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A method of predicting the magnitude and failure time of a forthcoming earthquake

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Abstract—Let $T$ and $M$ be, respectively, the precursor time of a certain precursor and the magnitude of a forthcoming earthquake. Observations may lead to a relationship of $T$ versus $M$ in a form of $\log(T)=a+bM$. In this study, we will explore the physical basis of the relationship. Based on the $\log(T)−M$ relationships of two different precursors, we propose a method to predict the magnitude and failure time of the forthcoming earthquake. A testing example based on the $\log(T)−M$ relationships of presiemic radon concentration anomalies and gamma-ray emission changes observed at respective monitoring stations in Taiwan is given in this study. Results strongly confirm a high possibility of predicting the magnitude and failure time of a forthcoming earthquake just from the observed occurrence times of two different precursors based on their $\log(T)−M$ relationships.

Key words: Earthquake prediction, earthquake precursor, precursor time, earthquake magnitude, failure time, relationship of precursor time versus magnitude
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One of the significant ways to reduce seismic hazards is the successful prediction of forthcoming earthquakes from observations of reliable precursors. Aki (1989, 2009) assumed that earthquakes are predictable and also suggested that earthquake scientists would inform the public the probability of the occurrence of an earthquake with a specified magnitude, place, and time window $T$. An earthquake, especially for the large one, is usually preceded by complex physical and chemical processes which may behave as the precursors (e.g., Atkinson, 1984; Main and Meredith, 1989). For a certain precursor, the time window is merely the precursor time of the precursor (e.g., Wang et al., 2016; Wang, 2021a,b). Of course, the precursor times may be different for distinct precursors.

A common characteristic of observed precursors is the linear relationship between $\log(T)$ and the magnitude, $M$, of a forthcoming earthquake. Such a relationship has been recognized for a long time by numerous authors (e.g., Scholz et al., 1973; Whitcomb et al., 1973; and Rikitake, 1975a). From the plot of $T$ (in days) versus $M$ for five precursors (crustal movements, electric resistivity, radon (denoted as Rn) emission, $v_p/v_s$ anomaly, and $b$-value) from 30 world-wide earthquakes, Scholz et al. (1973) inferred a regression relationship: $M=-5.81+1.55\log(T)$ ($T$ in days) which gives $\log(T)=3.75+0.65M$. For the precursors of crustal deformations and seismic-wave velocities, Whitcomb et al. (1973) obtained $\log(T)=-1.92+0.80M$ ($T$ in days). They assumed that $T$ is positively proportional to $L^\kappa$ where $L$ is the fault length of the forthcoming earthquake and $\kappa$ is the scaling exponent. This gives $\log(T) \sim \kappa \log(L)$. The value of $\kappa$ is 1.6 by Aggarwal et al. (1973) and $\sim 2.0$ by both Scholz et al. (1973) and Whitcomb et al. (1973). That $M$ is proportional to $\log(L)$ yields the linear relationship between $\log(T)$ and $M$ in the form: $\log(T)=a+bM$ where $a$ and $b$ are two constants. Dieterich (1978) applied the preseismic processes of $\sigma$ and $u$ to interpret the $\log(T)$−$M$ relationship inferred by Scholz et al. (1973).

The existence of such a positive correlation for different precursors indicates that all preseismic phenomena would obey the same physical law. This strongly suggests the existence of a same preseismic mechanism that controls the physical and chemical parameters in the fault zone prior to an earthquake. This could lead to an opportunity of successfully predicting earthquakes. Although it seems successful for few large
events, including the 1975 Haicheng, China, earthquake (e.g., Wang et al., 2006),
earthquake prediction is still debatable. Numerous scientists do not believe that
earthquakes can be predicted (e.g., Geller, 1996, 1997; Geller et al., 1997). The
problem might be caused by a reason as described below. The relationships of log(T)
versus M as reported by Scholz et al. (1973) and Whitcomb et al. (1973) are not
universal. The log(T)−M relationship may be dependent upon the types of precursors.
From a plot of log(T) versus M for Rn concentration anomalies observed in different
tectonic provinces, Hauksson (1981) could not infer a linear log(T)−M relationship
like those reported by Scholz et al. (1973) and Whitcomb et al. (1973) due to large
dispersion of data points even though log(T) increases with M. This suggests regional-
dependence of the log(T)−M relationship.

In Japan, Tsubokawa (1969, 1973) first obtained a linear relation between the
precursor time of crustal movement and magnitude of mainshock in the form:
log(T)=−1.88+0.79M. After analyzed the data of various earthquake precursors
(including land deformation, tilt and strain, foreshocks, b-value, micro-
seismicity, source mechanism, fault creep anomaly, v_p/v_s, v_p and v_s, geomagnetism, earth current,
resistivity, radon, underground water, and oil flow) amounting to 418 in number,
Rikitake (1975b, 1976) related the precursor time of a precursor to the magnitude, M,
of the forthcoming mainshock in the following equation: log(T)=−1.83+0.76M. He
also stressed that the relationships of T versus M are different for different groups of
precursors. Rikitake (1979, 1984) collected a large data set of 391 cases of earthquake
precursors. He divided the data into three classes. Excluding the third class for
foreshocks, tilt and strain, and earth’s currents, the log(T)−M relationships from his
data set of 192 cases are log(T)=−1.01+0.60M (T in days) for the first class and
log(T)=−1.0 for the second class. Clearly, the second class of precursors is almost the
imminent precursor that appeared about one day immediately before earthquakes. For
the third class, the frequencies of log(T) are distributed in a very wide range with two
peaks: one at log(T)=1.0 and the other at log(T)=−1.0. From 30 large Japanese
earthquakes with 4.7≤M≤7.9, Rikitake and Yamazaki (1985) inferred the function of
log(T) (T in days) in terms of M and R (the hypocentral distance, in km) for the
preseismic earth resistivity changes as: log(T)=0.41M−1.6log(R). In New Zealand,
Smith (1981, 1986) obtained the following relationship: log(T)=1.42+0.30M from the
data of precursor times of abnormal b-values. In China, Ding et al. (1985) inferred the
following relationship: log(T)=−0.34+0.38M for various precursors proceeding large
Chinese earthquakes. In Taiwan, from more than one hundred earthquakes with 
$4 \leq M_L < 8$ ($M_L =$ the local magnitude, magnitude) Wang (2021b) inferred the $\log(T) - M_L$ relationship ($T$ in days) for the Rn concentration anomalies as $\log(T) = (-2.05 \pm 0.40) + (0.58 \pm 0.01) M_L$ for the events with focal depths $\leq 40$ km and $\log(T) = (-0.40 \pm 0.42) + (0.26 \pm 0.01) M_L$ for the events with focal depths $> 40$ km. For six earthquakes with $M_w = 5.0 - 6.8$ and $d = 7.0 - 35.6$ km occurred in southeastern Taiwan, Kuo et al. (2020) inferred the following relationship: $\log(T) = 1.456 + 0.053 M_w$ for Rn concentration anomalies. From 45 world-wide earthquakes with $3 \leq M_S \leq 9$ ($M_S =$ the surface-wave magnitude), Wang et al. (2016) inferred the $\log(T) - M_S$ relationship ($T$ in years) for the $b$-value anomalies as: $\log(T) = (2.02 \pm 0.49) + (0.15 \pm 0.07) M_S$.

