Statistics of simultaneous macroscopic detection of electrons at both slits of a double-slit experiment

Satish Ramakrishna (sr1087@physics.rutgers.edu)
Rutgers, The State University of New Jersey

Research Article

Keywords: quantum physics, quantum measurement, random walks, double-slit experiment

Posted Date: October 12th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-2104657/v2

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License
Statistics of simultaneous macroscopic detection of electrons at both slits of a double-slit experiment

Satish Ramakrishna
Department of Physics & Astronomy,
Rutgers, the State University of New Jersey,
136 Frelinghuysen Road Piscataway, NJ 08854-8019

ABSTRACT

In the standard double-slit experiment, the interaction between the electron and a detector at either of the slits is supposed to be local. However, one never notices a situation where the electron is detected at both slits simultaneously in the frame of the observer. This is usually explained by the statement that there is only one electron that is being located and it obviously cannot actually be located in two places at the same time in the observer’s frame of reference. However, this is a global constraint, which is hard to understand when the interaction between the electron and a measurement device at a particular slit is completely local - how would the measuring device realize that there was only one electron that was to be detected? It would be useful to deduce this from purely local interactions at each slit, with detectors at each slit, without having to adduce assumptions about long-range correlations between (potentially) space-like separated events.

In this paper, we start with a Hamiltonian that was studied in a previous paper on the measurement problem, with a local interaction between the electron and a detector - here we assume that there is a detector at each slit. We study this problem and make a theoretical argument that the detection problem reduces to equilibration of a macroscopic number of coupled harmonic oscillators. At each detector, as the measurement happens, the amplitude for each of the states of the measuring apparatus relaxes to an equilibrium configuration akin to the equilibration of a thermodynamic system.

The problem of the detection of the electron at each slit is then mapped to the motion of a random walker in a line with reflecting barriers at both ends, where the ends are separated by a macroscopic number of steps. This connection to a well-studied mathematical problem has a well-understood consequence - two (or a finite number of) random walkers in this space will be expected to be separated by a macroscopic number of steps. Hence, only one of the detectors will see a macroscopic number of its detection degrees of freedom affected - ergo, only one of the detectors, selected completely at random, will "see" the electron. For a large, macroscopic, though finite-sized measurement device, there will be a vanishingly small chance of detecting the electron simultaneously at both slits. Indeed, in the limit of an infinitely massive detector, we will never detect the electron at two places at the same time.

Similar problems have been studied in the context of non-linear additions to Schrodinger’s equation [26]. In contrast to that work, we argue that such effects are precisely akin to amplification at photomultiplier tubes. Indeed, this problem illustrates further that the systematic application of Schrödinger’s equation along with a local interaction Hamiltonian that effectively represents the measurement process helps solve some curious issues in the quantum measurement problem.
INTRODUCTION

The study of quantum measurement has seen many approaches \([1–22]\) applied to various situations for several years now. In particular, the double slit experimental set-up has been a common test-bed to analyze \([23, 24]\), due to the especially straightforward demonstration of the peculiarity of quantum effects.

The experiment has been analyzed with a variety of mathematical methods \([3, 19]\) and in particular the latter approach uses a cascading series of small interactions to produce decoherence starting from a coherent state. The unitary interaction between the electron and a macroscopic number of degrees of freedom in the measuring apparatus is equivalent to the response of a collection of harmonic oscillators to an initial impulse. The interaction is microscopically deterministic and reversible, but due to the immense number of interacting degrees of freedom, the decoherence is increasingly hard to reverse as the number of interactions gets bigger.

This paper studies the following question in the above framework. If the interactions at each slit are local (between the electron and the measuring apparatus at the slit), why is it that we never see a situation where, for sufficiently separated slits, there is a simultaneous detection of the electron at both slits by macroscopic detectors at each slit? The usual answer to this question is that there is only one electron sent towards the slit, so they cannot possibly see more than one electron. However, as a practical matter, the detector at each slit does not know that - the detection is occurring at each slit independently of the other slit (as is reasonable to assume) and it would be useful to deduce the result from the mechanics of the problem.

