being sought, according to the CMI Millennium problem.

In our opinion, it is necessary to separate two concepts: the Yang-Mills field and the gap of the strong fundamental interaction. It is also necessary to determine what is meant by the Higgs phase transition. A specific phase transition described in [22], or a phase transition associated with the breach of the gauge symmetry of the interaction field when unobservable interaction fields become observable.

In the future, the Higgs transition will be understood as the phase transition in which the minimal interaction disappears, the gauge or gradient symmetry is broken and the compensating field becomes observable.

At the moment, only one phase transition with a gap in the low-symmetric state is described. This is the phase transition to a superconducting state with the Meissner effect. In the monograph [11] such a superconducting phase transition is called the Higgs transition. Note that in this phase transition, the compensating fields become not just observable, but proportional to the conjugate observable fields. This is due to the breach of the gauge symmetry of the minimal interaction. For superconductivity, this is due to a breach of the gauge symmetry of the Ginzburg-Landau potential [23] or the gradient symmetry of the Maxwell equations.

Indeed, in the London equation, the electromagnetic potential is proportional to the current density: $A_i = -\delta'^2 j_i$. As it is shown above, a similar situation occurs when in a continuous medium the distortion tensor becomes proportional to the conjugate stress tensor: (16). In essence, expression $\sigma_{ij} = \rho c^2 A_{ij}$ (16) is a generalization of Hooke's law for the distortion tensor.

In this regard, the question arises, what interaction describes the quantum momentum as a charge and the distortion tensor as a compensating field of minimal interaction.

In our opinion this is a strong fundamental interaction, because above we managed to describe a gap in a continuous elastic medium with nonzero density ρ (17). After all, in addition to the gap associated with sound (14, 15, 20), and the superconducting gap [11], there are no other gaps due to the symmetry of space-time.

In fact, it is not important how to call the minimal interaction induced by the subgroup of spatial translations (1-4). This interaction can be called a phonon interaction in accordance with the electron-phonon interaction introduced in the BCS theory [10], since the waves of the distortion tensor describe sound in a continuous medium [19]. This interaction can be called quantum, since the quantum momentum is the charge of this interaction [9]. This interaction can be called strong, because at short distances, the quantum momentum, as a charge, sets a very strong interaction.

From the practical point of view, the most important is not the name of this interaction, nor the description of the gap (17) in the low-symmetric state of the distortion tensor, but the interaction itself and the equations of state (5,6) for the tensions of the distortion tensor (7,8).

After all, the minimal interaction (1,3) induced by the translation subgroup with the tensor compensating field has not been previously investigated in the field theory [2,4]. Since only vector compensating fields have been investigated in gauge field theory so far. This is clearly stated in the introduction to the monograph [2], see [6].

We know that the minimal interaction (1-4) has a gap (17) in the low-symmetric state and that the distortion tensor waves describe sound in a continuous medium. But if there is the low-symmetric state, then there is also the "normal" high-symmetric state of the distortion tensor. There is the complete analogy with the "normal" state of the electromagnetic field and the superconducting state with the Meissner effect.

In the low-symmetric state, the twelve equations of state (5, 6) or (12, 13) degenerate into the same massive wave equation (14,15). In this case, the minimal interaction (1-4) disappears and the momentum becomes proportional to the velocity: $p_i = \rho \upsilon_i$, and the distortion tensor becomes proportional to the stress tensor $\sigma_{ij} = \rho c^2 A_{ij}$ (16), and elasticity appears.

In order to investigate the high-symmetric state of the distortion tensor, it is necessary to destroy the low-symmetric state of the continuous medium. Then the momentum will "separate" from the velocity and become the quantum momentum, and the distortion tensor will "separate" from the stress tensor and there will be plastic deformations with dislocations. In this case, the elasticity disappears, and the distortion tensor justifies its name-the destruction tensor, which was given to it to describe plastic deformations [7].

The next article will be devoted to the description of the phase transition of destruction of a continuous elastic medium as a low-symmetric state of the distortion tensor. But it is already clear that being in the low-symmetric state – in the continuous elastic medium, it is impossible to understand how the tensions (7, 8) will behave in a high-symmetric state. Because there is no minimal interaction in a continuous elastic medium, since there is no gradient symmetry in a continuous medium.

Moreover, the vortex tension of the distortion tensor ρ_{ij} (8) is pushed out of the continuous medium in the same way as the magnetic field is pushed out of the superconductor.

Indeed, the vortex tension ρ_{ij} in the solid state is the linear defect (8) [7]. The elastic continuous medium does not admit dislocation density (8), since there is no emptiness in the continuous medium. There are no dislocations in a continuous medium. As it is known, dislocations lead to cracks or linear defects, as a result of which the continuous medium is destroyed.

