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Adaptive finite-time command filter tracking control of nonlinear system with multiple high-order coupling terms and disturbances

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Abstract: This paper is devoted to the research of adaptive command filter tracking control for a class of uncertain nonlinear systems. The coexistence of multiple high-order coupling terms, unknown coefficients of time-varying function and uncertain external disturbances makes the studied system essentially different from the existing results. By skillfully combining adaptive technology, command filter control and backstepping method, a new type of adaptive command filter tracking control algorithm is designed. This controller not only solves the problem of complex explosion, but also introduces compensation signals to achieve higher precision tracking effect. Ultimately, the validity of the control algorithm is verified by numerical simulation and practical application model simulation experiments.

Keywords: Uncertain nonlinear systems, tracking control, unknown coefficients of time-varying function, adaptive control, command filter, radial basis function-neural networks (RBF).

1. Introduction

In recent decades, the high-precision control in the changing environment of science and technology is the key to realize high automation. Due to the complexity of the control target and its environment, its performance parameter will be affected by multifarious internal or external factors, which makes the traditional control methods receive great constraints in the design process. Therefore, the intelligent control algorithm has gradually entered the scholars’ field of vision, and has made many achievements in intelligent driving, manipulator control, unmanned aerial vehicle control, ship control and other fields, see references [1]-[4].

It is noted that in engineering practice, it is difficult in many cases even that is not possible to get the accurate dynamical model which is totally conform to the the actual system. In addition, some uncertainties and nonlinearities in the actual system must be considered, such as external disturbances, uncertain parameters, model errors and other uncertain factors in the control system. Therefore, timely acquisition of the dynamic characteristics and errors of the system is very important for precise control. In this case, the control algorithm for nonlinear systems with uncertain factors has attracted a large number of scholars’ research, and has made unprecedented development and penetrated into our lives. For example, In [5], a class of tracking control problems with state constraints and disturbances is studied based on neural networks. In [6]-[8], based on neural network (fuzzy control) and backstepping method, an intelligent control algorithm adapted to some nonlinear terms in the system is designed, which further relaxes the assumptions of nonlinear terms and has been effectively verified by simulation in smart grid and multi-agent. In [9], the adaptive fuzzy neural network sliding mode control algorithm is applied to uncertain systems with input actuator and saturation effectiveness faults. However, in the controller design process of the above results, the repeated derivation of virtual control will increase the computational burden, while the amount of computation will become larger with the increase of system dimensions, that is, the complexity explosion problem is not considered.

For solving the problem of complexity explosion, the command filter control technology is introduced, which does not need to compute a derivative of the virtual controller, and a new parameter can be obtained...
from the first-order filter to replace it. Because this technology makes the calculation more convenient and simple, it has been widely used. For example, In [10], the model adaptive control of nonlinear transport aircraft affected by actuators and external disturbances is studied. In [11]-[13], some adaptive neural network (fuzzy) command filter controllers are designed, and good system performance is achieved. However, the above results are all obtained when time is approaching infinity. For some of the actual systems, the finite-time control strategy is increasingly important.

It should be underlined here that compared with the above results of infinite time stability, finite time stability has faster convergence speed, higher accuracy and better robustness. Due to this advantage, many scholars pay more attention to the finite-time and get a mass of research results. In [14], the finite-time controller for a class of nonlinear systems with actuator failures is studied. In [15], a novel finite-time control algorithm is designed for the system with full-state constraints. In [16]-[17], the finite-time control scheme of high-order nonlinear system is studied by combining neural network and backstepping method. However, none of the above literatures have studied the existence of multiple high-order coupling terms in the system.

Based on the problems mentioned above, we first study the finite-time command filter tracking control issue for a type of high-order nonlinear systems with multiple high-order coupling terms, external disturbances and time-varying parameter coefficients. The contribution of this paper is highlighted from the following three aspects

(i) So far as we know, the command filter tracking control of neural network for nonlinear systems with multi-order coupling terms, unknown coefficients of time-varying function and external disturbances is studied for the first time.

(ii) By introducing the compensation signal in each step of the design of the virtual controller, the designed tracking controller has a higher precision tracking effect.

(iii) Combining adaptive control technology, neural network and dynamic surface control technology, the tracking control problem of time-varying parameter coefficient, unknown nonlinear term and complex explosion problem is solved. The designed command filter controller can make the tracking error converge to a small area near zero in a finite time.

The organizational form of the article is as below. The second part gives the basic knowledge of this paper and the structure of the system. The third part gives the detailed design process and stability analysis. Part four makes a simulation experiment of the system, and gives the verification results of the control algorithm. Finally, the research contents are summarized and prospected.

2. Preliminaries and system description

2.1. Preliminaries

As part of the control design process, radial basis functions neural networks (RBFNN) are used to approximate continuous nonlinear functions: \( f(Z) : R^n \to R \),

\[
f(Z) = W^T \Phi(Z),
\]

where \( Z \in \Omega_Z \) is called the input vector, the weight vector \( W = [w_1, \cdots, w_l] \in R_l, l \geq 1 \) is node number, and the vector \( \Phi(Z) = [\phi_1(Z), \cdots, \phi_l(Z)] \in R^l \) is the radial basis function vector where \( \phi_i(Z) \) is selected as the Gaussian function

\[
\phi_i(Z) = e^{-\frac{(Z-v_i)^T(Z-v_i)}{\nu^2}}, i = 1, 2, \cdots, l,
\]
where \( v_i = [v_{i1}, \cdots, v_{iq}] \) are the center of the receptive domain and \( \nu \) denotes the width of the Gaussian function.

