A New Multi-objective Comprehensive Optimization Model for Design of Anti-slide Piles

Chao Xu
Institute of Geology and Geophysics Chinese Academy of Sciences

Lei Xue (✉ xuelei@mail.iggcas.ac.cn)
Institute of Geology and Geophysics Chinese Academy of Sciences

Yuan Cui
Institute of Geology and Geophysics Chinese Academy of Sciences

Songfeng Guo
Institute of Geology and Geophysics Chinese Academy of Sciences

Mengyang Zhai
Institute of Geology and Geophysics Chinese Academy of Sciences

Fengchang Bu
Institute of Geology and Geophysics Chinese Academy of Sciences

Haoyu Wang
Institute of Geology and Geophysics Chinese Academy of Sciences

Keywords: Anti-slide piles, Multi-objective comprehensive optimization model, Optimization index system, Optimization schemes, Numerical simulation, FLAC3D

Posted Date: June 28th, 2021

DOI: https://doi.org/10.21203/rs.3.rs-206508/v1

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A New Multi-objective Comprehensive Optimization Model
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Chao Xu1,2,3, Lei Xue1,2,3*, Yuan Cui1,2,3, Songfeng Guo1,2,3, Mengyang Zhai1,2,3,
Fengchang Bu1,2,3, Haoyu Wang1,2,3

*Corresponding author at:
Key Laboratory of Shale Gas and Geoengineering, Institute of Geology and Geophysics,
Chinese Academy of Sciences, Beijing 100029, China.
Tel. /fax: +86 010 82998295.
E-mail address: xuelei@mail.iggcas.ac.cn (Lei Xue).

Email addresses of all co-authors:
xuchao@mail.iggcas.ac.cn (Chao Xu);
xuelei@mail.iggcas.ac.cn (Lei Xue);
cuiyuan@mail.iggcas.ac.cn (Yuan Cui);
guosongfeng@mail.iggcas.ac.cn (Songfeng Guo);
zhaimengyang@mail.iggcas.ac.cn (Mengyang Zhai);
bfc@mail.iggcas.ac.cn (Fengchang Bu)
wanghaoyu2020@mails.ucas.ac.cn (Haoyu Wang)

1. Key Laboratory of Shale Gas and Geoengineering, Institute of Geology and
Geophysics, Chinese Academy of Sciences, Beijing, 100029, China
2. Innovation Academy for Earth Science, Chinese Academy of Sciences, Beijing,
100029, China
3. College of Earth and Planetary Sciences, University of Chinese Academy of Sciences,
Beijing, 100049, China
Abstract

Landslides have posed a huge threat to the ecological environment and human society all over the world. As the most conventional reinforcement method, anti-slide piles are widely used in the reinforcement of slopes. Currently, more and more attentions have been paid to the low-cost and high-efficiency optimal design of anti-slide piles. However, limitations in the method of the optimization design for slope reinforced with piles still exist. In this paper, a new multi-objective comprehensive optimization method was proposed for the optimization of the slope reinforced with anti-slide piles. The factor of safety, internal force and deflection of piles were selected as the optimization indexes and the optimization index weight was determined by integrating the subjective and objective weight. The influence of the pile location, pile length and pile spacing on the reinforcement effect was analyzed by the numerical simulation. Through the simulation case analysis, the proposed model had achieved good effects on the optimization design of anti-slide piles, which could effectively reduce the engineering costs. The optimization results showed that the best reinforcement effect for the homogeneous slope could be obtained when the anti-slide piles with the critical pile length and small pile spacing was located in the middle of the slope. This provides a new solution for the optimization design of other types of complex slopes, and has broad application prospects.

Keywords Anti-slide piles · Multi-objective comprehensive optimization model · Optimization index system · Optimization schemes · Numerical simulation · FLAC^3D
Declarations

Funding
This research was funded by the National Key Research and Development Program of China under Grant Nos. 2018YFC1505302 and 2019YFC1509701, the Strategic Priority Research Program of the Chinese Academy of Sciences under Grant No. XDA23090402, and the National Natural Science Foundation of China under Grant Nos. 41977249 and 42090052.

Conflicts of interest/Competing interests
The authors declare that they have no conflict of interest.

Availability of data and material
Availability. If necessary, please email to the corresponding author.
(Data transparency.)

Code availability
Availability. If necessary, please email to the corresponding author.
(Software application or custom code not applicable.)

