Power Spectrum Inception of F2 Layer at Solar-Terrestrial Region of Pakistan

Bulbul Jan (bulbul@duet.edu.pk)  
Dawood University of Engineering and Technology

Faisal Ahmed Khan Afridi  
University of Karachi

Muhammad Ayub Khan Yousufzai  
University of Karachi

Faisal Nawaz  
Dawood University of Engineering and Technology

Arsalan Ahmed  
Dawood University of Engineering and Technology

Faraz Mehmood  
Dawood University of Engineering and Technology

Mirza Jawwad Baig  
University of Karachi

Research Article

Keywords: Ionosphere, F2 layer, Nonlinear Dynamics, Wavelet analysis, Power Spectrum

Posted Date: October 20th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1832642/v2

License: This work is licensed under a Creative Commons Attribution 4.0 International License. Read Full License
Power Spectrum Inception of F$ _2 $ Layer at Solar-Terrestrial Region of Pakistan

Bulbul Jan$ ^1 $, Faisal Ahmed Khan Afridi$ ^2 $, Muhammad Ayub Khan Yousufzai$ ^2 $,$ ^3 $, Faisal Nawaz$ ^1 $, Arsalan Ahmed$ ^1 $, Faraz Mehmood$ ^1 $, Mirza Jawwad Baig$ ^2 $

1. Department of Mathematics, Dawood University of Engineering and Technology, Karachi
2. Institute of Space Science and Technology, University of Karachi, Karachi Pakistan
3. Department of Applied Physics University of Karachi, Karachi Pakistan

Corresponding Author: Bulbul Jan (bulbul@duet.edu.pk)

Abstract

In this study, we introduce a recent approach to implement the wavelet analysis of ionospheric parameters and anomaly detection during ionospheric irregularities over Pakistan. The most important aspect of the wavelet analysis is the timefrequency distribution representation of the signal. This study investigates the variability of ionospheric F$ _2 $ layer and utilizes critical frequency ($f_0$F$ _2 $) of F$ _2 $ layer during the year 2008-2017 in Pakistan region. The results of this study identify the most important bands of low frequency variations, which are very relevant for study of periodicities measurements of F$ _2 $ layer. Moreover, Morlet wavelet has revealed that generally the global wavelet power spectrum (GWPS) is the most prominent in the annual cycles. The multifaceted interactions of ionospheric fluctuations with different time scales, e.g. annual, biannual, become significant due to higher peaks appear during the periods 128-256, 256-512 and 512-1024 days in daily data sets. The wavelet trend reveals a noticeable power range between the 2–16 day bands, representing an annual fluctuation for daily observations of F$ _2 $ layer. The low deviation in a frequency range highlights the existence of seasonality in the F$ _2 $ layer during the study period. These results emphasize that $f_0$F$ _2 $ observations exhibit irregular disturbances with non-stationary power at different frequencies.

The study will be helpful for stakeholders and government authorities because high frequency sky-waves users schedule their programs one month in advance, but they also need day by day information about the ionospheric conditions in order to revise programs.

Keywords: Ionosphere, F$ _2 $ layer, Nonlinear Dynamics, Wavelet analysis, Power Spectrum

1. Introduction:

It has been known for a long time that the ionospheric plasma is formed by photo-ionization of the neutral atmosphere by extreme ultraviolet (EUV) rays and high energy cosmic particles (Davies, 1965). The temporal behavior of the ionosphere depends on the geographic, geomagnetic position, and solar activity (Belehaki et al. 2009; Katamzi and McKinnell 2011; Lastovicka 2017).

It has been observed that the ultraviolet radiation from the Sun causes the ionization of the extra terrestrial part of the atmosphere creating an ionosphere. This region comprises different layers including F2 layer under discussion. This is the highest layer that influences the sky wave propagation of radio signals (Schunk and Nagy 2000). The ionospheric region is the part of Earth’s atmosphere, where the dangerous emission from the Sun can disturb communication and navigation system (Priya and Parameswari 2003; Belehaki et al. 2009; Zubairet al. 2011; Chakravarty 2014).
The dynamics of the ionosphere in equatorial latitudes is extremely remarkable and as a result affects significantly radio signals propagating in the ionosphere, thus deteriorating the performance of satellite navigation systems. It has been investigated that a large number of phenomena are occurring to disturb ionospheric activities in the space such as Solar wind, bursts of the celestial objects (Brum et al. 2011; Cnossen et al. 2016; Li et al. 2016; Lyashenko 2016; Lastovicka 2017; Liu et al. 2020). This is a well known fact that F2 layer of the ionosphere can be responsible as one of the most essential variable of our atmosphere. The variability of this entirely dependent particular region on dynamics of sunspots and solar bursts (Szwabowski 2016; Wang et al. 2019, Liu et al. 2011; Li et al. 2016). It has been declared that the temperature of the atmosphere increases tremendously within the layer that intern produces the enhancement in the electron density of the same region (Abdullah 2011). The most important causes of ionization of Earth’s ionospheric F2 layer are solar EUV radiations (de Haro Barbá and Elias 2020). The thermal structure of the atmosphere consists of various portions with variability’s in electron temperature, heights, degree of ionization. These regions are troposphere, stratosphere, mesosphere, thermosphere and exosphere (Khorshed et al. 2008).

