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Coupled analytical solutions for high slopes considering rock creep effects and changes of anchoring force in prestressed cables

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Abstract

The unloading effects induced by rock excavation on high slopes are significant, and prestressed anchor cable is an effective reinforcement method for high slope safety. In this work, we consider the interaction between rock creep in high slopes and the changing anchoring force of prestressed cables. We then derive theoretical solutions for the unloading rock creep and anchoring force of prestressed cables considering the coupling effect, and verify the solutions using numerical simulation. First, based on the Boussinesq problem in elastic mechanics, we simplify the problem of slope reinforcement with a single prestressed anchor cable to the problem of concentrated force acting on a boundary of a semi-infinite medium. The concentrated force is affected by the excavation unloading effect from the slope and the anchoring forces from the anchor cables. Based on this simplification, we derive elastic solutions for the slope unloading displacement after excavation and for the anchoring force of prestressed cables. Second, considering rock creep behavior and varying anchoring force, Burgers model is used for rock masses and elastic model is used for anchor cables. According to the coordinated deformation between rock masses and anchor cables, we obtain the analytical solutions for the rock displacement and for the anchoring force of the cables under the coupling action in the Laplace space, based on which the viscoelastic solutions for rock displacement and for anchoring force considering the coupling effect are solved by the Laplace inverse transform. Finally, we validate the analytical solutions by comparing against numerical simulation results with FLAC$^{3D}$. A good agreement is achieved, suggesting the fidelity of the analytical solutions. The theoretical model provides a reference for studying slope reinforcement, analyzing slope rock creep behavior and the long-term prestress of the reinforcement structure.

Keywords: Prestressed anchor cable; rock creep; coupling effect; slope reinforcement; viscoelasticity

1 Introduction

High rock slopes are often encountered during the construction of hydropower, mining, and transportation projects. In high slopes in Southwest China, rock masses are often characterized by strong unloading effects after excavation due to the influence of regional tectonics or deep canyons. Prestressed anchor cable is the primary measure for reinforcing high slopes, as shown in Fig. 1. Therefore, the long-term evolution of the anchoring force in these cables has an essential influence on the overall safety of the slopes (Liu et al. 2012; Shreedharan et al. 2016; Fan et al. 2018; Hu et al. 2020).
Fig. 1. A high slope of a hydropower station reinforced with prestressed anchor cables.

Creep is a type of behavior in which the strain continues to evolve under the imposition of a constant stress. Many laboratory tests and field measurements have shown that rock materials often exhibit creep characteristics (Sun et al. 2007; Falaleev et al. 2010). As one of the important mechanical properties of rock masses, creep has a significant impact on the safety and long-term displacement of slopes. The instability and failure of slopes is closely related to time, which has long been observed in many field examples and experimental studies (Cruden et al. 1987; Forlati et al. 2001; Yang et al. 2014). As one of the most direct, efficient, and economical solutions to geotechnical engineering safety problems, anchorage reinforcement technology has been widely used in the field, among which prestressed anchor cable is an effective means (Yang et al. 2015; Thanh-Canh et al. 2018; Shi et al. 2018). Moreover, prestressed anchor cable is an active reinforcement technology, which can improve the stress state of rock masses, limit the deformation of the slope surface. The reinforcement effect of prestressed anchor cables on slopes has been confirmed by many engineering applications (Kumar et al. 2010; Tiwari et al. 2016; Bi et al. 2019; Yan et al. 2019).

The research of using prestressed anchor cables to reinforce the creeping rock masses has attracted much attention from scholars around the world. Many studies using field monitoring data have shown that time-dependent effect exists in the loss of prestress of anchor cables in anchorage engineering applications (Clough et al. 1971; Benmokrane et al. 1991; Sun et al. 2010; Tao et al. 2019). The long-term variations of anchorage force in anchor cables make it difficult to fully realize the anchorage effect. Based on the monitoring data of a field project, Zhang et al. (2003) explored the change process and characteristics of the anchorage prestress after anchor cables were applied to reinforce a slope. Li et al. (2010) studied the variation of prestress value of anchor cables and the failure mechanism of prestressed anchorage system in long-term anchorage projects. Fan et al. (2015) used statistics to study the prestress of anchor cables and analyzed the time-dependent prestress loss rate. Qian and Zhou (2018) observed the anchor cables applied to the underground cavern of Jinping I Hydropower Station for an extended period of time and found that the forces of anchor cables increased with time, and some cables even exceeded the design value. Shi et al. (2019) analyzed the time-dependence and degree of anchor force loss using statistical monitoring data of anchor cables, and proposed a coupled calculation method for the changes in anchorage force.
In terms of experimental studies and numerical simulation, Chen et al. (2002) investigated the evolution of the anchoring force in anchor cables with time in soft rocks through physical model tests. Using a large-scale vibration apparatus, Fu et al. (2017) and Fan et al. (2019) both successfully simulated the prestress loss in anchor cables considering dynamic characteristics. Yu et al. (2019) and Feng et al. (2021) studied the prestress loss of anchor cables in anchored rock masses and its long-term evolution using experimental tests. Based on the finite element method, Deng et al. (2016) analyzed the prestress loss of anchor cables caused by the excavation of cavern group and the creep of rock masses. Wu et al. (2016) examined the effect of creep deformation of a reinforced dam on the anchor cable pretension loss using numerical simulation. Using the finite difference simulator FLAC3D, Li et al. (2019) analyzed the influence of prestress of anchor cables on the creep of slope rock masses.

In terms of theoretical research, numerous studies have been conducted on the viscoelastic and viscoplastic solutions of stress and displacement after excavation for rheological rock masses (Sulem et al. 1987; Bobet et al. 2006; Fritz et al. 2010; Wang et al. 2015; Han et al. 2018; Gao et al. 2021). However, few theoretical studies considered the addition of anchors to creep rock masses (Oreste et al. 2003; Park et al. 2006; Fahimifar et al. 2010; Nomikos et al. 2011), especially considering the coupling between the creep of rock masses and the anchoring forces of prestressed anchor cables. A large body of work has been devoted to the theoretical study of coupled creep effect of anchored rock masses, and the relevant research progress is listed in Table 1. In addition, the creep models used in the theoretical study are shown in Fig. 2. For example, aiming at understanding the failure and loss of prestressed anchor cables in underground caverns, Jiang et al. (2013) and Wang et al. (2018) used the generalized Kelvin model for rock masses and the elastic model for anchored cables to examine the coupling effect between the rock and prestressed anchor cables in one-dimensional case. Chen et al. (2018) applied the Nishihara model for rock masses and the elastic model for anchored cables to explore the creep behavior of rock masses and the evolution of anchoring force of prestressed anchor cables in one-dimensional case. In summary, most studies have used the generalized Kelvin model to characterize rock masses of slopes and conducted theoretical analysis under one-dimensional condition for the qualitative analysis of slope deformation. However, the generalized Kelvin model with three parameters cannot fully characterize rock masses with steady-state creep properties.
Fig. 2. Typical creep models.

Table 1. Theoretical study progress on the coupling effect between rock masses and prestressed anchor cables

<table>
<thead>
<tr>
<th>Working condition</th>
<th>Constitutive model</th>
<th>Major developments</th>
<th>Dimension</th>
<th>References</th>
<th>Year(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunnel</td>
<td>Generalized Kelvin</td>
<td>Time-dependent solution for reinforced circular tunnels</td>
<td>3</td>
<td>Shunsuke</td>
<td>1978</td>
</tr>
<tr>
<td>Slope</td>
<td>Generalized Kelvin</td>
<td>Analytical solutions for long-term effective prestress of anchor cables under coupled action</td>
<td>1</td>
<td>Zhu et al.</td>
<td>2005</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Generalized Kelvin</td>
<td>Coupling effect of surrounding rock and prestressed anchor cables in underground engineering practices</td>
<td>1</td>
<td>Jiang et al. Wang et al.</td>
<td>2013-2018</td>
</tr>
<tr>
<td>Slope</td>
<td>Generalized Kelvin</td>
<td>Establishment of a coupling effect calculation model based on strain equilibrium</td>
<td>1</td>
<td>Wang et al.</td>
<td>2014</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Maxwell /Kelvin</td>
<td>Viscoelastic analytical solutions for rock masses supported by DMFC bolts</td>
<td>3</td>
<td>Wang et al.</td>
<td>2015-2019</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Generalized Kelvin</td>
<td>Displacement and stress analysis of deeply buried twin tunnels</td>
<td>3</td>
<td>Wang et al.</td>
<td>2017</td>
</tr>
<tr>
<td>Slope</td>
<td>Nishihara</td>
<td>Relationship between the changes of anchoring forces of prestressed cables and the creep of rock masses</td>
<td>1</td>
<td>Chen et al.</td>
<td>2018</td>
</tr>
<tr>
<td>Slope</td>
<td>Generalized Kelvin</td>
<td>New coupled calculation model for predicting the loss of anchoring force</td>
<td>1</td>
<td>Shi et al.</td>
<td>2019</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Burgers</td>
<td>Explicit expressions of rock displacement and stress</td>
<td>3</td>
<td>Chu et al.</td>
<td>2019</td>
</tr>
</tbody>
</table>
As mentioned above, many viscoelastic analytical solutions have been proposed for deep-buried circular tunnels. However, only a limited studies have considered the interaction between the creep of rock masses of three-dimensional slopes and the change in anchoring force of prestressed anchor cables. The main difficulty in this problem lies in the fact that the 3D theoretical formula derivation is much more complicated than that under 1D condition. Especially, when the rock masses are represented by elements, the more elements the model has, the more comprehensive the creep characteristics of rock masses can be expressed. At the same time, the corresponding increment of parameters leads to the difficulty of theoretical analysis, and in some cases, theoretical solutions cannot be obtained if there are too many parameters.

Based on the influence of strong unloading on the change of anchoring force of prestressed anchor cables, we establish a theoretical model for unloading creep of rock masses and anchoring force of prestressed anchor cables considering the coupling effect between these two processes for high slopes. Subsequently, we derive the viscoelastic theoretical solutions for the coupled model, in which the rock masses (Burgers) and the prestressed anchor cables (Elastic) are regarded as independent models. Finally, we compare and analyze the analytical and numerical solutions of the displacement of rock masses and the anchoring force of anchor cables to verify the fidelity of the developed model.

