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Numerical prediction of transient electrohydrodynamic instabilities under an alternating current electric field and unipolar injection

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Abstract: This paper presents a two-dimensional direct numerical simulation (DNS) of dielectric fluid flow subjected to unipolar injection under an alternating current (AC) electric field. The effect of frequency \( f \) of pulsed direct current (PDC) and AC on the transient evolution of electroconvection and their subcritical bifurcations are investigated for the first time. Electroconvection under PDC or AC tends to exhibit oscillating flow due to the periodic boundary condition of electric potential and charge density compared with the direct current (DC) case. The results demonstrate that the linear stability criterion \( T_c \) decreases as the frequency increases under a PDC field, while the nonlinear stability criterion \( T_f \) is hardly affected. Under the AC field, a critical frequency \( f_c = 0.0316 \) is found, which separates electroconvection into two typical flow regimes—periodic flow regime \( (f < f_c) \) and inhibited flow regime \( (f \geq f_c) \)—depending on whether free charges can reach the collector electrode before electric field inversion. These mechanisms of electroconvection under PDC/AC field offer possibilities in the field of flow control.

I. INTRODUCTION

Electrohydrodynamics (EHD) is an interdisciplinary field that studies the interaction between hydrodynamic and electric fields.1 Electroconvection flow (ECF) induced by charge injection is a fundamental problem in EHD. The Coulomb force from the electric field that was exerted on the space charges injected from the electrodes induced the flow motion. Owing to its low noise and energy consumption and lack of mechanical movement, the EHD has unique advantages in active flow control,2 microchannels transport,3 heat transfer enhancement,4 EHD drag pumps,5, 6 alternating current (AC)–EHD fluid micromixing and microfluidic systems,7-9 EHD printing driven by pulsed and AC voltages,10-12 atomization technology,13 and so on.

Coulomb force-driven hydrodynamic instabilities, also called electroconvection flow (ECF) instabilities, are an important issue in EHDs and have drawn much attention in the past few decades.14 Under an external applied direct current (DC) field, the bifurcation of the ECF is subcritical, characterized by an abrupt jump of...
flow motion from the rest state to the regular convection state when the driving parameter \( T \) (the ratio of the destabilizing Coulomb force to the stabilizing viscous one) reaches the linear critical value \( T_c \).\(^{15, 16}\) When \( T \) is lower than another critical value \( T_f \) (the nonlinear or finite-amplitude stability criterion), the flow recovers back to a zero state.\(^{17}\) Because \( T_f \) is smaller than \( T_c \), the two criteria are linked with a hysteresis loop.\(^{17}\) This subcritical ECF bifurcation with a hysteresis loop was proven in several later works\(^{16, 18-20}\) inside a model of an infinite liquid layer between two parallel electrodes under a DC field. Félici et al.\(^{19}\) simplified a hydraulic model of two-dimensional (2D) rolls and first deduced the physical mechanism for the subcritical bifurcation under a weak injection regime. Later, Atten and Lacroix\(^{21, 22}\) extended the model of Félici et al. to the case of hexagonal cells and studied nonlinear instability problems. The existence of a nonlinear stability criterion and hysteresis loop associated with discontinuities in the current and the liquid velocity was successfully predicted. Castellanos and Atten,\(^{23}\) Chicón et al.,\(^{24}\) Vázquez et al.,\(^{25-27}\) and Traoré et al.\(^{17, 27, 28}\) confirmed the existence of this hysteresis loop via numerical and experimental methods.\(^{22}\) Then, the influence of various factors, including the thermal effects,\(^{29}\) rigid wall effects,\(^{30}\) injection strength, injection configurations [three-dimensional (3D) and two coaxial cylinders], and viscoelastic fluid on flow structures and linear stability criteria were investigated numerically.\(^{31-35}\)

However, most existing works on electroconvective instabilities are based on DC assumptions. The unsteady AC field will introduce additional instabilities increasing the complexity of the problem. The consideration of EHD under the AC field is necessary from the aspects of both fundamental study and application. The EHD instabilities with rich flow patterns under an AC field are still unexplored in detail; concurrently, EHD equipment under an AC field avoids a large and expensive DC power system in real applications.

Previous studies on EHD under an AC field have mainly focused on nematic liquid crystals.\(^{36-39}\) The anisotropic properties of these materials appeared to account for the instability mechanisms and various observations associated with them. Electrokinetic flow under an AC field also exhibits many unique mechanisms that have been investigated widely.\(^{40-44}\) However, experimental, numerical, and theoretical studies on the electroconvection of dielectric liquids under AC and pulsed DC (PDC) electric fields are still limited.\(^{45}\) Atten extended a previous study of DC electroconvection between parallel plates to the AC field.\(^{46}\) They mainly focused on the influence of frequency on the linear stability criterion. Daaboul et al. studied the transient velocity field of dielectric liquids subjected to an AC square wave signal with different frequencies.\(^{47}\) They considered the blade–plate configuration, and a high voltage (±30.0 kV) was applied. In addition, the AC electric field with low free frequency may show strong fluency in both single-phase and multiphase electrothermal systems.\(^{48-50}\)

