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Classification tasks using input driven nonlinear magnetization dynamics in spin Hall oscillator

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Abstract

The inherent nonlinear magnetization dynamics in spintronic devices makes them suitable candidates for neuromorphic hardware. Among spintronic devices, spin torque oscillators such as spin-toque oscillators and spin Hall oscillators have shown the capability to perform recognition tasks. In this paper, we demonstrate that the magnetization dynamics of a single spin Hall oscillator can be nonlinearly transformed by harnessing input pulse streams and can be utilized for classification tasks. The spin Hall oscillator utilizes the microwave spectral characteristics of its magnetization dynamics for processing a binary data input. The spectral change due to the nonlinear magnetization dynamics assists in real-time feature extraction and classification of 4-bit binary digits. The performance is tested for the classification of the standard MNIST handwritten digit data set and achieved an accuracy of 83.1%. Our results suggest that modulating time driven input data can generate diverse magnetization dynamics in the spin Hall oscillator, which is suitable for temporal or sequential information processing.
Introduction

Machine learning using artificial neural networks (ANNs) has become an integral part of modern day computations\textsuperscript{1,2}. Classification and pattern recognition tasks are achieved in ANNs through nonlinear transformation of input data into a higher-dimensional space. Such nonlinear transformation involves weight (connection strength) optimization between multiple connected layers of the network, resulting in a complex learning process and large volume of data\textsuperscript{3}. In conventional von Neumann architecture, since the logic and memory units are physically separated and connected through a single bus, the sequential transfer of data results in limited data throughput and processing. This leads to significant costs in latency, processing speed and power consumption\textsuperscript{4}. In this regard, neuromorphic architectures based on photonics and spintronics hardware, having processing and memory functionalities within the same unit, are considered for performing computational tasks\textsuperscript{5–9}.

Although significant results have been demonstrated for the neuromorphic hardware, on-device inference and classification of multibit input data is scarce\textsuperscript{10–12}. Spintronic oscillators such as spin-torque oscillators (STO) and spin Hall oscillators (SHO) have been widely studied for feature filters and computation tasks such as pattern recognition and classification\textsuperscript{13–17}. In STOs, a spin transfer torque (STT) is induced by the electric current flowing through a multilayer structure, consisting of a free magnetic layer and a fixed magnetic layer acting as spin-polarizer, separated by a nonmagnetic conductor or an insulator\textsuperscript{18,19}. The SHO consists of a ferromagnetic (FM) and nonmagnetic (NM) bilayer structure in which generation of a pure spin current via the spin Hall effect (SHE) or Rashba-Edelstein effect induces spin orbit torque\textsuperscript{20–23}. Although the oscillatory output power is higher in STOs, SHOs offer several other advantages. Firstly, they are easier to fabricate due to their simpler bilayer structure\textsuperscript{24–26}. Secondly, a spin orbit torque (SOT) which is caused by the pure spin current from the NM electrode can be exerted over extended areas in SHOs, whereas the STT in STOs is a localized effect\textsuperscript{26,27}. The excitation and control of various magnetization oscillations can be effectively achieved with SOT\textsuperscript{28–31}. Moreover, improvement of charge-to-spin conversion efficiency by material engineering and a significant field-like term contribution
in SOT due to interfacial effects offers an interesting opportunity to explore SHO-based neuromorphic hardware\textsuperscript{32–35}.

In this article, we use micromagnetic simulations to model a SHO and utilize the microwave spectral characteristics of its magnetization dynamics for processing a binary data input. We demonstrate a direct classification of multibit data via filtering spectral characteristics. The key motivation is to realize fast information processing at low training costs, by reducing the dimension of the output data. Controlling the magnetization dynamics of the SHO by configuring the input pattern, we are able to classify the input binary data up to 4 bits. Subsequently, handwritten digit recognition is demonstrated with the Modified National Institute of Standards and Testing (MNIST) handwritten digit database. The SHO device, with modified input pulse pattern requires only a static readout and training of linear network for the classification of handwritten digits.