Authors' contributions
Conceptualization: Chao Xu, LeiXue; Data curation: Chao Xu, Yuan Cui, Songfeng Guo; Formal analysis: Chao Xu, Lei Xue, Fengchang Bu; Funding acquisition: Lei Xue; Investigation: Chao Xu, Fengchang Bu, Haoyu Wang; Methodology: Chao Xu, LeiXue; Project administration: LeiXue; Resources: LeiXue, Songfeng Guo; Software: Chao Xu, Songfeng Guo; Supervision: LeiXue, Songfeng Guo; Validation: Yuan Cui, Mengyang Zhai, Haoyu Wang; Visualization Chao Xu, Mengyang Zhai; Writing – original draft: Chao Xu; Writing – review & editing: Chao Xu, LeiXue.
1 Introduction

With the rapid development of global engineering construction, slope stability has become a worldwide significant problem in engineering practice (Hassiotis et al. 1997; Li et al. 2020). The anti-slide pile as the most conventional reinforcement method is widely used due to the advantages of strong anti-sliding ability and convenient construction (Cai and Ugai 2000; Kanagasabai et al. 2011; Liu et al. 2020; Wei and Cheng 2009). Therefore, the design of anti-slide pile is crucial. At present, the mainstream design methods for anti-slide pile include loading-structure method (Price and Morgenstern 1967), Viggiani method (Viggiani 1981), Ito Tomio method (Ito and Matsui 1975) and Poulos method (Poulos 1973), which are based on limit equilibrium or displacement compatibility. The basic idea for these methods is to firstly determine the residual pushing force and anti-slide force satisfied with the stability of the slope, then calculate the bending moment and shear force for each pile, and finally give the suitable design parameters such as the pile length, the pile spacing and the pile location (Kanagasabai et al. 2011). However, these methods fail to consider the interaction between pile and soil, which can not accurately reflect the true stability of the slope reinforced with piles.

Recently, more and more scholars (Ausilio et al. 2001; Cai and Ugai 2000; Li et al. 2016; Nian et al. 2008; Won et al. 2005; Yang and Zhang 2020) have realized the fact that the pile length, pile location, pile spacing and other design parameters have a significant impact on the reinforcement of the slope via limit equilibrium methods and finite element methods. For example, the optimization results of pile length show that there is a critical pile length which can achieve the optimal reinforcement effect, and excessive pile length can not increase the factor of safety of the slope but will cause construction waste (Chen et al. 2020; Griffiths et al. 2010; Shooshpasha and Amirdehi 2015; Won et al. 2005). The researches about pile location reveal the fact that the best slope reinforcement effect can be obtained when the anti-slide pile is located in the middle of the slope (Ausilio et al. 2001; Cai and Ugai 2000; Hajiazizi et al. 2017a; Li et al. 2012; Li et al. 2020; Zhang et al. 2017b), Wei and Cheng (2009) even gave the
precise pile location which is 0.2m above the middle of the slope. The studies about
optimal pile spacing show that the smaller spacing of the anti-slide pile is, the better
integrity of reinforced slope is, which is more conducive to the stability of the
reinforced slope (Cai and Ugai 2000; Hajiazizi et al. 2017b; Jeong et al. 2003;
Kourkoulis et al. 2011; Sun et al. 2016; Zhang et al. 2017a), but the determination of
pile spacing mainly depends on the soil arching effect between piles in practical
engineering. Although these studies have a guiding significance for the optimization
design of anti-slide piles, the optimization results may have some certain deviations
which are because the only the factor of safety of reinforced slope is taken as the only
optimization objective.

In fact, the slope reinforced with piles is a complete organism composed by the slope
and anti-slide piles. Numerous cases of the reinforced slope instability (Hu et al. 2020;
Li et al. 2010; Li et al. 2013; Liu et al. 2016) show that the optimization design for anti-
slide piles need to consider not only the stability of the slope, but also the safety of anti-
slide pile itself. Therefore, it is particularly important to consider the internal force and
deformation of the pile in the optimization design process. Yang et al. (2011b) and Wang
et al. (2015) revealed the internal force and deformation characteristics of pile under
various reinforcement schemes, and pointed out that the pile may not be under a safe
state when the slope obtained the maximum factor of safety. Zhu et al. (2017) fully
considered the change of pile head displacement and established the deformation
prediction model for anti-slide piles, which provided theoretical guidance for optimal
design. However, their studies failed to consider the coordination and contradiction of
various pile elements and give qualitative optimization results. Moreover, the analytic
hierarchy process (AHP) (Xu 2013) and multi-objective comprehensive evaluation
method (Li and Wei 2018) were used to optimize the design of anti-slide piles based on
the evaluation indexes considered the factor of safety and internal force of piles
comprehensively, which is concordant with practical situation.

In conclusion, it is unreasonable to ignore the safety state of anti-slide piles to
evaluate the stability of the slope reinforced. Therefore, in this paper the multi-objective
comprehensive optimization model based on improved fuzzy comprehensive
evaluation was developed to optimize the design of anti-slide pile, and the optimization indexes system and the comprehensive index weight was established, the factor of safety, bending moment, shear force and deflection were selected as the optimization indexes. FLAC$^{3D}$ software was used to establish a three-dimensional numerical model which could reflect the interaction between the slope and piles to analyze the changes of factor of safety and the internal force and deformation of anti-slide piles under various reinforcement schemes, and the proposed method was used to optimize. Thus, the proposed optimization model is expected to provide a reference for the optimization design of anti-slide pile engineering.

2 Multi-objective comprehensive optimization model

2.1 Feasibility of the method

Generally, most of the engineering optimization design problems are multi-objective optimization problems and there are usually contradictions between various optimization objectives. Therefore, the results of optimization design based on single factor merely are unreliable. The multi-objective comprehensive optimization model takes the research object as a whole, which has following advantages: 1) it can comprehensively consider the mutual influence between various factors; 2) it can quantify the impact of indexes on optimization goals. In fact, the optimization design of anti-slide piles to strengthen the slope is a multi-objective optimization problem, which could be solved reliably by the multi-objective comprehensive optimization model.