This situation is well known in the science of materials resistance. However, the destruction of materials was not previously described as a phase transition. In the following work it will be shown that the destruction of solids as a continuous elastic medium, linear defects are formed in the form of dislocations and cracks, and the destruction of gas as a continuous elastic medium, an explosion occurs and a high-temperature plasma is formed.

All these phenomena are associated with the manifestation of force tensions ε_{ij}, ρ_{ij} (7,8) the distortion tensor, which are analogous to the electric and magnetic fields in electrodynamics [6]. In a continuous elastic medium, these tensions are either absent, for example, $\varepsilon_{ij} = 0$ in a sound wave, or limited ρ_{ij} and manifest themselves in the form of a sound. According to (15), the vortex tension ρ_{ij} penetrates into the continuous elastic medium only to the certain depth of penetration, just like the magnetic field in the superconductor.

Therefore, the study of the force tensions of the distortion tensor ε_{ij}, ρ_{ij} in the "normal" highly symmetric state is the main task of this theory. Note that the centrally symmetric stress of the distortion tensor ε_{ij} (7) has never been investigated before, and the vortex stress ρ_{ij} (8) has been investigated but only in the solid state [7, 15, 16]. In the solid state the Peach-Kohler force is associated with the vortex tension ρ_{ij} [24], which for the "continuous distribution of dislocations" has the form: $f_j = e_{jnm}\rho_{in}\sigma_{im}$ [15, 16]. Note that in this form: $f_j = e_{jmn}p_i v_m \rho_{in}$, obtained from the action minimum [6], the Peach-Kohler force is analogous to the Lorentz force [17].

The expression "continuous distribution of dislocations" is taken in quotation marks due to the fact that there is no continuous distribution of dislocations since dislocation is a discrete concept. This topic was discussed in detail in the methodological article [6]. Of course ρ_{ij} this is the force characteristic of the minimal interaction (1) - vortex tension (8), similar to the magnetic field in electrodynamics. For example, in [6, 20] it was shown that in a continuous medium the force $f_j = e_{jmn}p_i v_m \rho_{in}$ is proportional to the pressure gradient.

In the presence of minimal interaction (1), the tension flux ρ_{ij} is quantized and sets the Burgers vector [6], just as the magnetic field flux in the Abrikosov vortices is quantized [8] for the Ginzburg-Landau interaction [23]. But this does not mean that the vortex tension of the compensating interaction field is always quantized, which is well known in the example of the magnetic field.

Conclusion

Thus, sound waves in the continuous medium (14, 15, 20) are not associated with the electromagnetic interaction, as previously thought [10], but are associated with the minimal interaction (1, 3) induced by the translation subgroup (2, 4). Sound waves were obtained as "massive" waves of the distortion tensor in the form of velocity and pressure waves in a continuous elastic medium (14, 20), where the gradient symmetry (18) of the equations of state (5-8) for the distortion tensor is broken.

In this case, the spectrum of the waves of the distortion tensor in a continuous medium (14, 15, 20) has a minimal frequency $\omega_0 = c^2\sqrt{\rho/\gamma}$ (17). This means that the distortion tensor, as an interaction field, has the gap in the low-symmetric state. In addition, the quantum momentum, as the charge [9], at short distances describes a very strong interaction. Since the smaller the wavelength of the particle is the greater of the quantum momentum and the stronger the interaction between them.

In [9] it was proved that the attraction of oppositely directed quantum momenta leads to the formation of Cooper pairs in the superconducting state. It is known that the shorter the coherence length of paired electrons is the higher the phase transition temperature in HTSC [25]. Therefore, there is a reason to believe that this gap corresponds to the gap of strong fundamental interaction, which is corresponds to the Millennium problem announced by CMI in 2000.

The Millennium problem can't be overemphasized. It does not matter how to call the interaction – strong, as it is formulated in the CMI problem, phonon, according to the theory of BCS [10], or quantum, according to the charge - quantum momentum [9]. It is important that the gap is an indicator of the phase transition to the low-symmetric state. In this paper, it is shown that the continuous medium is the low-symmetric state of the distortion tensor. In the continuous medium, the distortion tensor is elastic and describes the sound.

But if there is a low-symmetric state, there must be a high-symmetric state. The following work is devoted to the description of the phase transition of destruction of a continuous elastic medium as a low-symmetric state of the distortion tensor.

