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**P. Veerasha**

Department of Mathematics, Karnatak University, Dharwad-580003, India

**D. G. Prakasha**

Department of Mathematics, Davangere University, Shivagangothri, Davangere-577007, India

**Naveen Sanju Malagi**

Department of Mathematics, Davangere University, Shivagangothri, Davangere-577007, India

**Haci Mehmet Baskonus** (✉ [hmbaskonus@gmail.com](mailto:hmbaskonus@gmail.com))

Department of Mathematics and Science Education, Faculty of Education, Harran University, Sanliurfa, Turkey

**Wei Gao**

School of Information Science and Technology, Yunnan Normal University, Yunnan, China

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## Research Article

**Keywords:** Coronavirus, Atangana-Baleanu derivative, q-Homotopy analysis transform method, Fixed point theorem.

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# New dynamical behaviour of the coronavirus (COVID-19) infection system with nonlocal operator from reservoirs to people

P. Veerasha<sup>1</sup>, D. G. Prakasha<sup>2</sup>, Naveen Sanju Malagi<sup>2</sup>, H.M.Baskonus<sup>3\*</sup>, Wei Gao<sup>3,4</sup>

[viru0913@gmail.com](mailto:viru0913@gmail.com), [prakashadg@gmail.com](mailto:prakashadg@gmail.com), [naveen2018m@gmail.com](mailto:naveen2018m@gmail.com),  
[hmbaskonus@gmail.com](mailto:hmbaskonus@gmail.com), [gaowei@ynnu.edu.cn](mailto:gaowei@ynnu.edu.cn)

## Abstract

The fundamental aim of the present study is to analyse and find the solution for the system of nonlinear ordinary differential equations describing the deadly and most dangerous virus from the last three months called coronavirus. The mathematical model consisting of six nonlinear ordinary differential equations are exemplified and the corresponding solution is studied within the frame of *q-homotopy analysis transform method* (*q*-HATM). Moreover, a newly defined fractional operator is employed in order to understand more effectively, known as Atangana-Baleanu (AB) operator. For the obtained results, the fixed point theorem is hired to present the exactness as well as uniqueness. For diverse arbitrary order, the behaviour of the outcomes is presented in terms of plots. Finally, the present study may help to examine the wild class of real-world models and also aid to predict their behaviour with respect to parameters considered in the models.

**Keywords:** Coronavirus; Atangana-Baleanu derivative; *q*-Homotopy analysis transform method; Fixed point theorem.

## 1. Introduction

The evolution of the human population rapidly is increasing in associated with new cultures and lifestyles. Mankind have developed many types of equipments to lead to their beautiful life. To lead a happy life in their own directions, humans are breaking limitations and boundaries developed by nature.

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**\*Corresponding Author Contact No.: +905548714846**

<sup>1</sup>Department of Mathematics, Karnatak University, Dharwad-580003, India

<sup>2</sup>Department of Mathematics, Davangere University, Shivagangothri, Davangere-577007, India

<sup>3</sup>Department of Mathematics and Science Education, Faculty of Education, Harran University, Sanliurfa, Turkey

<sup>4</sup>School of Information Science and Technology, Yunnan Normal University, Yunnan, China

Particularly, the utilization of food, vehicles, mobiles, cosmetics, electrical and petroleum equipment and many others, made the highly polluted and virus filled environment. Due to this, new type and class of virus have been developed and made a diseased environment. Particularly, from the last three months, humans are afraid and timid to come out from their home, a new novel virus called coronavirus (COVID-19). The history of this virus traced back to 1965 when Tyrrell and Bynoe have identified when they passaged a virus named B814 [1]. This virus is found in human embryonic tracheal organ cultures acquired from the respiratory tract of an adult [2].

The outbreak of a deadly and highly infected virus of the present era is a coronavirus and it is identified in the Wuhan (Chinese city) on December 31, 2019 [3-4]. Since then it killed over 9,391 on March 19 over the infected case of 2,23,082 peoples in more than 180 countries. As from the beginning up to this day, there is no particular treatment, medicine or vaccine to completely cure infected patients and from the day by day, there is always exponential increase in the death of the deceased peoples. More precisely, the economy of each infected country is decreasing due to this cruel virus. Initially, every infected person has high fever, cough and shortness in the breath. This virus transmitted by touching the body of diseased patients to the uninfected person to his/her eyes, nose, mouth and some other parts. In order to control the spread of the virus, each country has taken all most all initializations and spends a huge amount to prevent or avoided its effects on humankind.

On the other hand, in order to study, analyse, examine, predict and capture the behaviour of virus, diseases, threads and others, the mathematics is the only tool that can help us in systematic, effective and accurate manner without too much expense. This tool is considered to propose a model with some cases study and examine with the help of its constituent subjects. When the model depends on other parameters and it contains the processes of the rate of change, we should and always referred a novel concept called calculus. Even though the concept of calculus with differential and integral operators are initiated in the 16<sup>th</sup> century all most all phenomena necessitates, and the role of nature is effectively and accurately illustrated with the help of calculus.

In this paper, we considered the concept of generalization of the calculus of integer order to fractional order called fractional calculus. This concept is originated soon after the classical one, but recently it attracted by many researchers. Particularly, when we examining and capturing the behaviour of the phenomena associated complexity, memory

consequences, hereditary possessions and other essential and stimulating properties, the concepts of fractional calculus are very effective and more useful [5-14]. We consider the newly defined fractional operator called Atangana-Baleanu operator contracted with the aid of function called the queen of fractional calculus known as Mittag-Leffler function [15-16]. From last three years, this derivative is expansively applied by many researchers to investigate numerous real-world models [17-21].