As described in [18], if \( f(Z) \) is continuous and \( Z \in \Omega_Z \) with \( \Omega_Z \) being a compact set, then for any given constant \( \varepsilon^* > 0 \), there is a neural network \( W^* \Phi(Z) \) such that

\[
f(Z) = W^* \Phi(Z) + \varepsilon(Z), \quad \forall Z \in \Omega_Z,
\]

(1)

where the ideal weight vector \( W^* \) is defined as

\[
W^* = \arg \min_{W \in \mathbb{R}^l} \left[ \sup_{Z \in \Omega_Z} |f(Z) - W^T \Phi(Z)| \right],
\]

(2)
\( \varepsilon(Z) \) denotes the approximation error that satisfies the inequality \( |\varepsilon(Z)| \leq \varepsilon^* \). Fig 1 shows the adaptive mechanism based on RBF-NNs.

**Lemma 1** [19]: The radial basis function of the neural network is presented as follows: \( \Phi(X_n) = [\phi_1(X_n), \cdots, \phi_l(X_n)]^T \), and the input vector is represented by \( X_n = [x_1, \cdots, x_n] \). Then, the following inequality holds

\[
\| \Phi(X_n) \|^2 \leq \| \Phi(X_m) \|^2,
\]

(3)

where \( m \leq n \).

**Lemma 2** [20]: For any \( z_j \in R, j = 1, 2, \cdots, n \), if \( l \geq 1 \), then there holds

\[
\sum_{j=1}^{n} |z_j|^l \leq \left( \sum_{j=1}^{n} |z_j|^l \right) \leq n^{l-1} \sum_{j=1}^{n} |z_j|^l,
\]

(4)

for \( \forall z_j \in R, j = 1, 2, \cdots, n \).

**Definition 1** [21]: The origin \( \xi = 0 \) of nonlinear system (6) is semi-global practical finite-time stable (SGPFTS), if for all initial values \( \xi(t_0) = \xi_0 \), there exists a constant \( \varepsilon \geq 0 \) and a setting time \( T(\varepsilon, \xi_0) \leq \infty \) to make \( \| \xi(t) \| \leq \varepsilon \), for all \( t \geq t_0 + T \).

**Lemma 3** [21]: Consider the nonlinear system \( \dot{\xi} = f(\xi) \) and the Lyapunov function \( V_n(\xi) \). If there exist \( \varepsilon > 0, \gamma > 0 \), and \( 0 < \varsigma < 1 \) such that

\[
\dot{V}_n(\xi) \leq -cV_n^\varsigma(\xi) + \gamma,
\]

(5)

then the nonlinear systems are semi-globally practically finite-time stable (SGPFTS).
2.2. System description

In this paper, the following nonlinear system with multiple high-order coupling terms, unknown coefficients of time-varying function and uncertain external disturbances is considered:

\[
\begin{align*}
\dot{x}_i &= \sum_{r=1}^{s} g_{i,r}(x)x_{i+1}^{p_{r,i}} + f_i(x) + d_i(t), \\
\dot{x}_n &= \sum_{r=1}^{s} g_{n,r}(x)u_{n}^{p_{n,r}} + f_n(x) + d_n(t), \\
y &= x_1,
\end{align*}
\]

where \( x = (x_1, x_2, \ldots, x_n) \) denotes the system state, \( y \) and \( u \) represent system output and control input respectively. \( f_i \) represents a unknown nonlinear function, \( p_r \) is a ratio of two positive odd integers, \( g_{i,r} \) represents a unknown control gain, \( d_i \) represents the bounded but unknown external disturbance.

**Assumption 1.** The sign of \( g_{i,r}(x_i) \), \((i = 1, 2, \ldots, n, r = 1, 2, \ldots, s) \) is known and there exist positive constants \( g_m \) and \( g_M \) such that system (6) satisfies \( g_m \leq |g_{i,j}| \leq g_M \). In order not to lose generality, we shall assume that \( g_m \leq g_{i,r} \leq g_M \).

**Assumption 2.** There is positive constants \( \bar{d}_i \) such that \(|d_i(t)| \leq \bar{d}_i \).

**Assumption 3.** There are positive numbers \( B_0 \) and \( B_1 \) such that the following condition holds

\[ |y_r| \leq B_0, \|\dot{y}_r\| \leq B_1. \]  

**Remark 1.** Assumption 3 relaxes the conditions in some references that require the desired tracking signal and higher-order derivatives to be bounded. In the backstepping method, some results such as [22]-[24] required the desired tracking signal \( y_r \) and its derivatives \( y_r^{(i)} \) to have an upper bound, i.e., \( |y_r^{(i)}| < B_i \), \( i = 0, 1, \ldots, n \). While in [25]-[27] where the dynamic surface control method is introduced, it is required that the desired tracking signal \( y_r \) together with its time derivatives \( \dot{y}_r \) and \( \ddot{y}_r \) are known and bounded or its time derivatives up to \( n \)-th order remain bounded. For the further relaxation, only \( \dot{y}_r \) besides \( y_r \) is demanded to be bounded, see [28]-[30]. However, all above results studied system (6) with \( p_r = 1 \). In this paper, for system (6) with \( p_r \geq 1 \), with the weakest Assumption 3, the tracking control issue is considered for the first time being.

**Remark 2.** When there are unknown nonlinear terms in the system, many scholars deal with them by fuzzy control or neural network control, and some achievements have been made. However, they ignored the influence of the learning time of neural network on the system, so by introducing dynamic surface technology here we can not only solve the differential explosion issue in backstepping, but also reduce the neural network inputs. This results in a shorter learning time for the neural networks and improved performance.

3. Adaptive command filter controller design and stability analysis

The control goal is to devise command filter controller for system (6), such that the output should be able to track the desired tracking signal, and all signals of the closed-loop system are semi-globally bounded. This diagram shows the system’s control structure in Fig 2.