Therefore, in this work, we conduct numerical simulations on the instability of EHD flows in a dielectric fluid subjected to unipolar injection under AC fields. In addition to the typical sinusoidal alternating current, PDC is also considered as an intermediate step from the DC field to the AC field. The main objectives include two aspects: (1) to determine the relationship between the finite amplitude criterion and frequency and (2) to find the electroconvection structures under PDC and AC fields. This paper is
organized as follows. The formulation of the physical problem and the numerical methods are described in Section II, with a brief introduction to the finite volume method (FVM). In Section III, model verifications are performed with two benchmarks. The equations and boundary conditions are solved numerically by means of the FVM. In Section IV our numerical findings are reported and discussed, and conclusions are given in Section V.

II. PROBLEM FORMULATION

A system with a fluid layer of width $H$ enclosed between two parallel metallic electrodes of length $L$ subject to a PDC or AC voltage difference is considered in this paper, as shown in Fig. 1. The aspect ratio $A$ is defined as $A = L/H$. The potential of the bottom emitter electrode with a unipolar charge injected from it is held at $\phi_b = |\phi_{\text{max}} \sin(2\pi ft)|$ for the PDC case or at $\phi_b = \phi_{\text{max}} \sin(2\pi ft)$ for the AC case. In contrast, the potential of the top collector electrode holds at $\phi_0$. The fluid is assumed to be incompressible and perfectly insulating with constant physical properties. Charge transport occurs via convection, ionic drift, and diffusion mechanisms.

FIG. 1. Schematic diagram of the electrohydrodynamic system. This dielectric liquid layer is enclosed between two electrodes and subjected to unipolar injection under pulsed direct current and alternating current voltages. The distance between the two electrodes is characterized as $H$.

A. Base governing equations

The incompressible hydrodynamic equations\(^1\) are the momentum equation and the continuity equation,

$$
\rho_0 \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \eta \Delta \mathbf{u} + \mathbf{f}_e
$$

(1)

$$\nabla \cdot \mathbf{u} = 0
$$

(2)

where $\mathbf{u}$ is the fluid velocity, $t$ is the time, $p$ is the pressure, $\eta$ is the dynamic viscosity, $\rho_0$ is the density, and $\mathbf{f}_e$ is the density of the electric force. In the general form, the electric force that acts on a unit volume of
the dielectric liquid can be expressed as:

\[ f_e = qE - \frac{E^2}{2} \nabla E + \sqrt{\frac{E^2}{2} \frac{\partial E}{\partial \rho}} \]  

(3)

where \( q \) is the charge density, \( E \) is the electric field, \( \varepsilon = \varepsilon_r \varepsilon_0 \) is the fluid permittivity, and \( \varepsilon_0 \) is the vacuum permittivity. In an isothermal and homogeneous fluid \( \nabla E = 0 \), the second term of the electric force, that is, the dielectric force, vanishes. By including a third term, the electrostriction force in the pressure gradient term \( p + \left[ \frac{E^2}{2} \frac{\partial E}{\partial \rho} \right] = \beta \), Eq. (1) can be simplified to

\[ \rho_0 \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla f_\beta + \eta \Delta \mathbf{u} + q \mathbf{E} \]  

(4)

The electric field equation and the Poisson equation are given by,

\[ \mathbf{E} = -\nabla \phi \]  

(5)

\[ \nabla \cdot (\varepsilon \nabla \phi) = q \]  

(6)

where \( \phi \) is the electric potential.

The conservation of charge density is represented by,

\[ \frac{\partial q}{\partial t} + \nabla \cdot \mathbf{j} = 0 \]  

(7)

Assuming a linear isotropic medium and the four classical distinct charge transfer mechanisms—convection, migration, diffusion, and conduction—we may write the expression of the current density \( \mathbf{j} \):

\[ \mathbf{j} = q \mathbf{u} + q \varepsilon \mathbf{E} - D \nabla q \]  

(8)

where \( K \) and \( D \) are the ionic mobility and charge-diffusion coefficient, respectively. Viscous dissipation and Joule heating are fully negligible because the liquid is considered to be isothermal. In general, the magnetic effects and the Joule heating can be neglected because the electric current through dielectric liquids is rather weak. The diffusion current is considered here. There are three components for current density: the conduction current density \( q \varepsilon \mathbf{E} \) due to ion drift, the convection term \( q \mathbf{u} \) due to fluid motion, and the diffusion current density \( -D \nabla q \) due to molecular diffusion.

**B. Dimensionless parameters and equations**

For universality in the description of the problem, it is particularly convenient to work with nondimensional equations. To transform the last set of equations into nondimensional ones, we introduce the following dimensionless quantities, denoted with an asterisk:
In our equations, \( x_i^* (i = x, y) \) represents the spatial coordinate in the \( i \)-direction, and \( t^* \) represents time. The reference velocity \( u_0 \) is scaled to the electrical drift velocity \( \mathcal{K} ((\phi_0)_{rms} - \phi_h) / H \), \( \mathcal{E} ((\phi_0)_{rms} - \phi_h) / H^2 \) for charge density, \((\phi_0)_{rms} - \phi_h \) for electric potential, and \( (\phi_0)_{rms} \) represents the root-mean-square (RMS) electric potential of \( \phi_h \). The time scale is the transit ionic time \( H / u_0 \), and the frequency scale is the inverse of the transit ionic time.