In the present investigation, we consider the epidemic model proposed by Khan and Atangana [22]. In [22], the authors presented and derived some interesting results for the projected model by comparison with some practical values and also cited interesting results. In this epidemic model, the total number of people is symbolised as  $N$ . Further, the susceptible people are denoted as  $S(t)$ , the exposed population is symbolised by  $E(t)$ , total infected strength is represented by  $I(t)$ , the asymptotically infected population is presented as  $A(t)$ , the total number of humans recovered is indicated by  $R(t)$  and  $M(t)$  is considered as a reservoir. Now, the nonlinear differential system considering the above components is considered as follows [22]

$$\begin{aligned}
\frac{dS(t)}{dt} &= \mathcal{b} - \gamma S - \frac{\delta S(I + \beta A)}{N} - \varepsilon SM, \\
\frac{dE(t)}{dt} &= \frac{\delta S(I + \beta A)}{N} + \varepsilon SM - (1 - \vartheta)\theta E - \vartheta\mu E - \gamma E, \\
\frac{dI(t)}{dt} &= (1 - \vartheta)\theta E - (\rho + \gamma)I, \\
\frac{dA(t)}{dt} &= \vartheta\mu E - (\sigma + \gamma)A, \\
\frac{dR(t)}{dt} &= \rho I + \sigma A - \gamma R, \\
\frac{dM(t)}{dt} &= \tau I + \kappa A - \omega M.
\end{aligned} \tag{1}$$

Here,  $\mathcal{b}$  denote the rate of birth and  $\gamma$  is a rate of death of the infected population. The symbol  $\delta$  represents the transmission coefficient,  $\beta$  is transmissibility multiple,  $\alpha$  is the transmission rate becomes infected and  $\theta$  is that of the incubation period, the amount of asymptomatic infection is denoted by  $\vartheta$ , the disease transmission coefficient is represented as  $\varepsilon$ . In the considered system,  $\rho$  and  $\sigma$  are respectively defines the recovery rate of the infected and asymptotically infected population. Further,  $\tau$  and  $\kappa$  respectively symbolise the influence of the virus to  $M$  by  $I$  and  $A$ . The parameter  $\omega$  defines the rate of virus removing from  $M$ .

**Table 1:** Parameters cited in Eq. (1) and their corresponding value [22].

| Parameter     | Value                        |
|---------------|------------------------------|
| $\gamma$      | $\frac{1}{76.79 \times 365}$ |
| $\delta$      | 0.05                         |
| $\beta$       | 0.02                         |
| $\varepsilon$ | 0.000001231                  |
| $\vartheta$   | 0.1243                       |
| $\theta$      | 0.00047876                   |
| $\alpha$      | 0.005                        |
| $\rho$        | 0.09871                      |
| $\sigma$      | 0.854302                     |
| $\tau$        | 0.000398                     |
| $\kappa$      | 0.001                        |
| $\omega$      | 0.01                         |

In this paper, we consider the fractional order of Eq. (1) with AB derivative and which as follows

$$\begin{aligned}
 {}_0^{ABC}D_t^\mu S(t) &= \mathcal{b} - \gamma S - \frac{\delta S(I + \beta A)}{N} - \varepsilon SM, \\
 {}_0^{ABC}D_t^\mu E(t) &= \frac{\delta S(I + \beta A)}{N} + \varepsilon SM - (1 - \vartheta)\theta E - \vartheta\alpha E - \gamma E, \\
 {}_0^{ABC}D_t^\mu I(t) &= (1 - \vartheta)\theta E - (\rho + \gamma)I, \\
 {}_0^{ABC}D_t^\mu A(t) &= \vartheta\alpha E - (\sigma + \gamma)A, \\
 {}_0^{ABC}D_t^\mu R(t) &= \rho I + \sigma A - \gamma R, \\
 {}_0^{ABC}D_t^\mu M(t) &= \tau I + \kappa A - \omega M,
 \end{aligned} \tag{2}$$

## 2. Preliminaries

Here, basic notations are recalled for the FC and Laplace transform.

**Definition 1.** For a function  $f \in H^1(a, b)$  the fractional Atangana-Baleanu-Caputo derivative is presented as follows:

$${}_a^{ABC}D_t^\mu (f(t)) = \frac{\mathcal{B}[\mu]}{1 - \mu} \int_a^t f'(\vartheta) E_\mu \left[ \mu \frac{(t - \vartheta)^\mu}{\mu - 1} \right] d\vartheta, \quad b > a. \tag{3}$$

**Definition 2.** The fractional AB integral defined as

$${}^{AB}I_a^\mu(f(t)) = \frac{1-\mu}{\mathcal{B}[\mu]}f(t) + \frac{\mu}{\mathcal{B}[\mu]\Gamma(\mu)}\int_a^t f(\vartheta)(t-\vartheta)^{\mu-1}d\vartheta. \quad (4)$$

**Definition 3.** Associated to AB operator, the Laplace transform (LT) is presented as

$$L[{}_0^{ABR}D_t^\mu(f(t))] = \frac{\mathcal{B}[\mu]s^\mu L[f(t)] - s^{\mu-1}f(0)}{1-\mu - s^{\mu+(\mu/(1-\mu))}}. \quad (5)$$

**Theorem 1.** For the Riemann-Liouville and AB derivatives, the following Lipschitz conditions satisfy respectively [15]

$$\|{}^{ABC}D_t^\mu f_1(t) - {}^{ABC}D_t^\mu f_2(t)\| < K_1\|f_1(x) - f_2(x)\|, \quad (6)$$

and

$$\|{}^{ABC}D_t^\mu f_1(t) - {}^{ABC}D_t^\mu f_2(t)\| < K_2\|f_1(x) - f_2(x)\|. \quad (7)$$

**Theorem 2.** The fractional differential equation  ${}^{ABC}D_t^\mu f_1(t) = s(t)$  has a unique solution is given by [15]

$$f(t) = 1 - \frac{\mu}{\mathcal{B}[\mu]}s(t) + \frac{\mu}{\mathcal{B}[\mu]\Gamma(\mu)}\int_a^t s(\varsigma)(t-\varsigma)^{\mu-1}d\varsigma. \quad (8)$$

### 3. $q$ -HATM Solution for Considered Model

In this section, we present the solution procedure of  $q$ -HATM proposed by Singh et al. [23] with aid of  $q$ -HAM and LT. Later, it has been hired by many researchers to find the solution for various class of nonlinear differential equations describing various phenomena including fluid mechanics, optics, chaotic behaviour, human disease, biological models, economic growth, chemical and others [24-27], and also presented some interesting simulating consequences with comparison of other traditional and modified techniques.

