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Individual versus simultaneous estimation of two parameters in a correlated noisy environment

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Recently, the phase parametric estimation for a single qubit system driven by a phase noisy laser under non-Markovian dynamics has been reported. We here generalize the single-parameter estimation to the two-parameter scenario where a two-qubit system is influenced by correlated phase noisy laser, and compare the performances of two-parameter estimation in both individual and simultaneous strategies by using the quantum Fisher information and the quantum Fisher information matrix, respectively. Our results show the phase parametric estimation precision can be improved due to the memory effect arising from successive applications of the noisy laser. With the memory coefficient µ increasing, the precision of parameter estimation becomes more accuracy. Besides, the phase parameter estimation precision can be well enhanced by engineering the ratios of classical phase noisy laser rate and system-environment coupling strength dependent of Markovian or non-Markovian regions. What’s more, we find the simultaneous estimation of two-phase parameters is not always advantageous over the individual strategy.

I. INTRODUCTION

Estimating an unknown parameter precisely is one of the main contents in theoretical quantum metrology and parameter estimation theory [1, 2]. In classical estimation theory, due to the existence of the vacuum quantum fluctuation, one cannot estimate parameter precisely. Theoretically, the precision of parameter estimation is bounded by the so called standard quantum limit (1/√N, where N denotes the number of photons). However, benefit from the quantum technique such as the squeezed state and quantum entanglement, the ultimate estimated parameter precision can beat the standard quantum limit and even achieve the Heisenberg limit (1/N) [3–6].

Quantum Fisher information (QFI), extending the estimation theory from the classical regime to the quantum one, lies at the heart of estimation theory and provides us the precision (accuracy) of the parameter estimation based on the Cramér-Rao inequality [7]: δφ ≥ 1/√QFI. A larger QFI represents a higher estimation accuracy. Therefore, how to maximize the QFI to enhance the precision of parameter becomes a main task in estimation theory. Unfortunately, any realistic quantum system unavoidably interacts with an uncontrollable environment which severely influences metrological protocols. It is commonly believed that Markovian type environments have been proven to be harmful to quantum metrology due to the decay of QFI [8–11]. In contrast, non-Markovian type environments owing to memory effect which leads to revivals of QFI can guarantee the advantage of quantum metrological strategies [12–16]. It would be of great interest in engineering adverse environment to improve the accuracy of parameter estimation. More recently, by regulating external field driving and non-Markovian effects, the parameter estimation for a two-level system in a zero-temperature non-Markovian reservoir has been remarkably improved [17, 18]. Making use of the control pulses as well as the initial correlations, the estimation of environment parameters can be increased by orders of magnitude [19].

However, the work above mentioned are mainly focused on the estimation of a single parameter. In many actual estimation tasks, there are more than one parameters to be estimated simultaneously and the joint estimation of multiple parameters becomes necessary. Recently, multiparameter estimation has attracted a great deal of interest due to the high efficiency in quantum metrology [20–26]. It has been shown that simultaneous estimation of multiparameter can give a better precision compared with individual estimation case where all the parameters are measured individually [27–29]. An interesting question and challenge is raised whether the multiparameter estimation always allows to reduce the total error and surpasses the individual strategy even in the presence of decoherence.

Motivated by the above considerations, we generalize the parametric estimation from a single qubit system to a two-qubit system subjected to a classical phase noisy laser, and explore the two-parameter estimation. Our goal is to address the performance of individual parameter estimation with simultaneous estimation. Compared to the results obtain in Refs. [27–29], our results indicate the phase parametric estimation in simultaneous scenario provides less precise result than that in individual parameter scheme. On the other hand, we focus our attention on how the classical noisy channels with memory affect the precision of parameter estimation. Our results show, the memory effect arising from successive applications of the noisy laser can improve the precision of
parametric estimation. With the memory coefficient $\mu$ increasing, the precision of parameter estimation becomes more accurate. In particular, for the memory coefficient $\mu = 1$, the phase parametric estimation is immune from the noisy environment. Interestingly, the two-parameter estimation precision can be well improved by engineering the ratios of classical phase noisy laser rate and system-environment coupling strength in both Markovian and non-Markovian regions.