If we drop the asterisk indicator for clarity, the basic dimensionless EHD equations for Newtonian fluids are expressed as follows:

\[
\nabla \cdot \mathbf{u} = 0
\]

\[
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \mathbf{p} + \frac{M^2}{T} \nabla^2 \mathbf{u} + M^2 \mathbf{q} \mathbf{E}
\]

\[
\frac{\partial q}{\partial t} + \nabla \cdot (q \mathbf{E} - \alpha \nabla q + q \mathbf{u}) = 0
\]

\[
\mathbf{E} = -\nabla \phi
\]

\[
\nabla^2 \phi = -q
\]

The relevant dimensionless parameters in these equations are as follows: \( T = \mathcal{E} ((\phi_0)_{rms} - \phi_h)/(\mu \mathcal{K}) \) is the ratio of Coulomb and viscous forces. \( C = q_0 H^2 / \mathcal{E} ((\phi_0)_{rms} - \phi_h) \) is the dimensionless measure of the injection level. \( M = (\mathcal{E} / \rho_0)^{1/2} / \mathcal{K} \) is the ratio between the so-called hydrodynamic mobility and the true mobility of ions. \( \alpha = D / \mathcal{K} ((\phi_0)_{rms} - \phi_h) \) is the ratio of the diffusion term and drift term. Comprehensive descriptions of the equations and the parameters involved can be found in Refs. 51 and 52.

The above problem is characterized by a particular time, the ionic transit time \( t_r \) from one electrode to the other under a constant uniform field: \( t_r = H / u_0 = H / KE \). Thus, the characteristic frequency \( f_c = 1 / t_r = 0.0316 \). We assume that each electrode injects unipolar ions (cations) into the insulating liquid during one half-cycle and collects those ions during the other half-cycle for the AC electrodynamics system. For the system frequency greater than \( f_c \), the ions cannot reach the other electrode during the half-cycle, which separates into two regimes: the periodic electroconvective flow regime \( (f < f_c) \) and the inhibited electroconvective flow regime \( (f \geq f_c) \).

**C. Numerical implementation and boundary conditions**

As our set of equations is time dependent, we have to provide the initial conditions. As a general rule,
we start either from the fluid at rest or from a steady state obtained from a previous simulation. Under the electric field, a thin electric double layer (EDL) will form at the charged surfaces. The size of the EDL is

$$\lambda_D = \sqrt{\frac{\varepsilon k_e \theta}{2(ze)^2 c_0}}$$  \hspace{1cm} (16)$$

where \( e \) is the elementary charge, \( z \) is the ionic valence, \( k_e \) is the Boltzmann constant, \( c_0 \) is the charge concentration of injection electrode and \( \theta \) is the temperature.

A common concentration \( c_0 = o(10^{-5}) \) results in \( \lambda_D = o(10^{-3}) \) in this paper and the size of system \( H \) is \( o(1) \). Therefore, the effect of EDL on flow can be ignored in this work and boundary conditions on each boundary for the governing equations (11)–(15) are described as follows:

In both PDC and AC cases, the symmetrical boundary conditions for velocity, cations, pressure, and potential have been considered on the lateral borders to simulate an infinitely long cavity:

\[ u = 0, \frac{\partial v}{\partial x} = 0, \frac{\partial \phi}{\partial x} = 0, \frac{\partial q}{\partial x} = 0, \frac{\partial \phi_c}{\partial x} = 0 \]  \hspace{1cm} (17)

For the velocity field, the no-slip and no-penetration conditions are assumed for two solid electrodes,

\[ y = 0, H \quad u = 0 \]  \hspace{1cm} (18)

For the electric potential of the PDC or AC cases, because a PDC or AC potential difference is applied, the conditions are required for both electrodes, given as

\[ y = 0, \phi_0 = \phi_{max} \sin (2\pi ft) \] \hspace{1cm} for the PDC case; \[ \phi_0 = \phi_{max} \sin (2\pi ft) \] \hspace{1cm} for the AC case

\[ y = H, \phi_1 = 0 \] \hspace{1cm} for both the PDC and AC cases

The boundary for charge density follows the injection law, a simplified formula applicable to the usual dielectric liquids for the injection charge density \( q \):

\[ q = \frac{q_{i0}}{bK_i(b)} \]  \hspace{1cm} (20)

where \( q_{i0} \) is the injected charge density at zero field, \( K_i \) is the modified Bessel function of the second kind and first order, and \( q_i \) is the injected charge density that represents a single ion species’ charge. \( bK_i(b) \) can be expanded as follows:\[54\]

\[ bK_i(b) = 1 + \frac{1}{2} b^2 \ln(b/2) + \left( \frac{\gamma_E}{2} - \frac{1}{4} \right) b^2 + ... \]  \hspace{1cm} (21)

where \( \gamma_E = 0.5772 \) is the Euler constant, \( b = l_b / l_o \) with the Bjerrum distance and Onsager distance given as follows:\[52,55\]

\[ l_b = e^2 / 4\pi\varepsilon k_b \theta \cdot l_o = \sqrt{e / 4\pi\varepsilon |E|} \]  \hspace{1cm} (22)

where \( b^2 = 4\gamma E \) and \( \gamma = e^2 / 16\pi\varepsilon k_b^2 \theta^2 \) is the Onsager constant, \( e \) is the elementary charge and \(|E|\) is the
electric intensity.