2.2 Optimization design process and method

Figure 1 shows the flowchart of the multi-objective comprehensive optimization model based on an improved fuzzy comprehensive evaluation method.

![Flowchart of the multi-objective comprehensive optimization model.](Fig. 1)

2.2.1 Determination of the optimal goals

In the design of slope reinforced with anti-slide piles, factors such as pile location,
pile length, and pile spacing are usually considered in order to achieve a good reinforcement effect. However, the overly conservative design has led to high engineering costs in most cases (Hu et al. 2020; Zhao et al. 2006). Therefore, the optimum design of anti-slide piles aims to reduce as many engineering costs as possible while satisfying the safety of supporting structure without affecting the stability of reinforced slope.

2.2.2 Construction of the optimization index system

The stability of slope, the safety of supporting structure and the economy must be taken into account in the selection of evaluating index system, which will determine the accuracy of the optimization model. The index of safety factor reflects the stability of slope reinforced with piles; the indexes of bending moment, internal force and displacement reflect the safety of anti-slide piles. The changes of the above indicators correspond to different optimization schemes (such as different pile locations, pile lengths and pile spacings), and will have an obvious impact on the construction difficulty and engineering cost. Therefore, this paper selects the factor of safety, bending moment, shear force and deflection of anti-slide piles as the main optimization indexes.

2.2.3 Construction of the index function

Supposing that there are \( n \) optimization indexes to compose a sample set of optimization indexes \( \{a_{i,j} \mid i = 1 \sim n, j = 1 \sim m\} \) for all \( m \) schemes. In order to make the data highly comparable and the modeling universal, this study adopts the percentage system (Liu et al. 2019) and the maximum-minimum standardization method to standardize the evaluating indexes \( a_{i,j} \).

The standardized formula for the positive optimization indexes that are positively correlated with the results, such as the factor of safety, can be taken as follows:

\[
    r_{i,j} = \alpha + \beta \cdot e^{\frac{a_{i,j} - a_{i,j}\max}{a_{i,j}\max - a_{i,j}\min}}
\]  

(1)

The standardized formula for the negative optimization indexes that are negatively correlated with the results, such as internal force and deflection of piles, can be taken
as follows:

\[ r_{i,j} = \alpha + \beta \cdot e^{-\frac{a_{i,j_{\min}} - a_{i,j_{\max}}}{a_{i,j_{\max}} - a_{i,j_{\min}}}} \]  

(2)

where \( a_{i,j_{\min}} \) and \( a_{i,j_{\max}} \) are the minimum and maximum values of the \( i \)th index in the

\( j \)th scheme, respectively; \( r_{i,j} \) is the standardized optimization value, that is, the

relative membership value of the \( i \)th index in the \( j \)th scheme is subordinate to the

optimal value; \( \alpha \) and \( \beta \) are constant indicators for constructing the percentile system

which meet \( \alpha + \beta = 100 \).

Thus, the fuzzy matrix can be determined as follows:

\[
R = \begin{bmatrix}
    r_{1,1} & r_{1,2} & \cdots & r_{1,m} \\
    r_{2,1} & r_{2,2} & \cdots & r_{2,m} \\
    \vdots   & \vdots   & \ddots & \vdots   \\
    r_{n,1} & r_{n,2} & \cdots & r_{n,m}
\end{bmatrix}_{m \times n}
\]

(3)

2.2.4 Determination of index weight

The subjective weight determined by the AHP and objective weight determined by
the entropy method are used to establish the comprehensive weight of optimization
indexes. Thus, the comprehensive weight of evaluating index can be obtained as
follows:

\[
w_{(i)} = \frac{w_{s(i)} \cdot w_{o(i)}}{\sum_{i=1}^{n} w_{s(i)} \cdot w_{o(i)}}
\]

(4)

where \( w_{s(i)} \) and \( w_{o(i)} \) are the subjective weight and objective weight, respectively.

The objective weight \( w_{o(i)} \) can be calculated as follows:

\[
w_{o(i)} = \frac{1-e_{(i)}}{\sum_{i=1}^{n}(1-e_{(i)})}
\]

(5)

where \( e_{(i)} \) is the entropy of the \( i \)th optimization index.

2.2.5 Analysis and comparison of the optimization results

The fuzzy comprehensive optimization value \( k_{(j)} \) can be obtained by synthesizing the
weight of each optimization index \( w_{(i)} \) and the relative membership value \( r_{(i,j)} \) of the
corresponding optimization index in different schemes.

\[ k_{(j)} = \sum_{i=1}^{n} \sum_{j=1}^{m} w_{(i) \cdot r_{(i,j)}} \]  

(6)

The value of \( k_{(j)} \) determines the optimal membership degree of different schemes. Generally, the larger the value \( k_{(j)} \) is, the more reasonable the scheme is.

