

Appendix

- ❖ The proposed programming model based on the DEA model employed in the research conducted by Klimberg and Ratick (2008) is as follows.

$$\mathbf{Max} Z_1 = \sum_{i \in I} (1 - u_i) \quad (51)$$

$$\mathbf{Max} Z_2, \mathbf{Min} Z_3$$

S. T. :

Eqs. (5) – (7) , (11) – (15) and (17) – (19)

$$\sum_{n \in N} \vartheta_{ni} E_{ni} = \sum_{l \in L} \sum_{r \in R} \sum_{t \in T} x_{ilrt} \quad \forall i \in I \quad (52)$$

$$\sum_{m \in M} \tau_{mi} O_{mi} + u_i = \sum_{l \in L} \sum_{r \in R} \sum_{t \in T} x_{ilrt} \quad \forall i \in I \quad (53)$$

$$\sum_{m \in M} \tau_{mi} O_{mi} - \sum_{n \in N} \vartheta_{ni} E_{ni} \leq 0 \quad \forall i, i' \in I; i \neq i' \quad (54)$$

$$\tau_{mi} O_{mi} \leq \sum_{l \in L} \sum_{r \in R} \sum_{t \in T} x_{ilrt} \quad \forall i \in I, m \in M \quad (55)$$

$$\vartheta_{ni} \geq \varepsilon (\sum_{l \in L} \sum_{r \in R} \sum_{t \in T} x_{ilrt}) \quad \forall i \in I, n \in N \quad (56)$$

$$\tau_{mi} \geq \varepsilon (\sum_{l \in L} \sum_{r \in R} \sum_{t \in T} x_{ilrt}) \quad \forall i \in I, m \in M \quad (57)$$

Where variables u_i , τ_{mi} , ϑ_{ni} indicate the level of inefficiency of warehouse i , the weight allocated to the m^{th} output criterion for warehouse i , and the weight allocated to the n^{th} input criterion for warehouse i , respectively. Eq. (51) maximizes the summation of the efficiencies of warehouses. Eq. (52) considers the weighted sum of the input criteria of the established and unestablished warehouse to be equal to one and zero, respectively. Eq. (53) indicates the level of inefficiency of the warehouse. Eq. (54) determines that the weighted sum of output criteria is less than or equal to the weighted sum of input criteria. Eq. (55) requires the weighted output criteria to be less than or equal to 1. Eqs. (56) and (57) express the range of the variables.

- ❖ The proposed programming model based on the DEA model used in the research conducted by Afsharian (2019) is as follows.

$$\mathbf{Max} Z_1 = \sum_{m \in M} \sum_{i \in I} \sum_{l \in L} \sum_{r \in R} \sum_{t \in T} x_{ilrt} \tau_m O_{mi} \quad (58)$$

$$\mathbf{Max} Z_2, \mathbf{Min} Z_3$$

S. T. :

Eqs. (5) – (8) and (11) – (19)

$$\sum_{n \in N} \sum_{i \in I} \sum_{l \in L} \sum_{r \in R} \sum_{t \in T} x_{ilrt} \vartheta_n E_{ni} = 1 \quad \forall i \in I \quad (59)$$

Eq. (58) maximizes the weighted sum of the output criteria of warehouses. Eq. (59) considers the weighted sum of the input criteria of an established warehouse to be equal to one.

- ❖ The proposed DEA model according to the DEA model introduced by Sun et al. (2013) is formulated as follows.

Sets and indices:

K : Set of alternatives, indexed by k .

N : Set of inputs criteria, indexed by n .

M : Set of outputs criteria, indexed by m .

Parameters:

ω_{nk} : Amount of input criterion n for alternative k .

φ_{mk} : Amount of output criterion m for alternative k .

ω_{nIDEAL} : Amount of input criterion n for virtual ideal alternative; $\omega_{nIDEAL} = \min_k \{\omega_{nk}\}$

φ_{mIDEAL} : Amount of output criterion m for virtual ideal alternative; $\varphi_{mIDEAL} = \max_k \{\varphi_{mk}\}$

α : Weight factor in the objective function.

$$\mathbf{Min} Z_I = \alpha \left\{ \sum_{k \in K} \left(\sum_{n \in N} \vartheta_n (\omega_{nk} - \omega_{nIDEAL}) + \sum_{m \in M} \tau_m (\varphi_{mIDEAL} - \varphi_{mk}) \right) \right\} + (1 - \alpha) \left(\max_{k \in K} \left\{ \sum_{n \in N} \vartheta_n (\omega_{nk} - \omega_{nIDEAL}) + \sum_{m \in M} \tau_m (\varphi_{mIDEAL} - \varphi_{mk}) \right\} \right) \quad (60)$$

$$\sum_{m \in M} \tau_m \varphi_{mk} \leq \sum_{n \in N} \vartheta_n \omega_{nk} \quad \forall k \in K \quad (61)$$

$$\sum_{n \in N} \vartheta_n \omega_{nIDEAL} = 1 \quad (62)$$

$$\sum_{m \in M} \tau_m \varphi_{mIDEAL} = 1 \quad (63)$$

$$\vartheta_n, \tau_m \geq \varepsilon \quad (64)$$

Eq. (60) minimizes the weighted sum of the summation of the weighted coordinate distances between the virtual ideal alternative and all alternatives and maximum weighted coordinate distances between the virtual ideal alternative and all alternatives. Eq. (61) determines that the weighted sum of output criteria is less than or equal to the weighted sum of input criteria. Eq. (62) and Eq. (63) take into account the weighted summation of the input criteria of the virtual ideal alternative and the weighted sum of the output criteria of the virtual ideal alternative equal to 1, respectively. Finally, Eq. (64) specify the eligible domain of decision variables.

The efficiency score of the alternative k is calculated as follows:

$$ES_k = \frac{\sum_{m \in M} \tau_m^* \varphi_{mk}}{\sum_{n \in N} \vartheta_n^* \omega_{nk}} \quad \forall k \in K \quad (65)$$

ϑ_n^* , τ_m^* represent the optimal value of the weight assigned to the n^{th} input criterion and the m^{th} output criterion, respectively, which are obtained by solving the DEA model presented above.