The paper is structured as follows. In section II, we introduce the definition of the QFI for both the single- and multi-parameter cases. In section III, the performances of individual parameter estimation with simultaneous estimation for a two-qubit system exposed in the classical phase noisy environment which is modeled by depolarizing effect are revealed. The two-parameter estimation scenario in a pure dephasing noisy environment is also discussed in Sec. IV. Finally, we give the conclusion in section V.

II. QUANTUM FISHER INFORMATION

Before investigating the two-parameter estimation, we first review the definition of quantum Fisher information (QFI) which quantifies the ultimate precision of an unknown parameter $\theta$ via the quantum Cramér-Rao bound [7]

$$\delta^2\theta = \Delta\theta \geq \frac{1}{N F_\theta},$$  \tag{1}

where $\Delta\theta = (\langle\hat{\theta}_{\text{est}} - \theta\rangle^2)^{1/2}$ denotes the variance of any unbiased estimator $\hat{\theta}$, and $N$ is the number of independent measurements. Here $F_\theta$ denotes the QFI which is defined as

$$F_\theta = Tr[\rho(\theta)L^2],$$  \tag{2}

in which $\rho(\theta)$ is the density matrix of the system, and the symmetric logarithmic derivation $L$ is determined by

$$\frac{\partial\rho(\theta)}{\partial\theta} = L\rho(\theta) + \rho(\theta)L.$$  \tag{3}

Obviously, according to Eq.(1), the inverse of QFI provides the lower bound of the error of the estimation. The larger QFI is, the higher precision of the estimation is.

Generalize a single parameter estimation to the multiparameter scenario, the limit of measurement precision of the parameters $\theta = (\theta_1, \theta_2, \ldots, \theta_n)$ is also bound by the quantum Cramér-Rao [30]

$$\delta^2(\theta_1, \theta_2, \ldots, \theta_n) = C(\hat{\theta}) \geq \frac{1}{N} Tr[F M^{-1}_\theta],$$  \tag{4}

where $C(\hat{\theta})$ is the estimation error covariance matrix of the parameters with elements $C_{\mu\nu} = \frac{1}{2}\langle(\theta_\mu \partial_{\nu} + \theta_\nu \partial_{\mu}) - \langle\theta_\mu\rangle \langle\theta_\nu\rangle\rangle$, and the QFI is substituted by quantum Fisher information matrix (QFIM) whose elements $F_{\mu\nu}$ are expressed as

$$F_{\mu\nu} = Tr[\rho(\theta) L_\mu L_\nu + L_\nu L_\mu] / 2.$$  \tag{5}

here $L_\mu$ and $L_\nu$ are the symmetric logarithmic derivations with respect to $\theta_\mu$ and $\theta_\nu$, respectively. In particular, if the estimated parameter $\theta_\mu$ is equivalent to $\theta_\nu$, Eq.(4) reduces to the case of a single parameter case.

In this paper, we focus on two parameters estimation including the simultaneous and individual estimation strategies for a bipartite system which is suffered from a decoherence environment. For this purpose, we suppose the estimated parameter $\phi_i$ is embedded into the initial two-qubit state $\rho$ through unitary operation $U(\phi) = |0\rangle_j<0| + \exp(i\phi_j)|1\rangle_j<1|$. More specifically, we consider a class of special Bell-diagonal states which are initially prepared in

$$\rho = \frac{1}{4}[I \otimes I + \sum_{i=1}^3 c_i \sigma_i \otimes \sigma_i]$$  \tag{6}

with $c_i \in [-1,1]$. After the phase gate operation, the density matrix $\rho(\phi)$ depending on parameter $\phi$ is expressed as

$$\rho_{AB}(\phi) = U_A(\phi_1)U_B(\phi_2)\rho_{AB}(0)U_A^\dagger(\phi_1)U_B^\dagger(\phi_2).$$  \tag{7}

Take $\phi_i$ as the parameters to be estimated, QFI respect to a single parameter $\phi_i(\phi_2)$ can be directly calculated

$$F_{\phi_1} = F_{\phi_2} = \frac{c_i^2 + c_j^2 + 2c_1c_2c_3}{1 - c_3^2}.$$  \tag{8}

According to Eq.(2), the low bound of the error for individual estimation becomes $\delta^2\phi_i \geq F_{\phi_i}^{-1}$. The inverse of QFI provides the lower bound of the error in estimation.