## 2 Coupled mechanistic analysis of prestressed anchor cables and slope rock masses

After the excavation of high slope rock masses, the unloading causes outward displacement perpendicular to the slope surface, while the anchoring force of prestressed anchor cables limits the outward deformation of rock masses. For the excavated and anchored slope, the deformation of the rock masses is mainly controlled by two factors: one is the unloading after the excavation, and the other is the anchoring force of the prestressed anchor cables.

Fig. 3 illustrates the reinforcement of a high rock slope with prestressed anchor cables for a hydropower station. Table 2 provides the parameters of anchor cables used in the slope reinforcement. In this work, the reinforcement area of one prestressed anchor cable is selected for the force analysis. Fig. 4 shows a schematic diagram of the equivalent force of a typical anchoring unit of the slope. The length of the anchor cable is $L_1$, the spacing of anchor cables is $L_s$, and the representative elementary volume (REV) is $L_1 \times L_s \times L_2$. In addition, the blue lines in the figure represent the prestressed anchor cables, and the range of the red dashed line represents the REV of the anchor cable.

<table>
<thead>
<tr>
<th>Tunnel</th>
<th>Generalized Kelvin</th>
<th>Elastic</th>
<th>Displacement and stress analysis of noncircular tunnel</th>
<th>3</th>
<th>Wang et al. 2018 Zeng et al. 2020</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope</td>
<td>Generalized Kelvin</td>
<td>Elastic</td>
<td>Establishment of two coupling models of prestressed anchor cables and slope rock masses</td>
<td>1</td>
<td>Xu et al. 2020</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Burgers</td>
<td>Elastic</td>
<td>Time-dependent analysis of a deep tunnel in the viscoelastic rock</td>
<td>3</td>
<td>Do et al. 2020</td>
</tr>
</tbody>
</table>
Fig. 3. Prestressed anchor cable layout and anchor cable details of a typical section of a hydropower station.

Table 2. Parameters of typical anchor cables

<table>
<thead>
<tr>
<th>Type of anchor cable</th>
<th>Designed pretension load (kN)</th>
<th>Total length of anchor cable (m)</th>
<th>Length of free section of anchor cable (m)</th>
<th>Anchor cable spacing (m)</th>
<th>Anchor cable diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DKDF-2000</td>
<td>2000</td>
<td>50</td>
<td>40</td>
<td>5x5</td>
<td>150</td>
</tr>
</tbody>
</table>

Fig. 4. Schematic diagram of the anchoring force of an anchorage unit.

As shown in Fig. 4, $T$ is the anchoring force of the anchor cable, $P$ is the equivalent force induced by excavation unloading, and $P = qL_x^2$. $q$ is the unloading strength of the rock masses within the anchoring range.
The resultant force on the anchor cable is \( F = P - T \). The direction of the force is defined positive when it points outward to the slope surface and negative when it points inward to the slope surface.

2.1 Mechanistic model of coupling between the prestressed anchor cable and slope rock masses

Considering the creep characteristics, the rock displacement increases with time due to the excavation unloading effect. The anchoring force of prestressed anchor cables is affected by the displacement of rock masses. When the displacement faces outward perpendicular to the slope surface, the anchoring force of anchor cables will increase. Conversely, when facing inward perpendicular to the slope surface, the anchoring force will decrease and prestress loss will occur. Therefore, the changes in the anchoring force of the prestressed anchor cables and the creep of the rock masses are mutually dependent. The excavation unloading of slope rock masses produces displacement, and the creep further increases the displacement with time. Assuming that the rock masses of the slope and the anchor cables have deformation compatibility, the creep of rock masses induces corresponding deformation of the anchor cables, which causes the axial force of the anchor cables to change accordingly. Therefore, the creep of rock masses interacts with the anchoring force of the prestressed anchor cables, based on which we establish a mechanistic model to couple the creep of rock masses on high slopes and the anchoring force of prestressed anchor cable.

The following assumptions are made in this work: (1) The rock masses of high slopes have creep characteristics; (2) The excavation unloading effect is equivalent to the uniformly distributed load acting on the slope surface; (3) The prestressed anchor cables are arranged perpendicular to the slope surface; and (4) The deformation of the prestressed anchor cables and the deformation of the rock masses are coordinated.

We simplify the problem of rock masses reinforced by prestressed anchor cables as the problem of concentrated normal force locally exerted on the boundary of a semi-infinite elastic medium, which is the Boussinesq problem (Timoshenko et al. 1951). Fig. 5 shows a schematic diagram of the concentrated normal force \( F \) acting on the boundary of a semi-infinite elastic medium (the surface of the rock masses).

![Fig. 5. Schematic diagram of a semi-infinite elastic medium subjected to a concentrated normal force on the boundary.](image-url)
According to Fig. 5, the vertical displacement $\omega$ at any point in the depth of rock masses is expressed as (Poulos et al. 1974),

$$\omega = \frac{F (1+\mu)}{2\pi E_r l} \left[ 2(1-\mu) + \frac{z^2}{l^2} \right]$$  \hspace{1cm} (1)

where $E_r$ is the elastic modulus of the rock; $\mu$ is Poisson’s ratio of the rock; $r$ is the distance between the measuring point and the center point; $z$ is the vertical depth of the measuring point; and $l$ is the distance from the concentrated force to the measuring point.

Fig. 4 shows the diagram of displacement calculation points of the rock masses and the prestressed anchor cable, where $P$ is the equivalent force induced by excavation unloading, and $P = qL_r^2$. $q$ is the unloading strength of rock masses within the anchorage range, with the direction pointing out of the slope surface. $T$ is the anchoring force of the anchor cable, and $T = q_0L_r^2$. $q_0$ is the equivalent stress of the anchoring force in the anchor cable within the anchoring range. The initial anchoring force of the cable is the initial pretension of the anchor cable, which is recorded as $T_0$.

The prestressed anchor cable is subjected to the combined action of the unloading force after the excavation of the slope and the anchoring force of the cable. Then, the force is transmitted to the surface of the rock mass through the anchor pier and the direction of the force on the rock surface is adjusted accordingly. The top and tail ends of the anchor cable are marked as points $A$ and $B$. The radius of the anchor pier is $R_0$ and the contact stress at the base is $q_f$. Suppose that a micro element has an area of $dA = rdrd\theta$ within the load action range, and the distance between the micro element and the anchor pier center point $A$ is $r$. The vertical displacement of any point in the depth of the slope along the anchor cable direction can be expressed as

$$\omega = \int \int q_f \frac{(1+\mu)}{2\pi E_r l} \left[ 2(1-\mu) + \frac{z^2}{l^2} \right] dA$$  \hspace{1cm} (2)

Integrating Equation (2) yields,

$$\omega = \frac{F R_0 (1-\mu^2)}{2\pi R_0^2 E_r} \left( \frac{\pi}{2} - \arcsin \frac{z^2 - R_0^2}{z^2 + R_0^2} \right) + \frac{F z (1+\mu)}{2\pi R_0^2 E_r} \frac{R_0^2}{z^2 + R_0^2}$$

$$\omega = \frac{F R_0 (1-\mu^2)}{2\pi R_0^2 E_r} \left( \frac{\pi}{2} - \arcsin \frac{z^2 - R_0^2}{z^2 + R_0^2} \right) + \frac{F L (1+\mu)}{2\pi E_r} \frac{1}{L^2 + R_0^2}$$

$$\omega_a = \frac{F (1-\mu^2)}{2R_0 E_r} \left( \frac{\pi}{2} - \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right) + \frac{F L (1+\mu)}{2\pi E_r} \frac{1}{L^2 + R_0^2}$$

$$\omega_b = \frac{F (1-\mu^2)}{2R_0 E_r} \left( \frac{\pi}{2} - \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right) + \frac{F L (1+\mu)}{2\pi E_r} \frac{1}{L^2 + R_0^2}$$

As a result, the vertical displacements at point $A$ and point $B$ are expressed as

$$\omega_a = \frac{F (1-\mu^2)}{2R_0 E_r} \left( \frac{\pi}{2} - \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right) + \frac{F L (1+\mu)}{2\pi E_r} \frac{1}{L^2 + R_0^2}$$

$$\omega_b = \frac{F (1-\mu^2)}{2R_0 E_r} \left( \frac{\pi}{2} - \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right) + \frac{F L (1+\mu)}{2\pi E_r} \frac{1}{L^2 + R_0^2}$$
where $\omega_A$ and $\omega_B$ are the vertical displacements at point $A$ and point $B$, respectively; $L$ is the length of the free section of the anchor cable; $F$ is the resultant force on the anchor cable unit; and $R_0$ is the radius of the anchor pier.

2.2 Elastic solution of the coupled mechanistic model

As mentioned above, the rock masses are subjected to the combined action of the unloading force due to the excavation and the reinforcing force of the anchor cable. In this work, we systematically analyze the elastic solutions of the coupled model of prestressed anchor cable and rock masses from two aspects: case (i) the unloading forces of the slope rock masses after being excavated are greater than the anchoring force of the cable, and case (ii) the unloading forces of the slope rock masses after being excavated are smaller than the anchoring force of the cable.

Case (i): the unloading forces of the rock masses after being excavated are greater than the anchoring force of the cable.

According to Fig. 4, when $P > T_0$, by definition, the resultant force on the anchor cable unit is

$$F = (q - q_0)L_s^2 = qL_s^2 - T$$

(6)

Without considering the influence of rock gravity and other environmental stresses, we assume that the prestress of the anchor cable acts uniformly on the surrounding rock. Then, the anchoring force of the prestressed cable is

$$T = \delta_0 + \frac{\Delta L}{L} E_b A_b$$

(7)

where $T$ is the anchoring force of the anchor cable; $\delta_0$ is the total strain of the anchor cable; $E_b$ is elastic modulus of the anchor cable; $A_b$ is the cross-sectional area of the anchor cable; $\delta_0$ is the pre-tension length of the anchor cable; and $\Delta L$ is the deformation of the anchor cable during coordinated deformation.