The recent literature shows locations where different layers are situated with variable heights. These regions are termed as: troposphere, stratosphere, mesosphere, thermosphere and exosphere. The ionosphere is found with the combination of the ions and electron that constitutes Space plasma. It usually consists of three layers; D layer is from 60 km to 80 km above the surface of the earth whereas, E layer is from about 80-150 km that responsible to scatter radio signals of low frequencies. F1 and F2 are higher than and E responsible to communicate signals of high frequencies. The uniqueness of F1 and F2 layers is that they remained active during night because of high degree of ionization according to Saha’s preposition (Gurevich 1978; Kelley 2009; Zubair et al. 2011; Piel and Brown 2011; Yamazaki and Maute 2017; Wang et al. 2019). Moreover, the characteristics of ionospheric layers Es and F2 including F1 at Karachi station studied during the solar cycles 21, 22 and 23 (Talha et al. 2019). They have shown that F2 layer exhibits small positive variations during day time from the first half of year 2008, whereas for the other half of 2008, variations are large due to small values of observed foF2 set of observations during September equinox.

In this study, we intend to illustrate the irregular behavior of F2 layer and manifested the hidden periodicities in the ionospheric signals. During the last decades, the wavelet based approaches have been broadly applied to multiple fields of signal processing and related areas (Mandrikova et al. 2015). Many investigators like Grossman and Morlet 1984; Santos et al. 2001; Domingues, et al. 2005 applied the same approach to tackle the problems on various fields of Science and Engineering. It was implemented to analyze the periodic and non-periodic signals based multiscaled phenomena, where Fourier ideas were placed and permit to inspect various scales of variability in time plots ahead (Santos and Ideião 2006). The past investigators conceived wavelet as a power full tool to apply for a local events deviation of power time series (Grossman and Morlet 1984). Albeit, the approach wavelet investigation is utilized to monitor fuzzy signals, therefore, the choice of the best wavelet manifestation for the inspection signals is important. A literature survey has been done for the same topic that can explain the various kinds of wavelet functions frequently applied in atmospheric science, these are termed as Mexican hat, Morlet, Daubechies and Haar (Jan et al. 2019). Conclusively, wavelet functions have depicted a characteristic that exist in the data set (Santos et al. 2003). Moreover, the smooth deviation in the continuous type wavelet is used for smoothly nature function, for example a Morlet wavelet (Jan et al. 2019). This concept
utilized to highlight the hidden points of the signal in uninterrupted wavelet and gives more localization in time and frequency that are explained by the Fourier transform (Rahman et al. 2018). On the other hand, this study focus on the amplitude and time that is a convolution functions of the Morlet wavelet that can be selected to justify the cyclic character of the observations (Adewale et al. 2017). Two scholars Santos and Moris (2013) were utilized the wavelet power spectrum (WPS) on domain of rainfall observations of Sao Francisco River catchment and recognized the high annual frequencies along the catchment area.

Lopez-Montes et al. (2014) has applied the same technique on the ionospheric phenomenon to study the total electron content (TEC) and geomagnetic storms. Their results have confirmed that the geomagnetic storm produces strong influence on TEC at mid latitude ionosphere. In our study, we have applied wavelet based approach, the Morlet wavelet for the continuous wavelet (CW) that can be properly useful for the parameter estimation and anomaly detection during the ionospheric perturbation (Domingues et al. 2005; Mandrikova et al. 2015).

The wavelet approach required usually nonstationarity or trending character of the average values, and alteration in the inconsistency for the definite time interval. Furthermore, in atmospheric time series, the irregular dispersed events may occur with nonstationarity power over various diverse frequencies. Many investigators studied the trend behavior of F₂ layer and shown the decreasing trend (Danilov et al. 2001; Mahajan et al. 2007; Gordiyenko et al. 2014; Roininen et al. 2015; Santos et al. 2016; Mendillo et al. 2018). The trend analysis is a familiar method used in same data points and exploring the event in terms of the influential effect of advancement in environmental and other events (Machiwal and Jha 2008; Jan et al. 2014). This study has classified into four sections. In the first place, we have described the exploratory data analysis (EDA); secondly the selection of best wavelet approach, thirdly, the brief interpretation of wavelet analysis while in the final section, concludes the computed results.