On the other hand, for simultaneous estimation parameters $\phi_1$ and $\phi_2$, the low bound of the error for simultaneous estimation is obtain by replaced QFI with the QFIM

$$\delta^2(\phi_1, \phi_2) = \frac{2(c_1^2 + c_2^2 + 2c_1c_2c_3)}{(c_1^2 - c_2^2)^2}.$$  \tag{9}

In order to reveal the performances of two-parameter estimation in both the individual and simultaneous estimation strategies, we introduce a ratio of the individual and simultaneous estimation

$$R = \frac{\delta^2\phi_1 + \delta^2\phi_2}{\delta^2(\phi_1, \phi_2)} = \frac{(c_1^2 - c_2^2)(1 - c_3^2)}{(c_1^2 + c_2^2 + 2c_1c_2c_3)^2}.$$  \tag{10}

satisfying $0 \leq R \leq 1$ for any $c_i$ of initial state. This result indicates the error limit of the individual parameter estimation is smaller than that of the simultaneous case. Namely, the phase parameter estimation in individual scenario would provide more precise result compared to simultaneous parameter scheme. We emphasize
our conclusion that is completely different from the results obtained [27–29] where the simultaneous strategy is always advantageous than the individual one. In what follows, we wonder what happens in the decoherence environment, specifically how the noisy environment affects on the two-phase estimation.

III. QUANTUM FISHER INFORMATION IN A CORRELATED CLASSICAL NOISY ENVIRONMENT

For estimating two-phase parameters in a noisy environment, we generalize Abdel-Khalek’s results [18] where the QFI for a single qubit system driven by a phase noisy laser under non-Markovian dynamics has been reported. We here focus on a two-qubit system with two consecutive uses of classical phase noisy laser. The Hamiltonian of interest system suffered from a classical noisy laser is worked out [18, 32].

\[ H(t) = \frac{1}{2} \omega_0 \sigma_z + \lambda [\sigma_- e^{i(\omega t + \Phi(t))} + \sigma_+ e^{-i(\omega t + \Phi(t))}] \]  

(11)

where \( \sigma_+ = |1\rangle \langle 0| \) and \( \sigma_- = |0\rangle \langle 1| \) are the raising and lowering operators, respectively. \( \lambda \) is the coupling strength between the qubit and its classical noisy environment. For simplicity, we assume that the classical phase noisy laser with a randomly fluctuating phase \( \Phi(t) \) is on resonance with the atom transition (\( \omega = \omega_0 \)), and it is a Wiener process in the standard phase diffusion model, i.e., \( \Phi(t) \) is a white noise, whose correlation function is a delta function \( \langle \Phi(t) \Phi(\tau) \rangle = C(t - \tau) \equiv D \delta(t - \tau) \), here \( C(t - \tau) \) is an arbitrary correlation function with the classical noisy rate \( D \equiv 2 \int_0^\infty dt C(t - \tau) \).

Under these assumptions, the dynamics of Hamiltonian \( H(t) \) is described by a depolarizing effect master equation

\[ \frac{d\rho}{dt} = \sum_{i=1}^{3} (\gamma_i \sigma_i \rho \sigma_i - \gamma_i \rho) \]  

(12)

where \( \gamma_1 = \gamma_2 = -\frac{d\Gamma_1(t)/dt}{4\Gamma_1(t)} \) and \( \gamma_3 = \frac{d\Gamma_1(t)/dt}{4\Gamma_1(t)} - \frac{d\Gamma_2(t)/dt}{4\Gamma_2(t)} \). \( \Gamma_1(t) \) and \( \Gamma_2(t) \) are given by the inverse Laplace transforms of \( \Gamma_1(s) \) and \( \Gamma_2(s) \), respectively

\[ \Gamma_1(s) = \frac{s + D}{s^2 + sD + 4\lambda^2}, \]  

(13)

\[ \Gamma_2(s) = \frac{(s + D)(s + 4D) + 2\lambda^2}{s((s + D)(s + 4D) + 2\lambda^2) + 2\lambda^2(s + 4D)}. \]  

(14)

There are two different dynamical regimes divided by the ratio of the phase diffusion rate \( D \) and the system-noisy laser coupling strength \( \lambda \) [18, 32]. For \( D/\lambda > 4 \) where the system dynamics is corresponding to the Markovian classical noisy region and exhibits an exponential decay behavior. However, for \( D/\lambda < 4 \) in which the system dynamics belongs to the non-Markovian classical noisy region and displays an oscillatory decay behavior.