That is exactly what this paper does. The upshot of the analysis is that the interaction between the electron and the degrees of freedom at the measuring apparatus lead to a situation akin to that of a random walker between two reflecting barriers - one at zero interactions and the second at the maximum number of degrees of freedom affected (notation \(Q\) is used in the paper). The fact that two independent random walkers in such a situation will be expected to be separated by a distance of the order of \(Q\) implies immediately that it is extremely unlikely (and in the limit of an infinitely large measuring apparatus, impossible) for the detectors at the slits to see the electron simultaneously at both slits with macroscopic detectors. However and we make this precise in the below, if we look for sub-macroscopic perturbations, it is possible that we might see simultaneous effects at both slits.

An approach similar to this has been studied by Kaplan & Rajendran\([26]\). They add small non-linear terms into the Hamiltonian that produce effects (of the sort discussed above) that could be measured. We argue that even without an explicit non-linear term in Schrödinger’s equation, such terms are indeed introduced into the Hamiltonian by a suitable amplification (photomultiplier) device, which we model in this paper. Indeed, then, one needs to define what a measurement means - we do so in what follows in Section III (B).

What is novel about this? The novelty here is connecting the microscopics of how detection occurs and the mathematics of random walks to outcomes of a quantum measurement. The analysis can be expanded to a countably infinite set of outcomes; so a particle at one of several slits is never detected at several slits at once, as also a spin measurement (as opposed to a position measurement), without requiring some non-local correlations through a quantum state that collapses across the whole universe at one instant.

This analysis is not in conflict with Bell’s theorem \([27, 28]\). There are no hidden variables - the time evolution follows Schrödinger’s equation and measurement still retains its non-deterministic features. It simply explains the local findings with local analysis of the probabilities of outcomes.

THE EXPERIMENT

We use the description of the apparatus slightly modified from the earlier paper\([19]\). A source of electrons is shown on the left in Fig. 1. It shoots electrons, at an extremely slow rate, at an absorbing screen (on the right). Interposed between the screen and the electron gun is another absorbing screen, upon which are cut two slits (labeled 1 and 2). Between the slits and assumed to be aimed at both slits, there is an intense source of high-energy “locator” photons, that can scatter (if they interact) off the electrons. Most of the photons that scatter off electrons are picked up by the input of two photo-multiplier tubes - one at each slit. The energy of the photons that scatter off the electrons is \(\omega_p\), while the energy of the incident electrons is \(\omega_e\) (we use natural units throughout the calculation, so \(\hbar = 1\)).

These “locator” photons are captured by two idealized photomultiplier tubes, one close to each of the two slits, after scattering. For the purposes of the calculation, we assume that one photon, when absorbed, produces \(N\) (\(N\) is a multiplication fraction, usually \(2 – 3\)) more photons and the process continues in this fashion inside each photomultiplier tube. The end result is a bright spot, that emits several - in fact, a macroscopic number of) photons of several frequencies, from a point on a display tube. This point on the display tube is, therefore, directly connected to the reception of a single locator photon at the input end of the amplifier.
As in the previous paper, we start with the Hamiltonian shown in Equation (1). There is a “free - electron” term, with energy $\omega_e$. This electron could be either at slit 1 or at slit 2, these are represented by subscripts on the number operator for the electrons. There is also a “free - photon” term, with frequency $\omega_p$ (for the locator photon that directly scatters off the electron at either slit 1 or slit 2), while $\omega_i$ represents the frequencies of the photons that are subsequently produced by the initially scattered photon (and thence) at the photomultiplier tube. We ignore photons from the intense source that do not scatter off the electron, since the photomultiplier input ignores the direct beam and only looks for scattered photons. We are modeling the process where the electron interacts with one photon at the slit, which, then at the photomultiplier tube, cascades, producing $N$ other photons in each state $i$ with each subsequent interaction.

This is an idealized photomultiplier tube. In a real photo-multiplier tube, the locator photons excite a few electrons, each of which then produces a few more electrons. The process cascades and a large current (millions of times larger than the initially excited current) is produced. In our case, we assume the locator photons excite a few photons, which each excite a few more photons and so on, to produce a macroscopic number of photons in a continuum of photon energy states. This constitutes “measurement” by the device (we make this notion a little more precise in Section II-B).

As usual, if the photomultiplier tube at slit 1 shows a macroscopic number of photons affected, we assume that the electron indeed passed through slit 1. Our observation that the interference pattern on the screen gets replaced by a “clump” pattern confirms that this assignment is right. In the density-matrix formalism, this just means that we have zero off-diagonal elements after a measurement[4].