There are only four fundamental interactions in nature. Two of them have gaps: the electromagnetic interaction and the strong interaction. There are also two Abelian models associated with translational space-time symmetry. The first model is related to the lagrangian invariance with respect to time translations and leads to electromagnetic interaction [13]. The second model is related to the lagrangian invariance with respect to spatial translations and leads to a minimal interaction (1-4) induced by the quantum momentum as the charge and the tensor compensating interaction field.

In the next paper, the tensions of the distortion tensor: ε_{ij}, ρ_{ij} (7,8), similar to the electric and magnetic tensions of the electromagnetic field will be investigated. They satisfy the equations of state (5, 6), which are analogous to Maxwell's equations, and give forces analogous to Coulomb's and Lorentz's forces [6]. These equations of state (5, 6) and force [6] have never been fully investigated before. In the following work it will be shown that the critical vortex stress ρ_{ij} (8) destroys the continuous elastic medium, and the centrally symmetric stress ε_{ij} (7) transforms gas into high-temperature plasma.

Reference

1. Yang C.N., Mills R. Conservation of Isotopic Spin and Isotopic Gauge Invariance. *Phys. Rev*. **96**, 191 (1954).

2. Slavnov A.A., Fadeev L.D. Introduction to quantum theory of gauge fields. Nauka, Moscow (1988).

3. Braginsky A.Y. Phenomenological theory of the phase transitions to states with the local symmetry. *Fiz. Tverd. Tela*. **32** (7), 2121 (1990).

4. Bogolyubov N.N., Shirkov D.V. Quantum fields. Nauka, Moscow (1980).

5. Pauli W. Relativistic field theories of elementary particles // *Revs. Mod. Phys*. **13**, 203 (1941).

6. Braginsky A.Y. The computation of the Kadić-Edelen forces using the principle of least action. *AJMPA*. **01** (02), 1 (2019).

7. Landau L.D., Lifshitz E.M. Elasticity Theory, Volume 7. Nauka, Moscow (1987).

8. Abrikosov A.A. On magnetic properties of superconductors of the second group. *Zh. Exp. Teor. Fiz*. **32**, 1442 (1957).

9. Braginsky A.Y. Electrons Cooper pairing as the attraction of the oppositely directed quantum momenta. *Moscow University Physics Bulletin*. **6**, 16 (2019).

10. Bardeen J., Cooper L.N., Schrieffer J.R. Theory of Superconductivity. *Phys. Rev*. **108**, 1175 (1957).

11. Itzkson K., Zuber J.B. Quantum Field Theory. New York, McGraw-Hill (1980).

12. London F. Superfluids. **1**. New York, Wiley. (1950).

13. Braginsky A.Y. Inhomogeneous Ordered States and Translational Nature of the Gauge Group in the Landau Continuum Theory: II. Applications of the General Theory. *Zh. Exp. Teor. Fiz.* **132**, 45 (2007).

14. Landau L.D., Lifshitz E.M. Hydrodynamics, Volume 6. Nauka, Moscow (1988).

15. Kadic A., Edelen D.G. A Gauge Theory of Dislocations and Disclinations. *Lecture Notes in Physics.* Heidelberg, Springer. **174**, 168 (1983).

16. Lazar M. On the fundamentals of the three-dimensional translation gauge theory of dislocations. *Math. Mech. Solids.* **16**, 253 (2011).

17. Landau L.D., Lifshitz E.M. Field Theory, Volume 2. Nauka, Moscow (1987).

18. Fletcher H., Munson W.A. Loudness, its definition, measurement and calculation. *J. Acoust. Soc. Am.* **5**, 82 (1933).

19. Braginsky A.Y. Exact solution of the equations of state for sound in the form of a "massive" wave. Explanation of the Fletcher-Manson diagram. (under review) https://www.researchgate.net/publication/332411248

20. Braginsky A.Y. The vortex motion of the continuous medium, depending on the pressure change. *PTEP* **2020 (6)**, 063J03. (2020).

21. Braginsky A.Y. Plane vortex motion of a continuous medium. Description of air rotation in cyclones and anticyclones. *Z. Angew. Math. Phys*. **70,** 177 (2019).

22. Higgs P.W. Broken Symmetries and the Masses of Gauge Bosons. *Phys. Rev. Lett.* **13**, 508 (1964).

23. Ginzburg V.L., Landau L.D. On the theory of superconductivity. *Zh. Exp. Teor. Fiz*. **20** (2), 1064 (1950).

24. Peach M.O., Kohler J.S. The forces exerted on dislocations and the stress fields produced by them. *Phys. Rev.* **80**, 436 (1950).

25. Ginzburg V.L. High temperature superconductivity. *Usp. Fiz. Nauk*. **161** (4), 1 (1991).