Results and discussion

SHO device model

A conceptual schematization of the hardware is shown in Fig. 1a, consisting of pulse input circuit, SHO, and an electrical output circuit. The modelled SHO is composed of platinum / permalloy bilayer (Pt/NiFe) with lateral size of 100 nm $\times$ 100 nm. Each layer has a thickness of 5 nm. The input binary data is in the form of a current pulse, where bit “1” and “0” are coded as two different current values $I_1$ and $I_0$. An in-plane-magnetic field ($H_{\text{ext}}$) of 100 mT is applied at angle $\varphi = 90^\circ$ (+Y direction), in order to align the magnetization perpendicular to the current direction. The working principle of the SHO is as follows, upon the flow of current ($I_c$) in +X direction, the SHE in Pt results in spin dependent scattering of electrons to the top and bottom surfaces of Pt. This leads to spin accumulation at the interface of Pt/NiFe and a subsequent transfer of spin angular momentum to NiFe results in a transverse flow (+Z direction) of spin current ($I_s$). The ratio between charge current density ($J_c$) and spin current density ($J_s$) is characterized by the spin Hall conversion efficiency (spin Hall angle) $\theta_{\text{SH}}$\textsuperscript{20,21}. The $J_s$ gives rise to two SOTs namely,
damping like torque (DLT) and field like torque (FLT). While DLT is a result of bulk phenomena of the SHE, FLT is an interfacial effect due to the strength of spin accumulation at the interface\textsuperscript{36,37}. Here, we have considered only the role of DLT on the magnetization dynamics due to the negligible effect of FLT for the 5 nm thick Pt Layer. By increasing $J_c$, the natural damping of NiFe can be manipulated by DLT and can be completely compensated to achieve auto-oscillations in the gigahertz frequency range\textsuperscript{24,29}. The simulation method and material parameters are given in the Methods section.

The electrical detection of oscillatory dynamics of the SHO depends on the oscillating anisotropic magnetoresistance (AMR) of the ferromagnet\textsuperscript{19,38}. The AMR effect is a spin dependent transport property of the FM in which the electrical resistance depends on the relative orientation of the magnetization and the current\textsuperscript{39}. The dependence of the FM resistance on the angle $\theta_M$ between current and magnetization is $R(\theta_M) = R_\perp + (R_\parallel - R_\perp)\cos^2\theta_M = R_0 + \Delta R_{AMR} \cos^2\theta_M$, where $R_{\perp(\parallel)}$ is the device resistance when the current and the magnetization are oriented perpendicular (parallel) to each other. $\Delta R_{AMR} = R_\parallel - R_\perp$ is the AMR resistance and $R_0$ is the minimum resistance of the device. The spectral characteristics of SHOs are normally measured using a spectrum analyzer while the time dependent oscillation amplitudes are detected using an oscilloscope\textsuperscript{16,40}. The microwave voltage signal across the device is,

\begin{equation}
V(t) = I_c \times R(t) = I_c \times [R_0 + \Delta R_{AMR} \cos^2\theta_M(t)].
\end{equation}

Since the magnetization dynamics occur in unit sphere and precess around the effective field ($H_{eff}$), the time dependent $\theta_M(t)$ consists of an in-plane component $\theta_{in}$ and an out-of-plane component $\theta_{out}$ and can be decomposed as\textsuperscript{41},

\begin{align}
\cos(\theta_M(t)) &= \cos(\theta_{in}(t))\cos(\theta_{out}(t)), \\
\theta_{in}(t) &= \varphi_0 + \varphi_c \cos(2\pi f), \\
\theta_{out}(t) &= \theta_c \sin(2\pi f),
\end{align}

where $\varphi_0$ is the static angle between the current and the magnetization $\mathbf{M}$ defined by the effective field $H_{eff}$, $\varphi_c$ and $\theta_c$ are the in-plane and out-of-plane precession cone angles of the magnetization, respectively. $f$ is the frequency of oscillation in gigahertz. For the Cartesian coordinate axes in Fig. 1a, $\varphi_0 = \varphi$ (when
$H_{\text{eff}} = H_{\text{ext}}$ and $\sin(\theta_{\text{in}}) \approx \frac{M_X}{M_0}$ (normalized magnetization component), such that the $M_X^2$ is related to the detected output voltage $V(t)$. The simulated time dependent $M_X$ is converted into frequency spectra via fast Fourier transform (FFT) to represent the collective behavior of the SHO for the given input signal. The main advantage of frequency domain analysis is the reduction in the amount of output data for further computations. A filtering mechanism can be applied to perform feature extraction and a simple linear classification of input data, which will be described below.