3.1. Adaptive command filter controller design

The detailed design steps of adaptive command filter tracking controller for system (6) are given below.
Step 1. Define the following tracking error:

$$z_1 = y - y_d,$$

where $y_d$ is the signal to be tracked, and the derivative is

$$\dot{z}_1 = \dot{y} - \dot{y}_d = \sum_{r=1}^{s} g_{1,r} x_{2,p}^r + f_1 + d_1 - \dot{y}_d.$$  

(9)

Because $f_1$ is unknown in the system (6), the traditional design method cannot be used. So we need to use RBF neural network to approximate it. Define

$$\tilde{f}_1 = f_1 - \dot{y},$$

where $Z_1 = (x_1, \ldots, x_n, \dot{y})$ and $\sigma_1$ is the maximum approximation error, then substituting (10) into (9) yields

$$\dot{z}_1 = \dot{y} - \dot{y}_d = \sum_{r=1}^{s} g_{1,r} x_{2,p}^r + W_1^T \Phi_1(Z_1) + \epsilon_1(Z_1) + d_1.$$  

(11)

In order to overcome the complexity explosion problem, we introduce command filter control technology to solve this problem, and let the first virtual control signal $\alpha_1$ pass through a first-order filter with a constant $T_2 \geq 0$, that is,

$$\begin{cases} T_2 \dot{x}_{2,v} + x_{2,v} = \alpha_1, \\ x_{2,v}(0) = \alpha_1(0). \end{cases}$$  

(12)

Because the introduction of the first-order filter will lead to an filter error $(x_{2,v} - \alpha_1)$ in the system (6), so we introduce a compensation variable $\eta_1$ by

$$\begin{cases} \dot{\eta}_1 = -k_1 g_{M} \eta_1^{\alpha_2} - 1 + \sum_{r=1}^{s} g_{1,r} \eta_2^r + \sum_{r=1}^{s} g_{1,r} (x_{2,v} - \alpha_1)^r, \\ \eta_1(0) = 0, \end{cases}$$  

(13)

where $\eta_2$ is defined in the next step, $0 < \varsigma < 1$ is a constant to be designed.

Define the first compensation error variable

$$\lambda_1 = z_1 - \eta_1,$$

and select the first Lyapunov function

$$V_1 = \frac{1}{2} \lambda_1^2 + \frac{1}{2T_1} \tilde{\theta}_1^2,$$  

(15)
where \( \hat{\theta}_1 = \theta_1 - \hat{\theta}_1, \hat{\theta}_1 \) denote the estimate of \( \theta_1 \), and \( \Gamma_1 > 0 \) is a parameters to be designed. Then, deriving the above equation and bring (11)-(14) into \( \dot{V}_1 \), one obtains

\[
\dot{V}_1 = \lambda_1 \lambda_1 - \hat{\theta}_1 \Gamma_1^{-1} \hat{\theta}_1
\]

\[
= \lambda_1 \left( \sum_{r=1}^{s} g_{1,r}x_{2,\nu}^{p_r} + W_1^T \Phi_1(Z_1) + \epsilon_1(Z_1) + d_1 + k_1 g_M \eta_1^{2,\nu - 1}
\right.
\]

\[
- \sum_{r=1}^{s} g_{1,r} \eta_{2,\nu}^{p_r} - \sum_{r=1}^{s} g_{1,r} (x_{2,\nu} - \alpha_1)^{p_r}) - \hat{\theta}_1 \Gamma_1^{-1} \hat{\theta}_1
\]

\[
\leq \lambda_1 \left( W_1^T \Phi_1(Z_1) + \epsilon_1(Z_1) + d_1 + k_1 g_M \eta_1^{2,\nu - 1} + \sum_{r=1}^{s} g_{1,r} \lambda_2^{p_r} + \sum_{r=1}^{s} g_{1,r} \alpha_1^{p_r}) \right) - \hat{\theta}_1 \Gamma_1^{-1} \hat{\theta}_1.
\]  

(16)

In the light of Young’s inequality \(^1\) and Lemma 1, one obtains

\[
\lambda_1 (W_1^T \Phi_1(Z_1) + \epsilon_1(Z_1)) \leq \frac{g_M}{2a_1} \lambda_2 \lambda_1 \Phi_1(X_1) \Phi_1(X_1) + \frac{a_1^2}{2g_M} + \frac{g_M}{2} \lambda_1^2 + \frac{1}{2g_M} \sigma_1^2,
\]

\[
\lambda_1 d_1 \leq \frac{g_M}{2} \lambda_1^2 + \frac{1}{2g_M} d_1^2,
\]  

(17)

where \( \theta_1 = \|W_1^*\|^2, X_1 = (x_1, y)^T. \)

By bringing (17) into (16), we can get

\[
\dot{V}_1 = \lambda_1 \lambda_1 - \hat{\theta}_1 \Gamma_1^{-1} \hat{\theta}_1
\]

\[
= \lambda_1 \left( \sum_{r=1}^{s} g_{1,r}x_{2,\nu}^{p_r} + W_1^T \Phi_1(Z_1) + \epsilon_1(Z_1) + d_1 + k_1 g_M \eta_1^{2,\nu - 1}
\right.
\]

\[
- \sum_{r=1}^{s} g_{1,r} \eta_{2,\nu}^{p_r} - \sum_{r=1}^{s} g_{1,r} (x_{2,\nu} - \alpha_1)^{p_r}) - \hat{\theta}_1 \Gamma_1^{-1} \hat{\theta}_1
\]

\[
\leq \lambda_1 \left( W_1^T \Phi_1(Z_1) + \epsilon_1(Z_1) + d_1 + k_1 g_M \eta_1^{2,\nu - 1} + \sum_{r=1}^{s} g_{1,r} \lambda_2^{p_r} + \sum_{r=1}^{s} g_{1,r} \alpha_1^{p_r}) \right) - \hat{\theta}_1 \Gamma_1^{-1} \hat{\theta}_1
\]

\[
\leq \lambda_1 \left( \frac{g_M}{2a_1} \lambda_1 \theta_1 \Phi_1^T(X_1) \Phi_1(X_1) + k_1 g_M \eta_1^{2,\nu - 1} + \sum_{r=1}^{s} g_{1,r} \lambda_2^{p_r} + \sum_{r=1}^{s} g_{1,r} \alpha_1^{p_r}) \right) + \frac{a_1^2}{2g_M} + \frac{g_M}{2} \lambda_1^2
\]

\[
+ \frac{1}{2g_M} \sigma_1^2 + \frac{g_M}{2} \lambda_1^2 + \frac{1}{2g_M} d_1^2 - \hat{\theta}_1 \Gamma_1^{-1} \hat{\theta}_1.
\]  