3 Determination of optimization indexes and values by numerical simulation

As mentioned above, the factor of safety, internal forces and deflection of the pile are selected as the target value for the optimization design of anti-slide piles, which will be significantly affected by the reinforcement options such as pile lengths, pile location and pile spacing (Cai and Ugai 2000). Therefore, the acquisition of optimization values is the key premise of comprehensive optimization. For this reason, numerical simulation method is selected to obtain the accurate optimization index value and verify the reliability of the proposed model.

3.1 Establishment of numerical model

The homogeneous slope models of different reinforcement schemes with anti-slide piles have been established by the finite difference software FLAC³D as shown in Fig. 2, and the reinforced slope model considered by many researchers (Cai and Ugai 2000; Wei and Cheng 2009; Won et al. 2005) is adopted excepts some changes in the dimensions and gradients of slope. The uniform boundary conditions follow: the displacement of bottom boundary is completely fixed, the horizontal displacement of left and right boundary is restrained, and the upper boundary is free to move. Mohr-Coulomb constitutive model is selected to simulate the deformation and failure behavior of slope soil. The initial stress field only considers the self-weight stress field. The entirely run-through of plastic zone is regarded as the criterion of slope instability. Details about the parameters of soil are shown in Table 1. The factor of safety is calculated via the strength reduction method (SRM) (Griffiths and Lane 1999; Matsui and San 1992; Wei et al. 2009; Zheng and Zhao 2004; Zienkiewicz et al. 1975).
Fig. 2 Numerical model of slope reinforced with anti-slide piles. $L_x$, $L_p$, $L$ and $S$ stand for the distance from the pile to the slope toe, the horizontal length of the slope, the pile length and the pile spacing, respectively. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

Fig. 3 Mechanical model of pile (modified after Wang et al. (2015)). Spring A, spring B stand for the normal coupling spring and the shear coupling spring, respectively. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

Table 1 Physical and mechanical parameters of the slope

<table>
<thead>
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<td>$L_x$</td>
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Considering that the internal force of any section can not be obtained directly with the solid element piles, and the accuracy of the calculating results is affected by the mesh size (Chen et al. 2019), the structural element pile was used to simulate the anti-slide pile due to the advantages of easy modeling, high calculation efficiency and guaranteed accuracy (Griffiths et al. 2010; Lee et al. 2014). The mechanical model of pile structural element is shown in Fig. 3, the transfer of force and bending moment between the pile element and the mesh element could be realized by the normal coupling spring (Spring A) and the shear coupling spring (Spring B) at the position of structural element node, which realizes the coupling effect between pile and soil. Details about the parameters of anti-slide piles are shown in Table 2.

Table 2 Physical and mechanical parameters of anti-slide piles

<table>
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3.2 Influence of anti-slide pile location on slope reinforcement

The influence of the anti-slide pile reinforcement location on the optimization indexes is studied with the pile spacing of 5m. The pile location is defined by the ratio of pile horizontal distance from the slope toe ($L_x$) to the horizontal length of the slope ($L_p$), which is shown in Fig. 2. The effects of various pile locations and pile lengths on the factor of safety of the slope reinforced with piles are shown in Fig. 4, and the
maximum factor of safety for each pile location is shown in Fig. 5. It can be obtained from the Figs. 4 and 5 that the factor of safety is the largest and the reinforcement effect is the best when the anti-slide pile is located in the middle of the slope \((L_x/L_p=0.5)\); on the contrary, the factor of safety is the smallest and the reinforcement effect is poor when the pile location is at the toe of the slope \((L_x/L_p=0.1)\). The results obtained in present research are similar to those of Cai and Ugai (2000), Griffiths et al. (2010) and Yang et al. (2011b).

Fig. 4 Factors of safety for various pile lengths and pile locations. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

Fig. 5 Maximum factor of safety for various pile locations

The distribution of shear strain increment zone is consistent with the large deformation area of the slope, which can reflect the position of the critical slip surface precisely (Zheng 2012). The effect of various pile locations on the maximum shear strain increment and the position of critical slip surface obtained by FISH language is shown in Figs. 6 and 7, respectively. It is seen from Figs. 6 and 7 that the change of pile location has a significant impact on the distribution of critical slip surface. In other word, the concentration region of shear strain increment and the run-through critical slip surface tends to form behind the anti-slide piles gradually when the pile location is located at the lower-middle part of the slope \((L_x/L_p=0.1, 0.3)\), however, the concentration region of shear strain increment and the run-through critical slip surface tends to form in front of the anti-slide piles eventually when the pile location is located at the upper-middle part of the slope \((L_x/L_p=0.7, 0.9)\). Therefore, it is concluded that there are three different failure modes of slope reinforced with piles. These failure modes are as follows: (1) slide will originate from posterior surface of the piles when the pile location is located in the lower-middle part; (2) the critical slip surface is divided into two disconnected parts and slide is not easy to originate when the pile location is locked in the middle part; (3) slide will originate from anterior of the piles...
when the pile location is located in the upper-middle part.