**Inherent magnetization dynamics excited by SOT**

We start by investigating the magnetization dynamics of the modelled SHO device, as a function of $I_1$, for a single current pulse of pulse width ($\tau$) 3 ns including pulse rise and fall times which are 1 ns respectively. The time evolution of the magnetization component $M_X$ for $I_1 = 5.0$, 5.5, 6.0 and 6.5 mA with $I_0 = 0$ mA are shown in Fig. 1b. For $I_1 = 5.0$ and 5.5 mA, the magnetization oscillations correspond to the excitation of a ferromagnetic resonance mode by SOT. They show small angle precession in which cone angle increases with the increase of $I_1$. Depending on the magnitude of $I_1$, the initiation and relaxation times vary as depicted by colored circles in the Fig. 1b. This indicates the manipulation of effective damping by the SOT. These excitations can be converted into self-sustained auto-oscillations with increase of $\tau$ as the precession amplitude increases gradually and saturates at the limit cycle of oscillations. For $I_1 = 6.0$ and 6.5 mA, the oscillations correspond to the in-plane and out-of-plane auto-oscillation modes, respectively. The trajectories of each mode (single cycle) are illustrated in Fig. 1c. The limit cycle of the auto-oscillation is governed by the equilibrium energy, which is determined by the device geometry, mode of excitation and the direction of effective field. For the single magnetic domain model, the auto-oscillation orbit is a clamshell in the in-plane direction and circular in the out of-plane direction.

The FFT amplitude of the $M_X$ spectrum for a range of $I_1$ (5.0 – 6.5 mA) is shown in Fig. 1d. For $I_1 < 5.5$ mA, the FFT amplitude increases linearly with increasing $I_1$, but the oscillation frequency which can be seen from the peak position of the FFT amplitude in the spectra is constant at $\sim 9.0$ GHz. This
constant frequency corresponds to the ferromagnetic resonance frequency (Supporting Figure S1). For 5.5 mA < \( I_1 < 6.0 \) mA, the frequency undergoes a red shift due to the large angle motion of magnetization components as shown in the Fig. 1c \( (I_1 = 6.0 \) mA), which reduces the effective demagnetization field in the NiFe layer. The magnetization component transverse to \( H_{\text{ext}} \) undergoes oscillations at twice the oscillation frequency in order to maintain a constant magnitude and thus reduces the frequency. Upon further increase of \( I_1 \), the frequency reduces and reaches \( \sim 7.1 \) GHz for \( I_1 = 6.0 \) mA. This frequency shift is attributed to the complex coupling of oscillatory amplitude and phase as predicted by the nonlinear auto-oscillator theory for STOs\(^{46} \). For \( I_1 = 6.5 \) mA, the frequency increases to 9.1 GHz and the FFT amplitude reduces due to the out-of-plane oscillation, as can be seen in Fig. 1c \( (I_1 = 6.5 \) mA). Therefore, the magnetization dynamics in the SHO can be classified into three regimes as shown in Fig. 1e: the linear excitation regime for \( I_1 < 5.5 \) mA, where the frequency is constant with increasing precession amplitudes along \( H_{\text{ext}} \) as a function of \( I_1 \), the nonlinear excitation regime for \( 5.5 \) mA < \( I_1 < 6.0 \) mA, where the FFT amplitude is saturated and the frequency decreases drastically with increasing \( I_1 \) and the out-of-plane oscillation regime for \( I_1 > 6.5 \) mA where the frequency increases and the FFT amplitude decreases.