(18)

Choose the first virtual control and adaptive law

\[
\alpha_1 = \left( -k_1 z_1^{2,\nu - 1} - \frac{1}{2a_1} \lambda_1 \hat{\theta}_1 \Phi_1^T \Phi_1 \right) \frac{1}{s},
\]  

(19)

\[
\hat{\theta}_1 = \frac{\Gamma_1 g_M}{2a_1} \lambda_1 \Phi_1^T \Phi_1 - \psi_1 \hat{\theta}_1,
\]  

(20)

where \( \alpha_1, k_1, \psi_1 \) are constants to be designed and \( \rho = \min\{r_1, r_2, \cdots, r_s\}. \)

By bringing (19) and (20) into (18) and applying the inequality \( \hat{\theta}_1 \hat{\theta}_1 \leq -\frac{1}{2} \|\hat{\theta}_1\|^2 + \frac{1}{2} \|\theta_1\|^2 \), one has

\[
\dot{V}_1 \leq -(k_1 - 1) g_M \lambda_1^2 - \frac{\psi_1}{2} \|\hat{\theta}_1\|^2 + \lambda_1 \sum_{r=1}^{s} g_{1,r} \lambda_2^{p_r} + \rho_1
\]  

(21)

where \( \rho_1 = \frac{1}{2g_M} (a_1^2 + \sigma_1^2 + d_1^2) + \frac{\psi_1}{2a_1} \|\theta_1\|^2. \)

\(^1\)For any variables \( \kappa \) and \( \omega \), and positive constant \( a, b, c \), one has \( |\kappa|^a |\omega|^b \leq \frac{a}{a+b} c |\kappa|^{a+b} + \frac{b}{a+b} c^{-\frac{b}{a+b}} |\omega|^{a+b}. \)
Step i: Define \( z_i = x_i - x_{i,v} \), then

\[
\dot{z}_i = \dot{x}_i - \dot{x}_{i,v} = \sum_{r=1}^{s} g_{i,r}(x_i)x_{r+1}^{pr} + f_i(x_i) + d_i(t) - \dot{x}_{i,v}. \tag{22}
\]

By defining \( \bar{f}_i = f_i - \dot{x}_{i,v} = \sum_{r=1}^{s} g_{i,r}x_{r+1}^{pr} + W_i^{*T}\Phi_i(Z_i) + \epsilon_i(Z_i), \) \(|\epsilon_i| \leq \sigma_i \), where \( \sigma_i \) is the maximum approximation error, \( Z_i = (x_1, x_2, \cdots, x_n, \dot{x}_{i,v}) \), and substituting it into (22), one can get

\[
\dot{z}_i = \dot{z}_i - \dot{x}_{i,v} = \sum_{r=1}^{s} g_{i,r}x_{r+1}^{pr} + W_i^{*T}\Phi_i(Z) + \epsilon_i(Z) + d_i. \tag{23}
\]

Same as the first step, the virtual control signal \( \alpha_i \) passes through the following first-order filter with a constant \( T_{i+1} \geq 0 \):

\[
\begin{aligned}
T_{i+1}\dot{x}_{i+1,v} + x_{i+1,v} &= \alpha_i, \\
x_{i+1,v}(0) &= \alpha_i(0).
\end{aligned} \tag{24}
\]

In order to estimate this filter error, this compensation signal \( \eta_i \) is designed

\[
\begin{aligned}
\dot{\eta}_i &= -k_i g M\eta_i^{2\kappa-1} + \sum_{r=1}^{s} g_{i,r}\eta_i^{pr} + \sum_{r=1}^{s} g_{i-1,r}\lambda_i^{pr} + \sum_{r=1}^{s} g_{i,r}(x_{i+1,v} - \alpha_i)^{pr}, \\
\eta_i(0) &= 0,
\end{aligned} \tag{25}
\]

where \( \eta_{i+1} \) is defined at \( n \) step.

The \( i \) compensation error is introduced as

\[
\lambda_i = z_i - \eta_i, \tag{26}
\]

and select the \( i \) Lyapunov function

\[
V_i = V_{i-1} + \frac{1}{2}\lambda_i^2 + \frac{1}{2}\theta_i^2 \tag{27}
\]

Deriving (27) and bring (23)-(26) into \( \dot{V}_i \), one obtains

\[
\dot{V}_i = \dot{V}_{i-1} + \lambda_i\dot{\lambda}_i - \theta_i \dot{\theta}_i \dot{i}
\]

\[
= \dot{V}_{i-1} + \lambda_i \left( \sum_{r=1}^{s} g_{i,r}x_{r+1}^{pr} + W_i^{*T}\Phi_i(Z_i) + \epsilon_i(Z_i) + d_i + k_i g M\eta_i^{2\kappa-1} - \sum_{r=1}^{s} g_{i-1,r}\lambda_i^{pr} - \sum_{r=1}^{s} g_{i,r}(x_{i+1,v} - \alpha_i)^{pr} \right) - \theta_i \Gamma_i^{-1}\dot{\theta}_i
\]

\[
\leq \dot{V}_{i-1} + \lambda_i \left( \sum_{r=1}^{s} g_{i,r}x_{r+1}^{pr} + W_i^{*T}\Phi_i(Z_i) + \epsilon_i(Z_i) + d_i + k_i g M\eta_i^{2\kappa-1} - \sum_{r=1}^{s} g_{i-1,r}\lambda_i^{pr} - \sum_{r=1}^{s} g_{i,r}\lambda_i^{pr} + \sum_{r=1}^{s} g_{i,r}\alpha_i^{pr} \right)
\]

\[
- \theta_i \Gamma_i^{-1}\dot{\theta}_i. \tag{28}
\]