Clearly, the relationships inferred by Rikitake (1979, 1984), Wang et al. (2016), Kuo et al. (2020), and Wang (2021b) are remarkably different from one another and also different from those obtained by Scholz et al. (1973) and Whitcomb et al. (1973). This again strongly suggests that the $\log(T) - M$ relationship is region-dependent and varies for different types of precursors. In other word, the time window of earthquake prediction should be dependent on the observed precursor and may change when one more precursor is observed. However, up to date the $\log(T) - M$ relationships have been inferred only for very few precursors in few seismically active regions.

The existence of distinct $\log(T) - M$ relationships for different types of precursors are significant for practical earthquake prediction. In this study, we will explore the physical basis to predict the magnitude and failure time, $t_r$, of a forthcoming earthquake based on the $\log(T) - M$ relationships of two different types of precursors. In addition, the $\log(T) - M$ relationships of presiemic radon (denoted by Rn hereafter) concentration anomalies and gamma-ray (written as $\gamma$-ray hereafter) emission changes observed in Taiwan will be taken as an example below.

2. Physics of the Relationship of $T$ versus $M$

In order to study the correlation between events of an earthquake sequence, it is necessary to further ask a question: Is the correlation between earthquakes long-term or short-term? Let $n(t)$ be the number of earthquakes in an area at time $t$. When the changing rate of $n(t)$ at time $t$, $\frac{\delta n(t)}{\delta t}$, is controlled only by $n(t)$, the relationship between $\frac{\delta n(t)}{\delta t}$ and $n(t)$ can be represented by a linear 1-D differential equation:
\[ \frac{dn}{dt} = -\lambda n(t) \]. This equation gives a solution in the form of the exponential function, \( n(t) \sim \exp(-t/\lambda) \), to show its temporal behavior. When \( \delta n(t)/\delta t \) is controlled not only by \( n(t) \) but also by the previous numbers, for example, \( n(t-\delta t) \), a memory effect exists in earthquakes. The relationship between \( \delta n(t)/\delta t \) and \( n(t) \) can be represented by a 1-D difference equation: \( \delta n(t)/\delta t = -\alpha n(t)n(t-\delta t) \). Hence, we have the non-linear 1-D differential equation \( \frac{dn}{dt} = -\alpha n^2(t) \) as \( \delta t \) approaches zero. This gives a solution in the form of the power-law function, \( n(t) \sim \alpha t^{-1} \), to show its temporal behavior. Hence, power-law behavior of an earthquake sequence suggests the possible existence of a memory effect between earthquakes.

The Gutenberg-Richter’s energy-magnitude law of earthquakes (Gutenberg and Richter, 1942, 1956) is:

\[ \log(E_s) = 11.8 + 1.5M \]  \hspace{1cm} (1)

which \( E_s \) is the seismic-wave energy and \( M \) is commonly the surface-wave magnitude, \( M_s \). Equation (1) gives \( M \sim (2/3)\log(E_s) \). From \( \log(T) = a + bM \), we have

\[ \log(T) \sim bM \sim (2b/3)\log(E_s). \]  \hspace{1cm} (2)

Since \( E_s = \eta \Delta E \) where \( \Delta E \) is the strain energy of an earthquake and \( \eta \ (<1) \) is the seismic efficiency (cf. Wang, 2004), we have

\[ T \sim \Delta E^{2b\eta/3}. \]  \hspace{1cm} (3)

Equation (3) indicates the precursor time is dependent on the strain energy of the forthcoming earthquake. Of course, the different precursors with distinct values of \( b \) may have different precursor times from Equation (3). The seismic efficiency that depends on the physical and chemical properties of the fault zone may influence \( T \).

Aki (1966) defined the seismic moment as \( M_o = \mu \bar{u} A \) where \( \mu \), \( \bar{u} \), and \( A \) are the rigidity of fault rocks, the average displacement on the source area, and the source area, respectively. Purcaru and Berckhemer (1978) obtained a relationship between \( M_o \) and \( M \) as described below
\[ \log(M_o) = 16.1 + 1.5M. \] (4)

The scaling law of \( M_o \) versus \( L \) that is the fault length is (e.g., Kanamori and Anderson, 1975; Scholz, 1990; Wang, 2018): \( M_o \sim L^n \) where \( n \) is 2 for small and medium-sized events that rupture in the 2-D domain and 1 for larger-sized events that ruptures mainly along the 1-D domain usually in the horizontal direction. Inserting this scaling law into Equation (4) leads to \( M \sim (2n/3)\log(L) \). Letting \( \kappa \) be \( 2bn/3 \), we have

\[ T \sim L^\kappa \] (5)

as mentioned above. Equation (5) indicates the precursor time is dependent on the fault length of the forthcoming earthquake. Of course, different precursors with distinct values of \( b \) may have different precursor times from Equation (5). The scaling exponent \( n \) between \( M_o \) and \( L \) is also a factor in influencing \( T \). As mentioned above, \( n \) is 2 times bigger for small and medium-sized events than for larger-sized ones. The value of \( T \) for a certain precursor may still be longer for the latters than for the formers, because the fault length is usually much longer for the latters than for the formers.

As mentioned above, \( \kappa \) is 1.6 by Scholz et al. (1973) or 2 by Whitcomb et al. (1973). Hence, when \( \kappa = 1.6 \) the value of \( b = 3\kappa/2n \) should be 2.4 for \( n = 1 \) and 1.2 for \( n = 2 \). Clearly, \( b \) is larger than 1. From the observed results obtained by Scholz et al. (1973), Whitcomb et al. (1973), Rikitake (1979, 1984), Wang et al. (2016), and Wang (2021b), the values of \( b \) are all smaller than 1.0. Hence, the values of \( \kappa \) inferred by Scholz et al. (1973) or 2 by Whitcomb et al. (1973) cannot be applied to a certain precursor or a certain class of precursors. Dieterich (1978) assumed that \( \kappa = 2 \) inferred by Whitcomb et al. (1973) is due to a use of the \( M-L \) relationship for medium and small earthquakes given by Wyss and Brune (1968) to evaluate \( L \). When Dieterich (1978) used the \( M-L \) relationship inferred by Press (1967) to evaluate \( L \), the value of \( \kappa \) becomes 1. This will reduce the estimated value of \( b \). Since \( b \) is commonly smaller than 1, \( \kappa \) should also be smaller than 1. Clearly, the inference of the \( M-L \) relationship
is quite important for the evaluation of $\kappa$. The reliable inference of the $\log(T) - M$ relationship is more important than that of the $M - L$ relationship in this study.