**4-bit binary digit classification**

Having explored magnetization dynamics, we investigate the ability of the SHO in classifying \( n \)-bit binary input data. The encoded input signal \( h(t) \) is a representation of the pulse stream, with current values \( I_0 \) and \( I_1 \) for bit 0 and bit 1, respectively. The pulse width \( (\tau) \) and pulse period \( (\Delta t) \) are 3 ns and 4 ns, respectively. The pulse width \( \tau \) includes rise and fall time of 1 ns each. Figure 2 represents 4-bit input pulse patterns, \( Mx \) responses and FFT amplitude spectra (Frequency) with input current values, \( I_1 = 3.5 \) mA and \( I_0 = 0 \) mA, which lies in the linear excitation regime. For the input pattern 1111 in Fig. 2a, the \( Mx \) response in the Fig. 2b shows the magnetization oscillations with varying amplitude for each bit 1 inputs and the corresponding FFT spectrum in Fig. 2c. For the 1001 pattern in Fig.2d, the magnetization relaxes to its initial state in the time between the two bit 1 inputs as shown in Fig. 2e and the FFT spectrum in Fig. 2f. The difference in magnetization dynamics for the input patterns of 1111 and 1001 can be clearly seen.
from the corresponding amplitudes in the FFT spectrum (Figs. 2c and 2f). In order to classify the input patterns, and for feature extraction from the outputs, we resort to the classical tool of filtering in the area of signal processing. Inspired by the filter neuron concept in recurrent neural network for feature extraction and processing time varying outputs, we utilize the same in filtering a particular feature in the output data\textsuperscript{47,48}.

To separate the input bit patterns in the output spectrum, we fix the filter characteristic as the FFT amplitude value at the linear excitation regime frequency, 9.0 GHz. The filtered amplitude values at 9.0 GHz for the input patterns of 1111 and 1001 are 0.0042 and 0.00002 as shown with the guide lines in Figs. 2c and 2f, respectively. Note that this filtering approach is different from the commonly used bit slicing methods in the computing paradigm. In the bit slicing method, \( n \)-bit input elements are mapped as \( n \) output values and further computed for weight optimization\textsuperscript{49}. However, the method followed here is the quantization of output values, where \( n \)-bit input elements are mapped to a single output value. This approach is well suited for the reduction of output data and does not require a weight optimization for the classification task of input patterns, which can reduce the computation costs\textsuperscript{50,51}.

The relaxation of magnetization precession within the duration between two consecutive pulses poses a challenge in the classification task. For the input pattern of 0101 in Fig. 2g, the Mx response is shown in Fig. 2h where the individual bit 1 inputs show the same oscillating amplitude since the magnetization has relaxed to its initial state before the arrival of the bit 1 input. The corresponding FFT spectrum is shown in Fig. 2i. Similar to the input pattern of 1001, the magnetization oscillates at the same amplitude for each bit 1 input. The resulting FFT spectra show the same amplitude, as can be seen in Figs. 2f and 2i. In such a scenario, the different 4-bit input patterns cannot be separated from the output of the SHO. The input pulse parameters such as \( I_1 \) and \( \tau \) can be varied to influence the magnetization dynamics, however, patterns such as 1000 and 0001 still result in the same FFT spectra in the linear excitation regime. This is due to the same magnetization dynamics for each bit 1 input, but in a different time frame. We term this type of input pulse stream as the regular pulse scheme. An analysis of the magnetization dynamics,
resultant FFT spectra and the inability to separate 4-bit input patterns for different values of \( I_1 \) and \( \tau \) are provided in Supporting Figures S2 and S3.

To tackle the challenge faced in the regular pulse scheme, we resort to modifying the input driven magnetization dynamics rather than the internal structure of the device\(^6\). Figure 3 shows magnetization dynamics for the use of an excitatory pulse. Figure 3a shows the excitatory pulse without an offset current, \( I_e = 3.5 \) mA, \( I_0 = 0 \) mA and excitatory pulse width \( (t_1) \) of 7 ns. The Mx oscillation reaches to an amplitude of 0.18 and relaxes to ground state within 2 ns as shown in Fig. 3b. This can be seen from the upper envelope plot of Mx shown after the pulse is off at 8 ns in Fig. 3c. However, with an offset value \( I_0 = 1.2 \) mA in the Fig. 3d, the oscillation amplitude reaches up to 0.38 due to the increasing precessional amplitude with current in Fig. 3e and the relaxation period is extended up to ~ 8 ns as can be seen in Fig. 3f. This allows us to modify the magnetization dynamics in the SHO during the inputs for different 4-bit patterns, as will be discussed below.