In the operator-sum representation, the solution of master equation given by Eq.(12) is expressed as \( \rho(t) = \sum_{i=1}^{3} A_i^\dagger \rho(0) A_i \), where the corresponding Kraus operators \( A_i = \sqrt{D} \sigma_i \) (\( i = 0, 1, 2, 3 \)), with \( q_0 = \frac{1}{4}(1 + \Gamma_1 + 2\Gamma_2) \), \( q_1 = q_2 = \frac{1}{4}(1 - \Gamma_1) \), and \( q_3 = \frac{1}{4}(1 + \Gamma_1 - 2\Gamma_2) \).

In what follows, we construct the dynamics of a two-qubit system for two consequent uses of the classical phase noise with the single-qubit reduced density matrix evolution. According to the reported by Macchiavello and Palma [33], the effect of correlated noise on the system of interest can be described using the operator-sum formalism which is defined

\[ \varepsilon(\rho) = \sum_{i,j} P_{ij} (\sigma_i \otimes \sigma_j) \rho (\sigma_i \otimes \sigma_j)^\dagger, \]  

(15)

where \( P_{ij} \) can be interpreted as a joint probability distribution that a random sequence of operations are applied to two consequent qubits transmitted through the noisy channels. For uncorrelated channels, \( P_{ij} = q_i q_j \) means that these probabilities can be factorized and these noisy channels are independent of each other. However, for correlated channels (also known as channels with memory in the literature), \( P_{ij} \) cannot be factorized and these noisy channels are correlated to each other, here \( P_{ij} = (1 - \mu) q_i q_j + \mu q_i \delta_{ij} \), with \( 0 \leq \mu \leq 1 \) is the memory coefficient of channel. Particularly, it is straightforward to recover the case of independent channels for \( \mu = 0 \), while these channels are maximally correlated in case of \( \mu = 1 \).

For a two-qubit probe given by eq.(6), the dynamics of a two-qubit system undergoing two consecutive uses of classical phase noisy laser is worked out
where \( c_\pm = \frac{1}{2}(c_1 \pm c_2), \) \( x_1 = (1 - \Gamma_1)\mu, \) \( x_2 = 2\Gamma_2^2(1 - \mu) + (1 + \Gamma_1)\mu, \) \( c_3 = c_3\delta \) with \( \delta = \Gamma_2^2(1 - \mu) + \mu. \)

Having introduced both the noisy environment with memory and its dynamical evolution, now we will examine how the memory effect arising from successive applications of the channel influences the estimated precision of \( \phi_i. \) In particular, we compare the performances of individual parameter estimation with simultaneous estimation subjected to the decoherence environment.

A. Individual estimation of a single parameter

First of all, let us consider the single-parameter estimation for a two-qubit probe in a correlated classical phase noisy environment. Starting from Eq. (2), and using spectrum decomposition of the density operator given in Eq. (15), the explicit analytic expression of the QFI respect to parameter \( \phi_i \) is derived in Appendix A1. We find the QFI strongly depends on the phase parameter \( \phi_i. \) To estimate the parameter precisely, one should find the optimal input states to maximize QFI. In Appendix A, we find that the optimal two-qubit probes with \((\phi_1, \phi_2) = (\pm \pi/2, 0)\) or \((0, \pm \pi/2)\) lead to the optimal value of the QFI. Under this condition, the explicit forms of the QFI yields

\[
\mathcal{F}_{\phi_1} = \frac{(c_1^2 + c_2^2 + 2c_1c_2\delta^2)\Delta^2}{1 - c_3^2\delta^2} = \mathcal{F}_{\phi_2},
\]

with \( \delta = \Gamma_2^2(1 - \mu) + \mu \) and \( \Delta = \Gamma_2^2(1 - \mu) + \mu. \)