Now, we consider the fractional-order system of equations illustrating the dynamics of the presented in Eq. (2)

$$\begin{aligned} {}_0^{ABC}D_t^\mu S(t) - \mathcal{b} + \gamma S - \frac{\delta S(I + \beta A)}{N} + \varepsilon SM &= 0, \\ {}_0^{ABC}D_t^\mu E(t) - \frac{\delta S(I + \beta A)}{N} - \varepsilon SM + (1 - \vartheta)\theta E + \vartheta\alpha E + \gamma E &= 0, \end{aligned} \quad (9)$$

$$\begin{aligned}
{}^{ABC}_0 D_t^\mu I(t) - (1 - \vartheta)\theta E + (\rho + \gamma)I &= 0, \\
{}^{ABC}_0 D_t^\mu A(t) - \vartheta\alpha E + (\sigma + \gamma)A &= 0, \\
{}^{ABC}_0 D_t^\mu R(t) - \rho I - \sigma A + \gamma R &= 0, \\
{}^{ABC}_0 D_t^\mu M(t) - \tau I - \kappa A + \omega M &= 0,
\end{aligned}$$

with initial conditions

$$S(0) = S_0, E(0) = E_0, I(0) = I_0, A(0) = A_0, R(0) = R_0 \text{ and } M(0) = M_0. \quad (10)$$

Now, applying LT on Eq. (9) and with the assist of Eq. (10), one can get

$$\begin{aligned}
L[S(t)] - \frac{1}{s}(S_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L \left\{ \wp - \gamma S - \frac{\delta S(I + \beta A)}{N} - \varepsilon SM \right\} &= 0, \\
L[E(t)] - \frac{1}{s}(E_0) \\
- \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L \left\{ \frac{\delta S(I + \beta A)}{N} + \varepsilon SM - (1 - \vartheta)\theta E - \vartheta\alpha E - \gamma E \right\} \\
&= 0, \\
L\{I(t)\} - \frac{1}{s}(I_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{(1 - \vartheta)\theta E - (\rho + \gamma)I\} &= 0, \\
L\{A(t)\} - \frac{1}{s}(A_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\vartheta\alpha E - (\sigma + \gamma)A\} &= 0, \\
L\{R(t)\} - \frac{1}{s}(R_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\rho I + \sigma A - \gamma R\} &= 0, \\
L\{M(t)\} - \frac{1}{s}(M_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\tau I + \kappa A - \omega M\} &= 0.
\end{aligned} \quad (11)$$

Now, the nonlinear operator is presented as

$$\begin{aligned}
N^1[\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6] &= L[\varphi_1(t; q)] - \frac{1}{s}(S_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\left\{ \wp - \gamma\varphi_1(t; q) \right. \\
&\quad \left. - \frac{\delta\varphi_1(t; q)(\varphi_3(t; q) + \beta\varphi_4(t; q))}{N} - \varepsilon\varphi_1(t; q)\varphi_6(t; q) \right\}, \\
N^2[\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6] &= L[\varphi_2(t; q)] - \frac{1}{s}(E_0) \\
- \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\left\{ \frac{\delta\varphi_1(t; q)(\varphi_3(t; q) + \beta\varphi_4(t; q))}{N} \right. \\
&\quad \left. + \varepsilon\varphi_1(t; q)\varphi_6(t; q) - (1 - \vartheta)\theta\varphi_2(t; q) \right. \\
&\quad \left. - \vartheta\alpha\varphi_2(t; q) - \gamma\varphi_2(t; q) \right\}, \\
N^3[\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6] &= L[\varphi_3(t; q)] - \frac{1}{s}(I_0)
\end{aligned} \quad (12)$$

$$\begin{aligned}
& -\frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{(1 - \vartheta)\theta\varphi_2(t; q) - (\rho + \gamma)\varphi_3(t; q)\}, \\
N^4[\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6] &= L[\varphi_4(t; q)] - \frac{1}{s}(A_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\vartheta\alpha\varphi_2(t; q) \\
& \quad - (\sigma + \gamma)\varphi_4(t; q)\}, \\
N^5[\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6] &= L[\varphi_5(t; q)] - \frac{1}{s}(R_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\rho\varphi_3(t; q) \\
& \quad + \sigma\varphi_4(t; q) - \gamma\varphi_5(t; q)\}, \\
N^6[\varphi_1, \varphi_2, \varphi_3, \varphi_4, \varphi_5, \varphi_6] &= L[\varphi_6(t; q)] - \frac{1}{s}(M_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\tau\varphi_3(t; q) \\
& \quad + \kappa\varphi_4(t; q) - \omega\varphi_6(t; q)\}.
\end{aligned}$$

By employing the projected scheme and for  $H(x, t) = 1$ , the  $m$ -th order deformation equation is presented as

$$\begin{aligned}
L[S_m(t) - k_m S_{m-1}(t)] &= \hbar \mathfrak{R}_{1,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}], \\
L[E_m(t) - k_m E_{m-1}(t)] &= \hbar \mathfrak{R}_{2,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}], \\
L[I_m(t) - k_m I_{m-1}(t)] &= \hbar \mathfrak{R}_{3,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}], \\
L[A_m(t) - k_m A_{m-1}(t)] &= \hbar \mathfrak{R}_{4,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}], \\
L[R_m(t) - k_m R_{m-1}(t)] &= \hbar \mathfrak{R}_{5,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}], \\
L[M_m(t) - k_m M_{m-1}(t)] &= \hbar \mathfrak{R}_{6,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}],
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
& \mathfrak{R}_{1,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}] \\
&= L[S_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right) \frac{1}{s}(S_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\mathcal{L} - \gamma S_{m-1} - \frac{\delta}{N} \left(\sum_{i=0}^{m-1} S_i I_{m-1-i}\right) \\
& \quad + \beta \sum_{i=0}^{m-1} S_i A_{m-1-i} - \varepsilon \sum_{i=0}^{m-1} S_i M_{m-1-i}\}, \\
& \mathfrak{R}_{2,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}] \\
&= L[E_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right) \frac{1}{s}(E_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\left\{\frac{\delta}{N} \left(\sum_{i=0}^{m-1} S_i I_{m-1-i} + \beta \sum_{i=0}^{m-1} S_i A_{m-1-i}\right) \right. \\
& \quad \left. + \varepsilon \sum_{i=0}^{m-1} S_i M_{m-1-i} - (1 - \vartheta)\theta E_{m-1} - \vartheta\alpha E_{m-1} - \gamma E_{m-1}\right\}, \\
& \mathfrak{R}_{3,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}]
\end{aligned} \tag{7}$$