In addition, because of the coordinated deformation between the anchor cable and the rock masses, the axial deformation of the anchor cable $\Delta L$ is equal to the deformation of the rock masses $\Delta \omega$ as

$$\Delta L = \Delta \omega$$

(8)

Subsequently, combining Equations (7) and (8) with Equation (6), the resultant force on the anchor cable unit can be expressed as

$$F = qL_s^2 - \frac{\delta_0 + \Delta \omega}{L} E_b A_b = P - T_0 - \frac{\Delta \omega}{L} E_b A_b$$

(9)

where $\Delta \omega$ is the deformation of the rock mass; $P$ is the equivalent force from excavation unloading; and $T_0$ is the initial pretension of the anchor cable.

From Equations (4) and (5), the deformation of the rock masses is equal to
\[ \Delta \omega = \omega_A - \omega_B = \frac{F \left(1 - \mu^2\right) \left(\frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2}\right)}{2\pi R_0 E_r} - \frac{FL(1 + \mu)}{2\pi E_r} \left(\frac{1}{L^2 + R_0^2}\right) \]  

(10)

Therefore, \( \Delta \omega \) and \( F \) can be easily obtained by substituting Equation (9) into Equation (10). Finally, the elastic solution of the deformation \( \Delta \omega \) of the rock masses and the elastic solution of the anchoring force \( T \) of the cable under the coupled effect are obtained as

\[ \Delta \omega = \frac{(P - T_0) L \left(\frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2}\right) \left(1 - \mu^2\right) - (P - T_0) \frac{R_0 L^2}{L^2 + R_0^2} (1 + \mu)}{2\pi R_0 E_r + E_b A_b} \left(\frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2}\right) \left(1 - \mu^2\right) - E_b A_b \frac{R_0 L}{L^2 + R_0^2} (1 + \mu) \]  

(11)

\[ T = T_0 + \frac{(P - T_0) L \left(\frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2}\right) \left(1 - \mu^2\right) E_b A_b - (P - T_0) \frac{R_0 L}{L^2 + R_0^2} (1 + \mu) E_b A_b}{2\pi R_0 E_r + E_b A_b} \left(\frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2}\right) \left(1 - \mu^2\right) - E_b A_b \frac{R_0 L}{L^2 + R_0^2} (1 + \mu) \]  

(12)

Case (ii): the unloading forces of the slope rock masses after being excavated are smaller than the anchoring force of the cable.

According to Fig. 4, when \( P < T_0 \), the resultant force on the anchor cable unit is written as

\[ F = (q_0 - q) L_s^2 = -T + qL_s^2 \]  

(13)

Meanwhile, the force on the prestressed anchor cable can be expressed as

\[ T = e_b E_b A_b = \frac{\delta_0 - \Delta L}{L} E_b A_b \]  

(14)

Combining Equations (8) and (14) with Equation (13) yields the resultant force on the anchor cable unit as

\[ F = -\frac{\delta_0 - \Delta \omega}{L} E_b A_b + qL_s^2 = -T_0 + P + \frac{\Delta \omega}{L} E_b A_b \]  

(15)

Therefore, combining Equation (10) with Equation (15), the elastic solution of the deformation \( \Delta \omega \) of the rock masses and the elastic solution of the anchoring force \( T \) of the cable under the coupled effect model are obtained as

\[ \Delta \omega = -\frac{(T_0 - P) L \left(\frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2}\right) \left(1 - \mu^2\right) - (T_0 - P) \frac{R_0 L^2}{L^2 + R_0^2} (1 + \mu)}{2\pi R_0 E_r + E_b A_b} \left(\frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2}\right) \left(1 - \mu^2\right) - E_b A_b \frac{R_0 L}{L^2 + R_0^2} (1 + \mu) \]  

(16)
\[ T = T_0 - \frac{(T_0 - P) L \left( \frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right) \left( 1 - \mu^2 \right) E_b A_b - (T_0 - P) \frac{R_0 L}{L^2 + R_0^2} \left( 1 + \mu \right) E_b A_b}{2\pi R_0 L E_r + E_b A_b \left( \frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right) \left( 1 - \mu^2 \right) - E_b A_b \frac{R_0 L}{L^2 + R_0^2} \left( 1 + \mu \right)} \] 

(17)

### 3 Viscoelastic analytical solutions of the coupled model

In the small deformation range of rock materials, the viscoelastic problem and the elastic problem are only different in constitutive relations, and the equilibrium equations, geometric equations, and boundary conditions are exactly the same. According to the corresponding principles of elasticity-viscoelasticity, the viscoelastic problems can be solved by the following procedure. First, the elastic parameters in the elastic solution of the theoretical model are replaced by viscoelastic parameters. Then, the operator function of the creep model is substituted to obtain the analytical solutions of the problem in the Laplace space. Finally, the inverse Laplace transform is applied to the analytical solutions to obtain the viscoelastic solution of the problem (Mogilevskaya et al. 2018).

#### 3.1 Selection and definition of the coupled model

##### 3.1.1 Selection of creep model for the anchor cable and definition of operator function

The anchor cable is described by the elastic model, and its constitutive equation satisfies

\[ \sigma = E_b \varepsilon \]  

(18)

where \( \sigma \) and \( \varepsilon \) are total stress and total strain.

The axial stiffness of the anchor cable is much larger than the tangential stiffness, so the anchor cable can be regarded as an ideal one-dimensional elastic material. The axial stress of the anchor cable is represented by \( \sigma_b \) and the axial strain is represented by \( \varepsilon_b \), and the generalized one-dimensional elastic constitutive equation of the anchor cable can be written as

\[ \sigma_b = \frac{Q_b(D)}{P_b(D)} \varepsilon_b \]  

(19)

\[ D = \frac{\partial}{\partial t}, P_b(D) = \sum_{k=0}^{m} p_k \frac{\partial^k}{\partial t^k}, Q_b(D) = \sum_{k=0}^{m} q_k \frac{\partial^k}{\partial t^k} \]

where \( D \) is the differential operator; \( P_b(D) \) and \( Q_b(D) \) are operator functions for the 1D constitutive equation of the anchor cable in Equation (19); and \( p_k \) and \( q_k \) are constants of the anchor cable material.

Hence, the parameter transformation in the Laplace domain is given by

\[ E_b(s) = \frac{\tilde{Q}_b(s)}{\tilde{P}_b(s)} \]  

(20)
where \( s \) is the Laplace variable; \( \bar{P}_b(s) \) and \( \bar{Q}_b(s) \) are the operator functions of 1D constitutive equation of the anchor cable after the Laplace transform. Specifically, the operator functions are written as
\[
\bar{P}_b(s) = 1 \\
\bar{Q}_b(s) = E_b
\] (21)

### 3.1.2 Selection of creep model of rock masses and definition of operator function

Burgers model is used to simulate the creep properties of the rock masses. The one-dimensional constitutive equation of rock masses satisfies
\[
\frac{\eta_{1r} \eta_{2r}}{E_{1r} E_{2r}} \frac{\mathcal{D}_{\sigma} + \frac{\eta_{1r} + \eta_{2r}}{E_{1r}}}{\mathcal{D}_e} + \eta_{1r} \eta_{2r} \mathcal{D}_{\sigma} = \eta_{1r} \eta_{2r} \mathcal{D}_e
\] (22)
where \( \mathcal{D}_{\sigma} \) and \( \mathcal{D}_e \) are the derivatives of \( \sigma \) and \( e \), respectively, and \( \mathcal{D}_{\sigma} \) and \( \mathcal{D}_e \) are the second derivatives of \( \sigma \) and \( e \), respectively. \( E_{1r} \) and \( E_{2r} \) are the visco-elastic parameters, \( \eta_{1r} \) and \( \eta_{2r} \) are the viscosity coefficient of rock masses.

It is well known that rock mechanics and engineering problems are often three-dimensional problems. The slope rock masses should be regarded as three-dimensional viscoelastic material, and therefore the one-dimensional constitutive equation should be expanded to three-dimensional. From the elastic theory, the one-dimensional form of the elastic constitutive relationship is \( \sigma = E_e e \), and the three-dimensional tensor form is expressed as
\[
S_{ij} = 2G e_{ij}, \sigma_{ij} = 3K e_{ij}
\] (23)
where \( G \) and \( K \) are the bulk modulus and shear modulus, respectively. \( S_{ij} \) and \( e_{ij} \) are the deviotoric stress and strain tensors, respectively. \( \sigma_{ij} \) and \( e_{ij} \) are the stress tensor and strain tensor, respectively.

The three-dimensional constitutive relationship of the constitutive models of elastic and viscoelastic materials can be expressed as
\[
S_{ij} = 2G e_{ij} = 2 \frac{Q'(D)}{P'(D)} e_{ij}, \sigma_{ij} = 3K e_{ij} = 3 \frac{Q''(D)}{P''(D)} e_{ij}
\] (24)
where \( P'(D), Q'(D), P''(D), Q''(D) \) are the operator functions of the viscoelastic constitutive model.

Therefore, the parameter transformation in the Laplace domain is given by
\[
G(s) = \frac{\bar{Q}'(s)}{\bar{P}'(s)} , K(s) = \frac{\bar{Q}''(s)}{\bar{P}''(s)}
\] (25)
where \( \bar{P}'(s), \bar{Q}'(s), \bar{P}''(s), \bar{Q}''(s) \) are the operator functions of the viscoelastic constitutive model after the Laplace transformation. In addition, the operator functions of Burgers model are
\[ \bar{P}'(s) = 1 + \left( \eta_{1r} + \eta_{2r}r + \eta_{1r}r \right) \frac{G}{G_{2r}} s + \frac{\eta_{1r} \eta_{2r} r}{G_{2r}} s^2 = 1 + p_{1r} s + p_{2r} s^2 \]

\[ \bar{Q}'(s) = \eta_{1r}, s + \frac{\eta_{1r} \eta_{2r} r}{G_{2r}} s^2 = q_{1r}, s + q_{2r} s^2 \] (26)

\[ \bar{P}''(s) = 1 \]
\[ \bar{Q}''(s) = K \]

where \( G_{1r} \) and \( G_{2r} \) are the elastic shear modulus and visco-elastic shear modulus of rock masses, respectively.