3. A Method of Earthquake Prediction

3.1 Theory

For two different types of precursors, the $\log(T) - M$ relationship for the $i$-th precursor ($i=1$ and $2$) may be represented as the following form:

$$\log(T_i) = a_i + b_i M$$

(6)

or

$$T_i = 10^{(a_i + b_i M)}$$

(7)

where $T_i = t_r - t_i$, $t_r$ is the failure time of a forthcoming earthquake, and $t_i$ is the occurrence time of the $i$-th precursor. As mentioned previously, the coefficients $a_i$ and $b_i$ are dependent on the type of precursors and also of regional dependence.

We here suggest a method to evaluate the two unknowns $t_r$ and $M$ from Equation (6) or Equation (7). For this method, we need two $\log(T) - M$ relationships of different precursors. Considering two different precursors whose occurrence times are $t_1$ and $t_2$, respectively, Equation (6) gives

$$\log(t_r - t_1) = a_1 + b_1 M.$$  

(8)

for the first precursor and

$$\log(t_r - t_2) = a_2 + b_2 M$$

(9)

for the second one. Equation (8) and Equation (9), respectively, lead to

$$t_r = t_1 + 10^{(a_1 + b_1 M)};$$

(10)
\[ t = t_2 + 10^{(a_2 + b_2 M)}. \]  
(11)

The equality of the two equations yields

\[ t_1 + 10^{(a_1 + b_1 M)} = t_2 + 10^{(a_2 + b_2 M)}. \]  
(12)

Defining \( F_i(M) = t_i + 10^{(a_i + b_i M)} \) and \( F_2(M) = t_2 + 10^{(a_2 + b_2 M)} \), Equation (12) gives \( F_i(M) = F_2(M) \). We may solve the value of \( M \) from this equality through the following technique because \( t_1, a_1, b_1, t_2, a_2 \) and \( b_2 \) are known. Considering that the occurrence time for the first precursor is equal to or earlier than that for the second one, we have \( t_2 \geq t_1 \).

According to the condition of \( t_2 \geq t_1 \) as mentioned above, Equation (12) gives

\[ 10^{(a_1 + b_1 M)} - 10^{(a_2 + b_2 M)} = t_2 - t_1 \geq 0. \]  
(13)

Equation (13) leads to \( 10^{(a_1 + b_1 M)} > 10^{(a_2 + b_2 M)} \) or \( a_1 + b_1 M > a_2 + b_2 M \). This yields

\[ (b_1 - b_2)M \geq a_2 - a_1. \]  
(14)

Although the value of \( M \) may be negative for very small natural events, only large, positive \( M \) is considered here because we are only interested in moderate and large earthquakes.

Examples of the two curves associated with the two functions that are normalized by the maximum value of either \( F_i(M) \) or \( F_2(M) \) are schematically plotted in Fig. 1 with \( t_1 = 10 \) days and \( t_2 = 15 \) days for \( M = 1-9 \). In Fig. 1, \( F_1(M) \) and \( F_2(M) \) are displayed by a solid line and a dashed line, respectively. Note that in the four panels, the difference between \( F_1(M=1) \) and \( F_2(M=1) \) is very small because the maximum value of either \( F_1(M=9) \) or \( F_2(M=9) \) is relatively very large. We may numerically evaluate the value of \( M \) from the intersection point between the two curves. When \( M \) has been estimated, the value of \( t_r \) is either \( t_1 + 10^{(a_1 + b_1 M)} \) or \( t_2 + 10^{(a_2 + b_2 M)} \). Hence, we may predict \( M \) and \( t_r \) of the forthcoming earthquake.

From Equation (14), if \( a_2 - a_1 > 0 \) or \( a_1 < a_2 \), \( b_1 - b_2 \) must be positive and thus \( b_1 > b_2 \).
Hence, the two inequalities, i.e., \(a_2 < a_1\) and \(b_1 > b_2\), form the first condition such that \(M\) can be solved from Equation (12). An example is displayed in Fig. 1a where the two curves intersect at a certain \(M\), thus leading the solution. If \(a_2-a_1=0\) or \(a_1=a_2\), \(b_1-b_2\) must be positive and thus \(b_1 > b_2\). An example is displayed in Fig. 1b. The two curves cannot intersect each other when \(M>1\) because the increasing rate is higher for \(F_1(M)\) than for \(F_2(M)\) due to \(b_1 > b_2\). If \(a_2-a_1<0\) or \(a_1>a_2\), \(b_1-b_2\) may be positive, i.e., \(b_1 > b_2\), or zero, i.e., \(b_1=b_2\), or negative, i.e., \(b_1 < b_2\). For \(b_1 > b_2\), an example is displayed in Fig. 1c. The two curves cannot intersect each other at a certain \(M\) when \(M>1\) because the increasing rate is higher for \(F_1(M)\) than for \(F_2(M)\) due to \(b_1 > b_2\). Like \(b_1 > b_2\), the two curves may intersect each other at a certain \(M\) when \(b_1=b_2\). For \(b_1<b_2\), there are two possibilities. The first possibility is that the value of \(M\) may be solved from Equation (12) for \(M<(a_1-a_2)/(b_2-b_1)\). An example for this condition is displayed in Fig. 1d. Clearly, the two curves may intersect each other at a large value of \(M\), thus leading to the solution. The second possibility is that the inequality \(M<(a_1-a_2)/(b_2-b_1)\) does not hold or holds only for either negative \(M\) or small \(M\). This does not make \(M\) be solved from Equation (12). Consequently, the value of \(M\) of a forthcoming mainshock may be solved from Equation (12) under either the first condition of \(a_1<a_2\) and \(b_1 > b_2\) or the second one of \(a_1>a_2\) and \(b_1 \leq b_2\). Then, the failure time, i.e., \(t_r\), of the forthcoming mainshock may be evaluated from either Equation (10) or Equation (11).

3.2 Three Requirements

Based on the above-mentioned theory, we may predict the magnitude and failure time of a forthcoming earthquake when the relationships of \(\log(T)\) versus \(M\) of two different precursors are reliable. Nevertheless, we should still pay attention to the following three important requirements that will influence the feasibility and reliability of the present theory for predicting earthquakes.

The first requirement is that we must collect related data and construct, at least, two different \(\log(T)-M\) relationships for a certain study region. According to the differences in tectonic and geological conditions, it is not appropriate to apply the relationships obtained from other regions or to use the average relationships inferred from world-wide earthquakes to a certain study region.