$$\begin{aligned}
&= L[I_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right) \frac{1}{s} (I_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{(1 - \vartheta)\theta E_{m-1} - (\rho + \gamma)I_{m-1}\}, \\
&\quad \mathfrak{R}_{4,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}] \\
&= L[A_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right) \frac{1}{s} (A_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\vartheta\alpha E_{m-1} - (\sigma + \gamma)A_{m-1}\}, \\
&\quad \mathfrak{R}_{5,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}] \\
&= L[R_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right) \frac{1}{s} (R_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\rho I_{m-1} + \sigma A_{m-1} - \gamma R_{m-1}\}, \\
&\quad \mathfrak{R}_{6,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}] \\
&= L[M_{m-1}(t)] - \left(1 - \frac{k_m}{n}\right) \frac{1}{s} (M_0) - \frac{1}{\mathcal{B}[\mu]} \left(1 - \mu + \frac{\mu}{s^\mu}\right) L\{\tau I_{m-1} + \kappa A_{m-1} - \omega M_{m-1}\}.
\end{aligned}$$

Eq. (6) simplifies by the help of inverse LT as follows

$$\begin{aligned}
S_m(t) &= k_m S_{m-1}(t) + \hbar L^{-1}\{\mathfrak{R}_{1,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}]\}, \\
E_m(t) &= k_m E_{m-1}(t) + \hbar L^{-1}\{\mathfrak{R}_{2,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}]\}, \\
I_m(t) &= k_m I_{m-1}(t) + \hbar L^{-1}\{\mathfrak{R}_{3,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}]\}, \\
A_m(t) &= k_m A_{m-1}(t) + \hbar L^{-1}\{\mathfrak{R}_{4,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}]\}, \\
R_m(t) &= k_m R_{m-1}(t) + \hbar L^{-1}\{\mathfrak{R}_{5,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}]\}, \\
M_m(t) &= k_m M_{m-1}(t) + \hbar L^{-1}\{\mathfrak{R}_{6,m} [\vec{S}_{m-1}, \vec{E}_{m-1}, \vec{I}_{m-1}, \vec{A}_{m-1}, \vec{R}_{m-1}, \vec{M}_{m-1}]\}.
\end{aligned} \tag{8}$$

On simplifying the above system and with the assist of initial values, we obtained the required series solution. Then for Eq. (2), the  $q$ -HATM series solution is defined as

$$\begin{aligned}
S(t) &= S_0(t) + \sum_{m=1}^{\infty} S_m(t) \left(\frac{1}{n}\right)^m, \\
E(t) &= E_0(t) + \sum_{m=1}^{\infty} E_m(t) \left(\frac{1}{n}\right)^m, \\
I(t) &= I_0(t) + \sum_{m=1}^{\infty} I_m(t) \left(\frac{1}{n}\right)^m, \\
A(t) &= A_0(t) + \sum_{m=1}^{\infty} A_m(t) \left(\frac{1}{n}\right)^m, \\
R(t) &= R_0(t) + \sum_{m=1}^{\infty} R_m(t) \left(\frac{1}{n}\right)^m, \\
M(t) &= M_0(t) + \sum_{m=1}^{\infty} M_m(t) \left(\frac{1}{n}\right)^m.
\end{aligned} \tag{9}$$

#### 4. Existence of solutions for the future model

Here, we demonstrate the existence and uniqueness by the aid of the fixed-point theory for the obtained results. Now, the system (2) is considered as follows

$$\begin{cases} {}_0^{ABC}D_t^\alpha[S(t)] = \mathcal{G}_1(t, S), \\ {}_0^{ABC}D_t^\alpha[E(t)] = \mathcal{G}_2(t, E), \\ {}_0^{ABC}D_t^\alpha[I(t)] = \mathcal{G}_3(t, I), \\ {}_0^{ABC}D_t^\alpha[A(t)] = \mathcal{G}_4(t, A), \\ {}_0^{ABC}D_t^\alpha[R(t)] = \mathcal{G}_5(t, R), \\ {}_0^{ABC}D_t^\alpha[M(t)] = \mathcal{G}_6(t, M). \end{cases} \quad (10)$$

The above system is transformed into the Volterra integral equation, then we get

$$\begin{cases} S(t) - S(0) = \frac{(1-\mu)}{\mathcal{B}(\mu)}\mathcal{G}_1(t, S) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\int_0^t \mathcal{G}_1(\zeta, S)(t-\zeta)^{\mu-1}d\zeta, \\ E(t) - E(0) = \frac{(1-\mu)}{\mathcal{B}(\mu)}\mathcal{G}_2(t, E) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\int_0^t \mathcal{G}_2(\zeta, E)(t-\zeta)^{\mu-1}d\zeta, \\ I(t) - I(0) = \frac{(1-\mu)}{\mathcal{B}(\mu)}\mathcal{G}_3(t, I) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\int_0^t \mathcal{G}_3(\zeta, I)(t-\zeta)^{\mu-1}d\zeta, \\ A(t) - A(0) = \frac{(1-\mu)}{\mathcal{B}(\mu)}\mathcal{G}_4(t, A) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\int_0^t \mathcal{G}_4(\zeta, A)(t-\zeta)^{\mu-1}d\zeta, \\ R(t) - R(0) = \frac{(1-\mu)}{\mathcal{B}(\mu)}\mathcal{G}_5(t, R) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\int_0^t \mathcal{G}_5(\zeta, R)(t-\zeta)^{\mu-1}d\zeta, \\ M(t) - M(0) = \frac{(1-\mu)}{\mathcal{B}(\mu)}\mathcal{G}_6(t, M) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\int_0^t \mathcal{G}_6(\zeta, M)(t-\zeta)^{\mu-1}d\zeta. \end{cases} \quad (11)$$

**Theorem 3.** The kernel  $\mathcal{G}_1$  admits the Lipschitz condition and contraction if the condition  $0 \leq \left( \mathcal{L} - \left( \gamma - \frac{\delta(\lambda_3 + \beta\lambda_4)}{N} - \varepsilon\lambda_6 \right) \right) < 1$  holds.