Fig. 1 illustrates the single-parameter QFI for two-qubit probe state as a function of \( \mu \) and \( \lambda t \) in both Markovian and non-Markovian classical noisy regions. In the Markovian classical noisy region, the QFI experiences an exponential decay behavior, with the memory coefficient \( \mu \) increasing, the QFI decays more slowly and a larger amount of QFI could be preserved. This result implies the large memory coefficient \( \mu \) helps to improve the precision of estimation. In particular, for the degree of memory coefficient \( \mu = 1 \) where the classical noisy channel is fully correlated, the QFI is totally unaffected by the Markovian environmental effect. On the other hand, an oscillatory behavior along with revivals of the QFI occurs in the non-Markovian classical noisy region as shown in Fig. 1(b). This result is to be expected and the revivals of QFI are usually attributed to the non-Markovian memory environmental effect where the information flows back from the environment to the system [17, 18]. However, different from the memory coefficient of noisy channel which is arising from successive applications of the channel. With increasing of the memory coefficient \( \mu \), the amount of QFI can be preserved more. Particularly, for the degree of memory \( \mu = 1 \) where the QFI is unaffected by the the non-Markovian effect.

To further understand the physical mechanism of the classical noisy environment effect on the precision of esti-
information, we explore the effect of the ratio $D/\lambda$ of the classical phase noisy laser rate and the system-environment coupling strength on the parameter estimation. Fig. 2 demonstrates the dynamics of the QFI as a function of $\lambda t$ in Markovian and non-Markovian regions for different the ratio $D/\lambda$ with the fixed memory coefficient $\mu = 0.5$. It is clearly shown the dynamics of QFI displays a monotonic decay behavior. The larger the ratio $D/\lambda$ is, the much more slowly the QFI decay as depicted Fig. 2(a). This implies that a sufficiently large ratio $D/\lambda$ can perfectly combat against decoherence even though the system dynamic is in Markovian regions. On the contrary, in non-Markovian regions for different the ratio $D/\lambda$ with the fixed memory coefficient $\mu = 0.5$ as shown in Fig.2(b), collapse and revival phenomena of QFI occur. The smaller the ratio $D/\lambda$ is, the more the amount of QFI revivals. This is because the smaller of ratio $D/\lambda$ leads to the stronger coupling of the system-environment where the more information can be fed back to the system.

B. Simultaneous estimation of two parameters

In what follows, we turn to the situation where two parameters ($\phi_1, \phi_2$) are estimated simultaneously. According to Eq. (4), the explicit analytic expression of the QFIM respect to parameter $\phi_i$ is derived in Appendix A3. For the purpose of estimating phase with a high precision, one should maximize QFIM by optimizing over all possible a two-qubit probe. In the case of ($\phi_1, \phi_2$) = ($\pm \pi/2, 0$) or (0, $\pm \pi/2$), the QFIM achieves its maximum and the explicit forms of the QFIM yields

$$\mathcal{F}_M = \left(\frac{(c_1^2 + c_2^2 + 2c_1c_2\delta)^2}{1 - c_3^2\delta^2}, \frac{(2c_1c_2 + (c_1^2 + c_2^2)c_3\delta)^2}{1 + c_3^2\delta^2}\right).$$

According to the quantum Cramér-Rao bound, the minimal variance of the simultaneous estimation of the parameters $\phi_1$ and $\phi_2$ can be obtained as

$$\delta^2(\phi_1, \phi_2) = \frac{2(c_1^2 + c_2^2 + 2c_1c_2\delta)}{(c_1^2 - c_2^2)^2\Delta^2},$$

For comparison, we have plotted the mean square error of individual and simultaneous estimation case in correlated classical phase noisy environment in Fig.3. It is clearly shown that the minimal total variance corresponding to the simultaneous strategy is always larger than the minimal total variance of the individual strategy. This indicates simultaneous estimation of two parameters leads to less precise estimation than that of the individual estimation. This result can be confirmed by the ratio

$$\mathcal{R} = \frac{(c_1^2 - c_2^2)(1 - c_3^2\delta^2)}{(c_1^2 + c_2^2 + 2c_1c_2\delta\delta^2)^2}$$

which satisfies $0 \leq \mathcal{R} \leq 1$ for any $0 \leq |c_i| \leq 1$ under correlated classical phase noisy environment.