The second requirement is that the precursor times of the two precursors in use must be in the same order of magnitude. It is inappropriate to compare the power-law
function of \( \log(T) \) versus \( M \) of a precursor whose value of \( T \) is in a unit of days with that of a precursor whose value of \( T \) is in a unit of years. Otherwise, this will yield a large difference between \( t_1 \) and \( t_2 \), thus being unable to make the two curves intersect each other as shown in Fig. 1 because of remarkable separation between them. In other word, the values of \( t_1 \) and \( t_2 \) should be reliable and their difference cannot be too big.

The third requirement is that it is necessary to consider the standard deviations \( \delta a_i \) and \( \delta b_i \), respectively, for \( a_i \) and \( b_i \) which commonly exist because the \( \log(T) - M \) relationship is inferred from the observations with errors as shown in the previous examples. If the values of \( \delta a_i \) and \( \delta b_i \) are larger due to insufficient or lowly reliable data, the errors of evaluated values of \( M \) and \( t_r \) should be bigger, thus leading to higher uncertainty of prediction.

### 4. A Testing Example and Discussion

#### 4.1 Evaluation of the Magnitude of a Forthcoming Earthquake

Numerous earthquake precursors have been long observed and studied in Taiwan (e.g., Tsai et al., 1983, 2004, 2018; Wang, 2021b,c). Rn concentration anomalies are usually taken as a significant precursor of earthquakes (e.g., Teng, 1980; Wakita et al., 1985; King, 1986; King et al., 2005; Woith, 2015; Paudel et al., 2018). Taiwan’s geochemists installed numerous automatically monitoring stations for measuring Rn concentrations in the field. One of the stations is the TPT station as illustrated by with an open triangle in Fig. 2. The gamma-ray (denoted as \( \gamma \)-ray hereafter) emission is mainly produced from the radioactive decay of Rn or from thunderstorms (e.g., Tsukuda, 2008; Minnehan, 2015). Four stations for automatically monitoring \( \gamma \)-ray emissions have been installed in Taiwan (Fu et al., 2015). One station is the YMSG station that is installed at the Taiwan Volcano Observatory (TVO) in Mt. Yangming, Northern Taiwan and illustrated by an open diamond symbol in Fig. 2. For numerous earthquakes, Rn concentration anomalies and \( \gamma \)-ray emission changes have been measured on the respective stations by local geochemists. Hence, the \( \log(T) - M \) relationships for the two precursors may be established.

In the followings, the source parameters of earthquakes in use are taken from the
data base provided by the Central Weather Bureau (CWB). The earthquake magnitude is the local magnitude, $M_L$ (Shin, 1992). The focal depth of an earthquake is denoted by $d$ (in km) and the epicentral distance from an event to an observation station is represented by $\Delta$ (in km).

At the YMSG station, Fu et al. (2019) observed $\gamma$-ray emission changes before 20 events with $M_L=2.8-6.7$ happened during July 1, 2014 to June 1, 2015. The precursor times are 2–20 days. Meanwhile, they also observed Rn concentration anomalies before 15 events with $M_L=2.3-6.7$ at the station TPT. The precursor times are 1–23 days. Totally, there are 25 events having either Rn concentration anomalies or $\gamma$-ray emission changes. Ten of the 25 events have the two precursors simultaneously. The related data of the 25 events are listed in Table 1 and their epicenters are plotted in Fig. 2 (open circles for the events with $d \leq 40$ km and solid circle for those with $d > 40$ km). Clearly, only one event is located near the YMSG station, three events close to the TPT station, and others far away from the two stations with $\Delta > 40$ km. At stations YMSG and TPT, Fu et al. (2019) reported that the temporal variations of $\gamma$-ray and Rn have similar patterns. They also mentioned three interesting points: First, some high $\gamma$-ray and Rn concentration peaks in the entire spectrum. Secondly, the increase of $\gamma$-ray emission changes usually come after the Rn concentration anomalies as listed in Table 1. Thirdly, although the duration time of Rn concentration anomalies is longer than that of $\gamma$-ray emission changes, the two types of precursors disappeared almost at the same time before the forthcoming earthquake.

Based on the data obtained by Fu et al. (2019), Wang (2021c) explored the correlation between the Rn concentration anomalies and $\gamma$-ray emission changes. He defined $T_{Rn}$ (in days) and $T_{gr}$ (in days) to be the precursor time of the former and that of the latter, respectively. From the plot of $T_{gr}$ versus $T_{Rn}$, he found an increase in $T_{gr}$ with $T_{Rn}$. From the plot of $T_{Rn}$-$T_{gr}$ versus $T_{Rn}$, he saw an increase in $T_{Rn}$-$T_{gr}$ with $T_{Rn}$ even though the data points are somewhat scattered. Results reveal that the $\gamma$-ray emission change is associated with the Rn concentration anomaly as mentioned by Fu et al. (2019). From the plot of $T_{gr}$ and $T_{Rn}$ versus $M_L$, Wang (2021c) found the increases in both $T_{Rn}$ and $T_{gr}$ with $M_L$, thus suggesting that the larger the forthcoming earthquake is, the earlier the occurrence times of the two precursors are. From the plot of $T_{Rn}$-$T_{gr}$ versus $M_L$, he also saw an increase in $T_{Rn}$-$T_{gr}$ with $M_L$. This suggests that when the occurrence time of $\gamma$-ray emission change after the Rn concentration
anomaly is longer, the forthcoming earthquake is bigger and its occurrence time is longer after the appearance of the two types of precursors. Based on the above-mentioned physical theory, we may predict the forthcoming mainshocks by using the observed $\log(T) - M$ relationships of Rn concentration anomalies and $\gamma$-ray emission changes.

As listed in Table 1, the $\gamma$-ray emission changes were recorded before 20 events. Clearly, $T_{\gamma} = T_{Rn} = 17$ days for Event 12 is questionable for the $\gamma$-ray emission changes. Event 16 could be the largest aftershock of Event 15, and thus it must be deleted. Event 20, 21, and 22 could be the aftershocks of Event 19, and thus they must be removed. Excluding the 5 events, the plots of $T_{\gamma}$ versus $M_L$ and $\log(T_{\gamma})$ versus $M_L$ for 15 events with $T_{\gamma}$ are displayed in Fig. 3a and Fig. 3b, respectively. From the data points, the inferred regression equations of $\log(T_{\gamma})$ versus $M_L$ is

$$\log(T_{\gamma}) = (-0.29 \pm 0.23) + (0.24 \pm 0.01)M_L \tag{15}$$

Equation (15) is displayed with a thin solid line in Fig. 3.

As listed in Table 1, the Rn concentration anomalies were recorded before 15 events. Clearly, Events 20, 21, and 22 are the aftershocks of Event 19, and thus they must be removed. Excluding the 3 events, The plots of $T_{Rn}$ versus $M_L$ and $\log(T_{Rn})$ versus $M_L$ for 12 events with $T_{Rn}$ are displayed in Fig. 4a and Fig. 4b, respectively. From the data points as shown in the figure, the inferred regression equations of $\log(T_{Rn})$ versus $M_L$ is

$$\log(T_{Rn}) = (-0.22 \pm 0.27) + (0.24 \pm 0.02)M_L \tag{16}$$

Equation (16) is displayed with a thin solid line in Fig. 4.