Using Young’s inequality and Lemma 1, we can get

\[
\lambda_i(W_i^{*T}\Phi_i(Z_i) + \epsilon_i(Z_i)) \leq \frac{g M}{2a_i} \lambda_i^2 \theta_i \Phi_i^T(X_i)\Phi_i(X_i) + \frac{a_i^2}{2g M} \lambda_i^2 + \frac{g M}{2} \lambda_i^2 + \frac{1}{2g M} \sigma_i^2,
\]

\[
\lambda_i d_i \leq \frac{g M}{2} \lambda_i^2 + \frac{1}{2g M} a_i^2, \tag{29}
\]

where \( \theta_i = ||W_i^*||^2, X_i = (x_1, \cdots, x_n, \dot{x}_{i,v})^T \).
By plugging (29) into (28), one has
\[
\dot{V}_i = \dot{V}_{i-1} + \lambda_i \hat{\alpha}_i - \tilde{\theta}_i \Gamma_i^{-1} \dot{\hat{\theta}}_i
\]
\[
= \dot{V}_{i-1} + \lambda_i \left( \sum_{r=1}^{s} g_{i,r} \rho_{ir}^{pr} + W_i^T \Phi_i(Z_i) + \epsilon_i(Z_i) + d_i + k_i g_M \eta_i^{2k-1} - \sum_{r=1}^{s} g_{i,r} \rho_{ir}^{pr} - \sum_{r=1}^{s} g_{i-1,r} \lambda_i^{pr} \right)
\]
\[
- \sum_{r=1}^{s} g_{i,r} (x_{i+1,v} - \alpha_i) - \tilde{\theta}_i \Gamma_i^{-1} \dot{\hat{\theta}}_i
\]
\[
\leq \dot{V}_{i-1} + \lambda_i \left( W_i^T \Phi_i(Z_i) + \epsilon_i(Z_i) + d_i + k_i g_M \eta_i^{2k-1} - \sum_{r=1}^{s} g_{i-1,r} \lambda_i^{pr} - \sum_{r=1}^{s} g_{i,r} \rho_{ir}^{pr} + \sum_{r=1}^{s} g_{i,r} \alpha_i^{pr} \right)
\]
\[
- \tilde{\theta}_i \Gamma_i^{-1} \dot{\hat{\theta}}_i
\]
\[
\leq \dot{V}_{i-1} + \lambda_i \left( \frac{2gM}{2a_i} \lambda_i \Phi_i^T(X_i) \Phi_i(X_i) + k_i g_M \eta_i^{2k-1} - \sum_{r=1}^{s} g_{i-1,r} \lambda_i^{pr} + \sum_{r=1}^{s} g_{i,r} \lambda_i^{pr} + \sum_{r=1}^{s} g_{i,r} \alpha_i^{pr} \right)
\]
\[
+ \frac{a_i^2}{2gM} + \frac{2g}{2} \lambda_i^2 + \frac{1}{2gM} \sigma_i^2 + \frac{gM}{2} \lambda_i^2 + \frac{1}{2gM} d_i^2 - \tilde{\theta}_i \Gamma_i^{-1} \dot{\hat{\theta}}_i.
\]
(30)

The virtual control signal \( \alpha_i \) and adaptive law \( \hat{\theta}_i \) are designed as
\[
\alpha_i = \left( \frac{-k_i \eta_i^{2k-1} - \frac{1}{2a_i} \lambda_i \Phi_i^T \Phi_i}{s} \right)^{\frac{1}{2}}
\]
(31)
\[
\dot{\hat{\theta}}_i = \frac{\Gamma_i g_M}{2a_i} \lambda_i \Phi_i^T \Phi_i - \psi_i \hat{\theta}_i,
\]
(32)
where \( a_i, k_i, \psi_i \) are constants to be designed.

By plugging (31) and (32) into (30), and employing the inequality \( \tilde{\theta}_i \hat{\theta}_i \leq -\frac{1}{2} || \tilde{\theta}_i ||^2 + \frac{1}{2} || \hat{\theta}_i ||^2 \), one has
\[
\dot{V}_i \leq - \sum_{\ell=1}^{i} (k_{\ell} - 1) g_M \lambda_i^{2k-1} - \sum_{\ell=1}^{i} \psi_i \ell \tilde{\theta}_i || \tilde{\theta}_i ||^2 + \lambda_i \sum_{r=1}^{s} g_{i,r} \lambda_i^{pr} + \sum_{\ell=1}^{i} \rho_{\ell},
\]
(33)
where \( \rho_{\ell} = \frac{1}{2gM} (a_i^2 + \sigma_i^2 + d_i^2) + \frac{\psi_i}{2a_i} \| \theta_i \|_e^2 \).