**Fig. 6** Contour of maximum shear strain increment for various pile locations.  
**a** without piles;  
**b** $L_x/L_p=0.1$, pile length $L=12m$;  
**c** $L_x/L_p=0.3$, pile length $L=18m$;  
**d** $L_x/L_p=0.5$, pile length $L=28m$;  
**e** $L_x/L_p=0.7$, pile length $L=30m$;  
**f** $L_x/L_p=0.9$, pile length $L=24m$. (Taking the pile spacing of $S=5m$ as an example). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

**Fig. 7** The critical slip surface for various pile lengths and pile locations.  
**a** $L_x/L_p=0.1$;  
**b** $L_x/L_p=0.3$;  
**c** $L_x/L_p=0.5$;  
**d** $L_x/L_p=0.7$;  
**e** $L_x/L_p=0.9$. (Taking the pile spacing of $S=5m$ as an example). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

The effect of various pile locations on anti-slide pile behaviors is shown in Fig. 8. The bending moment, the shear force and the deflection of the pile increase at first and then decrease with the pile location from the toe upwards the top of slope, and the maximum points of both pile behaviors appear in the pile located at the middle part of the slope. It should be noted that the depth of maximum bending moment or the shear force valued zero at each pile location has a good correspondence with the position of critical slip surface. Thus, the various pile locations not only affect the factor of safety of the slope reinforced, but also change the distribution of the inter force of pile.

**Fig. 8** Anti-slide pile behaviors for various pile locations.  
**a** Bending moment;  
**b** Shear force;  
**c** Deflection. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

**3.3 Influence of anti-slide pile length on slope reinforcement**

The determination of pile length is the key to the optimization design of anti-slide piles. Too short pile length is not conducive to the slope stability (Kourkoulis et al. 2011), and too long pile length will increase the engineering costs (Yang et al. 2011b).
Figure 4 shows the effect of the pile length on the factor of safety of the slope reinforced with anti-slide piles. Taking the pile location of $L_x/L_p=0.3$ as an example, as expected, the factor of safety of reinforced slope increase with the increasing of the pile length. However, when the pile length exceeds a certain length (18m in present study) which is named the critical pile length (Griffiths et al. 2010), the factor of safety is close to a constant gradually, and this is because the enough anti-sliding force can be provided by the embedded length of pile in stable stratum to resist the sliding force.

In order to further study the effect of various pile lengths on the slope reinforcement, the maximum shear strain increment and the critical slip surface are obtained at the pile location of $L_x/L_p=0.3$, as shown in Figs. 9 and 7b. It can be observed that with the increase of pile length, the zone of maximum shear strain increment is gradually divided into two parts which are not disconnected, but when the pile length exceeds 18m, the run-through zone of maximum shear strain increment is reformed (Fig. 9e). As shown in Fig. 7b, the critical slip surface becomes deeper with the increase of pile length, and the failure mode of reinforced slope changes from shallow sliding to deep sliding. This is mainly due to the complex structure formed by pile-soil interaction improves the strength of soil around the pile. However, when the pile length is more than 18m, the critical slip surface suddenly becomes shallow and passes through the top of pile. The main reason for this is that deep sliding needs more energy due to the reinforcement of anti-slide piles, while the shallow sliding only requires less energy to produce.

Fig. 9 Contour of maximum shear strain increment for various pile lengths. a Pile length $L=6m$; b Pile length $L=10m$; c Pile length $L=14m$; d Pile length $L=16m$; e Pile length $L=18m$; f Pile length $L=22m$ (Taking the pile location of $L_x/L_p=0.3$ and the pile spacing of $S=5m$ as an example). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

The effect of various pile lengths on the pile behaviors is shown in Fig. 10. It can be seen that, when the pile length is less than the critical pile length (18m), the bending moment (Fig. 10a) increases with the increase of the pile length, and the position of the
maximum bending moment is continuously away from the top of piles, which corresponds well to the position of the critical slip surface (Fig. 7b); the positive shear force of piles (Fig. 10b) increases as the pile length increases; the pile deflection increases with the increase of the pile length, but it should be noted that the distribution of deflection is almost linearly (Fig. 10c) when the pile length is short, which indicates that the pile is prone to overturning failure under too short pile length. When the pile length exceeds the critical pile length, the bending moment, shear force and deflection all tend to be a stable distribution, which is consistent with the change law of the factor of safety.

Fig. 10 Anti-slide pile behaviors for various pile lengths. a Bending moment; b Shear force; c Deflection. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

3.4 Influence of anti-slide pile spacing on slope reinforcement

Taking the pile location of $L_x/L_p=0.3$ and the critical pile length (18m) as an example, the effect of various pile spacings on reinforced slope is studied. Figure 11 shows the change of factors of safety under various pile spacings, it can be seen that the factor of safety of the reinforced slope decreases with the increase of pile spacing.

Fig. 11 Factors of safety for various pile spacings under the critical pile length. $Fos$, $S$ stand for the factor of safety and the pile spacing, respective. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

The effects of various pile spacings on the maximum shear strain increment and the critical slip surface is shown in Fig. 12. It can be seen that when the pile spacing is small, the critical slip surface between two anti-slide piles is shallow and almost passes over the top of the pile. With the increasing of the pile spacing, the critical slip surface gradually becomes deeper and the instability mode has changed. When the pile spacing is large enough (Fig. 12a4), a complete and run-through critical slip surface is formed.
gradually, which is nearly close to the critical slip surface of slope unreinforced (Fig. 6a). This may be related to the evolution of soil arch under various pile spacings.