To look into the correlated environment on the simultaneous estimation of two parameters, we explore the effect of the memory effect resulting from the correlated
environment on the estimation precision in both Markovian and non-Markovian classical noisy regions, respectively. As it can be seen from Fig. 4, the larger the memory coefficient is, the lower the estimation error becomes. These results are similar to that of individual estimation case. Especially, when the phase noise are full correlated, namely $\mu \to 1$, the two-phase simultaneous estimation is immune to the correlated environment.

On the other hand, we also explore the effect of the ratio $D/\lambda$ of the classical phase noisy laser rate and the system-environment coupling strength on the parameter estimation in Fig. 5. The error of simultaneous estimation of two parameters can be reduced drastically by increasing the ratio $D/\lambda$ in Markovian regions and decreasing the ratio $D/\lambda$ in non-Markovian regions. The minimum variances give the highest precision for the estimation of parameters $\phi_1$ and $\phi_2$.

IV. QUANTUM FISHER INFORMATION IN A PURE PHASE NOISE

In above section, we discuss the performance of individual and simultaneous estimation in the classical phase noisy environment which is modeled by depolarizing effect. However, for $\gamma_1 = \gamma_2 = 0$, namely $-\frac{d\rho_{ij}(t)}{dt} = 0$, we get the system turns to pure dephasing noise model where the master equation given by Eq. (12) reduces to $\frac{d\rho}{dt} = \gamma_0(\sigma_3\rho\sigma_3 - \rho)$ with $\gamma_0 = -\frac{1}{2\tau}$. Taking the classical phase noise with correlated into consideration, Eq. (15) can be equivalent to correlated dephasing channel by replacing $P_{ij}$ with $Q_{ij} = (1 - \mu)q_iq_j + \mu q_i\delta_{ij}$, in which $q_0 = \frac{1}{2}(1 + \Gamma_2)$, $q_1 = q_2 = 0$, and $q_3 = \frac{1}{2}(1 - \Gamma_2)$.

For a two-qubit probe given by eq. (6), the dynamics of a two-qubit system undergoing two consecutive uses of pure dephasing noisy laser becomes...
The calculations of the QFI and QFIM are similar to the approach from the previous section. The minimal variances of the parameters $\phi_1$ and $\phi_2$ estimation are also accessible, respectively:

$$\delta \phi_1 = \sqrt{\frac{1 - c_3^2}{(c_1^2 + c_2^2 + 2c_1c_2c_3)\Delta^2}}, \quad (22)$$

for the individual estimation phase parameter scenario,

$$\delta \phi_1\phi_2 = \frac{\sqrt{2(c_1^2 + c_2^2 + 2c_1c_2c_3)}}{|(c_1^2 - c_2^2)|\Delta}, \quad (23)$$

for the simultaneous estimation phase parameter. Obviously,

$$\mathcal{R} = \frac{(c_1^2 - c_2^2)(1 - c_3^2)}{(c_1^2 + c_2^2 + 2c_1c_2c_3)^2} \quad (24)$$

which is independent of $\mu$ and $\Gamma_2$, satisfying $0 \leq \mathcal{R} \leq 1$.

V. CONCLUSION

In conclusion, we have extended the parametric estimation from a single qubit system to a two-qubit system subjected to a classical phase noisy laser. We have addressed how the classical noisy channel with memory affects the two-parameter estimation, and compared the performance of individual parameter estimation with simultaneous estimation. Be different from the results obtained in Refs. [27–29] where simultaneous estimation of multiparameter can give a better precision compared to individual estimation case, our results provide the exact opposite. Taking the correlated noise into consideration, our results show the memory effect arising from successive applications of the noisy laser can improve the precision of parametric estimation. With the memory coefficient $\mu$ increasing, the precision of parameter estimation becomes more accuracy. Particularly, for the memory coefficient $\mu = 1$, the phase parametric estimation is immune from the noisy environment. However, for $\mu \neq 1$, the two-parameter estimation precision can be well improved by engineering the ratios of classical phase noisy laser rate and system-environment coupling strength dependent of Markovian or non-Markovian regions. Our results provide an active way to engineer adverse environment and enhance the parameter-estimation precision, which is rather significant in quantum precision measurement and quantum metrology.

Data availability: The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

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