This means that $q$ follows the injection law at the injector at all times and, hence, the injection rate is influenced by both the electric field and by the liquid motion with time so that these models are more accurate than the autonomous injection of a unipolar charge at the emitter electrode in most DC studies.\textsuperscript{32, 33, 35}

For the charge density of PDC cases:

$$y = 0, q = \frac{q_{i0}}{b K_i(b)}; \quad y = H, \frac{dq}{dy} = 0$$

(23)

For the charge density of AC cases:

$$y = 0, q = \frac{q_{i0}}{b K_i(b)}; \quad y = H, \frac{dq}{dy} = 0 \quad \text{ (Positive cycle of AC)}$$

(24a)

$$y = 0, \frac{dq}{dy} = 0; \quad y = H, q = \frac{q_{i0}}{b K_i(b)} \quad \text{ (Negative cycle of AC)}$$

(24b)

For the pressure of PDC and AC cases:

$$y = 0, H \cdot n \cdot \nabla \phi = 0$$

(25)

The set of partial differential equations (9)–(13) cannot be solved analytically for most problems due to the complexity and high nonlinearity. In this work, the numerical study is performed via the finite volume method (FVM) in the open-source OpenFOAM\textsuperscript{®} toolbox, which takes both stability and efficiency into account. Furthermore, the semi-implicit method for pressure-linked equations–consistent (SIMPLEC) algorithm\textsuperscript{56} is used to couple velocity and pressure, as the fluid is assumed to be incompressible. All time variables adopt the Euler discrete in the discrete formula, and the gradient term is arranged as a Gaussian linear. The diffusion flux is discretized by the second-order central difference (CD) scheme. The convection term of the charge transport equation is discretized by the convergent and universally bounded interpolation scheme for the treatment of advection (CUBISTA) scheme\textsuperscript{57}.

III. MODEL VERIFICATION

Although formal frameworks have been proposed, the correctness of the main models needs to be tested first. We verify the correctness of our implementation in two steps.

First, we run direct numerical simulation (DNS) in the hydrostatic regime, and numerical results are validated by the analytical expression of Ref. 58 and Refs. 59, 60 (lattice Boltzmann method). And the uniform grid size $50 \times 100$ is selected in this paper. The performance of our adopted methodology is satisfactory, with the numerical solution agreeing well with the analytical solution (Fig. 2).
FIG. 2. Comparison of the numerical and analytical solutions at the hydrostatic state under strong injection $C = 10$ for (a) charge density and (b) electric field.

As very few results are available for PDC and AC–ECF, we reproduced the well-established 2D DC flows\textsuperscript{17, 27} to verify our numerical model.

FIG. 3. Bifurcation diagram for $C=10$, $M=10$, and $\alpha=1\times10^{-4}$ in the DC cases.

In Fig. 3, we present the bifurcation diagram obtained by our code. The bifurcation of ECF is of a subcritical nature, which is characterized by a hysteresis loop linking the linear stability criterion $T_c$ and the finite-amplitude criterion $T_f$. Our numerical values of $T_c$ and $T_f$ are 162.2 and 110.5, respectively. The linear criterion should compare with 162.6, the value predicted by the linear stability criterion.\textsuperscript{58} The finite amplitude criterion should compare with 111.7, the benchmark determined by a finite volume method (FVM).\textsuperscript{19}

**IV. RESULTS AND DISCUSSION**

Next, different flow patterns and their subcritical bifurcations for different $T$ and frequencies of PDC and
AC field are investigated, and the numerical results are discussed. Some parameters are fixed: \( C = 10 \), \( M = 10 \), and \( \alpha = 1 \times 10^{-4} \).