**Proof.** To present the required result, we consider  $S$  and  $S_1$ , then

$$\begin{aligned} \|\mathcal{G}_1(t, S) - \mathcal{G}_1(t, S_1)\| &= \left\| \mathcal{L} - \gamma[S(t) - S(t_1)] - \frac{\delta(I + \beta A)}{N}[S(t) - S(t_1)] - \varepsilon M[S(t) - S(t_1)] \right\| \\ &= \left\| \mathcal{L} - \left( \gamma - \frac{\delta(I + \beta A)}{N} - \varepsilon M \right) [S(t) - S(t_1)] \right\| \\ &\leq \left\| \mathcal{L} - \left( \gamma - \frac{\delta(I + \beta A)}{N} - \varepsilon M \right) \right\| \|S(t) - S(t_1)\| \\ &\leq \left( \mathcal{L} - \left( \gamma - \frac{\delta(\lambda_3 + \beta\lambda_4)}{N} - \varepsilon\lambda_6 \right) \right) \|S(t) - S(t_1)\|, \quad (12) \end{aligned}$$

where  $\|I(t)\| \leq \lambda_3$ ,  $\|A(t)\| \leq \lambda_4$  and  $\|A(t)\| \leq \lambda_6$  are the bounded functions. Substituting  $\eta_1 = \mathcal{L} - \left( \gamma - \frac{\delta(\lambda_3 + \beta\lambda_4)}{N} - \varepsilon\lambda_6 \right)$  in Eq. (12), one can have

$$\|\mathcal{G}_1(t, S) - \mathcal{G}_1(t, S_1)\| \leq \eta_1 \|S(t) - S(t_1)\|. \quad (13)$$

Clearly, which is the Lipschitz condition is obtained for  $\mathcal{G}_1$ . Moreover, if  $0 \leq \left( \mathcal{L} - \gamma - \delta\lambda_3 + \beta\lambda_4 N - \varepsilon\lambda_6 \right) < 1$ , then it leads to contraction. Similarly, we can be shown for the rest of the cases, and hence

$$\begin{aligned} \|\mathcal{G}_2(t, E) - \mathcal{G}_2(t, E_1)\| &\leq \eta_2 \|E(t) - E(t_1)\|, \\ \|\mathcal{G}_3(t, I) - \mathcal{G}_3(t, I_1)\| &\leq \eta_3 \|I(t) - I(t_1)\|, \\ \|\mathcal{G}_4(t, A) - \mathcal{G}_4(t, A_1)\| &\leq \eta_4 \|A(t) - A(t_1)\|, \\ \|\mathcal{G}_5(t, R) - \mathcal{G}_5(t, R_1)\| &\leq \eta_5 \|R(t) - R(t_1)\|, \\ \|\mathcal{G}_6(t, M) - \mathcal{G}_6(t, M_1)\| &\leq \eta_6 \|M(t) - M(t_1)\|. \end{aligned} \quad (14)$$

The recursive form of Eq. (11) and associated initial conditions are respectively presented as

$$\left\{ \begin{aligned} S_n(t) &= \frac{(1-\mu)}{\mathcal{B}(\mu)} \mathcal{G}_1(t, S_{n-1}) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_1(\zeta, S_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\ E_n(t) &= \frac{(1-\mu)}{\mathcal{B}(\mu)} \mathcal{G}_2(t, E_{n-1}) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_2(\zeta, E_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\ I_n(t) &= \frac{(1-\mu)}{\mathcal{B}(\mu)} \mathcal{G}_3(t, I_{n-1}) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_3(\zeta, I_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\ A_n(t) &= \frac{(1-\mu)}{\mathcal{B}(\mu)} \mathcal{G}_4(t, A_{n-1}) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_4(\zeta, A_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\ R_n(t) &= \frac{(1-\mu)}{\mathcal{B}(\mu)} \mathcal{G}_5(t, R_{n-1}) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_5(\zeta, R_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\ M_n(t) &= \frac{(1-\mu)}{\mathcal{B}(\mu)} \mathcal{G}_6(t, M_{n-1}) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_6(\zeta, M_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \end{aligned} \right. \quad (15)$$

and

$$S(0) = S_0(t), \quad E(0) = E_0(t), \quad I(0) = I_0(t), \quad A(0) = A_0(t), \quad R(0) = R_0(t), \quad \text{and} \quad E(0) = E_0(t). \quad (16)$$

Between the terms, the successive difference is defined as

$$\left\{ \begin{array}{l}
\phi_{1n}(t) = S_n(t) - S_{n-1}(t) \\
= \frac{(1-\mu)}{\mathcal{B}(\mu)} (\mathcal{G}_1(t, S_{n-1}) - \mathcal{G}_1(t, S_{n-2})) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_1(\zeta, S_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\
\phi_{2n}(t) = E_n(t) - E_{n-1}(t) \\
= \frac{(1-\mu)}{\mathcal{B}(\mu)} (\mathcal{G}_2(t, E_{n-1}) - \mathcal{G}_2(t, E_{n-2})) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_2(\zeta, E_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\
\phi_{3n}(t) = I_n(t) - I_{n-1}(t) \\
= \frac{(1-\mu)}{\mathcal{B}(\mu)} (\mathcal{G}_3(t, I_{n-1}) - \mathcal{G}_3(t, I_{n-2})) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_3(\zeta, I_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\
\phi_{4n}(t) = A_n(t) - A_{n-1}(t) \\
= \frac{(1-\mu)}{\mathcal{B}(\mu)} (\mathcal{G}_4(t, A_{n-1}) - \mathcal{G}_4(t, A_{n-2})) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_4(\zeta, A_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\
\phi_{5n}(t) = R_n(t) - R_{n-1}(t) \\
= \frac{(1-\mu)}{\mathcal{B}(\mu)} (\mathcal{G}_5(t, R_{n-1}) - \mathcal{G}_5(t, R_{n-2})) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_5(\zeta, R_{n-1})(t-\zeta)^{\alpha-1} d\zeta, \\
\phi_{6n}(t) = M_n(t) - M_{n-1}(t) \\
= \frac{(1-\mu)}{\mathcal{B}(\mu)} (\mathcal{G}_6(t, M_{n-1}) - \mathcal{G}_6(t, M_{n-2})) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \mathcal{G}_6(\zeta, M_{n-1})(t-\zeta)^{\alpha-1} d\zeta.
\end{array} \right. \quad (17)$$