**Fig. 12** Contour of shear strain increment and the critical slip surface for various pile spacings under the critical pile length. \( \text{a1, b1} \) Pile spacing \( S=4\text{m} \); \( \text{a2, b2} \) Pile spacing \( S=5\text{m} \); \( \text{a3, b3} \) Pile spacing \( S=6\text{m} \); \( \text{a4, b4} \) Pile spacing \( S=7\text{m} \) (Taking the pile location of \( L_x/L_p=0.3 \) and the critical pile length \( L=18\text{m} \) as an example). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

The effect of various pile spacings on the pile behaviors is shown in Fig. 13. It can be concluded that the bending moment (Fig. 13a) and the shear force (Fig. 13b) increase with the increase of pile spacing. This can be explained by the fact that the anti-slide piles act as the retaining walls and the integrity and strength of the pile and soil are improved significantly while the pile spacing decreases, so that soil wouldn't reach the limit state until the soil with large deformation (Cai and Ugai 2000), which can be demonstrated by the pile deflection (Fig. 13c); soil arch between piles disappears gradually when the pile spacing increases and only a single anti-slide pile works at this moment.

**Fig. 13** Pile behaviors for various pile spacings under the critical pile length. \( \text{a} \) Bending moment; \( \text{b} \) Shear force; \( \text{c} \) Deflection. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

To sum up, the interaction between pile and soil is fully considered with the numerical simulation method and the factor of safety, internal force and deformation of piles obtained under various reinforcement options are more realistic, which is in good agreement with previous studies (Cai and Ugai 2000; Gao et al. 2015; Hajiazizi et al. 2017b; Jeong et al. 2003; Sun et al. 2016; Yang et al. 2011b). Therefore, it is feasible to obtain the optimization indexes values by the numerical simulation and to optimize designs combined with the proposed multi-objective comprehensive optimization
4 Results and discussion

The factor of safety and the bending moment, shear force and deflection of piles were obtained based on the numerical simulation under various reinforcement schemes, and results were analyzed with the proposed multi-objective comprehensive optimization model.

4.1 Results analysis

4.1.1 Calculation results of indicator value and weight

The indicator value of optimization system was determined by the numerical simulation. It should be noted that the factor of safety of reinforced slope belongs to the positive optimization index and the bending moment, shear force and deflection belong to the negative optimization indexes, so Eqs. (1) and (2) were used for normalization calculation, respectively. The calculation results of indicator value under different reinforcement schemes are shown in Tables 3-7.

Table 3 The standardized values of optimization indexes (Pile location $L_x/L_p=0.1$)

Table 4 The standardized values of optimization indexes (Pile location $L_x/L_p=0.3$)

Table 5 The standardized values of optimization indexes (Pile location $L_x/L_p=0.5$)

Table 6 The standardized values of optimization indexes (Pile location $L_x/L_p=0.7$)

Table 7 The standardized values of optimization indexes (Pile location $L_x/L_p=0.9$)

Decision-making AHP method was adopted to determine the subjective weight of optimization indexes. Considering intentions of decision makers, engineering experience and judgements of geological hazard experts, 1-9 ratio scaling method was taken to define the relative importance and subjective weight of each optimization index, as shown in Table 8. According to the principle of AHP (Brunelli 2015), the maximum
The eigenvalue of judgment matrix ($\lambda_{\text{max}}$) is 4.25, the consistency index (CI) is 0.08, the consistency ratio (CR) equals 0.09 and is less than 0.1, which meets the consistency requirements.

**Table 8** Subjective weight determination for optimization indexes

The objective weight was calculated by Eq. (5) based on the entropy method, and the results are shown in Table 9.

The comprehensive weight of each evaluating index was calculated via Eq. (4) as shown in Table 9. Among them, the weight of factor of safety is the largest and that of shear force of pile is the smallest.

**Table 9** Comprehensive weight determination for optimization indexes

4.1.2 Optimal results analysis

The fuzzy comprehensive optimization value $k(j)$ was calculated according to Eq. (6) (where $\alpha=60$ and $\beta=40$). The comprehensive optimization results is shown in Fig. 14. It can be seen that the results under various reinforcement schemes are significantly different. According to the principle of optimal judgement, the comprehensive optimization value corresponding to scheme 35 is the highest, that is, the anti-slide pile located in the middle of slope, with the pile length of 28m and pile location of 4m is the most reasonable choice. In addition, more details could be drawn as follows:

**Fig.14** Optimization results based on the multi-objective comprehensive optimization model. $S$, $L_x/L_p$ stand for the pile spacing and the pile location, respectively. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

(1) When the anti-slide pile is located in the middle or upper-middle part ($L_x/L_p=0.7$) of the slope, the effect of slope reinforced with piles is obviously better than that of the toe or shoulder of the slope, which is in good agreement with the numerical simulation
(Section 3.2) and results obtained by Hassiotis et al. (1997) and Yang et al. (2011a).