We write the hermitian Hamiltonian (in line with the Ref.[19])

$$\mathcal{H} = \mathcal{H}_{\text{electron}} + \mathcal{H}_{\text{photon}} + \mathcal{H}_{\text{int}}^{\text{detection}} + \mathcal{H}_{\text{int}}^{\text{initial}} + \mathcal{H}_{\text{int}}^{\text{PM tube}}$$

(1)

where each piece is explained in the previous paper. First, the free terms:

$$\mathcal{H}_{\text{electron}} = \omega_e \left( c_1^\dagger c_1 + c_2^\dagger c_2 \right)$$

$$\mathcal{H}_{\text{photon}} = \omega_p \gamma_0^\dagger \gamma_0 + \sum_{i=1}^{R} \omega_i \gamma_i^\dagger \gamma_i$$

(2)

Then we add interactions between the electron and the first locator photon (with a real interaction parameter $\Gamma_1$), as in the earlier paper. We have this interaction at slit 1 and 2. In order to ensure energy conservation, energy needs to be supplied through the interaction. One may consider the interaction parameter as a background field, with a time dependence, which allows the photon to be produced in the presence of the electron at slit 1 (or 2) and emerge with energy $\omega_p$. In Appendix III of the earlier paper, we describe how this interaction would come about from the usual
QED vertex and a background field.

\[ \mathcal{H}^{\text{detection}}_{\text{int}} = \left( \Gamma_1 e^{-i\omega_p t_0} \gamma_0 \frac{1}{2} c_1 + \Gamma_1 e^{i\omega_p t} \gamma_0 c_1 \right) + \left( \Gamma_1 e^{-i\omega_p t_0} \gamma_0 \frac{1}{2} c_2 + \Gamma_1 e^{i\omega_p t} \gamma_0 c_2 \right) \]  

(3)

Next, we add the interactions that lead to the creation of child photons (in any of \( Q \) photon states) from this first locator photon at either of the two photomultiplier tubes. Again, energy is inserted into the system through the interaction vertex, the first “enhancement” being modeled by another auxiliary field with time-dependence.

\[ \mathcal{H}^{\text{PM initial}}_{\text{int}} = \sum_{i=1}^{Q} \left( \Gamma_i^2 e^{-i\omega_i t + i\omega_p t} \gamma_i \gamma_0 + \Gamma_i^2 e^{i\omega_i t - i\omega_p t} \gamma_0 \gamma_i \right) \]  

(4)

Next, these photons create other photons in the available states, with similar multiplication factors (we use \( M_1 \)), again using the external measurement system to add energy.

\[ \mathcal{H}^{\text{PM tube}}_{\text{int}} = \sum_{i=1}^{Q} \left( s e^{i\omega_i t - iM_1 \omega_j t} \gamma_j \gamma_i + s e^{-i\omega_i t + iM_1 \omega_j t} \gamma_i \gamma_j \right) \]  

(5)

Thus far, apart from adding a photomultiplier tube to slit 2, this is the same Hamiltonian that we studied analytically and numerically in the previous paper.

This Hamiltonian is explicitly time-dependent and energy is inserted from the outside through the interaction. Apart from a choice in how we display the results, these interactions are entirely plausible approximations to what actually happens in the real-world.

The scale of the couplings and photo-multiplication is, \( \Gamma_2 \approx \Gamma_1 \approx s \), while \( N, M_1 \) are of \( O(1-10) \). The number of modes \( Q \) of photons that are subsequently produced, is macroscopic and large. Once a photon interacts with the electron, it produces more photons and the process cascades within the states available to the photons - subsequent processes distribute this energy into other states in the continuum.

**The States**

Expanding from the previous paper[19], we write down the states in the Fock space of the system. The total state of the system is the weighted sum of all possible states - first, an ‘unperturbed” state and next, several other “perturbed” states.

