Supplementary information to: *Global optimization of an encapsulated Si/SiO$_2$ L3 cavity with a 43 million quality factor*

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1 Optimal parameters of the encapsulated L3 cavity

The optimal 27 parameters of the Si/SiO$_2$ L3 cavity with an FDTD quality factor of $Q_c = 4.33 \times 10^7$ are reported in Table 1.

Table 1. Optimal parameters of the Si/SiO$_2$ L3 cavity with $Q_c = 4.33 \times 10^7$

Parameter/Hole	1	2	3	4	5	6	7	8	9	10
dx	0.4407	0.3817	0.3936	0.3352	0.3097	0.1385	×	×	×	0.0010
dy	×	×	×	×	×	×	0.0109	0.0107	0.0082	×
dr	-0.1500	-0.1500	-0.0772	-0.1075	-0.0690	-0.0672	×	×	×	×
Parameter/Hole	11	12	13	14	15	16	17	18	19	20
dx	×	×	-0.0044	×	×	0.0010	0.0018	0.0017	×	×
dy	0.0027	×	0.0121	-0.0010	×	×	-0.0044	-0.0081	-0.0071	×
dr	×	0.0001	×	×	×	×	×	×	×	×

2 Fourier transform of near-field components

The far-field projections of the mode components for the non-optimized and optimized cavities, are shown in Figs. 1(a) and 1(b), respectively, in log scale. This projection is obtained through the Fourier transform of the near-field components[1], recorded in a *xy* plane localized at 90 nm above the photonic crystal surface. The dashed circle represents the region where the cavity frequency crosses the light-line. The strong reduction of the field components inside the light cone (or equivalently, above the light-line) is clearly seen for the optimized design.

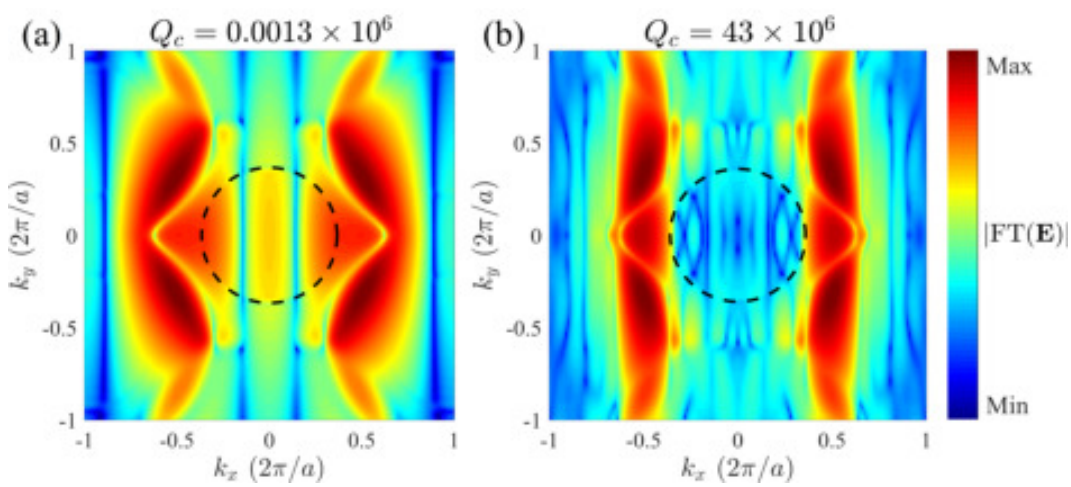

Figure 1. (a) Far-field of the non-optimized cavity. (b) same as (a) for the optimized design. The dashed circle represents the region where the cavity frequency crosses the light-line of the dielectric slab.

3 Results for the Si/Air L3 cavity

The Si/Air (air-bridge) L3 cavity is considered in a silicon PC with a hexagonal lattice of holes with radii $r = 100$ nm, lattice parameter $a = 400$ nm and slab thickness $d = 220$ nm. The PS optimization is carried out by considering the same 27 parameters of the Si/SiO_2 case.