(2) The increase of pile length can significantly improve the reinforcement effect of the slope, but it does not mean that the longer the pile length, the better the reinforcement effect. For example, when the pile is located in the lower-middle part ($L_x/L_p=0.3$) with the pile spacing of 5m, the value of $k_i$ increases slightly or even decreases when the pile length exceeds 18m (Fig. 14), this is mainly because excessive pile length leads to the increase of internal force of anti-slide piles under the premise of meeting design requirements of factor of safety, which is not conductive to the safety of piles.

(3) The smaller pile spacing is, the better reinforcement effect is. In addition, the reinforcement effect of piles located in the location of $L_x/L_p=0.7$ with pile spacing of 5m is significantly better than that in the location of $L_x/L_p=0.5$ with pile spacing of 6m, therefore, the pile spacing can be appropriately reduced to improve the reinforcement effect in actual engineering when the anti-pile is located in non-middle position.

4.2 Discussion

4.2.1 Comparisons with results under various $\alpha$ and $\beta$ values

In order to make the evaluating indexes more comparable and keep as much information as possible about the changes in the optimization index values, the constants $\alpha$ and $\beta$ were introduced to construct the normalization functions (Eqs. (1) and (2)) based on the percentage system. Figure 15 presents the effect of various combinations of $\alpha$ and $\beta$ on the optimization results of anti-slide piles. The result indicates that curves of the optimization result under various values of $\alpha$ and $\beta$ present the approximately parallel relationship and have exactly the same changing law. Besides, the larger the value of $\alpha$ is, the greater the optimization value for the corresponding scheme. However, the changes of $\alpha$ and $\beta$ only change the absolute value of optimization results and amplitude of variation of curves, but have no effects on the final optimization results.

**Fig.15** Comparison of optimization results with different $\alpha$ and $\beta$ values (Refer to Tables 3-7 or Fig. 14 for detailed reinforcement scheme corresponding to scheme number). For interpretation of the
4.2.2 Comparisons with results under various weights

The final optimization results under various index weight types are shown in Fig. 16. It can be seen that, the result (Red line in Fig. 16) by using the subjective weight only indicates that scheme 35 is the optimal reinforcement option, which is basically consistent with the conclusion drawn via the proposed method in this article, but the optimization values of each scheme have little difference, which is easy to make wrong decisions due to the human factors implications and inaccurate data. The optimization result (Blue line in Fig. 16) obtained only by the objective weight reveals that the reinforcement effect is the worst when the anti-slide pile is located in the middle part of the slope, which is totally at variance with the practical engineering experience, this may be the reason that the index weight of entropy method is determined according to the variation degree of the index, which ignores the importance of the index itself. Comparison of the three different optimization results shows that the comprehensive weight proposed in this article make the optimization results more scientific and reasonable. Therefore, with the continuous development of habitable earth construction, the proposed multi-objective comprehensive optimization model will play an important role in the design of slopes reinforcement.

Fig.16 Comparison of optimization results with different weights (Refer to Tables 3-7 or Fig. 14 for detailed reinforcement scheme corresponding to scheme number). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

4.2.3 Limitation of the proposed method

In the current study, the numerical simulation method was used to obtain the values of evaluating indexes in the multi-objective comprehensive optimization model. Although more reasonable optimization results had been achieved, more engineering cases and field monitoring data are still needed to further study to verify the accuracy and applicability of this proposed model. Besides, the comprehensive weight used in
this article considers the advantages of both subjective and objective weight, and minimizes the adverse effects of shortcomings of two on optimization results, but it is still unavoidable that the weight obtained goes against the actual situation, which leads to make an absurd decision-making. Therefore, it is necessary to further optimize the index weight based on methods of big data, machine learning and deep learning.

5 Conclusion

The multi-objective comprehensive optimization model for the design of slope reinforced with anti-slide piles was proposed based on the traditional fuzzy comprehensive evaluation method, and the reliability of the model was verified by finite element numerical simulation. The main conclusions are as follows:

(1) According to the numerical simulation results, various pile locations, pile lengths and pile spacings have significant effects on the slope reinforced. The best reinforcement effect could be obtained when the pile is located in the middle part of the slope. The increase of the pile length can increase the reinforcement effect obviously, but it will not continue to increase the slope stability when the pile length exceeds the critical pile length. The larger the pile spacing is, the worse the stability of the slope and safety of the anti-slide pile are.

(2) The factor of safety, internal force and deflection of the anti-slide pile were selected as the optimization index system to ensure that the optimized reinforcement scheme could meet the stability of the pile-slope system. Meanwhile, the comprehensive weight was determined combined with the subjective and objective, which is more in line with practical engineering cases.