In order to extend the magnetization relaxation dynamics for the entire time of the input digit pattern, a modified pulse scheme \((I_{mod})\) with an additional excitatory pulse \((I_e)\) added before the input binary pattern pulses \( b_i(t) \). The \( I_{mod} \) with input pattern \( b_i(t) \) is given after a pulse gap \( (\delta) \) such that the excitatory effect can be modulated to a range of input pulses. Hence the SHO responds to a combination of two input signals \( I_e \) and \( b_i(t) \) given by,

\[
I_{mod} = \begin{cases} 
I_e & ; 0 < t < t_1 \\
 b_i(t) & ; t > t_1 + \delta 
\end{cases}
\]  

Figures 4 shows the input patterns with modified pulse scheme with \( I_e = 3.0 \) mA, \( t_1 = 7 \) ns, \( \delta = 5 \) ns, \( I_0 = 1.2 \) mA, \( I_1 = 2.4 \) mA, \( \Delta t = 4 \) ns, \( \tau = 3 \) ns. For simplicity we denote these input patterns with the above mentioned parameters as IP\(_1\). Figure 4a shows the input pattern of 1010, Mx response in Fig. 4b and the FFT spectrum in Fig. 4c. Similarly, Fig. 4d shows the input pattern for 0101, Mx response in Fig. 4e and the FFT spectrum in Fig. 4f. As can be seen from the Mx responses in Figs. 4b and 4e, each digit 1 input shows different oscillating amplitude as the magnetization dynamics is influenced not only by their respective pulses in \( b_i(t) \), but also by the previous \( I_e \). The influence of \( I_e \) gradually decreases with time, and hence each input
has varying dynamics and influence of the past inputs as a function of time. As expected, the change in relaxation dynamics plays a key role in the oscillation amplitude. Since the magnetization dynamics for each digit 1 input is different, a large oscillation amplitude with a red shift of frequency in the nonlinear excitation regime is observed. Such a behavior is clearly seen in the FFT spectra of Figs. 4c and 4f. In case of the \( I_{\text{mod}} \) scheme, the FFTs were collected from the Mx in the range of input pulse patterns \( b_i(t) \) (Supporting Figure S4). The FFT amplitudes filtered at 9.0 GHz are 0.037 and 0.017 for 1010 and 0101, respectively. Therefore, these two input patterns can be easily classified with the \( I_{\text{mod}} \) scheme which the previous regular pulse scheme could not classify. Similarly, 1000 and 0001 patterns can also be separated by using the FFT amplitude due to the difference in the rate of relaxation. The 16 combinations of 4-bit digit input patterns and their respective magnetization dynamics are illustrated in Supporting Figure S5.

Figure 5a shows the filtered FFT amplitude for different 4-bit input patterns with \( I_{P1} \). Due to the distinction in the filtered amplitude values, a 4-bit digit pattern can be quantized and represented as a 1-D analog output. We further analyzed the influence of different input \( I_e \) and \( \delta \) values of \( I_{P1} \) on the separation of 4-bit digit patterns. The results are compared using a commonly used metric in pattern recognition problems, called separability index, (SI) which is a measure of the average difference between the outputs, for different classes of input parameters\(^{53,54} \). Figure 5b shows SI for \( I_{P1} \) with \( I_e = 1.4 - 6.0 \) mA and \( \delta = 5 \) ns. For \( I_e = 1.4 \) and 2.0 mA, the magnetization precession excited by a digit 1 input relaxes before the arrival of the next input. This results in the FFT amplitudes being very close to each other, leading to low SI. On the other hand, for \( I_e = 4.0, 5.0 \) and 6.0 mA, the magnetization precession excited by a digit 1 input shows a large amplitude of auto-oscillation, resulting in the consecutive \( b_i(t) \) pulses oscillating at the same amplitude. Hence, the input patterns such as 0111, 1011, 1101, 1110 and 1111 give the FFT amplitudes very close to each other. This results in similar SI values for \( I_e = 4.0, 5.0, 6.0 \) mA, indicating that the effectiveness of the excitatory pulse in distinguishing the different input patterns has been lost. We observe the maximum SI in case of \( I_e = 3.0 \) mA, which indicates a balance between the \( I_e \) pulse and \( b_i(t) \) pulses.

Figure 5c shows SI for \( I_{P1} \) with \( \delta = 1 - 25 \) ns and \( I_e = 3.0 \) mA. With an increase of \( \delta \), the influence of \( I_e \) on \( b_i(t) \) reduces. Therefore, the FFT amplitudes become closer to each other for the input patterns
such as 0010 and 0001. An appreciable influence of $I_e$ on $b(t)$ lasts up to $\delta = 20$ ns and is completely lost at $\delta = 25$ ns. For $\delta > 25$ ns, the magnetization dynamics are same as the regular pulse scheme. Thus, we can deduce that, with proper $I_e$ and $\delta$, the separation of 4-bit inputs can be optimized to obtain the highest possible separation at the filtered FFT amplitude. A comprehensive analysis of magnetization dynamics for various $\delta$ and $I_e$ are summarized in the Supporting Figures S6 and S7.