**Step n:** Same as the i-step, the derivative \( z_n \) is
\[
\dot{z}_n = \dot{x}_n - \dot{x}_{n,v} = \sum_{r=1}^{s} g_{n,r} (x_n) u^{pr} + f_n(x_n) + d_n(t) - \dot{x}_{n,v},
\]
(34)
and by defining \( \tilde{f}_n = f_n + d_n = W_n^T \Phi_n(Z_n) + \epsilon_n(Z_n), |\epsilon_n| \leq \sigma_n \), where \( \sigma_n \) is the maximum approximation error, \( Z_n = (x_1, x_2, \ldots, x_n, \dot{x}_{n,v}) \), then plugging it into (34), one has
\[
\dot{z}_n = \tilde{z}_n - \dot{x}_{n,v} = \sum_{r=1}^{s} g_{n,r} u^{pr} + W_n^T \Phi_n + \epsilon_n + d_n.
\]
(35)

This is the last step, and the last compensation below is introduced
\[
\begin{aligned}
\eta_n &= -k_n g_M \eta_n^{2k-1} + \sum_{r=1}^{s} g_{n,r} \lambda_n^{pr}, \\
\eta_n(0) &= 0.
\end{aligned}
\]
(36)
Define the n compensation error variable
\[
\lambda_n = z_n - \eta_n,
\]
(37)
and selecte the $n$ Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2} \dot{\lambda}_n^2 + \frac{1}{2} \ddot{\theta}_n^2,$$

thus, its derivative is

$$\dot{V}_n = \dot{V}_{n-1} + \lambda_n \dot{\lambda}_n - \ddot{\theta}_n \Gamma_n^{-1} \dot{\theta}_n$$

$$\leq \dot{V}_{n-1} + \lambda_n \sum_{r=1}^{s} g_{n,r} \dot{u}_p^r + W_{n}^* \Phi_n(Z_n) + \epsilon_n(Z_n) + d_n + k_n g_M \eta_n^{2c-1}$$

$$- \sum_{r=1}^{s} g_{n,r} \lambda_n^p - \ddot{\theta}_n \Gamma_n^{-1} \dot{\theta}_n. \quad (39)$$

By utilizing Young’s inequality and Lemma 1, one obtains

$$\lambda_n (W_{n}^* \Phi_n(Z_n) + \epsilon_n(Z_n)) \leq \frac{g_M}{2a_n} \lambda_n^2 \Phi_n^T(X_n) \Phi_n(X_n) + \frac{a_n^2}{2g_M} + \frac{g_M}{2} \lambda_n^2 + \frac{1}{2} \sigma_n^2,$$

$$\lambda_n d_n \leq \frac{g_M}{2} \lambda_n^2 + \frac{1}{2g_M} d_n^2. \quad (40)$$

where $\theta_n = \|W_n\|^2, X_n = (x_1, \ldots, x_n, \dot{x}_{n,v})^T$.

Dragging (40) in (39) yields

$$\dot{V}_n = \dot{V}_{n-1} + \lambda_n \dot{\lambda}_n - \ddot{\theta}_n \Gamma_n^{-1} \dot{\theta}_n$$

$$\leq \dot{V}_{n-1} + \lambda_n \left( \frac{g_M}{2a_n} \lambda_n \Phi_n^T(X_n) \Phi_n(X_n) + k_n g_M \eta_n^{2c-1} - \sum_{r=1}^{s} g_{n,r} \lambda_n^p - \sum_{r=1}^{s} g_{n,r} \lambda_n^p \right)$$

$$+ \frac{a_n^2}{2g_M} + \frac{g_M}{2} \lambda_n^2 + \frac{1}{2g_M} \sigma_n^2 + \frac{g_M}{2} \lambda_n^2 + \frac{1}{2g_M} d_n^2 - \ddot{\theta}_n \Gamma_n^{-1} \dot{\theta}_n. \quad (41)$$

Finally, Selecting actual controller and adaptive law

$$u = \left( \frac{-k_n \eta_n^{2c-1} - \frac{1}{2a_n} \lambda_n \Phi_n^T \Phi_n}{S} \right)^{\frac{1}{2}}, \quad (42)$$

$$\dot{\theta}_n = \frac{\Gamma_n g_M}{2a_n} \lambda_n \Phi_n \Phi_n - \psi_n \dot{\theta}_n, \quad (43)$$

where $\alpha_n, k_n, \psi_n$ are constants to be designed.

By plugging (42) and (43) into (41), and applying the inequality $\dot{\theta}_n \dot{\theta}_n \leq -\frac{1}{2} \| \dot{\theta}_n \|^2 + \frac{1}{2} \| \theta_n \|^2$, one has

$$\dot{V}_n \leq -\sum_{\ell=1}^{n} (k_\ell - 1) g_M \lambda_\ell^{2c} - \sum_{\ell=1}^{n} \psi_\ell \| \dot{\theta}_\ell \| + \sum_{\ell=1}^{n} \rho_\ell, \quad (44)$$

where $\rho_\ell = \frac{1}{2g_M} (a_\ell^2 + \sigma_\ell^2 + d_\ell^2) + \frac{\psi_\ell}{2g_M} \| \theta_\ell \|^2$.

Up to this point, the design process of the whole control is completed.

**Remark 3.** By introducing command filter control technology, the issue of differential explosion in backstepping method is avoidable settle and the design of control becomes simple. That’s because the differential of $\dot{\alpha}_i$ is replaced by $\alpha_{i+1,v}^\prime$ and $\dot{\alpha}_{i+1,v}^\prime$. $\alpha_{i+1,v}^\prime$ is the output of command filter (24) in the design process. Therefore, the repeated derivation of virtual control is effectively avoided.
3.2. Stability analysis

**Theorem 1.** For high-order nonlinear system (6) with Assumptions 1-3, under the designed controller (42) together with the intermediate control functions (19), (31) and adaptive laws (20), (32), (43), all signals of the closed-loop system are bounded, and the tracking error converges to a small neighborhood of zero in finite time.