Theoretical work determined that the critical wavelength \( k_c \), corresponding to the most unstable mode at the linear stability criterion \( T_c \), can be minimized.\(^{22-58} \) We conduct a series of numerical tests to determine \( k_c \) for the PDC and AC electric field by changing the wave number \( k \) and calculating the corresponding \( T_c \). Figure 4 shows the variation of \( T_c \) with the wave number \( k \) under the PDC \((f = 0.02)\) and AC \((f = 0.01)\) field. Symmetrical boundary conditions have been applied to the lateral walls in these computations. \( T_c \) reaches its minimum at \( k = 1.228 \) for PDC and AC fields in the range of the numerical tests. Concurrently, the calculated results at \( k_c = 1.228 \) have symmetry. Therefore, we can select half of the critical wavelength as the computational domain \( L = 0.614 \) \((A = 0.614)\) for three typical waveforms (DC, PDC, and AC voltages) to reduce the computational load.

\[ \text{FIG. 4. } T_c \text{ vs wave number } k \text{ for } C = 10 \text{ and } M = 10. \text{ Symmetrical boundary conditions have been applied to the lateral walls. (a) PDC field } (f = 0.02) \text{ and (b) AC field } (f = 0.01). \]

**A. Stability for pure unipolar injection under DC and PDC fields**

1. Electrodynamic structures

We first compare the temporal evolution of the maximum velocity and EHD structures between the DC and PDC fields. Figure 5(a) shows the temporal evolution of the corresponding maximum velocity \( V_{\text{max}} \) under the DC and PDC fields for \( T = 170 \), which is higher than the critical value \( T_c \). A similar evolution of fluid motion under DC field can be observed. In the initial stage, the periodic disturbance accumulates slowly. When the disturbance accumulates to a certain extent, the fluid motion goes through stages of rapid development, and the velocity amplitude increases exponentially. Eventually, the flow evolves into a periodic steady flow. In contrast to the DC case,\(^{31} \) oscillations occurs throughout the PDC evolution. The peak value and RMS for the nondimensional maximum velocity of PDC electroconvection are 4.462 and 2.45, respectively. The nondimensional maximum velocity of DC electroconvection is 3.17, between the peak and RMS values of PDC. From Fig. 5(b), we can see that the peak of PDC is \( \sqrt{2} \), which is higher than the 1 of DC. As for why the RMS value of PDC is lower than the maximum value of DC, one possible reason is that the external electric field varies periodically from zero to peak over time [Fig. 5(b)] and cannot provide a
steady driving force \((qE)\), unlike the DC field. The other possible reason is that periodic variation of the flow field produces energy dissipation because the DC voltage is the same as the RMS of PDC voltage.

![Graph showing temporal evolution of \(V_{\text{max}}\) for different fields.]

**FIG. 5.** (a) Temporal evolution of \(V_{\text{max}}\) for \(T = 170\) in the direct current (DC) and pulsed direct current (PDC) \((f = 0.02)\) cases and (b) external voltage variations of PDC \((f = 0.02)\) and DC with time.

ECF by an external DC field demonstrates distinct qualitative features represented by one convective cell structure and void charge region when \(T = 170\). A similar observation can be made for the present setup where an external PDC voltage is applied. Figure 6(a) shows the instantaneous contours of the charge density distribution and streamlines corresponding to five distinct times in an oscillation period. The electrode at \(y = 0\) stays positively charged (the upper electrode is grounded). The applied voltage at the lower electrode takes its maximum \(V_{\text{max}}\) in Fig. 6(c) between \(t_3\) and \(t_4\). The PDC frequency is \(f = 0.02\), which is below the characteristic frequency of this system.

As the instantaneous voltage difference across the channel increases [Fig. 6(b) between \(t_1\) and \(t_2\)], a charge layer comparable to 1/3 of the channel height is formed. When the voltage continues to increase [Fig. 6(b) between \(t_2\) and \(t_3\)], the typical void charge region is formed initially. At the maximum voltage difference [Fig. 6(b) between \(t_3\) and \(t_4\)], the complete void charge region is observed. Similarities to the findings of Wu\(^{31}\) are observed under the DC field. As the voltage drop returns to zero in Fig. 6(b) between \(t_4\) and \(t_5\), the void charge region is relaxed primarily because of a significant decrease in velocity and charge diffusion.

Figures 5 and 6 mainly consider the effect of PDC on ECF for low values of \(T\). We also consider the corresponding results for \(T = 400\) under PDC, which has different flow structures than those at \(T = 170\) because the second bifurcation occurs.\(^{17}\)
FIG. 6. Temporal evolution of charge density ($q$) distribution, voltage and maximum velocity ($V_{\text{max}}$) for $T = 170$ and $f = 0.02$, as a representative of the regular oscillation: (a) charge density distribution and streamlines at five distinct times, (b) external periodic voltages of pulsed direct current ($f = 0.02$) and (c) the periodic $V_{\text{max}}$ signal.
FIG. 7. Temporal evolution of $V_{\text{max}}$ for $T = 400$ in the direct current and pulsed direct current ($f = 0.02$) cases. Figure 7 plots the temporal evolution of $V_{\text{max}}$ for the DC and PDC cases for $T = 400$. For the PDC case, the flows first experience a one-cell pattern similar to Fig. 6(a), and then after a transition phase, a two-cell pattern with lower values of $V_{\text{max}}$ arises. Additionally, the velocity oscillates periodically throughout the whole process. The steady-state nondimensional maximum velocity of DC electroconvection is 4.21. The peak value and RMS of the steady-state nondimensional maximum velocity of PDC electroconvection are 7.12 and 3.75, respectively. The possible reasons are similar to those outlined for the $T = 170$ case.