2. Observed Data and Mathematical Approaches

In this study, the daily observed data of critical frequency of the ionospheric F₂ layer (Day time) obtained from United Kingdom Solar System Data Centre (UKSSDC) and space agency SUPARCO Pakistan for time interval from 2008 to 2017 for the stations located at Sonmiani (17.62°N, 141.5°E) and Karachi (24.95°N, 67.13°E) is studied (Ameen et al. 2019; Talha et al. 2019).

2.1 Mann-Kendall’s trend test:

It has been experienced that the MK test is a well-known approach to recognize the reality of trend found inside an observe series. The detail explanation of this test approach is presented by many investigators (Wang, 2006; Chattopadhyay et al. 2012; Hirsch and Slack 1984). The parameters for MK test can be expressed as:

\[ S = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} sgn(x_j - x_i), \]  
where \( sgn = \begin{cases} +1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases} \quad (1) \]
According the null hypothesis $H_0$ test the sequence $\{X_t : t = 1, 2, \ldots, n\}$ is independently and identically distributed (i.i.d.) and another hypothesis $H_1$ is that a signal trend present in $\{X_t\}$. The parameters mean ($E(S)$) and variance ($V(S)$) of the Kendall sum ($S$) can be represent by the formula when

$$E(s) = 0$$

And

$$V(S) = \frac{1}{18} \left[ n(n-1)(2n+5) - \sum_{i=1}^{m} t_i(t_i-1)(2t_i+5) \right] \quad \text{... (2)}$$

The symbol $t_i$ is the number of bands of $i$th values and $m$ is number of trail observations. Now, we defined the standard normal distribution $Z$ by the mathematical function:

$$Z = \begin{cases} 
\frac{s - 1}{\sigma}, & \text{if } S > 0 \\
0, & \text{if } S = 0 \\
\frac{s + 1}{\sigma}, & \text{if } S < 0 
\end{cases} \quad \text{... (3)}$$

According to the bilateral test for trend, the null hypothesis is that no trend is rejected if $Z > Z_{\alpha/2}$, where $\alpha$ described the level of significance. The Kendall’s tau is estimated as under:

$$\tau = \frac{2S}{n(n-1)} \quad \text{... (4)}$$

Where, ‘$S$’ is Kendall sum. Further explanations on Kendall’s tau are presented by Chattopadhyay et al. (2012) and Wang (2006).

### 2.2 Wavelet Approach:

In this communication, we have applied the approach that permits to the wavelet structure to evaluate the idea of scale and frequency. The function $f(t)$ can be given to represent the wavelet:

$$\text{wf}(p, t) = \int_{-\infty}^{\infty} f(u) \overline{\psi_{p,t}}(u) du, \quad p > 0 \quad \text{..... (5)}$$

Where,

$$\psi_{p,t}(u) = \frac{1}{\sqrt{p}} \psi\left(\frac{u-t}{p}\right) \quad \text{..... (6)}$$

The above expression exhibits, known as continuous wavelets. In this case the symbol $p$ and $t$ indicates the scale and location parameters respectively, and the function $\overline{\psi_{p,t}}(u)$ is known the complex conjugate of the above equation (Lopez-Montes et al. 2015; Ahmed et al. 2015; Jan et al. 2019). If we change the value of $s$ that has influenced the dilating $(p > 1)$ or decrease $(p < 1)$ of the function $\psi(t)$ and where the shifting of $t$ has the cause of examining the function $f(t)$ about different time scales. As the scale ($s$) increases, the wavelet turn into further enlarge and obtains the extensive nature of $f(t)$ depiction. Consequently, the
wavelet approach gives different time-scale aspects that shorten when pay attention on microscopic scale characteristics and widespread on large-scale phase. It has been observed that $\psi_{p,t}(u)$ has a similar structure for whole levels (Misiti et al. 2007). The representation of the function $\psi(t)$ is neither distinctive nor random and called wavelet because of the following two properties:

$$\int_{-\infty}^{\infty} |\psi(t)|^2 dt = 1 \ (\text{unit energy}) \quad \ldots \quad (7)$$

$$\int_{-\infty}^{\infty} \psi(t) dt = 0 \ (\text{zero energy}) \quad \ldots \quad (8)$$

So that it has compressed or sharply decomposed to obtain localization in space and zero mean, whereas the higher-order minutes could also be zero, i.e.

$$\int_{-\infty}^{\infty} t^k \psi(t) dt = 0 \quad k = 0, 1, N - 1 \quad \ldots \quad (9)$$

The requirement of zero mean is called the adequacy condition of the wavelet (Kumar and Georgiou, 1997). However, two popular wavelets for CW are known as (1) Morlet wavelet and (2) Mexican hat. Where the Morlet wavelet is termed as the Gaussian transformation of a plane complex wave and the different levels in given sequences found by converting and escalating the fundamental wavelet $\psi(t)$ and illustrated by the formula:

$$\psi_o(t) = \frac{1}{\sqrt{\pi}} e^{-\left(\omega_0 t + \frac{t^2}{\omega_0^2}\right)^2} \quad (\omega_o \geq 5) \quad \ldots \quad (10)$$