The unperturbed state of the system is

\[ |\psi\rangle_0 = \frac{a |1\rangle_e + u |2\rangle_e}{\sqrt{2}} \times \left| n_0^{(1)} = 0, n_1^{(1)} = 0, n_2^{(1)} = 0, ..., n_Q^{(1)} = 0; n_0^{(2)} = 0, n_1^{(2)} = 0, n_2^{(2)} = 0, ..., n_Q^{(2)} = 0 \right\rangle \]  

(6)

The kets \( |1\rangle_e, |2\rangle_e \) represent the state of the electron localized either to slit 1 or 2 respectively. The 0’s in the last (common) ket represents that there are no photons scattered either at slit 1 or slit 2, hence the electron is (initially) still in a superposition of two states (either at slit 1 or slit 2). \( a \) and \( u \) are parameters that initially are 1, i.e., at \( t = 0 \).

The various perturbed states are as follows. In all the states with a non-zero 1 superscript for the \( n_i \), a scattering event has occurred relative to slit 1. Hence, for that state, the electron is firmly in state \( |1\rangle_e \). Also, the state

\[ |2\rangle_e \left| n_0^{(1)} = 1; n_1^{(1)}, n_2^{(1)}, ..., n_Q^{(1)}; n_0^{(2)} = 0, n_1^{(2)} = 0, n_2^{(2)} = 0, ..., n_Q^{(2)} = 0 \right\rangle \]

is not a possible state - we cannot have a scattering event at slit 1 while the electron is at slit 2! We could, of course, expect noise, but that can be corrected for. Hence, the only possible state of the system with the electron at slit 2 is actually the second part of the state in Equation (6), with appropriate non-zero values for \( n_i^{(2)} \).

As was done in the previous paper, the first of the perturbed states is where the initial photon is scattered, while the others are where subsequent photomultiplier events have occurred with the further production of other photons.
These states are as follows - the superscripts (1) and (2) refer to the particular slit whose photomultiplier tube photons are affected, while the subscripts label different states.

\[ |\psi(1)_{1}\rangle = |\psi(1)_{1,0,0,0}\rangle = \frac{1}{\sqrt{2}} |n_0(1) = 0; n_1(1) = 0, n_2(1) = 0, \ldots, n_Q(1) = 0; n_0(2) = 0, \ldots, n_Q(2) = 0\rangle \]

\[ |\psi(1)_{2}\rangle = |\psi(1)_{1,0,0,0}\rangle = \frac{1}{\sqrt{2}} |n_0(1) = 0; n_1(1) = 0, n_2(1) = 0, \ldots, n_1(1) = N, \ldots, n_Q(1) = 0; n_0(2) = 0, \ldots, n_Q(2) = 0\rangle \]

\[ |\psi(1)_{3}\rangle = |\psi(1)_{1,0,0,0,0}\rangle = \frac{1}{\sqrt{2}} |n_0(1) = 0; n_1(1) = 0, \ldots, n_i(1) = N - 1, \ldots, n_j(1) = M_{1}, \ldots, n_1(1) = 0; n_0(2) = 0, \ldots, n_Q(2) = 0\rangle \]

\[ |\psi(1)_{4}\rangle = |\psi(1)_{1,0,0,0,0}\rangle = \frac{1}{\sqrt{2}} |n_0(1) = 0; n_1(1) = 0, \ldots, n_i(1) = N - 2, \ldots, n_j(1) = M_{1}, \ldots, n_k(1) = M_{1}, \ldots, n_1(1) = 0; n_0(2) = 0, \ldots, n_Q(2) = 0\rangle \]

There are an entirely equivalent set of states with the electron at slit 2 and photons in various photomultiplier tube states at slit 2. We assume, in what follows that the states are labeled by the macroscopic number \( Q \) of them. In order of size, every state for which \( d_{ij}^{(1,2)} \) is the coefficient, for high \( i, j \) contains \( M_{1} \) times the number of photons in the adjacent state whose coefficient is \( d_{ij}^{(1,2)} \), for instance. Since the states have, successively, multiples of the number of photons in “nearby” states, the last few of these states contain a macroscopic number of photons.