3.1 Optimal figures of merit

We show in Table 2 the linear and non-linear figures of merit of both, non-optimized and optimized designs. The quality factor is improved by four orders of magnitude with a final FDTD value of $Q_c = 1.91 \times 10^8$, which is around 20 times larger than the previous best, for the silicon L3 cavity, obtained with deep learning optimization techniques[2].

Table 2. Linear and non-linear figures of merit for the non-optimized and optimized Si/Air L3 cavities.

Si/Air – L3 cavity	f (Thz)	Q_c	$V_l\ (\lambda/n_{\mathrm{Si}})^3$	$V_{nl}\ (\lambda/n_{\mathrm{Si}})^3$	$Q_c/V_l\ (n_{\mathrm{Si}}/\lambda)^3$	$Q_c^2/V_{nl}^2\ (n_{\mathrm{Si}}/\lambda)^6$
Non-optimized	196.3	6.53×10^3	0.59	2.48	1.10×10^4	6.94×10^6
Optimized	193.6	1.91×10^8	1.07	4.30	1.78×10^8	1.97×10^{15}

3.2 Optimal parameters

The optimal parameters for the Si/Air L3 cavity with $Q_c = 1.91 \times 10^8$ are reported in Table 3

Table 3. Optimal parameters of the Si/Air L3 cavity with $Q_c = 1.91 \times 10^8$

Parameter/Hole	1	2	3	4	5	6	7	8	9	10
dx	0.3800	0.2954	0.2000	0.4032	0.2360	0.0475	×	×	×	-0.0179
dy	×	×	×	×	×	×	-0.0232	-0.0157	0.0028	×
dr	-0.0445	-0.0174	0.0033	-0.0433	-0.1500	-0.0805	×	×	×	×
Parameter/Hole	11	12	13	14	15	16	17	18	19	20
dx	×	×	-0.0341	×	×	0.0040	-0.0001	-0.0059	×	×
dy	-0.0600	×	-0.0141	-0.0078	×	×	0.0083	-0.0114	-0.0307	×
dr	×	-0.0427	×	×	×	×	×	×	×	×

3.3 Disorder analysis

Figure 2 show the disorder analysis for the optimal air-bridge L3 cavity. An averaged Q_c in the 4 million regime is predicted for typical tolerances, ranging between $\sigma = 0.001a$ and $\sigma = 0.002a$, in silicon fabrication techniques[3,4].

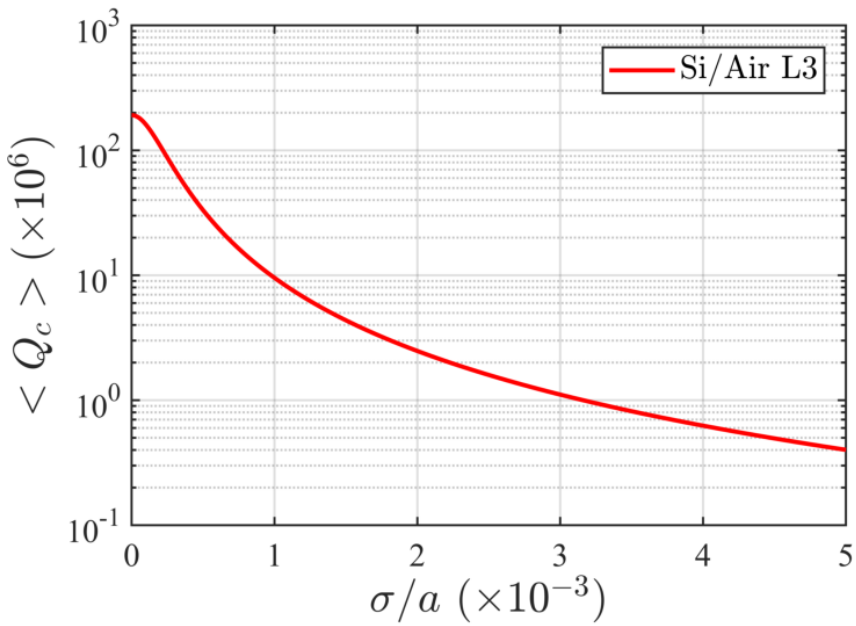

Figure 2. Averaged Q_c, computed over 100 independent disorder realizations of the optimal Si/Air cavity, as a function of the disorder parameter σ.

References

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