For 111 earthquakes, Liu et al. (1984), Chyi et al. (2001, 2005), Yang et al. (2005), Fu et al. (2017a,b,c, 2019), and Kuo et al. (2006a, 2010, 2017, 2018, 2019, 2020) observed the Rn concentration changes before these events on several automatically monitoring stations in Taiwan. The plots of $T_{Rn}$ versus $M_L$ and $\log(T_{Rn})$ versus $M_L$ are displayed in Fig. 5a and Fig. 5b, respectively. In Fig. 5, the open and solid circles are made, respectively, for the events with $d \leq 40$ km and $\Delta \leq 40$ km and for those with $d > 40$ km and $\Delta > 40$ km. From these data, Wang (2021b) inferred the $\log(T_{Rn}) - M_L$
relationships as:

\[
\log(T_{Rn}) = (-2.05 \pm 0.40) + (0.58 \pm 0.01)M_L \quad (17)
\]

for the events with \(d \leq 40\) km and \(\Delta \leq 40\) km and

\[
\log(T_{Rn}) = (-0.40 \pm 0.42) + (0.26 \pm 0.01)M_L \quad (18)
\]

for those with \(d > 40\) km or \(\Delta > 40\) km. Equations (17) and (18) are displayed with a dashed line and a dotted line, respectively, in Fig. 5 and also in Fig. 4. In Fig. 4, the dashed line is in parallel with and close to the thin solid line, thus indicating that Equation (17) could be almost an average \(\log(T_{Rn}) - M_L\) relationship for Taiwan’s earthquakes with \(d \leq 40\) km and \(\Delta \leq 40\) km. On the other hand, the dotted line is across the thin solid line with a large intersection angle. This means that Equation (18) cannot be the average \(\log(T_{Rn}) - M_L\) relationship for Taiwan’s earthquakes with \(d > 40\) km or \(\Delta > 40\) km.

In order to solve \(M_L\) from Equations (15) and (16) based on the physical basis as shown in Equation (12), the observed Rn concentration anomalies and \(\gamma\)-ray emission changes are, respectively, considered as the first and second precursors. Hence, we have \(a_1 = -0.22\), \(b_1 = 0.24\), \(a_2 = -0.29\), and \(b_2 = 0.24\). This exhibits the second condition with \(a_1 > a_2\) and \(b_1 \leq b_2\). Hence, \(M_L\) may be solved from Equation (12). From the third requirement as mentioned in Section 3, we must also take the standard deviations \(\delta a_i\) and \(\delta b_i\) of \(a_i\) and \(b_i\), respectively, into account. The values are \(\delta a_1 = 0.27\) and \(\delta b_1 = 0.02\) for \(F_1(M_L)\) from Equation (16) and \(\delta a_2 = 0.24\) and \(\delta b_2 = 0.02\) for \(F_2(M_L)\) from Equation (18). Hence, we have \(F_1(M_L) = t_{Rn} + 10^{(0.22 \pm 0.27) + (0.24 \pm 0.02)M_L}\) and \(F_2(M_L) = t_{gr} + 10^{(0.29 \pm 0.23) + (0.24 \pm 0.01)M_L}\) where \(t_{Rn}\) and \(t_{gr}\) are the appearance times of Rn concentration anomalies and \(\gamma\)-ray emission changes, respectively. Based on Equations (10) and (11), the values of \(F_1(M_L)\) and \(F_2(M_L)\) are both equal to \(t_r\). Considering \(t_{Rn} = 0\) as the initial time for each event for convenience, the value of \(t_{gr}\) is thus \(sT = T_{Rn} - T_{gr}\). This means that when the difference in occurrence times between two different precursors have been measured, we may evaluate the magnitude and also the failure time of a forthcoming earthquake based on the \(\log(T) - M_L\) relationships..

In order to include the effects of standard deviations, we will solve \(M_L\) through an
alterative way. We define three difference functions of \( F_2(M_L)-F_1(M_L) \): \( \delta F_M(M_L) = \delta T + 10^{-0.29+0.24M_L} \cdot 10^{-0.22+0.24M_L} \) without the standard deviations; \( \delta F_{lb}(M_L) = \delta T + 10^{-0.52+0.23M_L} \cdot 10^{-0.49+0.22M_L} \) with the negative standard deviations; and \( \delta F_{ub}(M_L) = \delta T + 10^{-0.06+0.25M_L} \cdot 10^{+0.05+0.26M_L} \) with the positive standard deviations. Clearly, \( \delta F_{lb}(M_L) \) and \( \delta F_{ub}(M_L) \) are, respectively, the lower-bound (denoted by ‘lb’) and upper-bound (denoted by ‘ub’) values of \( \delta F_M(M_L) \) at a certain \( M_L \).

As shown in Fig. 6, the lb curve for \((a-\delta a)+(b-\delta b)M\) and the ub curve for \((a+\delta a)+(b+\delta b)M\) are both illustrated by dashed lines. A vertical dotted line segment denoted by ‘\( T_{ub} \)’ at the upper end and by ‘\( T_{lb} \)’ at the lower end exhibits the range of estimated values of \( T \) at a certain \( M \). Hence, the ub and lb curves show the ub and lb values, respectively, for \( T \). On the other hand, a horizontal dotted line segment denoted by ‘\( M_{ub} \)’ at the right end and by ‘\( M_{lb} \)’ at the left end exhibits the range of estimated values of \( M \) at a certain \( T \). Hence, the ub and lb curves exhibit the lb and ub values, respectively, for \( M \).