Clearly

$$\left\{ \begin{array}{l}
S_n(t) = \sum_{i=1}^n \phi_{1i}(t), \\
E_n(t) = \sum_{i=1}^n \phi_{2i}(t), \\
I_n(t) = \sum_{i=1}^n \phi_{3i}(t), \\
A_n(t) = \sum_{i=1}^n \phi_{4i}(t), \\
R_n(t) = \sum_{i=1}^n \phi_{5i}(t), \\
M_n(t) = \sum_{i=1}^n \phi_{6i}(t).
\end{array} \right. \quad (18)$$

By the help of Eq. (13) after employing the norm on the second equation of Eq. (17), we have

$$\|\phi_{1n}(t)\| \leq \frac{(1-\mu)}{\mathcal{B}(\mu)} \eta_1 \|\phi_{1(n-1)}(t)\| + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \eta_1 \int_0^t \|\phi_{1(n-1)}(\zeta)\| d\zeta. \quad (19)$$

Similarly, we have

$$\begin{aligned}
\|\phi_{2n}(t)\| &\leq \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_2\|\phi_{2(n-1)}(t)\| + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_2\int_0^t\|\phi_{2(n-1)}(\zeta)\|d\zeta, \\
\|\phi_{3n}(t)\| &\leq \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_3\|\phi_{3(n-1)}(t)\| + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_3\int_0^t\|\phi_{3(n-1)}(\zeta)\|d\zeta, \\
\|\phi_{4n}(t)\| &\leq \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_4\|\phi_{4(n-1)}(t)\| + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_4\int_0^t\|\phi_{4(n-1)}(\zeta)\|d\zeta, \\
\|\phi_{5n}(t)\| &\leq \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_5\|\phi_{5(n-1)}(t)\| + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_5\int_0^t\|\phi_{5(n-1)}(\zeta)\|d\zeta, \\
\|\phi_{6n}(t)\| &\leq \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_6\|\phi_{6(n-1)}(t)\| + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_6\int_0^t\|\phi_{6(n-1)}(\zeta)\|d\zeta.
\end{aligned} \tag{20}$$

Now, by the help of forgoing results we prove the following important result:

**Theorem 4.** For the Eq. (2), the solution will exist and unique if we have particular  $t_0$ , then

$$\frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_i + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_i < 1.$$

for  $i = 1, 2, \dots, 6$ .

**Proof.** Let us consider  $S(t), E(t), I(t), A(t), R(t)$  and  $M(t)$  bounded functions admitting the Lipschitz condition. Then, we have by the help of Eqs. (18) and (20)

$$\begin{aligned}
\|\phi_{1i}(t)\| &\leq \|S_n(0)\| \left[ \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_1 + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_1 \right]^n, \\
\|\phi_{2i}(t)\| &\leq \|E_n(0)\| \left[ \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_2 + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_2 \right]^n, \\
\|\phi_{3i}(t)\| &\leq \|I_n(0)\| \left[ \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_3 + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_3 \right]^n, \\
\|\phi_{4i}(t)\| &\leq \|A_n(0)\| \left[ \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_4 + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_4 \right]^n, \\
\|\phi_{5i}(t)\| &\leq \|R_n(0)\| \left[ \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_5 + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_5 \right]^n, \\
\|\phi_{6i}(t)\| &\leq \|M_n(0)\| \left[ \frac{(1-\mu)}{\mathcal{B}(\mu)}\eta_6 + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)}\eta_6 \right]^n.
\end{aligned} \tag{21}$$

This proves existence for the attained solutions. Further, to prove Eq. (21) is a solution for the Eq. (2), we consider

$$\begin{aligned}
S(t) - S(0) &= S_n(t) - \mathcal{K}_{1n}(t), \\
E(t) - E(0) &= E_n(t) - \mathcal{K}_{2n}(t),
\end{aligned} \tag{22}$$

$$\begin{aligned}
I(t) - I(0) &= I_n(t) - \mathcal{K}_{3n}(t), \\
A(t) - A(0) &= A_n(t) - \mathcal{K}_{4n}(t), \\
R(t) - R(0) &= R_n(t) - \mathcal{K}_{5n}(t), \\
M(t) - M(0) &= M_n(t) - \mathcal{K}_{6n}(t).
\end{aligned}$$

Now, we consider the following condition in order obtain require a result

$$\begin{aligned}
\|\mathcal{K}_{1n}(t)\| &= \left\| \frac{(1-\mu)}{\mathcal{B}(\mu)} (\mathcal{G}_1(t, S) - \mathcal{G}_1(t, S_{n-1})) \right. \\
&\quad \left. + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t (t-\zeta)^{\mu-1} (\mathcal{G}_1(\zeta, S) - \mathcal{G}_1(\zeta, S_{n-1})) d\zeta \right\| \\
&\leq \frac{(1-\mu)}{\mathcal{B}(\mu)} \|(\mathcal{G}_1(t, S) - \mathcal{G}_1(t, S_{n-1}))\| + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t \|(\mathcal{G}_1(\zeta, S) - \mathcal{G}_1(\zeta, S_{n-1}))\| d\zeta \\
&\leq \frac{(1-\mu)}{\mathcal{B}(\mu)} \eta_1 \|S - S_{n-1}\| + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \eta_1 \|S - S_{n-1}\| t. \tag{23}
\end{aligned}$$

Similarly, at  $t_0$  we can obtain

$$\|\mathcal{K}_{1n}(t)\| \leq \left( \frac{1-\mu}{\mathcal{B}(\mu)} + \frac{\mu t_0}{\mathcal{B}(\mu)\Gamma(\mu)} \right)^{n+1} \eta_1^{n+1} M. \tag{24}$$