FIG. 8. Temporal evolution of voltage, $V_{\text{max}}$, and charge density, $q$, distribution for $T = 400$ and $f = 0.02$, as a representation of the regular oscillation: (a) external periodic voltages of pulsed direct current ($f = 0.02$), (b) the periodic $V_{\text{max}}$ signal and (c) charge density distribution and streamlines at five distinct times.

Similar processes like the case for $T = 170$ are observed at $T = 400$ under the PDC field. As the instantaneous voltage difference across the channel increases [Fig. 8(a) between $t_1$ and $t_2$], the maximum velocity $V_{\text{max}}$ increases slowly [Fig. 8(b) between $t_1$ and $t_2$] and a charge layer comparable to 1/10 of the channel height is formed [Fig. 8(c) between $t_1$ and $t_2$]. When the voltage continues to increase [Fig. 8(a) between $t_2$ and $t_3$], the $V_{\text{max}}$ increases rapidly [Fig. 8(b) between $t_2$ and $t_3$], the typical void charge region is formed initially, and charges are injected into the bulk region from both sides of the edge [Fig. 8(c) between
At the maximum voltage difference [Fig. 8(a) between $t_3$ and $t_4$], $V_{\text{max}}$ reaches its peak [Fig. 8(b) between $t_3$ and $t_4$], a complete void charge region is observed in the middle of the domain [Fig. 8(c) between $t_3$ and $t_4$], similar to the findings of Wu.\textsuperscript{31} As the voltage drop returns to zero in Fig. 8(a) between $t_4$ and $t_5$, $V_{\text{max}}$ declines [Fig. 8(b) between $t_4$ and $t_5$] and the void charge region is also relaxed [Fig. 8(c) between $t_4$ and $t_5$], primarily because of the decrease in velocity and charge diffusion.

\section*{2. Electrodynamic bifurcation effect}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig9.png}
\caption{Evolution of $V_{\text{max}}$ for $C = 10$ and $M = 10$ in the pulsed direct current ($f = 0.02$) field; cases for $T = 170$, $T = 180$, and $T = 190$.}
\end{figure}

Bifurcation is a characteristic feature in electroconvection that most experimental and numerical studies have focused on.\textsuperscript{17, 31, 58} Therefore, this section discusses the subcritical bifurcation in ECF under DC and PDC fields. We first briefly introduce a commonly used method to determine the line stability criterion from the result of the direct numerical simulation.\textsuperscript{61} Theoretically, when the driving parameter is set close to the linear instability threshold, the perturbations of every physical quantity $f$ at the initial stage follow an exponential law $f = f_0 e^{\sigma t}$, where $\sigma$ is the growth rate. The growth rates can be extracted from the time evolution curves of several different values of the driving parameter and then extrapolated to obtain the linear stability criterion corresponding to the zero-growth rate. In Fig. 9, we have plotted the time evolution curves of the peak velocity $V_{\text{max}}$ for $T = 170$, $T = 180$, and $T = 190$ under the PDC ($f = 0.02$) field. The exponential behavior is visually seen from the semilog plot. The growth rates were measured with linear fitting in the exponential growth stage. For each of the three cases, our numerical prediction of $T_c$ is approximately 152.
Figures 10(a) and 10(b) describe the complete bifurcation diagram of the DC case and PDC case with $f = 0.02$. When the flow field starts from a hydrostatic solution, the flow remains static as we increase the driving parameter $T$. When $T$ approaches 152, there is a sudden jump in the velocity, which exhibits the subcritical bifurcation characteristic that is typical for electroconvection. We found that the linear criterion $T_c$ of the PDC case ($f = 0.02$) is lower than that of the DC case and that the nonlinear criterion $T_f$ of the PDC case is very close to that of the DC case. A possible reason is that a varying electric field promotes fluid motion. However, PDC field has a minimal effect on the nonlinear criterion $T_f$.

Then, we examine the bifurcation process under the different frequencies of the PDC field to find the relationship between the bifurcation process and the frequency of PDC. Figures 10(b) and 10(c) show the electroconvection bifurcation diagram when $f = 0.02$ and $f = 0.1$. We find that the linear criterion $T_c$ increases with the frequency, which confirms the existence of a critical frequency. For the same $T$, the lower frequency voltage can promote fluid motion more than the higher frequency voltage. According to the results, we can
calculate the average kinetic energy as follows:

\[ E_k = \frac{1}{2V_t} \int |\mathbf{u}|^2 dV = \frac{1}{2V_t} \sum_{i=1}^{N} |u_i|^2 V_k = \frac{1}{2N} \sum_{i=1}^{N} |u_i|^2 \]  

(26)

where \( N \) is the number of cells of the mesh and \( V_t = NV_k \) for a uniform mesh.