3.2 General viscoelastic solution of the coupled model

In the three-dimensional space, the relationships between elastic modulus \( E \), Poisson ratio \( \mu \), elastic shear modulus \( G \), and bulk modulus \( K \) are

\[ E = \frac{9GK}{3K+G} \] (27)
\[ \mu = \frac{3K-2G}{2(3K+G)} \] (28)

Case (i): the unloading forces of the rock masses after being excavated are larger than the anchoring force of the anchor cable.

By substituting the expressions of \( E \) (see Equation (27)) and \( \mu \) (see Equation (28)) into Equations (11) and (12), the spatial solutions of the deformation \( \Delta \omega \) of rock masses and the anchoring force \( T \) of the cable can then be obtained as

\[ \Delta \omega = \frac{a_1 \left( 4G + 3K \right) - 2a_2 \left( G + 3K \right)}{4a_3 G \left( G + 3K \right) + \left( a_4 \left( 4G + 3K \right) - 2a_5 \left( G + 3K \right) \right) E_b} \] (29)

\[ T = b_1 + \frac{\left( b_2 \left( 4G + 3K \right) - 2b_3 \left( G + 3K \right) \right) E_b}{4b_4 G \left( G + 3K \right) + \left( b_5 \left( 4G + 3K \right) - 2b_6 \left( G + 3K \right) \right) E_b} \] (30)

where \( a_1 = (P-T_0) L \left( \frac{\pi}{2} + \text{arcsin} \left( \frac{L^2 - R_0^2}{L^2 + R_0^2} \right) \right), \ a_2 = (P-T_0) \frac{R_0 L^2}{L^2 + R_0^2}, \ a_3 = 2\pi R_0 L, \ a_4 = A, \ a_5 = A \frac{R_0 L}{L^2 + R_0^2}, \ b_1 = T_0, \ b_2 = (P-T_0) L \left( \frac{\pi}{2} + \text{arcsin} \left( \frac{L^2 - R_0^2}{L^2 + R_0^2} \right) \right) A, \ b_3 = (P-T_0) \frac{R_0 L^2}{L^2 + R_0^2} A, \ b_4 = 2\pi R_0 L, \ b_5 = A, \ b_6 = A \frac{R_0 L}{L^2 + R_0^2}.

In the next step, the equations for the spatial solutions, (e.g., Equations (29) and (30)), are solved using the Laplace transform by the following method. In the viscoelastic case, \( P \) is replaced with its Laplace
transform \( \frac{P}{s} \), \( T_0 \) is replaced with \( \frac{T_0}{s} \), \( G \) is replaced with \( \frac{\overline{Q}(s)}{\overline{P}(s)} \), \( K \) is replaced with \( \frac{\overline{Q}^*(s)}{\overline{P}^*(s)} \), and \( E_b \) is replaced with \( \frac{\overline{Q}_b(s)}{\overline{P}_b(s)} \), so that in the Laplace domain the general solutions of the deformation \( \Delta \omega \) of rock masses and the anchoring force \( T \) of the cable are

\[
\Delta \overline{\omega}(s) = \frac{1}{s} \frac{\overline{P} \overline{P}_b(2(2a_1 - a_2) \overline{P}^* \overline{Q}' + 3(a_1 - 2a_2) \overline{P}^* \overline{Q}^*)}{2 \overline{P}^* \overline{Q}' (2a_5 \overline{P}_b \overline{Q}' + (2a_4 - a_5) \overline{P}_b \overline{Q}) + 3 \overline{P}^* \overline{Q}^* (4a_3 \overline{P}_b \overline{Q}' + (a_4 - 2a_5) \overline{P}_b \overline{Q})}
\]

(31)

\[
\overline{T}(s) = \frac{b_1}{s} + \frac{1}{s} \frac{\overline{P} \overline{Q}_b(2(2a_1 - a_2) \overline{P}^* \overline{Q}' + 3(a_1 - 2a_2) \overline{P}^* \overline{Q}^*)}{2 \overline{P}^* \overline{Q}' (2a_5 \overline{P}_b \overline{Q}' + (2a_4 - a_5) \overline{P}_b \overline{Q}) + 3 \overline{P}^* \overline{Q}^* (4a_3 \overline{P}_b \overline{Q}' + (a_4 - 2a_5) \overline{P}_b \overline{Q})}
\]

(32)

Case (ii): the unloading forces of the rock masses after being excavated are smaller than the anchoring force of the anchor cable.

Similarly, by substituting the expressions of \( E \) (see Equation (27)) and \( \mu \) (see Equation (28)) into Equations (16) and (17), the spatial solutions of the deformation \( \Delta \omega \) of rock masses and the anchoring force \( T \) of the anchor cable can be obtained as

\[
\Delta \omega = - \frac{a_1 (4G + 3K) - 2a_2 (G + 3K)}{4a_5 G (G + 3K) + (a_4 (4G + 3K) - 2a_5 (G + 3K))} E_b
\]

(33)

\[
T = b_1 - \frac{(b_2 (4G + 3K) - 2b_3 (G + 3K)) E_b}{4b_4 G (G + 3K) + (b_5 (4G + 3K) - 2b_6 (G + 3K))} E_b
\]

(34)

where

\[
a_1 = (T_0 - P) L \left( \frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right), \quad a_2 = (T_0 - P) \frac{R_0 L^2}{L^2 + R_0^2}, \quad a_3 = 2\pi R_0 L.
\]

\[
a_4 = A_b \left( \frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right), \quad a_5 = A_b \frac{R_0 L}{L^2 + R_0^2}, \quad b_1 = T_0, \quad b_2 = (T_0 - P) L \left( \frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right) A_b,
\]

\[
b_3 = (T_0 - P) \frac{R_0 L^2}{L^2 + R_0^2} A_b, \quad b_4 = 2\pi R_0 L, \quad b_5 = A_b \left( \frac{\pi}{2} + \arcsin \frac{L^2 - R_0^2}{L^2 + R_0^2} \right), \quad b_6 = A_b \frac{R_0 L}{L^2 + R_0^2}.
\]

The Equations (33) and (34) for the spatial solutions are then solved using the Laplace transform. Therefore, in the Laplace domain the general solutions of the deformation \( \Delta \omega \) of rock masses and the anchoring force \( T \) of the cable are

\[
\Delta \overline{\omega}(s) = \frac{1}{s} \frac{\overline{P} \overline{P}_b(2(2a_1 - a_2) \overline{P}^* \overline{Q}' + 3(a_1 - 2a_2) \overline{P}^* \overline{Q}^*)}{2 \overline{P}^* \overline{Q}' (2a_5 \overline{P}_b \overline{Q}' + (2a_4 - a_5) \overline{P}_b \overline{Q}) + 3 \overline{P}^* \overline{Q}^* (4a_3 \overline{P}_b \overline{Q}' + (a_4 - 2a_5) \overline{P}_b \overline{Q})}
\]

(35)
\[
\bar{T}(s) = \frac{b_1}{s} - \frac{1}{s} \cdot \frac{P'Q_0}{2P^*Q'} \left( 2(a_1 - a_2)\frac{P'Q}{Q} + 3(a_1 - 2a_2)\frac{P'Q'}{Q} \right)
\]

(36)

which are identical to Equations (29) and (30), with \( P \) replaced by \( \frac{P}{s} \), \( T_0 \) replaced by \( \frac{T_0}{s} \), \( G \) replaced by \( \frac{G}{s} \), \( K \) replaced by \( \frac{K}{s} \) and \( E_b \) replaced by \( \frac{E_b}{s} \).

3.3 General viscoelastic solution of the coupled model

Case (i): the unloading forces of the rock masses of the slope after being excavated are larger than the anchoring force of the anchor cable.

Based on the theoretical models, Burgers model is used for the slope rock masses, and elastic model is used for the anchor cable. Substituting the expressions of the differential operator into Equations (31) and (32) yields the analytical solutions of the deformation \( \Delta \bar{\omega}(s) \) of the rock masses and the anchoring force \( \bar{T}(s) \) of the cable in the Laplace domain. Here, we first combine Equations (21) and (26) with Equations (31) and (32), and then the corresponding expressions of \( \Delta \bar{\omega}(s) \) and \( \bar{T}(s) \) can be obtained as

\[
\Delta \bar{\omega}(s) = \frac{1 + sp_{tr} + s^2p_{2r}}{s} \left( 2a_1 - a_2 \right) \left( sq_{1r} + s^2q_{2r} \right) + \frac{3}{s} \left( a_1 - 2a_2 \right) K \left( 1 + sp_{tr} + s^2p_{2r} \right) + \frac{2}{s} \left( sq_{1r} + s^2q_{2r} \right) \left( 2a_1 \left( sq_{1r} + s^2q_{2r} \right) \right) + \frac{3}{s} \left( 1 + sp_{tr} + s^2p_{2r} \right) E_b + \frac{3K}{s} \left( 1 + sp_{tr} + s^2p_{2r} \right) \left( 4a_3 \left( sq_{1r} + s^2q_{2r} \right) \right) + \frac{3K}{s} \left( 1 + sp_{tr} + s^2p_{2r} \right) E_b
\]

(37)

\[
\bar{T}(s) = \frac{b_1}{s} + \frac{1 + sp_{tr} + s^2p_{2r} \varepsilon_b}{s} \left( 2 \left( b_2 - b_3 \right) \left( sq_{1r} + s^2q_{2r} \right) + 3 \left( b_2 - 2b_3 \right) K \left( 1 + sp_{tr} + s^2p_{2r} \right) + 2 \left( sq_{1r} + s^2q_{2r} \right) \left( b_3 \left( b_2 - b_3 \right) \right) \left( sq_{1r} + s^2q_{2r} \right) + \left( b_3 - b_5 \right) \left( 1 + sp_{tr} + s^2p_{2r} \right) E_b \right)
\]

\[
\frac{3K}{s} \left( 1 + sp_{tr} + s^2p_{2r} \right) \left( 4b_3 \left( sq_{1r} + s^2q_{2r} \right) \right) + \left( b_5 - b_6 \right) \left( 1 + sp_{tr} + s^2p_{2r} \right) E_b
\]