Where, $\omega_o$ representing dimensionless frequency which is equal to 6 for this interpretation to induce the permissibility state and $\psi_o(t)$ is function of time (Ahmed et al. 2015; Peres et al. 2017). The next type of CW Mexican hat is famous second order differentiation of Gaussian wave function, given as:

$$\psi(t) = \frac{2}{\sqrt{3}} \pi^{-\frac{1}{4}}(1 - t^2)e^{-\frac{t^2}{\pi}} \quad \ldots \quad (11)$$

Now, suppose that the conversion of the wavelet of series data in discrete sequences $x_n$, at time index ‘n’ is termed as the complexity of $x_n$ with a size and conversion of $\psi_o(t)$. The definition of wavelet analysis is known as the cause of the inner product of wavelet analysis with specific observations ($x_n$). This concept is expressed mathematically (Torrence and Compo 1998; Yan et al. 2020):

$$W_n(p) = \sum_{n=0}^{N-1} x_n \bar{\psi} \left[ \frac{(n' - n) \Delta t}{p} \right] \quad \ldots \quad (12)$$

In the above relation, ‘n’ corresponds to confined time indicator; ‘p’ is known for the width of investigated wavelet, $\Delta t$ time difference, ‘N’ stand for total observations in the considered data sets and $\bar{\psi}$ revealed the inverse of the complex function $\psi(t)$ (Jan et al. 2019). The above relation can be computed for a large number of data points of the scale ‘s’ (generally used in the multiples of the smallest feasible frequency) with entire observations of ‘n’ among the starting and ending times. A two dimensional fluctuation function can be then demonstrated by scheming the wavelet amplitude and phase. After that, the observed data sets can be translated into time-frequency localization by using mother wavelet (Santos and Ideiao 2006).
It has been known that the wavelet function connected to intricate continuous wavelet, as a result the wavelet Scalograms, depicted by the absolute variance square \( |W_n(p)|^2 \), which is a fitting illustration of the variance at various frequencies (Goodwin 2008; Yan et al. 2020). When wavelet power spectrum (WPS) integrated with respect to time, gives the global wavelet spectrum (GWS) Scalograms, that can be defined by the wavelet variance that locates the usual phase of an event to be identified on the whole highest peak in the series correlated to a particular time intervals (Kirby 2005).

To view the fluctuations in powerful range over a sequence of a particular length, square-average wavelet spectrum defined above the global wavelet spectrum is utilized, which has demonstrated the variance in a particular scale range, is attained by taking the mean of the confined wavelet coefficients with the N-vertical gird of axis as:

\[
W_2 = \frac{1}{N} \sum_{n=0}^{N-1} |W_n(p)| \quad \ldots \quad (13)
\]

Where, \( n=1, 2, \ldots, N \) and this wavelet is a complex valued function, enabling one to extract information about the amplitude and phase of the signal, but now some new method called scaled wavelet will be required to transform the whole size with decrease the complete wavelet along in time (Peres et al. 2017). Therefore, the scaled wavelet can be described as:

\[
\psi \left( \frac{(n' - n)\Delta t}{p} \right) = \psi_o \left( \frac{(n' - n)\Delta t}{p} \right) \quad \ldots \quad (14)
\]

Where ‘s’ is known as the dilation parameter and ‘n’ is stands for the transformation factor. Furthermore, the term \( \frac{1}{\sqrt{p}} \) is a standardized part to stable the overall energy of the scaling wavelet (Peres et al. 2017). A significant level can be made in the 2-D plot, using red-noise background spectrum. Various atmospheric events can signify from white to red-noise. Therefore, the univariate lag-1 autoregressive method is applied to the investigation of the red-noise, where the lag-1 is the relationship between the observed data and itself, however transferred by unit time. In our study, it would be one day. The univariate lag-1 estimates the positive correlation of a variance from one day (or one hour) to after that day (Santos and Ideiaio 2006). The actual lag-1 \( \alpha \) value can be worked out by an estimation applying \( \alpha = \frac{\alpha_1 + \sqrt{\alpha_2}}{2} \), here \( \alpha_1 = \) lag-1 correlation, and \( \alpha_2 = \) lag-2 correlation, i.e. similar as lag-1 however now changed by two time units as an alternative to one. When \( \alpha_1 < 0.4 \), therefore, suggested to represent the observations as constant variance \((\alpha = 0)\) the null hypothesis declare for the WPS as imagine that the observed data values have a mean power spectrum. It has been observed that if a peak in the WPS is considerably over a cone of influence, after that it can be supposed to be a true aspect with a definite interval (95%). This shows 5% of the WPS must be over this place (Santos et al. 2018).