For Events 08, 10, 13, 17, 19, and 25 as listed in Table 1 and displayed in Fig. 2 with event number, we evaluate their magnitudes from the above-mentioned method. We calculate the values of \( \delta F_M(M_L) \), \( \delta F_{lb}(M_L) \), and \( \delta F_{ub}(M_L) \) from \( M_L=0 \) to \( M_L=9 \) for the six events. Results are plotted in Fig. 7 in which \( \delta F_{lb}(M_L) \), \( \delta F_M(M_L) \) and \( \delta F_{ub}(M_L) \) are normalized by the maximum value of either \( |\delta F_{lb}(M_L)| \) or \( |\delta F_{ub}(M_L)| \) in each panel for the related event. The three difference functions are illustrated by different curves: the solid line for \( \delta F_M(M_L) \), the dashed line for \( \delta F_{lb}(M_L) \), and the dotted line for \( \delta F_{ub}(M_L) \). In each panel of Fig. 7, a horizontal solid line represents \( F_2(M_L)-F_1(M_L)=0 \) and a vertical solid line denotes the observed value of \( M_L \) of the related event as listed in Table 1. If the curve of \( F_2(M_L)-F_1(M_L) \) intersects with the horizontal solid line at a certain point, we may evaluate the value of \( M_L \) at the intersection point. The expected, lower-bound, and upper-bound values of \( M_L \) are denoted by \( M_{L,0} \), \( M_{L,lb} \) (related to \( M_{lb} \) in Fig. 6), and \( M_{L,ub} \) (related to \( M_{ub} \) in Fig. 6), respectively, and they are evaluated from \( \delta F_M(M_L)=0 \), \( \delta F_{ub}(M_L)=0 \), and \( \delta F_{lb}(M_L)=0 \), respectively. Fig. 7 shows that the thin dashed line for \( \delta F_{lb}(M_L) \) increases with \( M_L \) from \( M_L=0 \) to \( M_L=9 \), but it does not intersect the horizontal line at a point. Hence, we cannot estimate the value of \( M_{L,ub} \). The thin solid line for \( \delta F_M(M_L) \) decreases with increasing \( M_L \) from \( M_L=0 \) to \( M_L=9 \) and intersects the horizontal line at a point with \( M_{L,0} \). Hence, we can estimate the values of
For the six events. The thin dotted line for \( \delta F_{ub} (M_L) \) decreases with increasing \( M_L \) from \( M_L=0 \) to \( M_L=9 \) and intersects the horizontal line at a point with \( M_{L,ub} \). Hence, we can estimate the values of \( M_{L,ub} \) for the six events. The estimated values of \( M_{L,ub} \) and \( M_{L,0} \) for the six events are listed in Table 2.

Table 2 shows that from Equation (15) plus Equation (16), \( M_{L,0} \) is larger than observed \( M_L \) for Events 8, 10, 13, 17, and 19. The values of \( M_{L,ub}-M_L \) range from 0.32 to 2.28. The values of \( \delta T \) are equal to or longer than 2 days for the five events as listed in the last column of Table 1. On the other hand, \( M_{L,0} \) is smaller than the observed \( M_L \) for Event 25. The value of \( M_{L,0}-M_L \) is -0.62. Meanwhile, the value of \( \delta T \) is 1 day for the event as listed in the last column of Table 1. In addition, \( M_{L,ub} \) is smaller than observed \( M_L \) for the six events. Consequently, observed \( M_L \) is in between \( M_{L,ub} \) and \( M_{L,0} \) for Events 8, 10, 13, 17, and 19 and slightly outside the range of \( M_{L,ub} \) to \( M_{L,0} \) for Events 25. This can be clearly seen in Fig. 7.

For the purpose of comparison, we also evaluate the values of \( M_{L,0} \), \( M_{L,ub} \), and \( M_{L,lab} \), from Equation (15) plus Equation (18). We have \( a_1=-0.40 \), \( b_1=0.26 \), \( a_2=-0.29 \), and \( b_2=0.24 \). This exhibits the first condition with \( a_1< a_2 \) and \( b_1>b_2 \). Hence, \( M_L \) may be solved from Equation (12). The values of standard deviations are: \( \delta a_1=0.02 \) and \( \delta b_1=0.01 \) for \( F_1(M_L) \) from Equation (18) and \( \delta a_2=0.24 \) and \( \delta b_2=0.02 \) for \( F_2(M_L) \) from Equation (15). The three difference functions of \( F_2(M_L)-F_1(M_L) \) are: \( \delta F_0(M_L)=\delta T+10^{(-0.29+0.24M_L)-10^{(-0.40+0.26M_L)}} \), \( \delta F_{lb}(M_L)=-\delta T+10^{(-0.52+0.23M_L)-10^{(-0.82+0.25M_L)}} \), and \( \delta F_{ub}(M_L)=\delta T+10^{(-0.06+0.25M_L)-10^{(-0.02+0.27M_L)}} \).

We calculate the values of three functions of differences between \( F_2(M_L) \) and \( F_1(M_L) \) from \( M_L=0 \) to \( M_L=9 \) for the above-mentioned six events, i.e., Events 08, 10, 13, 17, 19, and 25. Results are plotted in Fig. 8 in which \( \delta F_{lb}(M_L) \), \( \delta F_0(M_L) \), and \( \delta F_{ub}(M_L) \) are normalized by the maximum value of either \( |\delta F_{lb}(M_L)| \) or \( |\delta F_{ub}(M_L)| \) for each event. The three difference functions are shown with different curves: the solid line for \( \delta F_0(M_L) \), the dashed line for \( \delta F_{lb}(M_L) \), and the dotted line for \( \delta F_{ub}(M_L) \). In each panel of Fig. 8, a horizontal line represents \( F_2(M_L)-F_1(M_L)=0 \) and a vertical solid line denotes the observed value of \( M_L \) of the related event. Fig. 8 shows that the thin dashed line for \( \delta F_{lb}(M_L) \) increases with \( M_L \) from \( M_L=0 \) to \( M_L=9 \) and does not intersect the horizontal line at a point. Hence, we cannot estimate the value of \( M_{L,ub} \). The thin solid line for \( \delta F_0(M_L) \) decreases with increasing \( M_L \) from \( M_L=0 \) to \( M_L=9 \) and intersect the horizontal line at a point with \( M_{L,0} \). Hence, we can estimate the value of \( M_{L,0} \) for
the six events. The thin dotted line for $\delta F_{ub}(M_L)$ decreases with increasing $M_L$ from $M_L=0$ to $M_L=9$ and intersects the horizontal line at a point with $M_{Llb}$. Hence, we can estimate the values of $M_{Llb}$ for the six events. The values of $M_{Llb}$ and $M_{L0}$ for the six events are listed in Table 2.

Table 2 reveals that from Equation (15) plus Equation (18), $M_{L0}$ are larger than observed $M_L$ for the six events. The values of $M_{L0}$- $M_L$ are 1.57 to 2.98. Meanwhile, $M_{Llb}$ are smaller than observed $M_L$ for the six events. Hence, observed $M_L$ is in between $M_{Llb}$ and $M_{L0}$ for the six events. In addition, the values of $|M_{Llb}-M_L|$ are smaller from Equation (16) than from Equation (18). This seems to suggest that it is better to use Equation (15) plus Equation (16) than to use Equation (15) plus Equation (18) to evaluate the value of $M_L$ for a forthcoming earthquake.