(38)

Then, Equation (37) is further simplified as,

\[
\Delta \bar{\omega}(s) = \frac{c_1s^4 + c_2s^3 + c_3s^2 + c_4s + c_5}{c_6s^5 + c_7s^4 + c_8s^3 + c_9s^2 + c_{10}s}
\]

\[
= \frac{c_1}{c_6} \frac{s^4}{s-s_1} + \frac{c_2}{c_6} \frac{s^3}{s-s_2} + \frac{c_3}{c_6} \frac{s^2}{s-s_3} + \frac{c_4}{c_6} \frac{s}{s-s_4} + \frac{c_5}{c_6} \frac{1}{s-s_5}
\]

\[
= \frac{r_1}{s-s_1} + \frac{r_2}{s-s_2} + \frac{r_3}{s-s_3} + \frac{r_4}{s-s_4} + \frac{r_5}{s-s_5}
\]

(39)
where \( c_1 = 3(a_1 - 2a_2)Kp_{1r}^2 + 2(2a_1 - a_2)p_2q_{2r}, \)
\( c_2 = 6(a_1 - 2a_2)Kp_{1r}p_2 + 2(2a_1 - a_2)(p_2q_{1r} + p_1q_{2r}), \)
\( c_3 = 3(a_1 - 2a_2)(Kp_{1r}^2 + 2Kp_{2r}) + 2(2a_1 - a_2)(p_{1r}q_{1r} + 5q_{2r}), \)
\( c_4 = 6(a_1 - 2a_2)Kp_{1r} + 2(2a_1 - a_2)q_{1r}, \)
\( c_5 = 3(a_1 - 2a_2)K, \)
\( c_6 = 3(a_4 - 2a_5)Kp_{1r}^2 + 12a_1Kp_{2r}q_{2r} + 2(2a_4 - a_5)p_{2r}q_{2r}E_b + 4a_2q_{2r}^2, \)
\( c_7 = 6(a_4 - 2a_5)Kp_{1r}q_{2r}E_b + 12a_1K + 2(2a_4 - a_5)E_b)(p_{2r}q_{1r} + p_1q_{2r}) + 8a_1q_{1r}q_{2r}, \)
\( c_8 = 3(a_4 - 2a_5)KE_b(p_{1r}^2 + 2p_{2r}) + 12a_1K + 2(2a_4 - a_5)E_b(p_{1r}q_{1r} + q_{2r}) + 4a_2q_{1r}q_{2r}, \)
\( c_9 = 6(a_4 - 2a_5)Kp_{1r}E_b + 12a_1Kq_{1r} + 2(2a_4 - a_5)q_{1r}E_b, c_{10} = 3(a_4 - 2a_5)KE_b. \)

In Equation (39), \( s_1, s_2, s_3, s_4, \) and \( s_5 \) are the roots of the characteristic equation \( c_6s^5 + c_7s^4 + c_8s^3 + c_9s^2 + c_{10}s = 0. \) \( r_1, r_2, r_3, r_4, \) and \( r_5 \) are coefficients to be determined, which are called residues of Equation (39) at \( s_1, s_2, s_3, s_4, \) and \( s_5, \) and can be calculated according to the following formulas

\[
r_1 = \left[ s(t)(s - s_1) \right]_{s=s_1} = \frac{c_1s_1^4 + c_2s_1^3 + c_3s_1^2 + c_4s_1 + c_5}{c_6(s_1 - s_2)(s_1 - s_3)(s_1 - s_4)(s_1 - s_5)}
\]
\[
r_2 = \left[ s(t)(s - s_2) \right]_{s=s_2} = \frac{c_1s_2^4 + c_2s_2^3 + c_3s_2^2 + c_4s_2 + c_5}{c_6(s_2 - s_1)(s_2 - s_3)(s_2 - s_4)(s_2 - s_5)}
\]
\[
r_3 = \left[ s(t)(s - s_3) \right]_{s=s_3} = \frac{c_1s_3^4 + c_2s_3^3 + c_3s_3^2 + c_4s_3 + c_5}{c_6(s_3 - s_1)(s_3 - s_2)(s_3 - s_4)(s_3 - s_5)}
\]
\[
r_4 = \left[ s(t)(s - s_4) \right]_{s=s_4} = \frac{c_1s_4^4 + c_2s_4^3 + c_3s_4^2 + c_4s_4 + c_5}{c_6(s_4 - s_1)(s_4 - s_2)(s_4 - s_3)(s_4 - s_5)}
\]
\[
r_5 = \left[ s(t)(s - s_5) \right]_{s=s_5} = \frac{c_1s_5^4 + c_2s_5^3 + c_3s_5^2 + c_4s_5 + c_5}{c_6(s_5 - s_1)(s_5 - s_2)(s_5 - s_3)(s_5 - s_4)}
\]

Finally, these analytical solutions can be inverted back into the time domain using the inverse Laplace transform. Hence, using the inverse Laplace transform on Equation (39), the viscoelastic analytical solutions of rock deformation \( \Delta \omega \) under the coupled effect can be obtained as

\[
\Delta \omega(t) = r_1e^{s_1t} + r_2e^{s_2t} + r_3e^{s_3t} + r_4e^{s_4t} + r_5e^{s_5t}
\]

Subsequently, Equation (38) is further simplified as,
\[
\bar{T}(s) = \frac{d_1}{s} + \frac{d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}{s(\zeta s + 1)}
\]
\[
= \frac{d_1}{s} + \frac{d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}{s(s - s_1)(s - s_2)(s - s_3)(s - s_4)(s - s_5)}
\]
(46)

where \(d_1 = b_1, d_2 = (3(b_2 - 2b_3) K p_{2r}^2 + 2(2b_2 - b_3) p_{2r} q_{2r}) E_b\),

\(d_3 = (6(b_2 - 2b_3) K p_{2r} p_{2r} + 2(2b_2 - b_3) (p_{2r} q_{2r} + p_{2r} q_{2r})) E_b\),

\(d_4 = (3(b_2 - 2b_3) (K p_{2r}^2 + 2K p_{2r}) + 2(2b_2 - b_3) (p_{1r} q_{2r} + 5q_{2r})) E_b\),

\(d_5 = (6(b_2 - 2b_3) K p_{1r} q_{1r}) E_b\),  \(d_6 = 3(b_2 - 2b_3) KE_b\),

\(d_7 = 3(b_5 - 2b_6) K p_{2r}^2 E_b + 12b_4 K p_{2r} q_{2r} + 2(2b_5 - b_6) p_{2r} q_{2r} E_b + 4b_4 q_{2r}^2\),

\(d_8 = 6(b_5 - 2b_6) K p_{1r} q_{2r} E_b + (12b_1 K + 2(2b_5 - b_6) E_b) (p_{2r} q_{1r} + p_{2r} q_{2r}) + 8b_4 q_{1r} q_{2r}\),

\(d_9 = 3(b_5 - 2b_6) KE_b \left( p_{1r}^2 + 2p_{2r} \right) + (12b_1 K + 2(2b_5 - b_6) E_b) (p_{1r} q_{1r} + q_{2r}) + 4b_4 q_{2r}^2\),

\(d_{10} = 6(b_5 - 2b_6) K p_{1r} E_b + 12b_4 K q_{1r} + 2(2b_5 - b_6) q_{1r} E_b\),  \(d_{11} = 3(b_5 - 2b_6) KE_b\).

In Equation (46), \(s_1, s_2, s_3, s_4\), and \(s_5\) are the roots of the characteristic equation \(d_7 s^5 + d_6 s^4 + d_5 s^3 + d_4 s^2 + d_3 s + 1 = 0\).  \(r_1', r_2', r_3', r_4', \) and \(r_5'\) are coefficients to be determined, and are the residues of Equation (46) at \(s_1, s_2, s_3, s_4\), and \(s_5\), and can be calculated according to the following formulas

\[r_1' = \left[ s(t) (s - s_1) \right]_{s=s_1} = \frac{d_2}{d_7} s_1^4 + \frac{d_3}{d_7} s_1^3 + \frac{d_4}{d_7} s_1^2 + \frac{d_5}{d_7} s_1 + \frac{d_6}{d_7} \]
(47)

\[r_2' = \left[ s(t) (s - s_2) \right]_{s=s_2} = \frac{d_2}{d_7} s_2^4 + \frac{d_3}{d_7} s_2^3 + \frac{d_4}{d_7} s_2^2 + \frac{d_5}{d_7} s_2 + \frac{d_6}{d_7} \]
(48)

\[r_3' = \left[ s(t) (s - s_3) \right]_{s=s_3} = \frac{d_2}{d_7} s_3^4 + \frac{d_3}{d_7} s_3^3 + \frac{d_4}{d_7} s_3^2 + \frac{d_5}{d_7} s_3 + \frac{d_6}{d_7} \]
(49)

\[r_4' = \left[ s(t) (s - s_4) \right]_{s=s_4} = \frac{d_2}{d_7} s_4^4 + \frac{d_3}{d_7} s_4^3 + \frac{d_4}{d_7} s_4^2 + \frac{d_5}{d_7} s_4 + \frac{d_6}{d_7} \]
(50)
\[ r'_s = \left[ s(t)(s-s_5) \right]_{s=s_5} = \frac{d_2 s_5^4 + d_3 s_5^3 + d_4 s_5^2 + d_5 s_5 + d_6}{d_7 (s-s_1)(s-s_2)(s-s_3)(s-s_4)} \]  

(51)

Finally, we perform the inverse Laplace transform on Equation (46) to obtain the viscoelastic analytical solutions of the anchoring force \( T \) of the cable in the time domain under the coupled effect as

\[ T(t) = d_1 - \left( r'_1 e^{s_{1t}} + r'_2 e^{s_{2t}} + r'_3 e^{s_{3t}} + r'_4 e^{s_{4t}} + r'_5 e^{s_{5t}} \right) \]  

(52)

Case (ii): the unloading forces of the slope rock masses after being excavated are smaller than the anchoring force of the anchor cable.