The RMS of the average fluid kinetic energy for the PDC with \( f = 0.02 \) case is 1.153 and that of the case for \( f = 0.1 \) is 1.1014 when \( T = 180 \) is close to the linear criterion \( T_c \). We also calculate the DC case for \( T = 180 \); its average kinetic energy is 1.066, which is lower than the values of the PDC cases for \( f = 0.02 \) and \( f = 0.1 \). When \( T \) increases, the fluid has more kinetic energy under low-frequency rather than high-frequency cases, to support the fluid motion under the same external driving force. Therefore, we find that low-frequency cases have lower \( T_c \) values than high-frequency cases. The RMS of the average kinetic energy of fluid for the PDC with \( f = 0.02 \) case is 0.4058 and that of the case for \( f = 0.1 \) is 0.4064 when \( T = 120 \), which is close to the nonlinear criterion \( T_f \). From these results, we can infer that the average kinetic energy varies little as the PDC frequency increases when \( T \) is relatively low. However, the average kinetic energy decreases as the PDC frequency increases when \( T \) is relatively high. A possible reason is that higher frequency cases produce more viscous dissipation with the increase in \( T \).

![FIG. 11. Evolution of \( V_{\text{max}} \) for \( C = 10, M = 10, \) and \( T = 190 \) under the pulsed direct current field for \( f = 0.02 \) and \( f = 0.1 \).](image)

We analyze the relationship between the frequency \( f \) of PDC voltage and oscillation frequency of maximum velocity from Figure 11. Based on the fast Fourier transform (FFT), the oscillation frequency of maximum velocity under PDC field of \( f = 0.02 \) and \( f = 0.1 \) for \( T = 190 \) are 0.02 and 0.1, respectively. The results show that the motion of the fluid and the PDC field are in a state of resonance.
B. Stability for pure unipolar injection under an AC field

We quantitatively measure the influence of the AC field on electroconvection and numerically validate two regimes: periodic ECF and inhibited ECF, which were explained theoretically in Section II B. We restrict our attention to the strong injection case of $C = 10$, which has been extensively discussed for the DC and PDC cases. The aspect ratio of domain $A$ is still set to 0.614, which corresponds to the half wavelength of the most unstable mode that has been verified in Fig. 4.

1. Periodic eletroconvective flow regime ($f < f_c$)

First, we analyze a case with a moderately small AC frequency $f = 0.01$ ($f < f_c$). Figure 12(a) shows the time series of charge density and streamline for half an oscillation. Figure 12(b)-(c) shows temporal evolution of periodic voltages of AC and maximum velocity $V_{\text{max}}$. A similar observation like PDC case can be made for the present setup when an external AC is applied [Fig. 12(a)-(c)]. The difference is that the charge density distribution is reserved during the negative half-cycle because the opposite electrode becomes the injector. For $f < f_c$, the space charge can occupy the whole gap, and we see that the void-charge region is formed periodically. A similar observation can be found during the negative half-cycle. The RMS of the maximum velocity under an AC electric field for $T = 160$ is 1.967, which is lower than 2.212, the corresponding RMS value under PDC voltage with the same frequency. The flow occurs through secondary bifurcation; two vortex structures and a central void-charge region appear in the fluid when $T = 190$ in Fig. 13.
FIG. 12. Temporal evolution of charge density, $q$, distribution, voltage and $V_{\text{max}}$ for $T=160$ and $f=0.01$, as a representation of the regular oscillation: (a) charge density distribution and streamlines at four distinct times, (b) external periodic voltages of AC ($f=0.01$) and (c) the periodic $V_{\text{max}}$ signal.
FIG. 13. Temporal evolution of voltage, $V_{\text{max}}$, and charge density, $q$, distribution for $T = 190$ and $f = 0.01$, as a representation of the regular oscillation: (a) external periodic voltages of alternating current ($f = 0.01$), (b) the periodic $V_{\text{max}}$ signal, and (c) charge density distribution and streamlines at four distinct times.

FIG. 14. Bifurcation diagram for $C = 10$, $M = 10$, and $\alpha = 1 \times 10^{-4}$ under an alternating current field when $f = 0.01$.

Figure 14 shows the subcritical bifurcation of AC electroconvection where $T_c = 127.86$ and $T_f = 107.5$. We found that $T_c$ in AC cases, 127.86, is lower than the 162.3 found in DC cases and $T_f$, 107.5, is slightly lower than the 110.5 found in DC cases. In AC cases, the periodic electric field promotes fluid motion. Thus, lower driving parameters are needed to maintain the charge-free region in the flow field. We also find that secondary bifurcation occurs in the fluid when $T = 190$ and that the critical value $T_{c2}$ of the secondary bifurcation under DC ranges between 290 and 300. The AC field promotes subcritical bifurcation and secondary bifurcation.