The inverse frequency also called period is the estimated Fourier period that connected with the fluctuations contained by the wavelet. In wavelet analysis, there is a 1-1 relationship between scale and period. For Morlet, the period is more important as compared to others because it has many smooth oscillations (Jan, et al. 2019).
2.3 The Correlation coefficient ($\rho$) Test:

This test inspects correlation coefficient ($\rho$) that illustrates the measure of the degree of relationship between the wavelet coefficients and the original observed signals observations. Mathematically represented by the formula (Ameen et al. 2019):

$$
\rho = \frac{\sum_{i=0}^{n}(f_{obs} - \bar{f}_{obs})(f_{coef} - \bar{f}_{coef})}{\sqrt{\sum_{i=0}^{n}(f_{obs} - \bar{f}_{obs})^2 \sum_{i=0}^{n}(f_{coef} - \bar{f}_{coef})^2}}
$$

Where $f_{obs}$ denotes observed signals and $\bar{f}_{obs}$ for average of observed signal, whereas $f_{coef}$ denotes coefficient of wavelet and $\bar{f}_{coef}$ for average of coefficient of wavelet. The value of correlation coefficient fluctuates from 1 to -1. When, the given variables are in a strong linear combination than $\rho$ will be close to 1 or –1 (Ruigar and Golian 2015).

3. Results and Discussions

3.1 Analysis of the Physical Characteristics of F$_2$ Layer

In this section, we have described the physical features of critical frequency of F$_2$ layer using Exploratory Data Analysis (EDA) that has been carried out for the selected data sets which give quantitative aspects about time series and its characteristics. For this purpose, computational work based on descriptive investigation, including parameters such as mean, Standard deviation (Std.D), Variance (Var) etc. at given time intervals (Zai et al. 2013; Sharma 2016; Jan et al. 2018). These evaluated records have been summarized (as in the Table: 1).

The average critical frequency of F$_2$ layer is 9.45 MHz, while the maximum frequency is 14.2 MHz in October 2017 and minimum 4.2 MHz in January 2011 with Std. D = 1.74.

<table>
<thead>
<tr>
<th>Data</th>
<th>N</th>
<th>Mean</th>
<th>Std. D</th>
<th>Var.</th>
<th>Min.</th>
<th>Median</th>
<th>Max.</th>
<th>CV (%)</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008-2017</td>
<td>3653</td>
<td>9.45</td>
<td>1.74</td>
<td>3.03</td>
<td>4.2</td>
<td>9.4</td>
<td>14.2</td>
<td>18.44</td>
<td>0.03</td>
<td>-0.68</td>
</tr>
<tr>
<td>Jan</td>
<td>310</td>
<td>9.2</td>
<td>1.76</td>
<td>4.2</td>
<td>3.11</td>
<td>9.0</td>
<td>13.0</td>
<td>19.12</td>
<td>-0.02</td>
<td>-0.87</td>
</tr>
<tr>
<td>Feb</td>
<td>283</td>
<td>9.8</td>
<td>1.68</td>
<td>5.8</td>
<td>2.843</td>
<td>10.0</td>
<td>14.0</td>
<td>17.24</td>
<td>-0.39</td>
<td>-0.79</td>
</tr>
<tr>
<td>Mar</td>
<td>310</td>
<td>10.4</td>
<td>1.62</td>
<td>6.7</td>
<td>2.616</td>
<td>11.0</td>
<td>13.8</td>
<td>15.61</td>
<td>-0.29</td>
<td>-1.05</td>
</tr>
<tr>
<td>Apr</td>
<td>300</td>
<td>10.5</td>
<td>1.69</td>
<td>5.4</td>
<td>2.874</td>
<td>11.0</td>
<td>14.0</td>
<td>16.10</td>
<td>-0.08</td>
<td>-0.48</td>
</tr>
<tr>
<td>May</td>
<td>310</td>
<td>9.7</td>
<td>1.49</td>
<td>6.0</td>
<td>2.243</td>
<td>9.8</td>
<td>13.6</td>
<td>15.50</td>
<td>0.18</td>
<td>-0.64</td>
</tr>
<tr>
<td>Jun</td>
<td>300</td>
<td>8.9</td>
<td>1.70</td>
<td>5.2</td>
<td>2.893</td>
<td>9.0</td>
<td>13.0</td>
<td>19.08</td>
<td>0.16</td>
<td>-0.80</td>
</tr>
<tr>
<td>July</td>
<td>310</td>
<td>8.8</td>
<td>1.51</td>
<td>4.9</td>
<td>2.288</td>
<td>8.75</td>
<td>13.0</td>
<td>17.11</td>
<td>0.16</td>
<td>-0.56</td>
</tr>
<tr>
<td>Aug</td>
<td>310</td>
<td>8.8</td>
<td>1.67</td>
<td>5.0</td>
<td>2.426</td>
<td>9.0</td>
<td>19.70</td>
<td>17.75</td>
<td>0.76</td>
<td>4.47</td>
</tr>
<tr>
<td>Sep</td>
<td>300</td>
<td>9.6</td>
<td>1.73</td>
<td>5.9</td>
<td>3.006</td>
<td>9.90</td>
<td>14.0</td>
<td>18.06</td>
<td>0.13</td>
<td>-0.84</td>
</tr>
<tr>
<td>Oct</td>
<td>310</td>
<td>9.5</td>
<td>1.77</td>
<td>5.5</td>
<td>3.145</td>
<td>9.60</td>
<td>14.20</td>
<td>18.59</td>
<td>0.10</td>
<td>-0.59</td>
</tr>
<tr>
<td>Nov</td>
<td>300</td>
<td>9.1</td>
<td>1.65</td>
<td>5.7</td>
<td>2.752</td>
<td>9.0</td>
<td>13.0</td>
<td>18.14</td>
<td>0.09</td>
<td>-0.63</td>
</tr>
<tr>
<td>Dec</td>
<td>310</td>
<td>9.0</td>
<td>1.66</td>
<td>5.0</td>
<td>2.761</td>
<td>8.95</td>
<td>13.0</td>
<td>18.38</td>
<td>-0.00</td>
<td>-0.55</td>
</tr>
</tbody>
</table>