Similarly, Burgers model is used for the rock masses of the high slope, and elastic model is used for the anchor cable. Substituting the expressions of the differential operator into Equations (35) and (36), we obtain the analytical solutions of deformation \( \Delta \omega(s) \) of the rock masses and anchoring force \( \bar{T}(s) \) of the cable in the Laplace domain. Therefore, combining Equations (21) and (26) with Equations (35) and (36), the corresponding expressions of \( \Delta \omega(s) \) and \( \bar{T}(s) \) can be obtained as

\[
\Delta \omega(s) = -\frac{\left(1 + s p_{1r} + s^2 p_{2r}\right)\left(2 a_{y} - a_{z}\right)\left(s q_{1r} + s^2 q_{2r}\right) + 3\left(a_{r} - 2a_{z}\right)K\left(1 + s p_{1r} + s^2 p_{2r}\right)}{s\left(2 s q_{1r} + s^2 q_{2r}\right)\left(2 a_{y} - a_{z}\right)\left(s q_{1r} + s^2 q_{2r}\right) + \left(2 a_{y} - a_{z}\right)\left(1 + s p_{1r} + s^2 p_{2r}\right)E_{b}\right)} + \frac{3K\left(1 + s p_{1r} + s^2 p_{2r}\right)\left(4 a_{y} \left(s q_{1r} + s^2 q_{2r}\right) + \left(a_{r} - 2a_{y}\right)\left(1 + s p_{1r} + s^2 p_{2r}\right)E_{b}\right)}{s\left(3K\left(1 + s p_{1r} + s^2 p_{2r}\right)\left(4 a_{y} \left(s q_{1r} + s^2 q_{2r}\right) + \left(b_{r} - 2b_{y}\right)\left(1 + s p_{1r} + s^2 p_{2r}\right)E_{b}\right)\right)}
\]  

(53)

\[
\bar{T}(s) = \frac{b_{y}}{s} - \frac{\left(1 + s p_{1r} + s^2 p_{2r}\right)E_{b}\left(2 b_{y} - b_{z}\right)\left(s q_{1r} + s^2 q_{2r}\right) + 3\left(b_{r} - 2b_{y}\right)K\left(1 + s p_{1r} + s^2 p_{2r}\right)}{s\left(2 s q_{1r} + s^2 q_{2r}\right)\left(2 b_{y} - b_{z}\right)\left(s q_{1r} + s^2 q_{2r}\right) + \left(2 b_{y} - b_{z}\right)\left(1 + s p_{1r} + s^2 p_{2r}\right)E_{b}\right)} + \frac{\left(3K\left(1 + s p_{1r} + s^2 p_{2r}\right)\left(4 b_{y} \left(s q_{1r} + s^2 q_{2r}\right) + \left(b_{r} - 2b_{y}\right)\left(1 + s p_{1r} + s^2 p_{2r}\right)E_{b}\right)\right)}{s\left(3K\left(1 + s p_{1r} + s^2 p_{2r}\right)\left(4 b_{y} \left(s q_{1r} + s^2 q_{2r}\right) + \left(b_{r} - 2b_{y}\right)\left(1 + s p_{1r} + s^2 p_{2r}\right)E_{b}\right)\right)}
\]  

(54)

Equation (53) can be further simplified as,

\[
\Delta \omega(s) = -\frac{c_{1} s^{4} + c_{2} s^{3} + c_{3} s^{2} + c_{4} s + c_{5}}{c_{6} s^{5} + c_{7} s^{4} + c_{8} s^{3} + c_{9} s^{2} + c_{10} s} - \frac{c_{1} s^{4} + c_{2} s^{3} + c_{3} s^{2} + c_{4} s + c_{5}}{c_{6} s^{5} + c_{7} s^{4} + c_{8} s^{3} + c_{9} s^{2} + c_{10} s} = -\frac{r'_1}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} - \frac{r'_2}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} - \frac{r'_3}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} - \frac{r'_4}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} - \frac{r'_5}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)}
\]  

(55)

where \( c_{1} = 3\left(a_{r} - 2a_{z}\right)Kp_{2r}^{2} + 2\left(2a_{r} - a_{z}\right)p_{2r}q_{2r}, \)

\( c_{2} = 6\left(a_{r} - 2a_{z}\right)Kp_{1r}p_{2r} + 2\left(2a_{r} - a_{z}\right)\left(p_{2r}q_{1r} + p_{1r}q_{2r}\right), \)
\[ c_y = 3(a_y - 2a_z)\left( Kp_1, 2 + 2Kp_2, 2 \right) + 2(2a_y - a_z)\left(p_{1, tr} + 5q_{2, tr}\right), \]
\[ c_a = 6(a_y - 2a_z)Kp_{1, tr} + 2(2a_y - a_z)q_{1, tr}, \quad c_z = 3(a_y - 2a_z)K, \]
\[ c_\nu = 3(a_y - 2a_z)Kp_2, 2E_b + 12a_3Kp_2, 2q_{2, tr} + 2\left(2a_y - a_z\right)p_{2, tr}q_{2, tr}E_b + 4a_3q_{2, tr}^2, \]
\[ c_\theta = 6(a_y - 2a_z)Kp_{1, tr}q_{2, tr}E_b + \left(12a_3K + 2\left(2a_y - a_z\right)E_b\right)\left(p_{1, tr}q_{1, tr} + p_{1, tr}q_{2, tr}\right) + 8a_3q_{1, tr}q_{2, tr}, \]
\[ c_\nu = 3(a_y - 2a_z)KE_b \left( p_{1, tr}^2 + 2p_{2, tr} \right) + \left(12a_3K + 2\left(2a_y - a_z\right)E_b\right)\left(p_{1, tr}q_{1, tr} + q_{2, tr} \right) + 4a_3q_{1, tr}^2, \]
\[ c_\theta = 6(a_y - 2a_z)Kp_{1, tr}E_b + 12a_3Kq_{1, tr} + 2\left(2a_y - a_z\right)q_{1, tr}E_b, \quad c_{10} = 3(a_y - 2a_z)KE_b. \]

In Equation (55), \(s_1, s_2, s_3, s_4, s_5\) are the roots of the characteristic equation \(c_s^5 + c_y^3 + c_\nu^3 + c_\nu^2 + c_\theta^2 + c_{10}^s = 0\). \(r_1, r_2, r_3, r_4, r_5\) and \(s_5\) are coefficients to be determined, as the residues of Equation (55) at \(s_1, s_2, s_3, s_4, s_5\), and can be calculated from the following formulas

\[ r_1 = \left[ s(t)(s - s_1) \right]_{s=s_1} = \frac{c_\nu}{c_\nu} s_1^4 + \frac{c_\nu}{c_\nu} s_1^3 + \frac{c_\nu}{c_\nu} s_1^2 + \frac{c_\nu}{c_\nu} s_1 + \frac{c_\nu}{c_\nu} s_1 \left( \frac{c_\nu}{c_\nu} s_1 - s \right) \]

(56)

\[ r_2 = \left[ s(t)(s - s_2) \right]_{s=s_2} = \frac{c_\nu}{c_\nu} s_2^4 + \frac{c_\nu}{c_\nu} s_2^3 + \frac{c_\nu}{c_\nu} s_2^2 + \frac{c_\nu}{c_\nu} s_2 + \frac{c_\nu}{c_\nu} s_2 \left( \frac{c_\nu}{c_\nu} s_2 - s \right) \]

(57)

\[ r_3 = \left[ s(t)(s - s_3) \right]_{s=s_3} = \frac{c_\nu}{c_\nu} s_3^4 + \frac{c_\nu}{c_\nu} s_3^3 + \frac{c_\nu}{c_\nu} s_3^2 + \frac{c_\nu}{c_\nu} s_3 + \frac{c_\nu}{c_\nu} s_3 \left( \frac{c_\nu}{c_\nu} s_3 - s \right) \]

(58)

\[ r_4 = \left[ s(t)(s - s_4) \right]_{s=s_4} = \frac{c_\nu}{c_\nu} s_4^4 + \frac{c_\nu}{c_\nu} s_4^3 + \frac{c_\nu}{c_\nu} s_4^2 + \frac{c_\nu}{c_\nu} s_4 + \frac{c_\nu}{c_\nu} s_4 \left( \frac{c_\nu}{c_\nu} s_4 - s \right) \]

(59)

\[ r_5 = \left[ s(t)(s - s_5) \right]_{s=s_5} = \frac{c_\nu}{c_\nu} s_5^4 + \frac{c_\nu}{c_\nu} s_5^3 + \frac{c_\nu}{c_\nu} s_5^2 + \frac{c_\nu}{c_\nu} s_5 + \frac{c_\nu}{c_\nu} s_5 \left( \frac{c_\nu}{c_\nu} s_5 - s \right) \]

(60)

Finally, we use the inverse Laplace transform on Equation (55) to obtain the viscoelastic analytical solutions of rock deformation \(\Delta \omega\) under the coupled effect in the time domain as

\[ \Delta \omega(t) = - \left( r_1 e^{s_1 t} + r_2 e^{s_2 t} + r_3 e^{s_3 t} + r_4 e^{s_4 t} + r_5 e^{s_5 t} \right) \]

(61)

Equation (54) is then further simplified as,
\[ \bar{T}(s) = \frac{d_1}{s} + \frac{d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}{s (s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} \]

\[ = \frac{d_1}{s} + \frac{d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}{s (s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} \]

\[ = \frac{d_1}{s} + \left[ \frac{r_1'}{(s-s_1)} + \frac{r_2'}{(s-s_2)} + \frac{r_3'}{(s-s_3)} + \frac{r_4'}{(s-s_4)} + \frac{r_5'}{(s-s_5)} \right] \]

where \( d_1 = b_1, d_2 = (3(b_2 - 2b_1)Kp_{2r}^2 + 2(2b_2 - b_3)p_{2r}q_{2r})E_b, \)

\[ d_3 = (6(b_2 - 2b_1)Kp_{1r}p_{2r} + 2(2b_2 - b_3)(p_{2r}q_{1r} + p_{1r}q_{2r}))E_b, \]

\[ d_4 = (3(b_2 - 2b_1)(Kp_{1r}^2 + 2Kp_{2r}^2) + 2(2b_2 - b_3)(p_{2r}q_{1r} + 5q_{2r}))E_b, \]

\[ d_5 = (6(b_2 - 2b_1)Kp_{1r}q_{1r} + 12b_4 K + 2(2b_2 - b_3)E_b)(p_{2r}q_{1r} + p_{1r}q_{2r}) + 8b_5 q_{1r}q_{2r}, \]

\[ d_6 = 3(b_2 - 2b_1)K_{E_b} \left( p_{1r} + 2p_{2r} \right) + \left( 12b_4 K + 2(2b_2 - b_3)E_b \right)(p_{2r}q_{1r} + q_{2r}) + 4b_4 q_{1r}q_{2r}, \]

\[ d_{10} = 6(b_2 - 2b_1)Kp_{1r}E_b + 12b_4 Kq_{1r} + 2(2b_2 - b_3)q_{1r}E_b, \]

\[ d_{11} = 3(b_2 - 2b_1)K_{E_b}. \]