4.2 Evaluation of the Precursor Time of Rn Concentration Anomalies

We may evaluate the precursor times for both Rn concentration anomalies and $\gamma$-ray emission changes from the evaluated value of $M_L$, i.e., $M_{L0}$. Here, only the former is taken into account. The expected, lower-bound, and upper-bound values of precursor time are denoted by $T_{Rn0}$, $T_{Rnlb}$ (related to $T_{lb}$ in Fig. 6), and $T_{Rnub}$ (related to $T_{ub}$ in Fig. 6) are evaluated based on the value of $M_{L0}$. From Equation (15) plus Equation (16), the three quantities are computed from the following equations:

$T_{Rn0}(M_{L0})=10^{(-0.22+0.24M_{L0})}$, $T_{Rnlb}(M_{L0})=10^{(-0.49+0.22M_{L0})}$, and $T_{Rnub}(M_{L0})=10^{(0.05+0.26M_{L0})}$.

From Equation (15) plus Equation (18), the three quantities are computed from the following equations:

$T_{Rn0}(M_{L0})=10^{(-0.40+0.26M_{L0})}$, $T_{Rnlb}(M_{L0})=10^{(-0.82+0.25M_{L0})}$, and $T_{Rnub}(M_{L0})=10^{(0.02+0.27M_{L0})}$. For the 6 events, the values of $T_{Rnlb}$, $T_{Rn0}$, and $T_{Rnub}$ computed from the two ways are listed in Table 3. Note that from the observed data, the failure time, $t_r$, of the forthcoming earthquake will be $t_r=t_{Rn}+T_{Rn}$.

Table 3 reveals that from Equation (15) plus Equation (16), $T_{Rn0}$ is longer than observed $T_{Rn}$ for Events 8, 10, 13, 17, and 19 and slightly shorter than observed $T_{Rn}$ for Event 25. The values of $T_{Rn0}$- $T_{Rn}$ range from 0.5 days to 39.0 days for the former five events and -0.2 days for Event 25. Meanwhile, $T_{Rn}$- are shorter than observed $T_{Rn}$ for Events 8, 13, 17, and 25 and longer than observed $T_{Rn}$ for Events 10 and 19. In other word, observed $T_{Rn}$ is is between $T_{Rnlb}$ and $T_{Rnub}$ for Events 8, 13, 17, and 25 and slightly outside the range of $T_{Rnlb}$ to $T_{Rnub}$ for Events 10 and 19. From Equation (15)
plus Equation (18), $T_{Rn0}$ is longer than observed $T_{Rn}$ for the six events. The values of $T_{Rn0} - T_{Rn}$ range from 13.4 days to 46.5 days. Obviously, the values of $T_{Rn0} - T_{Rn}$ are large for the six events. Meanwhile, $T_{Rnlb}$ are shorter than or equal to observed $T_{Rn}$ for Events 08, 13, 17, and 25 and longer than observed $T_{Rn}$ for Events 10 and 19. In other words, except for Events 10 and 19, observed $T_{Rn}$ is in between $T_{Rnlb}$ and $T_{Rnub}$. Of course, the value of $T_{Rnlb} - T_{Rn} = 0.1$ days for Event 19 is tiny.

From Tables 2 and 3, we can see that the values of $|M_{L0} - M_{L}|$ and $|T_{Rn0} - T_{Rn}|$ are smaller from Equation (15) plus Equation (16) than from Equation (15) plus Equation (18). In addition, the ranges of $M_{Llb}$ to $M_{L0}$ and those of $T_{Rnlb}$ and $T_{Rnub}$ are larger from Equation (15) plus Equation (16) than from Equation (15) plus Equation (18). Hence, it is more appropriate to apply Equation (15) plus Equation (16) than to use Equation (15) plus Equation (18) for evaluating $M_{L}$ and $T_{Rn}$. This indicates that in order to predict a forthcoming earthquake, the $\log(T) - M_{L}$ relationship for Rn concentration anomalies inferred from the data observed in a limited area is better than that done from the data observed in the whole Taiwan region.

4.3 Discussion

Figs. 7 and 8 show that each curve of $\delta F(M_{L})$ does not intersect the horizontal solid line with $F_{2}(M_{L}) - F_{1}(M_{L}) = 0$ at a certain point. This makes us be unable to estimate the value of $M_{Lub}$ that is the upper-bound values of $M_{L}$. Nevertheless, the results are still acceptable because observed $M_{L}$ is almost in between $M_{Llb}$ and $M_{L0}$, thus implying that the magnitude of the forthcoming earthquake may be estimated in a reasonable range.

Tables 2 and 3 reveal that the values of $|M_{L0} - M_{L}|$ and those of $|T_{Rn0} - T_{Rn}|$ for Events 17 and 25 are much smaller than those for Events 08, 10, 13, and 19. This seems to say that the evaluations of magnitude and precursor time are better for the former two events than the latter four events. But, we have to pay attention to a problem. From Table 1, the values of $\delta T = T_{Rn} - T_{gr}$ for the former two events are shorter than 3 days; while those for the latter four events are longer than 3 days. The $\gamma$-ray emission is mainly produced from the radioactive decay of Rn (e.g., Tsukuda, 2008; Minnehan, 2015). $^{222}$Rn first decays, with a half time of 3.8 days, to $^{218}$Po. During the decaying processes, there are $\alpha$-particle ($^{4}$He) emissions with energy release of 5.49 MeV and...
energy release by $\gamma$-ray emissions. Hence, the value of $\delta T$ should be 3.8 days when
the two precursors are observed at the same site. Although the two precursors were
observed at two different sites, it is still necessary to further explore the reason why
$\delta T$ is shorter than 3 days for Events 17 and 25.

Tables 2 and 3 seem to suggest that it is better to evaluate the magnitude than to
estimate the failure time for a forthcoming earthquake based on the given $\log(T)$–$M_L$
relationships. This might be due to a fact that in the relationship, the precursor time
and the magnitude are represented, respectively, by $\log(T)$ and $M_L$. Hence, the high
standard deviations, especially for $a_i$ ($i=1$ and 2), of the $\log(T)$–$M_L$ relationships will
yield a remarkable difference in evaluations between $T$ and $M_L$. High standard
deviations will yield larger uncertainty of evaluated value of $T_{Rn}$, than that of $M_L$. It is
assumed that a reduction in standard deviations from a larger number of reliable data
will more substantially improve the evaluated vales of both $M_L$ and $T_{Rn}$.

In addition, Equations (15) and (16) are inferred from two data sets. In respective
data sets, the numbers of earthquakes are different. Some of the events are the same in
the two data sets and some are different. This is due to limited available data. Clearly,
the standard deviations are different between Equation (15) and Equation (16). This
might also yield influences on the evaluations of $M_L$. We assume that a single data set
consisting of a large number of same earthquakes with both $T_{gr}$ and $T_{Rn}$ will
substantially improve the evaluated vales of both $M_L$ and $T_{Rn}$.

In addition, there is a weak point about the present method. Clearly, the source area
cannot be predicted from the present method. It needs other precursors, for example,
crustal deformation pattern, $b$-value anomalies, foreshock activities, electromagnetic
anomalies etc., or other methods for predicting the possible source area of the
forthcoming earthquake. For example, Hayakawa and Hobara (2010) described the
goniometric method to detect the directions of ULF emissions from the observational
stations to the earthquake epicenter and then to infer the possible location of the
forthcoming event. This sounds a good way.