The Fig. 1 shows fluctuations of daily average of foF$_2$ frequency from 2008 to 2017. It depicts a significant variation in a given time interval. The Fig. 2 presents fluctuations of the daily average of foF$_2$ frequency for each month from 2008 to 2017. The overall Coefficient of variations (CV) observed 18.44% of the daily data.
set. On the other hand, for monthly data set of \( f_{oF_2} \) series the maximum coefficient of variation were observed in January (19.12%) whereas the minimum value of CV observed in the month of May (15.50%). These variations of CV \( f_{oF_2} \) discover that the measure of scatter or dispersion of time series of critical frequency of \( F_2 \) layer, relative to the size of the mean value.

![Temporal diurnal behavior of \( f_{oF_2} \) set of observations from 2008-2017](image1.png)

*Fig. 1* Temporal diurnal behavior of \( f_{oF_2} \) set of observations from 2008-2017

![Temporal characteristics of \( f_{oF_2} \) observations on monthly basis during the year 2008-2017](image2.png)

*Fig. 2* Temporal characteristics of \( f_{oF_2} \) observations on monthly basis during the year 2008-2017

In this communication, we have applied the Wavelet approach instead of Fourier analysis (FA) because the FA should be only utilized when observed data fulfill the following two important criteria. (1) Stationarity; that is, there is no change in the mean and variance throughout the observed data, (2) the time series can be expressed as the integration of different cyclic constituents presented via simple harmonic functions. Indeed, the space phenomena and atmospheric timescales demonstrate trending character or non-stationarity and the wavelet approach does require only nonstationarity observations (Santos and Galvao 2003; Wang 2006; Santos and Freire 2012; Santos et al. 2016). We have presented the trends for each month in the table .2. Off course, they are the causes of some effects that are found in deep space. The set of observations for \( f_{oF_2} \) particularly have fluctuations. In couple of months these variations are not indicated due to some diverse phenomena occurring in deep space that do not favor the trend analyses.
Table: 2 Monthly data representation for non-parametric values for Mann-Kendall trend tests on a diurnal basis from 2008 to 2017 (α = 0.05)

<table>
<thead>
<tr>
<th>Month</th>
<th>tau(τ)</th>
<th>p-value</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
<td>0.197</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>Feb</td>
<td>0.333</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>Mar</td>
<td>0.301</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>Apr</td>
<td>0.273</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>May</td>
<td>0.293</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>Jun</td>
<td>0.042</td>
<td>0.2810</td>
<td>No trend</td>
</tr>
<tr>
<td>July</td>
<td>0.153</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>Aug</td>
<td>0.267</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>Sep</td>
<td>0.328</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>Oct</td>
<td>0.304</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>Nov</td>
<td>0.221</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
<tr>
<td>Dec</td>
<td>-0.056</td>
<td>0.1480</td>
<td>No trend</td>
</tr>
<tr>
<td>Total</td>
<td>0.216</td>
<td>0.0001</td>
<td>Trend exist</td>
</tr>
</tbody>
</table>

3.2 Selection Criteria of Mother Wavelet:

A question that is constantly rising in the utilization of wavelet approaches is the selection of appropriate wavelet function to a particular series. In order to show the capability of the wavelet transformation and its correlation on the uniqueness of the preferred wavelet, the correlation coefficient test was applied to demonstrate this concept (Domingues et al. 2005). In our case, we have selected particular types of wavelet in order to carry on ionospheric signals for F\textsubscript{2} layer. These signals launched from the transmitter to detect the ionospheric sudden disturbances (SIDS) by receiving the echoes from the respective layer of the ionosphere. These echoes are considered as ionospheric signals. In view of the above, Guassian (gaus), Mexican hat (mexh) and Morlet are selected for the test process. Obviously, they are being utilized for ionospheric signal’s conditioning and processing.