**Proof.** Consider the following entire candidate Lyapunov function:

\[ V = V_n = \sum_{\ell=1}^{n} \left( V_\ell + \frac{1}{2T_\ell} \dot{\hat{\theta}}_\ell^2 \right). \]  

(45)

By differentiating \( V \), we can get

\[ \dot{V} = \sum_{\ell=1}^{n} \left( \dot{V}_\ell + \frac{1}{T_\ell} \dot{\hat{\theta}}_\ell \dot{\hat{\theta}}_\ell \right). \]  

(46)

Bring (19), (31) and (42) into (46), and applying (20), (32) and (43), it follows that

\[ \dot{V} \leq -\sum_{\ell=1}^{n} (k_\ell - 1)g_{M} \lambda_\ell^2 - \sum_{\ell=1}^{n} \frac{\psi_\ell}{2T_\ell} \| \dot{\hat{\theta}}_\ell \|^2 + \sum_{\ell=1}^{n} \rho_\ell. \]  

(47)

Using Young’s inequality, one has

\[ \left( \sum_{\ell=1}^{n} \frac{\dot{\hat{\theta}}_\ell^2}{2T_\ell} \right)^{\varsigma} \leq (1 - \varsigma) \varsigma^{\frac{1}{1-\varsigma}} + \sum_{\ell=1}^{n} \frac{\dot{\hat{\theta}}_\ell^2}{2T_\ell}. \]  

(48)

According to Theorem 3, combined with (47) and (48), one has

\[ \dot{V} \leq -2h \left( \sum_{\ell=1}^{n} \frac{\lambda_\ell^2}{2} \right)^{\varsigma} - h \left( \sum_{\ell=1}^{n} \frac{\dot{\hat{\theta}}_\ell^2}{2T_\ell} \right)^{\varsigma} + \rho, \]  

(49)

where \( \rho = \sum_{\ell=1}^{n} \rho_\ell + h(1 - \varsigma) \varsigma^{\frac{1}{1-\varsigma}} \), \( h = \min\{ (k_\ell - 1)g_{M}, \psi_\ell, \ell = 1, 2, \cdots, n \} \). Then from Lemma 3, one has

\[ \dot{V} \leq -hV^{\varsigma} + \rho. \]  

(50)

In the light of (50), \( \forall 0 < \tau < 1 \), one has

\[ \dot{V}(\chi) \leq -\tau hV^{\varsigma}(\chi) - (1 - \tau)hV^{\varsigma}(\chi) + \rho, \]  

(51)

where \( \chi = [z_1, \cdots, z_n, \hat{\theta}_1, \cdots, \hat{\theta}_n]^T \). Let \( \Phi_\chi = \left\{ \chi \mid V^{\varsigma}(\chi) \leq \frac{\rho}{(1-\tau)h} \right\} \) and \( \Phi_\chi^c = \left\{ \chi \mid V^{\varsigma}(\chi) > \frac{\rho}{(1-\tau)h} \right\} \).

If \( \chi \in \Phi_\chi \), then one has

\[ \dot{V}(\chi) \leq -\tau hV^{\varsigma}(\chi). \]  

(52)

Integrating both sides of (52) from 0 to \( T \) at the same time, one obtains

\[ \int_{0}^{T} \frac{\dot{V}(\chi)}{V^{\varsigma}(\chi)} \, dt \leq -\int_{0}^{T} \tau h \, dt, \]  

(53)
which yields
\[
\frac{1}{1-\zeta} V^{1-\zeta}(\chi(T)) - \frac{1}{1-\zeta} V^{1-\zeta}(\chi(0)) \leq -\tau h T.
\] (54)

Let
\[
T_r = \frac{1}{(1-\zeta)\tau h} \left[ V^{1-\zeta}(\chi(0)) - \left( \frac{\rho}{(1-\tau)h} \right)^{\frac{1}{1-\zeta}} \right].
\] (55)

According to (54) and (55), an upper limit time \( T_r \) can be obtained. Therefore, the closed-loop system is SGPFTS.

**Remark 4.** Adaptive command filter control algorithm integrates the preponderance of adaptive technology, neural network and command filter control, and this technique is capable of dealing with more complex nonlinear system problems and achieving better performance.

(i) Adaptive control method can learn all kinds of dynamic characteristics of system in real time, and then automatically update controls parameter according to the real-time changes of the controlled system. This leads to stronger robustness of the controller when there are various uncertain factors in the system.

(ii) Neural control algorithm is an intelligent control algorithm that can dispose of unknown items in the system. It has low requirements for the model of the system, so it is more widely used in practical applications. In addition, we skillfully use the characteristics of neural network to solve the difficulty of controller design for non-strict feedback systems, relax the assumptions and simplify the design of tracking controller for high-order nonlinear systems.

(iii) Although some literatures, such as [31]-[33], have solved the problem of unknown nonlinear terms in some systems by combining neural network control with backstepping and recursion technology, but, they have not considered the differential explosion problem of high-order systems and the problem of finite time convergence, and the tracking error accuracy caused by the introduction of filters has not been considered. Therefore, this paper not only introduces command filter to solve the differential explosion problem, but also redesigns the compensation signal to make the tracking accuracy very good. At the same time, the design controller can make that system converge quickly in a finite time.

4. Simulation result

**Example 1.** A simulation example of the second-order is presented in order to show the effectiveness of the method.

\[
\dot{x}_1 = (2 + \cos x_1 x_2)x_2 + x_2^5 + x_2 e^{-0.5x_1} + d_1(t),
\]
\[
\dot{x}_2 = (3 + \cos x_1 x_2)u + \sin(x_1 x_2) + d_2(t),
\]
\[
y = x_1,
\] (56)

where \( f_1 = x_2 e^{-0.5x_1}, f_2 = \sin(x_1 x_2), g_1 = 2 + \cos x_1 x_2, g_2 = 3 + \cos x_1 x_2 \). The unknown external disturbance is selected as \( d_1(t) = 0.5 \sin t, d_2(t) = 0.7 \cos 1.5t \). The expected tracking signal \( y_d = 1.2 \sin t \).