As  $n$  tendsto  $\infty$ ,  $\|\mathcal{K}_{1n}(t)\|$  tends to 0. Similarly, we can provefor  $\|\mathcal{K}_{2n}(t)\|$ ,  $\|\mathcal{K}_{3n}(t)\|$ ,  $\|\mathcal{K}_{4n}(t)\|$ ,  $\|\mathcal{K}_{5n}(t)\|$  and  $\|\mathcal{K}_{6n}(t)\|$ . Next, it is important to present uniqueness for the solution of the projected model. Suppose  $S^*(t)$ ,  $E^*(t)$ ,  $I^*(t)$ ,  $A^*(t)$ ,  $R^*(t)$  and  $M^*(t)$  be the set of other solutions, then we have

$$S(t) - S^*(t) = \frac{(1-\mu)}{\mathcal{B}(\mu)} (\mathcal{G}_1(t, S) - \mathcal{G}_1(t, S^*)) + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t (\mathcal{G}_1(\zeta, S) - \mathcal{G}_1(\zeta, S^*)) d\zeta. \tag{25}$$

On employing norm, the Eq. (25) reduces to

$$\begin{aligned}
\|S(t) - S^*(t)\| &= \left\| \frac{(1-\mu)}{\mathcal{B}(\mu)} (\mathcal{G}_1(t, S) - \mathcal{G}_1(t, S^*)) \right. \\
&\quad \left. + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \int_0^t (\mathcal{G}_1(\zeta, S) - \mathcal{G}_1(\zeta, S^*)) d\zeta \right\| \\
&\leq \frac{(1-\mu)}{\mathcal{B}(\mu)} \eta_1 \|S(t) - S^*(t)\| + \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \eta_1 t \|S(t) - S^*(t)\|.
\end{aligned} \tag{26}$$

On simplification

$$\|S(t) - S^*(t)\| \left( 1 - \frac{(1-\mu)}{\mathcal{B}(\mu)} \eta_1 - \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \eta_1 t \right) \leq 0. \tag{27}$$

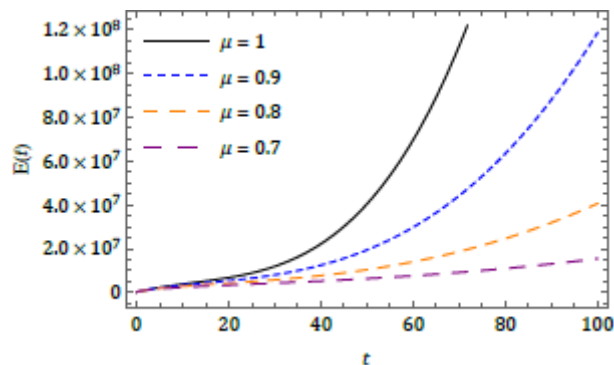
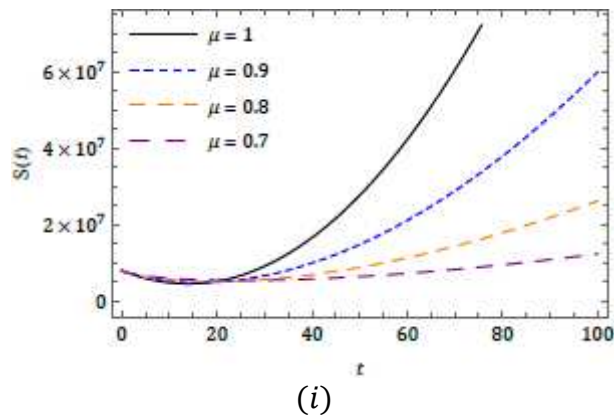
From the above condition, it is clear that  $S(t) - S^*(t) = 0$ , if

$$\left(1 - \frac{(1-\mu)}{\mathcal{B}(\mu)} \eta_1 - \frac{\mu}{\mathcal{B}(\mu)\Gamma(\mu)} \eta_1 t\right) \geq 0. \quad (28)$$

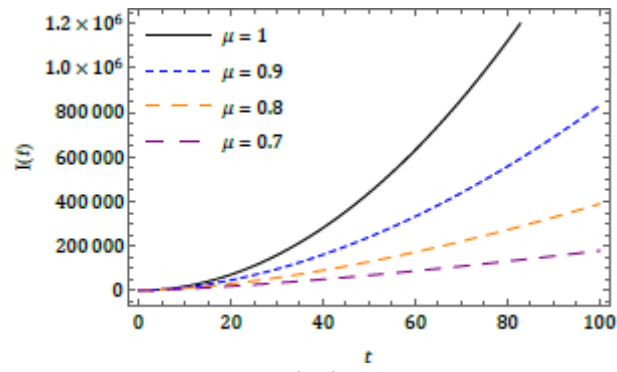
Hence, Eq. (28) evidence of our essential result.

## 5. Results and discussion

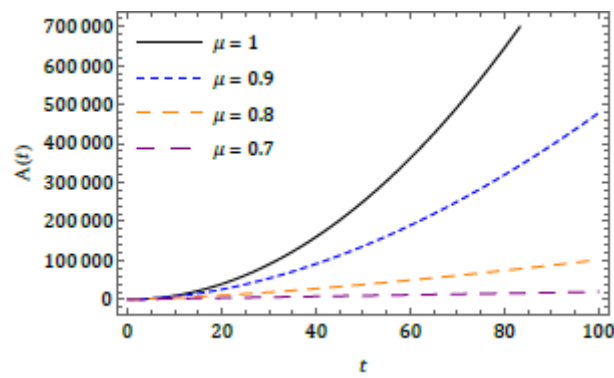
In this paper, we consider the initial conditions for the projected epidemic model as  $S(0) = S_0 = 8065518$ ,  $E(t) = E_0 = 200000$ ,  $I(0) = I_0 = 282$ ,  $A(0) = A_0 = 200$ ,  $R(0) = R_0 = 0$  and  $M(0) = M_0 = 50000$ . We evaluate up to a third-order series solution to capture the behaviour for the model. Figure 1 exemplifies the behaviour of achieved results by projected solution procedure for  $S(t)$ ,  $E(t)$ ,  $I(t)$ ,  $A(t)$ ,  $R(t)$  and  $M(t)$  for different fractional order ( $\mu$ ) with respect to time ( $t$ ) and we consider values of all the parameters with the help of Table 1. From the cited figures we can observe that the projected model extremely depends on the order and offers more degree of flexibility. Moreover, the considered fractional operator provides more interesting consequences to examine and predict the future of the considered model. The epidemic models are highly dependent on hereditary properties and non-Markovian phenomena and hence, the present investigation may help to understand the deadly virus.