Chosen this way, these states are themselves orthogonal to each other and since they are in the occupation number representation, can be enumerated and are complete. In general, we can write the state of the system at any time as

\[
|\psi\rangle = e^{-i\omega_{c} t} |\psi\rangle_{0} + \sum_{i=1}^{Q} c_{i}^{(1)} e^{-i(\omega_{c} + \omega_{t}) t} |\psi(1)_{i}\rangle + \sum_{i,j;i \neq j} d_{ij}^{(1)} |\psi(1)_{ij}\rangle + \sum_{i,j;k;i \neq j,k \neq i} d_{ijk}^{(1)} |\psi(1)_{ijk}\rangle + \ldots \]

\[
|\psi\rangle = e^{-i\omega_{c} t} |\psi\rangle_{0} + \sum_{i=1}^{Q} c_{i}^{(2)} e^{-i(\omega_{c} + \omega_{t}) t} |\psi_{ij}\rangle + \sum_{i,j;i \neq j} d_{ij}^{(2)} |\psi(2)_{ij}\rangle + \sum_{i,j;k;i \neq j,k \neq i} d_{ijk}^{(2)} |\psi(2)_{ijk}\rangle + \ldots
\]

where we have explicitly included the “free” time-dependence of the states in the exponentials multiplying every term. Note that the parameters \( a \) and \( u \) introduced in Equation (6) appears in \( |\psi\rangle_{0} \) itself.

In the above, \( a \) is the amplitude (and \( |a|^{2} \) the probability, by the Born rule) that the state is localized to slit 1 and nothing else is affected. Similarly, \( b^{(1)} \) is the amplitude (and \( |b^{(1)}|^{2} \) the probability) that the electron’s state is localized to slit 1 and one photon has been scattered into the idealized photomultiplier. Carrying on, \( c_{i}^{(1)} \) is the amplitude (and \( |c_{i}^{(1)}|^{2} \) the probability) that the electron’s state has localized to slit 1, while \( N \) photons have then been generated in state \( i \) by the initially scattered photon. We can carry this forward, \( viz. \), \( d_{ij}^{(1)} \) is the amplitude (and \( |d_{ij}^{(1)}|^{2} \) the probability) that the electron is localized to slit 1, while \( N - 1 \) photons are in state \( i \) in the photomultiplier tube and \( M_{1} \) photons are in state \( j \) in the tube, all produced by the amplification process that was sparked by the first scattered photon. The process goes on, involving increasing numbers of photons in the photomultiplier tube in various other states. The same can be said for the parameters \( u, b^{(2)}, c_{i}^{(2)}, d_{ij}^{(2)} \) and slit 2.
What is a measurement?

Schrödinger’s equation describes the time-evolution of the amplitudes

\[ a, b^{(1)}, c^{(1)}, d^{(1)}, \ldots, u, b^{(2)}, c^{(2)}, d^{(2)}, \ldots \]

We will shortly write down these equations. Initially, the system starts off with \( a = u = 1 \).

What, however, constitutes the experience (for us and for the electron) of being “measured” in a classical sense? The reasonable, local assumption to make is that each local state could settle into (“collapse”) into one of several possible states. In the context of this experimental set-up, one of the \( a, b^{(1)}, c^{(1)}, d^{(1)}, \ldots \) goes to 1 at slit 1 and one of \( u, b^{(2)}, c^{(2)}, d^{(2)}, \ldots \) goes to 1 at slit 2. Each of these states (that these are the coefficients of) contain different numbers of photons in a large number of different states. Since \( a \) and \( u \) start out at 1 and the interactions are not necessarily strong, the most likely result is that \( a \) and \( u \) stay equal to 1 - the electron goes through the slits without anything to impede it.

However, in line with general expectation, a classical measurement would be said to occur at a slit if the state corresponding to the coefficient that goes to 1 contains a macroscopic number of photons. A state whose coefficient goes to 1, that doesn’t contain a macroscopic number of photons affected, would not be counted as describing a “detection” of the electron.

The question one could ask is: what happens if two states, at different slits, with macroscopic numbers of photons (in them) were to have their coefficients go to 1 in the same measurement. Let us take the computation further to see how unlikely this is.