In Equation (62), \( s_1, s_2, s_3, s_4, s_5 \) are the roots of the characteristic equation \( d_7 s^3 + d_8 s^4 + d_9 s^3 + d_{10} s^2 + d_{11} s = 0 \), \( r_1', r_2', r_3', r_4', r_5' \) and \( r_6' \) are coefficients to be determined, as the residues of Equation (62) at \( s_1, s_2, s_3, s_4, s_5 \), and can be calculated according to the following formulas

\[ r_1' = \left[ s(t)(s-s_1) \right]_{s=s_1} = \frac{d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} \]

\[ r_2' = \left[ s(t)(s-s_2) \right]_{s=s_2} = \frac{d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} \]

\[ r_3' = \left[ s(t)(s-s_3) \right]_{s=s_3} = \frac{d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} \]

\[ r_4' = \left[ s(t)(s-s_4) \right]_{s=s_4} = \frac{d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} \]

\[ r_5' = \left[ s(t)(s-s_5) \right]_{s=s_5} = \frac{d_2 s^4 + d_3 s^3 + d_4 s^2 + d_5 s + d_6}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)} \]
Finally, we use the inverse Laplace transform on Equation (62) to obtain the viscoelastic analytical solutions of the anchoring force $T$ of the cable under the coupled effect in the time domain as

$$T(t) = d_1 - \left( r_1' e^{\xi t} + r_2' e^{\xi^2 t} + r_3' e^{\xi^3 t} + r_4' e^{\xi^4 t} + r_5' e^{\xi^5 t} \right)$$  \hspace{1cm} (68)

4 Verification of the theoretical model

In this section, we use numerical simulations with the finite difference software FLAC$^3$D to verify the fidelity of the analytical solutions proposed in this work.

4.1 Establishment of the numerical model

The finite difference software FLAC$^3$D is used to numerically simulate the coupling effect of the creep of the slope rock masses and the changes of anchoring force of prestressed anchor cables. We use the reinforcement area of one prestressed anchor cable to establish the simulation model, as shown in Fig. 6. The size of the numerical model is 5 m in length (X direction) and 5 m in width (Y direction) and 60 m in height (Z direction), which consists of 12,544 elements and 14,299 nodes. In addition, the anchor cable in the numerical model is simulated by the Cable structural element type with a length of 50 m.

Fig. 6. Dimension and grid of the numerical model.

In the numerical simulations, Burgers model is used for the rock, and the axial stress-strain relationship of the Cable unit follows the elastic model. We conduct numerical simulations of creep effect of the rock reinforced by prestressed anchor cables. To be as consistent as possible between analytical solutions and
numerical simulations, it is necessary to ensure that the anchoring force of the cable changes with the creep effect of the rock in each step of operation. Accordingly, the Fish function is used to calculate the deformation of the rock at each time step, and the anchoring force of the cable in the current creep state is calculated by using Equation (9) to meet the requirement of the creep coupling effect.

The mechanical parameters provided in Table 3 and Table 4 are used for numerical simulations and analysis of the theoretical model.

**Table 3.** Rock mechanical parameters used in the numerical model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_i$ (m)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$K$ (GPa)</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$P$ (kN)</td>
<td>10000</td>
<td>1100</td>
</tr>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>$G_{1r}$ (GPa)</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>$G_{2r}$ (GPa)</td>
<td>1.92</td>
<td>1.92</td>
</tr>
<tr>
<td>$\eta_1$ (GPa·h)</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>$\eta_2$ (GPa·h)</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

**Table 4.** Parameters of the anchor cable used in the numerical model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L$ (m)</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>$T_0$ (kN)</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td>$E_b$ (GPa)</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>$A_b$ (m$^2$)</td>
<td>0.0176</td>
<td>0.0176</td>
</tr>
<tr>
<td>$R_0$ (m)</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>$F_i$ (kN)</td>
<td>24000</td>
<td>24000</td>
</tr>
</tbody>
</table>

4.2 Comparison of analytical solutions and simulation results

Fig. 7 plots the comparison of the analytical and numerical solutions of the rock deformation and the anchoring force of the cable in the coupled model. The simulation results show that the deformation $\Delta \omega$ of the rock and the anchoring force $T$ of the cable increase with time.
(b) Simulation solution
(c) Analytical solution
Fig. 7. Comparison between analytical solutions and numerical simulation results. (a) Comparison between analytical solutions and numerical simulation results of rock deformation when $P > T_0$; (b) Comparison between analytical solutions and numerical simulation results of rock deformation when $P < T_0$; (c) Comparison between analytical solutions and numerical simulation results of the cable anchoring force when $P > T_0$; (d) Comparison between analytical solutions and numerical simulation results of the cable anchoring force when $P < T_0$.

Fig. 7 (a) and Fig. 7 (b) show the increasing rock deformation with time. Clearly, both numerical simulation results and analytical solutions of the rock deformation exhibit a time dependence. Specifically, when the unloading force $P$ after slope excavation is larger than the initial cable anchoring force $T_0$, an outward displacement perpendicular to the slope surface is created. Conversely, when the unloading force $P$ after slope excavation is smaller than the initial anchoring force $T_0$, a inward displacement perpendicular to the slope surface is created. Additionally, the rock deformation increases with time and eventually converges to a stable value. The reason is that the instability of the surrounding rock at the initial stage of the slope excavation and anchoring causes a larger rock deformation rate, and then the rate of deformation gradually decreases with time.

Fig. 7 (c) and Fig. 7 (d) show the evolution of cable anchoring force $T$ with time. Similarly, both the numerical simulation results and analytical solutions of the anchoring force show a time dependence. Specifically, when the unloading force $P$ after slope excavation is larger than the initial anchoring force $T_0$, the anchoring force gradually increases and finally converges to a stable value with time. When the unloading force
After slope excavation is smaller than the initial anchoring force $T_0$, the anchoring force gradually decreases and ultimately converges to a stable value with time.

From Fig. 7, it is found that when considering the coupling effect, the trends of the rock deformation and the cable anchoring force from the numerical simulation are similar to that of the analytical solutions, and the magnitudes are also in good agreement, which suggests the fidelity of the solution procedure and the results of the coupled model. In general, the final rock deformation from the numerical simulation is slightly smaller than that from the analytical solution. When $P > T_0$, the numerical simulation results of the anchoring force are slightly smaller than the analytical solutions. Conversely, when $P < T_0$, the numerical simulation results of the anchoring force are slightly larger than the analytical solutions. The main reasons for the differences between the analytical solutions and the numerical simulations are related to the size of the numerical model and the geometric distribution of the rock masses.

5 Project example

5.1 Project overview

Jinping I Hydropower Station is located in Liangshan, Sichuan Province, Southwest China. The total installed capacity of the hydropower station is 3.6 million kW, and the average annual power generation is 16.62 billion kW∙h. The arch dam of Jinping I Hydropower Station is 305 m high, and both sides of the dam have steep slopes resulting in a deep “V”-type valley. The natural slope height of the left abutment exceeds 1000 m, the overall excavation height of the slope is about 530 m, and the total excavation volume is about 5.5 million m$^3$. The left slope is affected by the geological structure, and the unloading effects due to large-scale excavation of the rock masses are significant. During the operation of the hydropower station, the rock deformation of the left slope is accompanied by time effects. Therefore, in order to ensure the long-term safety of Jinping I Hydropower Station, long-term monitoring of the slope displacement and support measurements are conducted. The monitoring data are used to analyze the evolution law of the rock creep in high slopes and the changing anchoring force of prestressed cables. The panorama and cross section of the left abutment slope anchoring project of the Jinping I hydropower station are shown in Fig. 8 and 9, respectively.
5.2 Model validation

Based on the study of the coupling effect between the rock creep and the changing anchoring force of the prestressed cable, the monitoring data of the displacement of the rock masses on the Jinping I Hydropower Station left slope and the anchoring force of the anchor cables are compared with the theoretical analytical solutions to verify the applicability of the theoretical model.
The anchor cables in this area were made of high-strength steel strand of 1860 MPa with Φ15.24 mm. The design length is 60m and 80m alternately, the spacing is set to 5m. The elastic modulus of the anchor cable is 210 GPa, and the design tonnage of the anchor cables is 300 tons. Three dynamometers (BL5-3, BK4-11 and AL2-2) and one multi-point displacement meter (M4-10) were installed near the area for real-time monitoring to ensure the safety of the cables during the period tension and operation. Based on the measured data of the M4-10 displacement meter and the BL5-3 anchorage force monitoring point, comparative analysis was carried out to verify the applicability of the coupling effect model between the rock creep and the changing anchoring force of the prestressed cable. The parameters of rock masses and prestressed anchor are shown in Table 5, and the comparison results are shown in Fig. 10.