The MNIST handwritten digit classification

Finally, the SHO device with the modified pulse scheme is evaluated for the classification of handwritten digits with the Modified National Institute of Standards and Technology (MNIST) handwritten database. The database contains 60000 training and 10000 testing images of digits 0 to 9. The assumed model network for classification is illustrated in the Fig. 6a, containing an input layer, the SHO layer and a readout layer. In the input layer, the images are preprocessed such that each of the original $28 \times 28$ pixel images are converted into binary valued pixels where 1 represents the white pixel and 0 represents black pixel. In the SHO layer, each of the 4-bit inputs are encoded and fed as the modified pulse scheme with $IP_1$ parameters. The output of the SHO layer is collected as the filtered FFT amplitude at 9.0 GHz as discussed in the previous section. A fully connected single layer perceptron based on linear regression is chosen for read-out layer training and classification of the handwritten digits. The classification results are compared with a single layer linear regression network without the SHO layer in which the inputs are 4-bit sliced binary values. The impact of the SHO layer for the classification task can be seen with the use of weight vectors color maps after the training process, in Figs. 6b and 6c. Figure 6b shows the weight vector matrix that consists of 7860 elements for the classifier without SHO, and Fig. 6c shows the weight vector matrix for classifier with SHO that contains 1960 elements. Due to the nonlinear dynamics of the SHO layer and ability to separate 4-bit input patterns, only 196 input features are required as input for the read-out layer. This shows the reduction of processing data by 4 times, when using the SHO layer.
The MNIST handwritten digit classification accuracy is evaluated as the ratio of the total number of correctly classified digits to the total number of digits. The classification accuracy of the network with the SHO layer for a testing of 10000 images is 83.1%. The confusion matrix (predicted vs true digit) in Fig. 6d shows the success of classification in a color map for the modified pulse scheme IP. The least successfully classified digit is digit 5, which affects the overall success rate. The accuracy without SHO by 4-bit slicing is 85.0% (not shown here). Moreover, we tested the classification accuracies with a computationally intense weight training method called gradient descent based on logistic regression, and achieved classification accuracies of 89.2% and 91.7% with \( n = 4 \) and without \( n = 1 \) the SHO layer, respectively. Figure 6e shows the classification accuracies for \( n \)-bit quantization without SHO \( (n = 1) \) and with SHO \( (n = 2,3,4) \) under the modified pulse scheme for linear and gradient descent training methods. An accuracy decreases of only 2% is observed in both the training methods for classification with SHO which shows that the simple linear training process can be utilized. The line plot corresponding to the left y axis shows the number of weight elements required for each training for \( n \)-bit data. The SHO based weight elements for 2-bit data is 3920 and that for 4-bit data is 1960 and the corresponding accuracies are 84.6% and 83.1% (linear regression), respectively. Hence the data reduction does not affect the classification accuracy significantly and the SHO can be successfully utilized for 4-bit classification with reduced number of computing elements in the read-out layer.

**Conclusion**

In this study, a single spin Hall oscillator capable of real-time classification of sixteen different binary 4-bit patterns (0000 to 1111) distinctly was demonstrated. Control and tuning of intrinsic magnetization oscillations of the spin Hall oscillator were performed by modifying an input digit pulse pattern. The performance of the sample was tested with the standard MNIST handwritten digit data set classification and achieved an accuracy of 83.1% with linear training network and reduced output layer
computations. Hence, these results are expected to provide an insight into the controlling and tuning of spintronic oscillators for real time and on-device neuromorphic computations.