We can now write down Schrödinger’s equation for this general state, remembering that the initial state is described by \( a = u = 1, b^{(1,2)} = 0, c^{(1,2)} = 0 \forall i, d^{(1,2)} = 0 \forall i, n \). The algebra is very similar to that in the previous paper [19] and we get, in a convenient matrix form, at slit 1

\[
\begin{align*}
\frac{i}{\partial t} a &= \Gamma_1 b^{(1)} \\
\frac{i}{\partial t} b^{(1)} &= \Gamma_1 a + \sqrt{N}! \bar{\Gamma}_2 \bar{c}^{(1)} \\
\frac{i}{\partial t} \bar{c}^{(1)} &= \bar{\Gamma}_2 \sqrt{N}! b^{(1)} + s \sqrt{\bar{N}} \sqrt{M_1}! \bar{B} \bar{d}^{(1)} \\
\frac{i}{\partial t} \bar{d}^{(1)} &= s \sqrt{\bar{N}} \sqrt{M_1}! \bar{B} \bar{c}^{(1)} + \sqrt{\bar{N}} - 1 \sqrt{M_1}! \bar{E} \bar{d}^{(1)} \\
\end{align*}
\]

where

1. \( \bar{\Gamma}_2 \) is a diagonal matrix, whose elements are the quantities \( \Gamma^{(i)}_2 \)
2. \( \bar{B} \) is a symmetric matrix, required by the unitarity conditions upon the time evolution.
3. \( \bar{E} \) is a symmetric matrix, again required by the unitarity conditions upon the time evolution.

We have used the notation \( \bar{c}^{(1)} = (c^{(1)}_1, c^{(1)}_2, \ldots, c^{(1)}_{R}) \) (the column vector) and \( \bar{d}^{(1)}_2 = (d^{(1)}_{i,j}) \), \( \bar{d}^{(1)}_3 = (d^{(1)}_{i,j,k}) \) (elements written as column vector whose elements are each distinct pair \((i,j)\) and distinct triplet \((i,j,k)\) respectively) etc. Essentially, we write all the distinct states in a column-vector format.

We also have exactly the same equations for slit 2, with

\[
\begin{align*}
a &\leftrightarrow u \\
b^{(1)} &\leftrightarrow b^{(2)} \\
c^{(1)} &\leftrightarrow \bar{c}^{(2)}
\end{align*}
\]

Note the structure of the equations; the coefficient of the term on the right side for every pair of parameters (say
\((a^{(1)}, b^{(1)})\) is the same. The structure of the above is \((m = 1, 2)\)

\[
\begin{pmatrix}
\tilde{a}^{(m)} \\
\tilde{b}^{(m)} \\
\tilde{c}^{(m)} \\
\tilde{d}^{(m)} \\
\vdots \\
\end{pmatrix}
\begin{array}{c}
\frac{i}{\hbar} \frac{\partial}{\partial t}
\end{array}
\begin{pmatrix}
\tilde{a}^{(m)} \\
\tilde{b}^{(m)} \\
\tilde{c}^{(m)} \\
\tilde{d}^{(m)} \\
\vdots \\
\end{pmatrix}
= Q_{\text{total}}
\end{align}
\]

where \(Q_{\text{total}}\) is a time-independent hermitian matrix.

**Theoretical analysis**

The above equations represent a large number of coupled oscillators, see [19], independently at each slit. In general, we should consider it as the reaction of a weakly coupled set of harmonic oscillators to an initial stimulus. One might consider the timescale relevant to the transmission of the stimulus through the forest of oscillators to be the timescale of the electromagnetic interaction, roughly the Compton wavelength of the electron divided by the speed of light, \(\sim 10^{-22}\) s. Hence, if we consider the typical transit time of the electron through the slits, \(\sim \frac{1\text{nm}}{3\times10^8}\text{m/s} \sim 10^{-11}\) s, we have ample time for a large number of interactions. With a large number of interactions, we can consider the system of oscillators to have a small probability (depending on whether the first interaction with the locator photon happens) of being in a state of equilibrium, with all the coefficients (of the states, \(a, b, \ldots\)) achieving roughly the same order of magnitude.

States with larger number of photons will be less likely than states with smaller number of photons, since they correspond to higher-orders of the electron-photon interaction. Generally speaking, if there are \(s\) photons in a state, and \(\Gamma\) is the interaction strength, the probability of this state being realized is equal to \(P(s) \sim \Gamma^{s} / s!\) if, for instance, every interaction produced three photons from one. Generically, this falls off as \(s\) gets large, however, \(\ln(s)\) goes over a much smaller range than \(s\). This generic behavior is important for what follows. To make a concrete example, we assume that \(P(\ln s)\) is constant as a function of \(\ln s\), so that \(P(s) \sim \frac{1}{s}\) - we are just looking at a much smaller range in \(\ln s\). The arguments below go through in a wide variety of distributions of this nature.