2. Inhibited electroconvective flow regime ($f > f_c$)

In this section, cases with AC frequencies greater than $f_c$ are analyzed. The ECF instability induced by external DC fields demonstrates two vortex structures and void charge regions. Figure 15 shows the evolution of charge density and streamlines for $f = 0.05$ ($f > f_c$) and $T = 170$ under the AC field. We find that the space charge remains in two layers that do not overlap, and the fluid fluctuates periodically with low velocity. Concurrently, there are obvious fluctuations in the charge layer. Figure 16 shows the evolution of the charge density and streamline diagram for $f = 0.05$ ($f > f_c$) and $T = 420$ under the AC field. The space charge remains in two layers with obvious fluctuations near the electrode, but the interface of the charge layer becomes more uneven than that seen for $T = 170$. Several irregular vortex flows appear in the fluid, and the irregular evolution of the maximum velocity fluctuates with time. The cases for $T = 170$ and $T = 420$ are consistent with the theoretical prediction in Section II. We also investigated the electric Rayleigh cases with larger $T$ values, and Fig. 17 shows the corresponding results. The flow remains irregular for $T = 800$, and the velocity fluctuates more acutely than for $T = 420$, leading to unsteady plume structures.
FIG. 15. (a) Evolution of maximum velocity for $T = 170$. (b) Evolution of charge density and streamline when $f = 0.05$ under an alternating current field.
FIG. 16. (a) Evolution of maximum velocity for $T = 420$. (b) Evolution of charge density and streamline when $f = 0.05$ under an alternating current field.
FIG. 17. (a) Evolution of maximum velocity for $T = 800$. (b) Evolution of charge density and streamline when $f = 0.05$ under an alternating current field.

C. Spectral analysis
FIG. 18. The plot of power spectral density decay under alternating current fields (a) $T = 160$ for $f = 0.01$, $f = 0.02$, and $f = 0.03$; (b) $T = 160$ for $f = 0.01$, $f = 0.04$, and $f = 0.05$; (c) $T = 420$ for $f = 0.01$, $f = 0.02$, and $f = 0.03$; (d) $T = 420$ for $f = 0.01$, $f = 0.04$, and $f = 0.05$.

One way to look at the state of the flow is to compute the power spectral density (PSD) of the time series. Figure 18 shows the PSDs of the maximum velocity for $T = 160$ and $T = 420$ with different frequencies $f$ of the AC field ranging from 0.01 to 0.05. When $T = 160$ for frequencies $f$ from 0.01 to 0.03, the flow motion is typically periodic, and the amplitude of the fundamental frequency $f_0$ is consistent with the corresponding AC frequency $f$ [Fig. 18(a)]. Moreover, for cases with frequencies $f$ greater than the critical frequency, only periodic small disturbances occur in the fluid due to the significant decay of the PSD compared with the cases for $f = 0.01$ to 0.03 and $T = 160$[Fig. 18(b)]. When $T = 420$ for frequencies $f$ from 0.01 to 0.03, we can obtain similar results to $T = 160$ [Fig. 18(c)]. However, for $f = 0.04$ and 0.05, which are larger than the critical frequency, the flow becomes complicated and irregular, and the power spectrum shows obvious broadband characteristics [Fig. 18(d)]. From these results, we can see that the motion of the fluid and the electric field are in a state of resonance.

V. CONCLUSIONS

In this work, the ECFs in a dielectric liquid layer subjected to unipolar injection under PDC and AC fields are numerically investigated by FVM for the first time. The PDC and AC waveforms are sinusoidal. The flow structures and bifurcation patterns have been studied in detail under various frequencies of PDC and AC fields.

The results show that the varying electric fields play an essential role in instability patterns. The flow structures and linear and nonlinear criteria may change drastically with the field type. In PDC cases, the flow loses its instability at the linear criterion $T_c$, and the bifurcation pattern is characteristic of subcritical bifurcation with hysteresis at different frequencies. The bifurcation pattern is the same as that of the ECF of DC cases. The linear criterion $T_c$ increases with increasing frequency $f$ of PDC field, while the nonlinear stability criterion $T_f$ is hardly affected by the PDC filed frequency. In AC cases, the periodic
electroconvection flow regime and the inhibited electroconvection flow regime are observed with an increase in the frequency of AC when the electric Rayleigh remains the same. When the frequencies of AC are lower than the critical frequency $f_c = 0.0316$, the flow is in the periodic regime, and the bifurcation is characterized by subcritical bifurcation. When the frequency of AC is larger than the critical frequency $f_c$, the flow is in the inhibited regime. At small electric Rayleigh, $T$, the flow is periodic with little disturbance, and the space charge remains in two layers near the electrode. With the increase in $T$, the flow becomes irregular and enters a chaotic state. The space charge layers near the electrode exhibit plume structures when $T$ increases. In addition, the varying electric fields promote a transition from steady state to unsteady convection under PDC and AC fields. Through a comprehensive comparison, the varying electric fields exhibit rich bifurcation and flow phenomena in ECF.

In summary, the varying electric fields lead to new instability patterns and richer dynamic behaviors in electroconvection that are absent when DC fields are considered. The interaction of alternating electric fields and fluid dynamics may have a potential effect on heat transfer enhancement because of enhanced or weakened oscillatory convection behavior. If it is used for flow control under AC fields, the applied frequency and voltage range of the EHD system should be given exceptional attention due to the typical regime characteristics in AC. In future work, we plan to explore the experimental validation in a lab and its further industrial applications. We will also extend this study to viscoelastic fluids or include thermal effects.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.
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