The relevant parameters of the simulation experiment are given below: \( k_1 = 25, k_2 = 220, \Gamma_1 = \Gamma_2 = 0.2, T_2 = 0.02, \psi_1 = 0.5, \psi_2 = 0.6 \). With the initial conditions: \( x_1 = 0.05, x_2 = 0.5, \hat{\theta}_1 = 5, \hat{\theta}_2 = -3, x_{2,v} = 0.1, \eta_1 = 0.5, \eta_2 = 2 \), simulation results are shown in the following Figs.3-8. Fig.3 shows that the output signal tracks a given expected signal. Fig.4 and Fig.5 shows the changes of controller \( u \) and state \( x_2 \), respectively. Fig.6 and Fig.7 shows the trajectory of adaptive laws \( \hat{\theta}_1, \hat{\theta}_2 \) and the compensation signal \( \eta_1, \eta_2 \), respectively. Fig.8 shows the changes of \( \alpha_1 \) and \( x_{2,v} \).
Example 2. To prove the correctness and effectiveness of our designed control, we provided a dynamic model of the aircraft in Fig 9. The dynamic equation of the model is given below:

\[ \dot{\tau} = \bar{A}_\beta \beta - \frac{g}{V_T} \cos \tau + \bar{A}_o, \]
\[ \dot{\beta} = q + \frac{g}{V_T} \cos \tau - \bar{A}_o - \bar{A}_\beta \beta, \]
\[ \dot{\omega}_p = q, \]
\[ \dot{q} = N_\sigma + N_\beta \beta, \]

(57)

where \( \bar{A}_o = \frac{A_0}{mV_T}, \bar{A}_\beta = \frac{A_\beta}{mV_T} \). \( \omega_p, \beta \) and \( \tau \) represent the pith angle, attack angle and the inclination angle.
of the flight path, respectively. $q$ and $V_T$ represent the change rate of the pitch angle of the aircraft and the flight speed of the aircraft, respectively. $A_{\beta}$, $A_o$, $N_{\sigma}$ and $N_{\beta}$ represent the slope of lift curve, another factors affecting lift, the control pitching torque and torque which from other aspects, respectively. $m$ and $g$ represent the mass and gravitational acceleration of high-speed aircraft, respectively. $\rho$ is a deflection angle which represents input control.

Choose $(\tau, \beta, q)$ as the state variables, and introduce state transition: $x_1 = \tau$, $x_2 = \beta$, $x_3 = q$, $u = \rho$. Then by introducing the external disturbance, (57) is converted into the following system by the above state transformation:

\[
\begin{align*}
\dot{x}_1 &= g_1 x_2 + f_1(x_1) + d_1, \\
\dot{x}_2 &= g_2 x_3 + f_2(x_2) + d_2, \\
\dot{x}_3 &= g_3 u + f_3(x_3) + d_3, \\
y &= x_1, 
\end{align*}
\]

where $f_1 = -\frac{q}{V_T} \cos(x_1) + \ddot{A_o} + \ddot{A}_{\beta} x_2$, $f_2 = \frac{q}{V_T} \cos(x_1) - \ddot{A}_o + \ddot{A}_{\beta} x_2$, $f_3 = N_{\beta} x_2 + N_q x_3$, $g_1 = \ddot{A}_o$, $g_2 = 1$, $g_3 = N_{\sigma}$. Let expected tracking signal $y_r = 6 \sin 0.3 t$, $d_1 = 0.01 \sin 0.5 t$, $d_2 = 0.01 \cos 2 t$, $d_3 = 0.05 \sin t \cos 2 t$.

The parameters of dynamics system $\ddot{A}_o = 1$, $\ddot{A}_o = 1$, $m_q = 0.1$, $N_{\beta} = -0.03$, $N_{\sigma} = 1$, $V_T = 200$, $g = 9.8$. The design parameters of this system $k_1 = 20$, $k_2 = k_3 = 25$, $\Gamma_1 = 0.5$, $\Gamma_2 = \Gamma_3 = 0.6$, $T_2 = 0.5$, $T_3 = 0.01$, $\psi_1 = 0.5$, $\psi_2 = 0.6$, $\psi_3 = 0.8$. With the initial conditions: $x_0 = [1, 0.2, 0.5]$, $\dot{\theta}_1 = 5$, $\dot{\theta}_2 = 6$, $\dot{\theta}_3 = 10$.

This simulation result is presented in Fig 10-16. Fig.10 shows that the output signal tracks a given expected signal. Fig.11 and Fig.12 shows the trajectory of the controller $u$ and the state $x_2, x_3$, respectively. Fig.13 and Fig.14 shows the trajectory of adaptive laws $\dot{\theta}_1, \dot{\theta}_2$ and $\dot{\theta}_3$ and the compensation signal $\eta_1, \eta_2$ and $\eta_3$, respectively. Fig.15-16 shows the output of the first-order filter and the tracking track of the virtual controller.

5. Conclusion

In this paper, an adaptive command filter controller is proposed for high-order nonlinear systems with multi-high-order coupling terms, unknown coefficients of time-varying function and uncertain external dis-
turbances. For the sake of overcoming the differential explosion problem, the DSC strategy is drawn to settle this issue. At the same time, the introduction of compensation signal can solve the error of the filter and obtain better tracking performance. In the end, the controller’s performance is verified by a numerical simulation and a model simulation. The control scheme can also be used for ship control and manipulator control. Our further research mainly focuses on how to combine event trigger control to achieve better control effect while saving signal channels.

**Date availability statement** The authors can confirm that all relevant data are included in this article.
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