This happens at each slit independently. However, for the observer at each slit, the chances are

1. Case 1: nothing happens, the electron passes through the slit without a scattering event
2. Case 2: scattering happens and the amplitudes of being in various states (of electron and photons) become equal very quickly

One might ask about the concordance between the observers at Slit 1 and Slit 2.

1. Could both see Case 1 happen - of course, both observers might see no scattering event, i.e., an electron is not observed at either slit.
2. Could one see Case 1 happen and the other see Case 2? Indeed, yes and in fact, after this, the observer that sees Case 2 would see the system collapse into one of the several states - if the state the system settles on were to contain a macroscopic number of photons, this would constitute a “measurement”. Otherwise, the system would collapse into a state that did not contain enough photons, this would not qualify as a measurement and the interference pattern on the screen would be slightly fuzzy, but not disturbed (by the noise in the system).
3. Could both observers see Case 2 and indeed see each system (at each slit) settle/collapse onto a macroscopic number of photons? The answer to this is, NO. The reason comes from a theorem about the standard deviation of expected separation of two random walkers in a confining space (of size \(Q\)) where the probability of occupying any location is uniform in the log of the coordinate. This happens in the (well-studied) situation of a random walk with reflecting barriers [22] - the standard deviation of the separation is proportional to the (log of the) size \(Q\). If \(Q\) is macroscopic, then one of the observers will see a macroscopic number of photons affected, will consider this a measurement, while the other will not see a macroscopic number of photons affected - the second observation will not constitute a classical measurement.
In addition to the above analysis, the theory of random walks also allows one to estimate the probability that one sees a macroscopic number of photons at both slits simultaneously. This would, again, essentially be the probability that two independent random walkers are found at the same (far) end of a 1-d line (of length $Q$) with reflecting barriers \[29\]. Very roughly speaking, the probability of such a coincidence is proportional to $1/Q$ where $Q \sim 10^{23}$ corresponds to a macroscopic number of photons affected. This number tends to 0 as $Q \to \infty$, as mentioned in the abstract. In addition, since this is the result of equilibration, we can use the solution to the problem of a random walker returning to the same point on a 1-d line with reflecting barriers - the return time is proportional to $Q$ and tends to infinity in the limit of an infinitely massive measuring apparatus. Essentially, we will not see the measuring devices at each slit simultaneously "see" an electron as $Q \to \infty$.

This might lend itself to some experimental consequences, which is the subject of some future work. Briefly, if we relax the notion of "measurement" to allow perturbations of a small number (even 1) of degrees of freedom in the measuring device, then we should expect to see microscopic effects at both slits simultaneously. This can be tested with mesoscopic devices of small size. These sorts of experiments have not been done in this framework and might be interesting to do.

CONCLUSIONS

We have demonstrated in a simplified model of the double slit experiment that it is extremely unlikely and in the limit of an infinitely massive measuring apparatus at each slit, impossible, that one would see a simultaneous detection of an electron at both slits in the reference frame of the observer where the slits are stationary. The result relies on some well-understood notions from the theory of Brownian motion. It is completely consistent with Bell’s theorem[27, 28], as well as the usual result we would expect with the Copenhagen interpretation.

ACKNOWLEDGMENTS

Useful discussions with Scott Thomas and comments from Surjeet Rajendran are acknowledged. The hospitality of the NHETC and the Department of Astronomy and Physics at Rutgers University are sincerely acknowledged.

REFERENCES

* ramakrishna@physics.rutgers.edu

[8] "dephasing" refers to the phenomenon that the states lose phase coherence; then the phases get randomized in interaction with a bath of other oscillators, which is referred to as "decoherence" . See Ref. 35 for a lucid description.
[16] Schlosshauer, Maximilian *Quantum Decoherence* arxiv: 9111.066282v1
[29] essentially, if $P(x) = \frac{1}{Q}(0 < x < Q)$, $\bar{x} = \frac{Q}{2}, \bar{x}^2 = \frac{Q^2}{4}, \text{stdev}(x) = \frac{Q}{\sqrt{12}}$