Methods

Micromagnetic simulations

The micromagnetic simulations were carried out using LLG micromagnetic simulator in the framework of a single domain model\textsuperscript{56-58}. The temporal dynamics of the ferromagnetic layer was solved by the Landau-Lifshitz-Gilbert (LLG) equation with spin transfer torque term,

$$\frac{d\hat{m}}{dt} = -\gamma \hat{m} \times \mu_0 H_{\text{eff}} + \alpha \hat{m} \times \frac{d\hat{m}}{dt} - \gamma \frac{h}{2|e|} \frac{\theta_{SH}}{M_s t_{\text{FM}}} \left( \hat{m} \times (\hat{m} \times \hat{\sigma}) \right),$$

(6)

where $\hat{m} = \frac{M}{M_s}$ is the normalized magnetization vector, $\gamma$ is the gyromagnetic ratio, $\alpha$ is the Gilbert damping parameter, $\mu_0$ is the vacuum permeability, $M_s$ is the saturation magnetization, $h$ is the reduced Planck constant, $e$ is the electron charge and $t_{\text{FM}}$ is the thickness of the magnetic layer. $H_{\text{eff}}$ is the effective field including external magnetic field $H_{\text{ext}}$, magneto-crystalline anisotropy field, exchange field and demagnetization field. $\theta_{SH}$ is the spin Hall angle which characterizes the conversion efficiency of charge current density $\hat{j}_c$ to spin current density $\hat{j}_s$ in the heavy metal layer. $\hat{\sigma} = -\text{sgn} \theta_{SH} (\hat{z} \times \hat{j}_c)$ is the orientation of spin injected into the ferromagnet, where $\hat{z}$ and $\hat{j}_c$ are the unit vectors in the direction of surface normal and the electrical current, respectively. In accordance with experiments, the material parameters used in the simulations, are: exchange constant $A_{\text{ex}} = 1.13 \times 10^{-12}$ J m$^{-3}$, $\mu_0 M_s = 1.0$ T, $\alpha = 0.02, \theta_{SH} = 0.07$, resistivities $\rho_{\text{NiFe}} = 4.5 \times 10^{-7}$ $\Omega$ m and $\rho_{\text{Pt}} = 2.0 \times 10^{-7}$ $\Omega$ m\textsuperscript{33,34}. The magnetic anisotropy is ignored and an in-plane external magnetic field of strength $\mu_0 H_{\text{ext}} = 100$ mT is applied to saturate the magnetization along Y direction.

4.2. Separability Index
We calculate the separability index between the N different outputs ($a_1$ to $a_N$) of the spin Hall oscillator, which are the filtered FFT amplitude values, corresponding to each of the N different input patterns. The separability index (SI) is given by,

$$SI = \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{a_i - a_j}{N^2},$$  \hspace{1cm} (7)

where i and j are the indices of summation and N = 16. The SI value of a particular input current scheme indicates how well the different input patterns can be distinguished from the closeness of output values. Thus, we can infer that, the input current scheme having highest SI value is the most suited for performing classification and recognition tasks.

4.3. Read-out layer training and classification

The original greyscale images are converted to binary black and white images, following which each image is divided into 4-bit patterns striding horizontally, resulting in a 196 $\times$ 4 matrix. In the readout layer, we use both linear and logistic regression functions executed in Matlab software. During learning process, for a particular image, the output amplitudes are mapped into a one dimensional feature vector of 196 elements. This process is repeated for all 60000 training images, creating a 60000 $\times$ 196 output matrix $O$. The 196 $\times$ 10 weight matrix $W$ (where each column corresponds to each digit from 0 to 9) is calculated using the output matrix $O$ and a label matrix $L$ (60000 $\times$ 10) containing the true labels for each training image. In each row of the label matrix, the $(l+1)^{th}$ column has a value 1 and the remaining columns have value 0, where $l$ is the true digit ($l$=0, 1...9).

In linear regression, assuming a linear relationship between the output matrix $O$ and the label matrix $L$,

$$OW = L.$$  \hspace{1cm} (8)

We find the weight matrix $W$ using the pseudo inverse $O^\dagger$. 

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In logistic regression, we assume the relation between \( O \) and \( L \) to be of the form,

\[
g_w(O) = \frac{1}{1 + e^{-OW}} = L. \tag{10}
\]

We find weight matrix \( W \) by the minimization of a cost function \( J(W) \) given by,

\[
J(W) = \frac{1}{m} \sum_{i=1}^{m} \left[ -L^{(i)} \log(g_{w(O^{(i)})}) - (1 - L^{(i)}) \log(1 - g_{w(O^{(i)})}) \right], \tag{11}
\]

where \( m \) is the number of iterations. We ran 12000 iterations for the weight matrix \( W \) optimization. The cost function \( J(W) \) is minimized by the method of gradient descent, given by,

\[
\frac{\partial J(W)}{\partial W_j} = \frac{1}{m} \sum_{i=1}^{m} (g_{w(O^{(i)})} - L^{(i)})O_j^{(i)}. \tag{12}
\]

**Data availability**

The datasets generated and analysed for this study is available from Kyushu Institute of Technology repository at http://hdl.handle.net/10228/00008940. The MNIST dataset\(^5\) is freely available at http://yann.lecun.com/exdb/mnist.