(3) Based on the three-dimensional slope numerical model, the proposed multi-objective comprehensive optimization model was applied to optimize various reinforcement schemes, which obtained reasonable optimization results. This provides a new solution for the optimization design of other types of complex slopes, and has broad application prospects.
Acknowledgements This research was funded by the National Key Research and Development Program of China under Grant Nos. 2018YFC1505302 and 2019YFC1509701, the Strategic Priority Research Program of the Chinese Academy of Sciences under Grant No. XDA23090402, and the National Natural Science Foundation of China under Grant Nos. 41977249 and 42090052.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.
References


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Table 8 Subjective weight determination for optimization indexes

Table 9 Comprehensive weight determination for optimization indexes
Table list

Table 1 Physical and mechanical parameters of the slope

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Table 6 The standardized values of optimization indexes (Pile location $L_x/L_p=0.7$)

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Table 7 The standardized values of optimization indexes (Pile location $L_x/L_p=0.9$)

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Table 8 Subjective weight determination for optimization indexes

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Table 9 Comprehensive weight determination for optimization indexes

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Figure captions

**Fig. 1** Flowchart of the multi-objective comprehensive optimization model.

**Fig. 2** Numerical model of slope reinforced with anti-slide piles. $L_x, L_p, L$ and $S$ stand for the distance from the pile to the slope toe, the horizontal length of the slope, the pile length and the pile spacing, respectively. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

**Fig. 3** Mechanical model of pile (modified after Wang et al. (2015)). Spring A, spring B stand for the normal coupling spring and the shear coupling spring, respectively. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

**Fig. 4** Factors of safety for various pile lengths and pile locations. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

**Fig. 5** Maximum factor of safety for various pile locations

**Fig. 6** Contour of maximum shear strain increment for various pile locations. a without piles; b $L_x/L_p=0.1$, pile length $L=12m$; c $L_x/L_p=0.3$, pile length $L=18m$; d $L_x/L_p=0.5$, pile length $L=28m$; e $L_x/L_p=0.7$, pile length $L=30m$; f $L_x/L_p=0.9$, pile length $L=24m$. (Taking the pile spacing of $S=5m$ as an example). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

**Fig. 7** The critical slip surface for various pile lengths and pile locations. a $L_x/L_p=0.1$; b $L_x/L_p=0.3$; c $L_x/L_p=0.5$; d $L_x/L_p=0.7$; e $L_x/L_p=0.9$. (Taking the pile spacing of $S=5m$ as an example). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

**Fig. 8** Anti-slide pile behaviors for various pile locations. a Bending moment; b Shear force; c Deflection. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

**Fig. 9** Contour of maximum shear strain increment for various pile lengths. a Pile length $L=6m$; b Pile length $L=10m$; c Pile length $L=14m$; d Pile length $L=16m$; e Pile length $L=18m$; f Pile length $L=22m$ (Taking the pile location of $L_x/L_p=0.3$ and the pile spacing of $S=5m$ as an example). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.
Fig. 10 Anti-slide pile behaviors for various pile lengths. a Bending moment; b Shear force; c Deflection. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

Fig. 11 Factors of safety for various pile spacings under the critical pile length. Fos, S stand for the factor of safety and the pile spacing, respective. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

Fig. 12 Contour of shear strain increment and the critical slip surface for various pile spacings under the critical pile length. a1, b1 Pile spacing S=4m; a2, b2 Pile spacing S=5m; a3, b3 Pile spacing S=6m; a4, b4 Pile spacing S=7m (Taking the pile location of \( L_x/L_p = 0.3 \) and the critical pile length \( L = 18 \text{m} \) as an example). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

Fig. 13 Pile behaviors for various pile spacings under the critical pile length. a Bending moment; b Shear force; c Deflection. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

Fig. 14 Optimization results based on the multi-objective comprehensive optimization model. S, \( L_x/L_p \) stand for the pile spacing and the pile location, respectively. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

Fig. 15 Comparison of optimization results with different \( \alpha \) and \( \beta \) values (Refer to Tables 3-7 or Fig. 14 for detailed reinforcement scheme corresponding to scheme number). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.

Fig. 16 Comparison of optimization results with different weights (Refer to Tables 3-7 or Fig. 14 for detailed reinforcement scheme corresponding to scheme number). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.
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Fig. 12 Contour of shear strain increment and the critical slip surface for various pile spacings under the critical pile length. a1, b1 Pile spacing $S=4\text{m}$; a2, b2 Pile spacing $S=5\text{m}$; a3, b3 Pile spacing $S=6\text{m}$; a4, b4 Pile spacing $S=7\text{m}$ (Taking the pile location of $L_x/L_y=0.3$ and the critical pile length ($L=18\text{m}$) as an example). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.
Fig. 13 Pile behaviors for various pile spacings under the critical pile length. a Bending moment; b Shear force; c Deflection. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.
Fig. 14 Optimization results based on the multi-objective comprehensive optimization model. $S$, $L_x/L_p$ stand for the pile spacing and the pile location, respectively. For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.
Fig. 15 Comparison of optimization results with different $\alpha$ and $\beta$ values (Refer to Tables 3-7 or Fig. 14 for detailed reinforcement scheme corresponding to scheme number). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.
Fig. 16 Comparison of optimization results with different weights (Refer to Tables 3-7 or Fig. 14 for detailed reinforcement scheme corresponding to scheme number). For interpretation of the references to color in this figure, the reader is referred to the electronic version of this page.