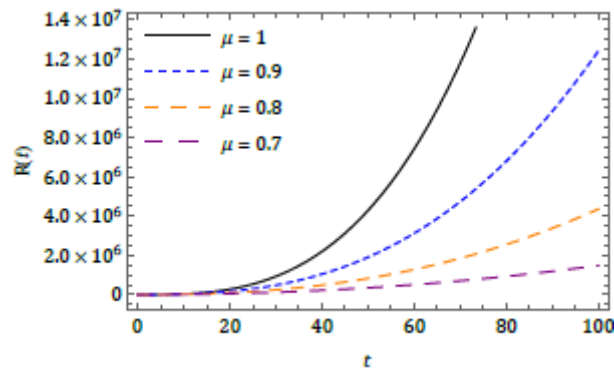
(ii)



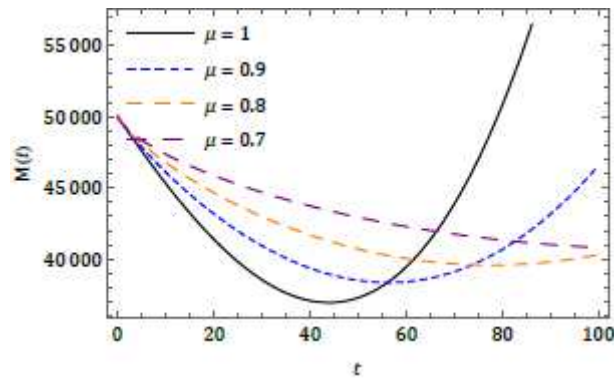
(iii)



(iv)



(v)





(vi)

**Figure 1** Nature of obtained solution for (i)  $S(t)$ , (ii)  $E(t)$ , (iii)  $I(t)$ , (iv)  $A(t)$ , (v)  $R(t)$  and (vi)  $M(t)$  for different  $\mu$  at  $\hbar = -1, n = 1$  and using Table 1.

## 6. Conclusion

The dynamic model of coronavirus is effectively analysed within the frame of fractional calculus using  $q$ -HATM in the present investigation and captured the behaviour of achieved results in terms of 2D plots. The projected model is described the evolution of deadly disease and highly effective virus in the current period due to which more than 9,391 peoples are dead till March 19, 2020. The existence and uniqueness for archived results have been demonstrated by the assist of fixed point theory. The obtained consequences show that, the considered method and fractional operator are highly methodical and effective to analyse the real world problems. Moreover, the projected solution procedure reduces computational time and it does not require any perturbation and new polynomial to find the solution for nonlinear models. Hence, we can conclude that, the considered scheme can be applied to any nonlinear and complex model describing biological, chemical and other phenomena.

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# Figures

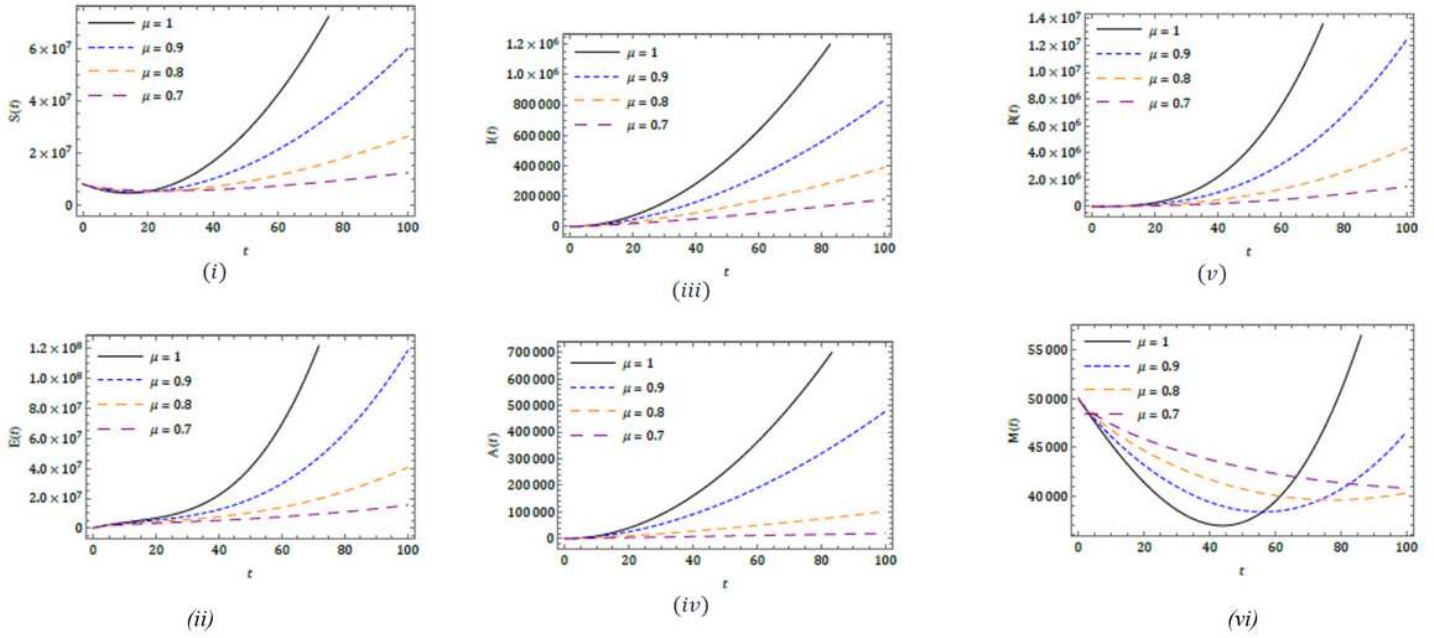


Figure 1

Nature of obtained solution for  $\mu = -1$ ,  $\mu = 1$  and using Table 1.