Table 5. Physical and mechanical parameters of rock masses on the left slop of Jinping I Hydropower Station

<table>
<thead>
<tr>
<th>$L_1$ (m)</th>
<th>$K$ (GPa)</th>
<th>$\nu$</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$G_{1r}$ (GPa)</th>
<th>$G_{2r}$ (GPa)</th>
<th>$\eta_{1r}$ (GPa·h)</th>
<th>$\eta_{2r}$ (GPa·h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.1</td>
<td>0.26</td>
<td>2700</td>
<td>0.52</td>
<td>1.47</td>
<td>523</td>
<td>200</td>
</tr>
</tbody>
</table>

(a)
It can be observed from Fig. 10 that the calculation result of theoretical model is consistent with the trend of on-site monitoring data. A good agreement is achieved, suggesting the fidelity the theoretical model. Specifically, an outward displacement of the slope perpendicular to the slope surface is created. Influenced by rock properties and in-situ stress, the initial rate of rock mass displacement is relatively large. The anchoring force of the anchor cable is lost, and the prestress loss rate of the anchor cable in the calculation results and monitoring data is 6.07% and 4.84%. Therefore, the above theoretical model can be used to analyze the interaction between rock mass and prestressed anchor cable in slope engineering.

6 Discussion

Based on the study of the coupling effect between the rock creep and the changing anchoring force of the prestressed cable, in this section we discuss and analyze the rock deformation with and without the coupling effect, so as to comprehensively and systematically understand the reinforcing effect of the theoretical model.

Fig. 11 shows the creep decomposing into elementary strains. If the rock creep strain can become stable after a long enough time, it is called stable creep. If the creep keeps increasing and cannot get stabilized, it is then called unstable creep. The generalized Kelvin model can describe stable creep, and most hard rocks exhibit...
stable creep behavior. Burgers model is often used to describe unstable creep and thus can describe the creep properties of some soft rocks.

Fig. 11. Creep decomposing into elementary strains.

Fig. 12 shows the rock displacement as a function of time with and without the coupling effect, when the unloading force $P$ after slope excavation is greater than the initial anchoring force $T_0$. The following conclusions can be drawn from Fig. 12. (i) When no anchor cables are applied to the rock after excavation and the rock shows an unstable creep behavior (green line), and the slope displacement will continue to increase. (ii) The anchor cables are applied to the rock after excavation, but the coupling effect between the changing anchoring force of the prestressed cables and the rock creep is not considered (blue line). When the rock exhibits unstable creep behavior, the slope displacement will be smaller than that without anchor cables (green line), but the displacement will also continue to increase, which cannot fully reflect the reinforcing effect of anchor cables. (iii) When the Burgers model is used for the rock and the coupling effect is considered (black line), although the rock exhibits unstable creep behavior, the displacement after excavation becomes stabilized with anchor cables eventually, which can well reflect the reinforcing effect of anchor cables. (iv) When the generalized Kelvin model is used for the rock and the coupling effect is considered (red line), the final displacement of the slope also reaches a stable value. Furthermore, the displacement is smaller than that of Burgers model (black line), which is related to the stable creep properties of the rock and the reinforcing effect of anchor cables.
It is worth noting that after considering the coupling effect between the rock creep and the changing anchoring force of prestressed cables, the rock with unstable creep properties also shows stable creep behavior (black line) after being strengthened with prestressed cables. This phenomenon fully reflects the reinforcing effect of anchor cables, consistent with observations from engineering practices.

### 7 Conclusions

In this work, we developed coupled analytical solutions for the unloading displacement of rock masses and the anchoring forces of prestressed cables that take into account the rock creep and the evolving anchoring forces. Subsequently, we validated the fidelity of the analytical solutions by comparing against numerical simulation results using the finite difference software FLAC$^3$D. The following conclusions can be drawn from this study.

1. Based on the Boussinesq problem in elastic mechanics, the problem of slope reinforcement with a single prestressed anchor cable is simplified to the problem of concentrated force acting on the boundary of a semi-infinite medium. We established a theoretical model considering the coupling effect between rock creep of high slopes and the changing anchoring force of prestressed cables. We further derived the elastic and viscoelastic analytical solutions for the rock displacement and the cable anchoring force under coupled action.

2. Based on the theoretical model and finite difference software FLAC$^3$D, we numerically simulated the creep effect of rock masses anchored by prestressed cables. The analytical solutions and the numerical results show good agreement, which suggests the fidelity of the analytical solutions considering the coupling effects.
effect. The model provides a theoretical reference for studying the slope reinforcement, analyzing the creep behavior of slope rock masses and the long-term prestress of anchoring reinforcement structure.

3. The displacement of rock masses exhibits a time dependence. When the unloading force $P$ after slope excavation is larger than the initial cable anchoring force $T_0$, an outward rock displacement perpendicular to the slope surface is created. Conversely, when the unloading force $P$ after slope excavation is smaller than the initial anchoring force $T_0$, a inward rock displacement perpendicular to the slope surface is created. The deformation of rock masses increases with time and finally converges to a stable value.

4. When the unloading force $P$ after slope excavation is larger than the initial anchoring force $T_0$, the anchoring force gradually increases and converges to a stable value with time. When the unloading force $P$ after slope excavation is smaller than the initial anchoring force $T_0$, the anchoring force gradually decreases and converges to a stable value with time.

5. Combined with engineering examples, it is proved that the new theoretical model is suitable for high slope engineering with strong unloading effect. Considering the coupling effect in the model, for rock masses with unstable creep properties, the rock displacement after excavation and reinforcement finally reaches to a stable value, which can well reflect the reinforcing effect of anchor cables.

Nomenclature

- $A_b$: Cross-sectional area of the anchor cable
- $D$: Differential operator
- $e_{ij}$: The deviatoric strain tensor
- $E$: Elastic modulus
- $E_b$: Elastic modulus of the anchor cable
- $E_r$: Elastic modulus of the rock mass
- $E_{1r}, E_{2r}$: Visco-elastic parameter
- $F$: Resultant force on the anchor cable unit
- $F_t$: Ultimate tensile load of the anchor cable
- $G$: Shear modulus of the rock mass
- $G_{1r}$: Elastic shear modulus of the rock mass
- $G_{2r}$: Visco-elastic shear modulus of the rock mass
- $K$: Bulk modulus of the rock mass
\( l \) Distance from the concentrated force to the measuring point
\( L \) Length of the free section of the anchor cable
\( L_s \) Spacing of anchor cables
\( \Delta L \) Deformation of the anchor cable during coordinated deformation
\( P \) Equivalent force of excavation unloading
\( P_b, Q_b \) Operator function of 1D constitutive equation of the anchor cable
\( \bar{P}_b, \bar{Q}_b \) Operator function of 1D constitutive equation of the anchor cable after Laplace transform
\( p_k, q_k \) Constant parameters of the anchor cable material
\( P', Q', P^*, Q^* \) Operator functions of the rock mass viscoelastic constitutive model
\( \bar{P}', \bar{Q}', \bar{P}^*, \bar{Q}^* \) Operator functions of the rock mass viscoelastic constitutive model after Laplace transform
\( q \) Unloading stress of the rock mass excavation within the anchorage range
\( q_0 \) Equivalent stress of the anchor cable anchoring force within the anchorage range
\( q_f \) Contact stress at the base
\( r \) Distance between the measuring point and the center point
\( R_0 \) Radius of the anchor pier
\( s \) Laplace variable
\( S_{ij} \) The deviatoric stress tensor
\( t \) Time
\( T_0 \) Initial prestress of the anchor cable
\( T \) Anchoring force of the anchor cable
\( z \) Vertical depth of the measuring point

**Greek symbols**

\( \sigma \) Total stress
\( \sigma' \) The derivative of \( \sigma \)
\( \sigma'' \) The second derivative of \( \sigma \)
\( \sigma_{ij} \) Stress tensor of the rock mass
\( \varepsilon \) Total strain
\( \varepsilon' \) The derivative of \( \varepsilon \)
\( \varepsilon'' \) The second derivative of \( \varepsilon \)
\( \varepsilon_b \) Total strain of the anchor cable
\( \varepsilon_{ij} \)  Strain tensor of the rock mass
\( \delta_0 \)  Pre-tension length of the anchor cable
\( \rho \)  Density of the rock mass
\( \eta_1, \eta_2 \)  Viscosity coefficient of the rock mass
\( \mu \)  Poisson ratio of the rock mass
\( \Delta \omega, \omega \)  Displacement parameters of the rock mass

**Authors’ contributions**
Wendong Yang and Xuepeng Wang wrote the main manuscript text. Ning Liu and Qi Wang validated the methods. All authors reviewed the manuscript.

**Declaration of competing Interest**
We declare no potential conflicts of interest with respect to the research, authorship, and publication of this article.

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**References**


Table Captions
Table 1. Theoretical study progress on the coupling effect between rock masses and prestressed anchor cables
Table 2. Parameters of typical anchor cables
Table 3. Rock mechanical parameters used in the numerical model
Table 4. Parameters of the anchor cable used in the numerical model
Table 5. Physical and mechanical parameters of rock masses on the left slope of Jinping I Hydropower Station

Figure Captions
Fig. 1. A high slope of a hydropower station reinforced with prestressed anchor cables
Fig. 2. Typical creep models
Fig. 3. Prestressed anchor cable layout and anchor cable details of a typical section of a hydropower station
Fig. 4. Schematic diagram of the anchoring force of an anchorage unit
Fig. 5. Schematic diagram of a semi-infinite elastic medium subjected to a concentrated normal force on the boundary
Fig. 6. Dimension and grid of the numerical model
Fig. 7. Comparison between analytical solutions and numerical simulation results. (a) Comparison between analytical solutions and numerical simulation results of rock deformation when \( P > T_0 \); (b) Comparison between analytical solutions and numerical simulation results of rock deformation when \( P < T_0 \); (c) Comparison between analytical solutions and numerical simulation results of the cable anchoring force when
$P > T_0$; (d) Comparison between analytical solutions and numerical simulation results of the cable anchoring force when $P < T_0$

**Fig. 8.** Panoramic view of the anchoring project for the left abutment slopes of the Jinping I Hydropower Station

**Fig. 9.** Engineering geological profile of left slope of Jinping I Hydropower Station [62]

**Fig. 10.** Comparisons of the curves between the theoretical value and the monitor data. (a) Comparison between the curves between the theoretical value and the monitor data of rock displacement; (b) Comparison between the curves between the theoretical value and the monitor data of the anchoring force

**Fig. 11.** Creep decomposing into elementary strains

**Fig. 12.** Comparison of rock displacement between coupled model and uncoupled model