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Author contributions

Y.F. conceptualized and supervised the study. J.R.M., A.J.M., K. N. and R.F. did the micromagnetic simulations and classification tasks. J.R.M., R.M. and S.G. analysed data and wrote the manuscript with the help of Y.F and R.S.R. All the authors discussed the results and commented to improve the quality of manuscript.

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Figure captions

Figure 1. a. Schematics of spin Hall oscillator microwave emission detection setup. b. Pulsed input current and magnetization dynamics of Mx component for $I_1 = 5.0 - 6.5$ mA, the initiation and relaxation time scales are indicated by colored circles. c. Transition of small angle precession to large angle oscillation trajectories as a function of $I_1$. d. Fast Fourier transformed amplitude spectrum of time domain Mx component as a function of $I_1$. e. Frequency response and peak amplitude level of FFT spectrum as a function of $I_1$.

Figure 2. a. Regular pulse scheme for 4-bit digit binary input pattern 1111 with pulse period ($\Delta t$) of 4 ns and pulse width ($\tau$) of 3 ns. b. Magnetization dynamics of Mx component and c. corresponding spectral characteristics. Guide line in FFT shows the amplitude value at 9.0 GHz. Similar input pattern, magnetization dynamics and FFT in d - f for 1001 pulse pattern, and in g – i for 0101 pulse pattern.

Figure 3. a. Excitatory pulse ($I_e = 3.0$ mA) without offset current ($I_0$). b. Magnetization dynamics of Mx component and c. relaxation characteristics plot from the upper envelope of the Mx time domain data after the pulse is off. d. Excitatory pulse ($I_e$) with an offset current value ($I_0 = 1.2$ mA). e. Magnetization dynamics of Mx component and f. relaxation characteristics plot of the Mx time domain data.

Figure 4. a Modified pulse scheme with an excitatory pulse $I_e = 3.0$ mA of pulse width $t_1 = 7$ ns, for 4-bit digit binary pattern 1010 with pulse period ($\Delta t$) of 4 ns and pulse width ($\tau$) of 3 ns. The pulse gap of $\delta = 5$ ns between $I_e$ and 4-bit patterns is shown with the grey box. b. Magnetization dynamics of Mx component and c. corresponding spectral characteristic FFT plot. d – f. Similar plots of input pattern, magnetization dynamics and FFT for 0101 pulse pattern. Guide line in FFT plots show the amplitude value at 9.0 GHz.

Figure 5. a. Filtered amplitude at frequency of 9.0 GHz for all 16 4-bit binary digit inputs for the modified pulse scheme with input parameters $I_e = 3.0$ mA, $t_1 = 5$ ns, $\delta = 5$ ns, $I_0 =1.2$ mA, $I_1 = 2.4$ mA, $\Delta t = 4$ ns, $\tau = 3$ ns. b. Separability index for different excitatory pulse current values $I_e = 1.4$
- 6.0 mA, with $\delta = 5$ ns. d. Separability index for different pulse gap values $\delta = 1 - 25$ ns with $I_e = 3.0$ mA.

**Figure 6.** a. Fully connected network model for classification of MNIST handwritten digit datasets with input layer, SHO layer and read-out layer. b. Color map of read-out layer weight elements required for 784 inputs without SHO. c. Color map of read-out layer weights required for 196 inputs with SHO. d. Confusion matrix of predicted vs true digit for classification of 10000 test images with SHO. e. Classification accuracies for different $n$-bit patterns with two training methods, linear regression and gradient descent. The bit pattern of $n = 1$ corresponds to the direct software classification without SHO, $n = 2-4$ corresponds to the SHO based classification with the modified pulse scheme where the outputs are the filtered FFT responses of the magnetization component Mx. The line plot corresponding to the left axis shows the required number of weight elements in the read-out layer.
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