5. Conclusions

Observations of precursor time, $T$, of a certain precursor and the magnitude, $M$, of
the forthcoming earthquake lead to a relationship of \( \log(T) = a + bM \). This is due to a physical basis that there is a power-law relation between \( T \) and seismic-wave energy, \( E_s \), of the forthcoming earthquake or the length, \( L \), of the related earthquake fault. Based on the \( \log(T) - M \) relationships of two different precursors, we propose a method to predict the magnitude and failure time of a forthcoming earthquake. Let \( a_i \) and \( b_i \) are the coefficients of the relationship for the \( i \)-th precursors. The first precursor appear earlier than the second one. Theoretical analyses reveal that the value of \( M \) of a forthcoming mainshock may be solved under two conditions: (1) \( a_1 < a_2 \) and \( b_1 > b_2 \) and (2) \( a_1 > a_2 \) and \( b_1 \leq b_2 \).

The \( \log(T) - M_L \) relationships of Rn concentration anomalies and gamma-ray emission changes that were observed on respective monitoring stations in Taiwan are taken as an testing example in this study. Results provide us an opportunity of reliably predicting a forthcoming earthquake based on the observed precursor times of two different precursors: \( T_{Rn} \) for Rn concentration anomalies and \( T_{gr} \) for \( \gamma \)-ray emission changes. Since we have the \( \log(T) - M_L \) relationships for two different precursors, we may predict \( M_L \) and \( T_{Rn} \) of a forthcoming earthquake. The failure time of the forthcoming earthquake is \( t_r = t_{Rn} + T_{Rn} \) in which \( t_{Rn} \) is the occurrence time of Rn concentration anomalies. Testing results reveal that it is better to apply the \( \log(T) - M_L \) relationship inferred from the data observed in a limited area than that done from the data observed in the whole Taiwan region for the evaluations of both \( M_L \) and \( T_{Rn} \) in a certain area. The standard deviations of the coefficients of the \( \log(T) - M_L \) relationships can also influence the evaluations of \( M_L \) and \( T_{Rn} \). To reduce the standard deviations, especially for \( a_i \) (\( i = 1 \) and 2), from a large number of reliable data will substantially improve the evaluated values of \( M_L \) and \( T_{Rn} \) (as well as \( t_r \)). In addition, it will be much useful when the \( \log(T) - M_L \) relationships for two different precursors are inferred from a data set consisting of the same events.

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Figure 1

The non-scaled curves of for $F_1(M)$ and $F_2(M)$ with $t_2>t_1$: (a) for $a_1<a_2$ and $b_1>b_2$; (b) for $a_1=a_2$ and $b_1>b_2$; (c) for $a_1>a_2$ and $b_1>b_2$; and (d) for $a_1>a_2$ and $b_1<b_2$ under the constrain: $M<(a_1-a_2)/(b_2-b_1)$. 
Figure 2

The figure shows the epicenters (open circles for $d \leq 40$ km and solid circles for $d > 40$ m) of the earthquakes as listed in Table 1. The radon monitoring station YMSG is shown by an open diamond symbol. The $\gamma$-ray monitoring station TPT is shown by an open triangle. A thin line marked with ‘LV’ in eastern Taiwan represents the Longitudinal Valley.
Figure 3

(a) Plot of $T$ versus $ML$ and (b) plot of $\log(T)$ versus $ML$ for $\gamma$-ray emission changes for 15 events that are explained in the text and listed in Table 1. The thin solid line represents Equation (15) listed in the text.
Figure 4

(a) Plot of $T$ versus $M_L$ and (b) plot of $\log(T)$ versus $M_L$ for Rn concentration anomalies for 12 events that are explained in the text and listed in Table 1. The solid line represents Eq. (16) listed in the text. The dashed line and dotted line represent, respectively, Eq. (17) and Eq. (18) listed in the text.
Figure 5

(a) Plot of $T$ versus $M_L$ and (b) plot of $\log(T)$ versus $M_L$ for Rn concentration anomalies (open circles for the events with $d \leq 40$ km and $\leq 40$ km and solid circles for those with $d > 40$ km and $> 40$ km). The dashed line and thin solid line represent, respectively, Eq. (17) and Eq. (18) listed in the text.
Figure 6

Plot of $T$ versus $M$: the solid line for $a+bM$; the lower dashed line for $(a-\delta a)+(b-\delta b)M$; and the upper dashed line for $(a+\delta a)+(b+\delta b)M$. A vertical dotted line segment denoted by ‘$T_{ub}$’ at the upper end and by ‘$T_{lb}$’ at the lower end exhibits the range of estimated values of $T$ at a certain $M$. A horizontal dotted line segment denoted by ‘$M_{ub}$’ at the right end and ‘$M_{lb}$’ at the left end exhibits the range of estimated values of $M$. 
Figure 7

The curves of for the differences between normalized $F_2(ML)$ (for $\gamma$-ray emission changes) and normalized $F_1(ML)$ (for Rn concentration anomalies) based on Eq. (16): (a) for event 08 with $ML=5.2$; (b) for event 10 with $ML=5.0$; (c) for event 13 with $ML=6.7$; (d) for event 17 with $ML=5.3$; (e) for event 19 with $ML=6.4$; and (f) for event 25 with $ML=5.0$. In each panel, the solid, dotted, and dashed curves represent the $\delta F(ML)$, $\delta F_{lb}(ML)$, and $\delta F_{ub}(ML)$, respectively. The vertical line denotes the observed value of $ML$ for each event; the vertical thin solid line for the estimated value of $ML_0$ and the vertical thin dotted line for the estimated value of $ML_{lb}$. The estimated values of $ML_0$ and $ML_{lb}$ are listed in Table 2.
The curves of for the differences between normalized $F_2(ML)$ (for $\gamma$-ray emission changes) and normalized $F_1(ML)$ (for Rn concentration anomalies) based on Eq. (18): (a) for event 08 with $ML=5.2$; (b) for event 10 with $ML=5.0$; (c) for event 13 with $ML=6.7$; (d) for event 17 with $ML=5.3$; (e) for event 19 with $ML=6.4$; and (f) for event 25 with $ML=5.0$. In each panel, the thin solid, dotted, and dashed curves represent the $\delta F(ML)$, $\delta F_{lb}(ML)$, and $\delta F_{ub}(ML)$, respectively. The vertical line denotes the observed value of $ML$ for each event; the vertical thin solid line for the estimated value of $ML_0$ and the vertical thin dotted line for the estimated value of $ML_{lb}$. The estimated values of $ML_0$ and $ML_{lb}$ are listed in Table 2.

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