The test correlation coefficient (\(\rho\)) inspects the correspondence among the wavelet coefficients and the original \(foF_2\) records. The mother wavelet which has a maximum value of ‘\(\rho\)’ can be chosen as the appropriate wavelet. Table: 3 has exposed the values of ‘\(\rho\)’ for different wavelets. It has been observed that Morlet wavelet has a maximum correlation coefficient of 0.815, selected as the best type of continuous wavelet.

Table: 3 Evaluation the correlation coefficient (\(\rho\)) of the extracted F\textsubscript{2} signals for families of continuous wavelets

<table>
<thead>
<tr>
<th>Mother Wavelet</th>
<th>Correlation Coefficient ((\rho))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morlet</td>
<td>0.815</td>
</tr>
<tr>
<td>mexh</td>
<td>0.457</td>
</tr>
<tr>
<td>gaus</td>
<td>0.297</td>
</tr>
</tbody>
</table>

3.3 Interpretation of Wavelet Approach:

To review the wavelet power spectrum (WPS), we have used 11 years observed data of F\textsubscript{2} layer that consist of daily time series. Therefore, the parameters for the wavelet approach is rearranged by the time \(\Delta t = 1\) day, the
initial scale \( p_0 = 2 \) days, the reason is that \( p = 2\Delta t \) and the scale width taken \( \Delta i = 0.25 \), which will further creates four sub-cycles per eight sets of cycles (octave), and the parameter \( i_1 = \frac{10}{\Delta i} \) used to make 10 powers of two with \( \Delta_1 \) sub-cycles each. In our case, this will be 10 power of two, i.e., \( 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^9 \), and \( 2^{10} \).

Wavelet spectrum and periodic perturbations of the \( F_2 \) layer has been computed using Morlet scheme. In a general way, we have normalized the data values to observe the variability as represented by the variable critical frequency of \( F_2 \) layer at the Pakistan terrestrial space as revealed in Fig. 3 (a).

![Fig.3](image)

**Fig.3** (a) Plot of daily wavelet for \( F_2 \) layer in the year 2008-2017 in Pakistan atmospheric region. (b) The Scalograms for energy WPS using the Morlet wavelet. The curve shows the cone of influence, the blue contours are significant levels. (c) The Scalograms of GWPS. The dashed line is 5% significance level. (d) Scale-average wavelet power Scalograms 2-16 day’s band. The dashed line also indicates 95% confidence level, assuming red noise.

### 3.4 Wavelet Scalograms and Variance Analysis

It has been known that the wavelet power spectrum (WPS) also named as wavelet Scalograms that have shown the signal’s energy content at a particular scale and position in time (Gunkel 2016). Albeit, the wavelet approach describes a wavelet of accurate and similar contour, but the range scales increase or decrease with the data length (Santos and Freire 2012). Larger amplitudes of variations are identified as cycles per day, which is clearly identified from \( F_2 \) signals exhibit as in the Fig. 3 (b).
The Scalograms are a feasible representation of the WPS that manifest temporal enhancement of the signal power along the particular scale. However, in the graphical representation the red and blue colors in WPS plots have shown the strong and weak intensities of the energy. Moreover, the Fig.3 (b) demonstrates the WPS via Morlet wavelet for critical frequency of $F_2$ layer and Fig.4 revealed for seasonal data sets.

![Scalograms](image)

**Fig.4.** Scalograms (WPS) for Ionospheric $F_2$ layer starting from January-December in the time interval 2008-2017 at ionospheric region of Pakistan, where the U-shaped curve stand for cone of influence.

These plots have real interpretation bestowed by physical concepts of time frequency and time energy. It is obvious from the statements that Wavelet Power Spectrum does exhibit essential features regarding the power and energy at a particular scale-time, where the color codes for the $\nu F_2$ ranges are between blue (low energy band) and red (high energy band) are significant from physical point of views. This indicates that the
oscillation of the individual wavelets as an alternative of magnitude. These figures interpret composite salient
inception of the power-energy that can be predictable in the domain of time-frequency. The oscillations in
periodic fashion in the wavelet power spectrum are illustrated by the observations from 2008-2017. In these
interpretations the discontinuities are represented by parabolic structure (Santos, and Ideiao 2006; Liu, et al.
2007).
Actually, what we have depicted in the figures that indicate the Scalograms are a practicable representation of
the WPS that evident temporal enhancement of the signal power along the particular scale. However, in the
graphical representation the red and blue colors in WPS plots have depicted significantly the strong and weak
intensities of the energy.

3.5 Global Wavelet Scalograms for F_2 layer

In this entire paragraph all the results illustrate the interpretation are indicated by the mathematical statistical
analyses of confidence level of the power and energy that are 95 %. The global wavelet power spectrum has
been computed for this particular layer using the average of power with respect to time simultaneously that
confirms the major portion of frequency patterns (Goodwin 2008). In observing data points, daily periodicities
of F_2 signals have been shown as in Fig.3 (c), that depicts the three prominent peaks larger than 95% confidence interval, considering the red-noise correspond to dashed lines, these peaks indicating from 128-256
days, 256-512 and 512-1024 days band foF_2 time series. All these dictate the fluctuations present in the set of
observations.

Fig.5 represented the GWS of F_2 layer for each month starting from Jan to Dec and depicts two significant
peaks in all plots showing the semiannual cycle at 95% significance level. The Table: 4. illustrates the
theoretical and fitted power spectrum for different scales. This bestows the accurate power scale of the
ionospheric observations and consequently, the GWS is a simple and powerful approach to illustrate the
dynamics of ionosphere and accommodating to defined spatial and temporal anomaly in the pattern of the F_2
region and compared with bordering regions. The GWS structure is initially prescribed using distribution of
different levels (Rahman et al. 2018).
Fig.5. Scalograms (GWPS) for ionospheric F\textsubscript{2} layer from January-December during the period 2008-2017 in Pakistan atmospheric region.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Global wavelet Spectrum</th>
<th>Global fit theoretical</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.409</td>
<td>0.163</td>
</tr>
<tr>
<td>2.380</td>
<td>0.582</td>
<td>0.177</td>
</tr>
<tr>
<td>2.831</td>
<td>0.659</td>
<td>0.208</td>
</tr>
<tr>
<td>3.362</td>
<td>0.744</td>
<td>0.259</td>
</tr>
</tbody>
</table>
3.6 Scalograms for Scale-Average Wavelet Spectra for F\textsubscript{2} Layer

The Fig. 3 (d) depicted the Scalograms scale-average wavelet spectrum (SAWS) for data recorded on the daily basis for F\textsubscript{2} layer. This figure illustrates the average variance in the interval of the ionospheric region in Pakistan air space. In this case, we have utilized the SAWS scheme for the scale 2 to 16 days.

To view the fluctuations in the process, we have found the cyclic variation. Moreover, the Fig. 3 (d) represents scales between 2-16 days, which characterize an estimation of the average day variation within the time interval. These Scalograms reveal individual periods that displays increase of the daily foF\textsubscript{2} variances in the Pakistan atmospheric region. From this figure three significant peaks in the range of F\textsubscript{2} layer might be viewed on 1274, 2675 and 3456 days that is clearly indicating different variations from normal days.
4 Conclusions

We have introduced wavelet characterization of ionospheric plasma F2 layer and compared different aspects of this approach. The wavelet approach used in this paper is the continuous wavelet to explore the hidden periodicities of plasma contents. As we know that the wavelet is considered as a mathematical microscope that can be utilized to magnify the hidden micro periodicities covered in the given signals. Moreover, we also intended to identify time frequency and the energy for continuous wavelet.

The results of Morlet wavelet for Pakistan air space revealed that generally GWS is most prominent in the annual and semiannual cycles. It has been confirmed that the deep space possesses highly nonlinear behavior. The interactions of launching signals from ground become rather complex so the energy peaks during the periods 128-256, 256-512 and 512-1024 are higher. Moreover some variables are controllable and most of the variations are short term and chaotic that cannot be easily approachable. Therefore, we must very careful to figure out their physical interpretation via various mathematical approaches such as mention in the manuscript.

Moreover, the wavelet trend investigates an influential power range between the 2–16 day bands, representing an annual fluctuation of 365 days for daily observations. This has been examined by the peak incorporation of scale-average range above the entire period that also manifests strong annual cycles. The low variance in a frequency range highlights the existence of seasonality in the F2 layer during the study period.

The wavelet energy of \( f_0 F_2 \) of F2 layer manifests the relationship between annual, semiannual, and seasonal ranges and interactions between ionospheric disturbances.

Acknowledgment:
We would like to acknowledge the services rendered by the Director, United Kingdom Solar System Data Centre (UKSSDC), Oxon, U.K. and Director, Space agency SUPARCO Pakistan to provide the necessary set of observations for this research. Also, it could be noted that this investigation is a part of the Ph.D dissertation of first author.

Author Contributions Statement
Bulbul Jan has written the main manuscript and analysis of the data and results. Muhammad Ayub Khan Yousuf Zai supervised the work and gave the main conceptual ideas of the manuscript. Faisal Nawaz and Faisal Ahmed Khan Afridi contributed to the design and completion of the research, and Arsalan Ahmed, Faraz Mehmood, and Mirza Jawwad Baig reviewed the manuscript.

Conflict of interest statement: On behalf of all authors, the corresponding author states that there is no conflict of interest.

References:


